Heat transfer
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Lecture - 51
View factor – Inside sphere method; Blackbody radiation exchange

So, we initiated discussion on view factors in the last lecture. So, we are going to continue forward with that and look at other methods, which are available to evaluate view factors.

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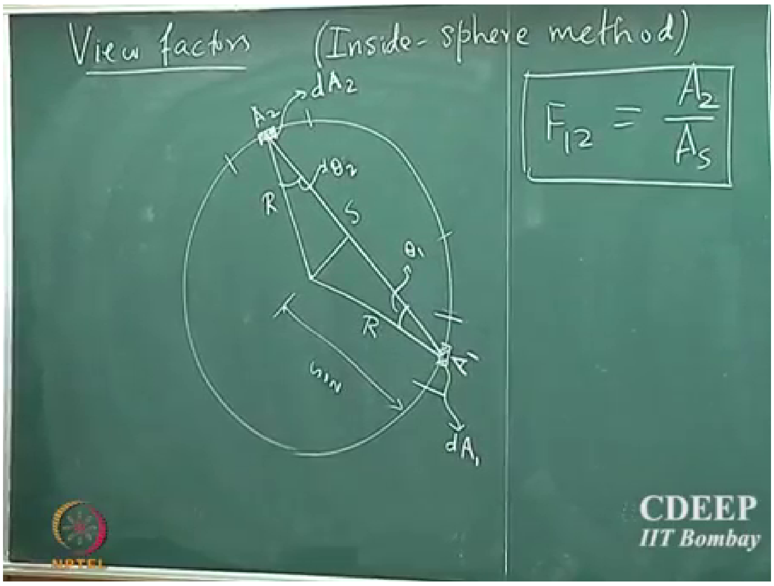

So, what we are going to see now is today's lecture is called the inside sphere method.

So, note that if you want to find the view factor for any system you can always integrate the expressions that we had. So, we looked at the summation rule and the reciprocity relationship, which will help you to calculate the view factors quickly we are going to look at another method called the inside sphere method. And this is a very very handy method to calculate view factors for various kinds of three dimensional geometry.

So, suppose ok; so I have a sphere and let us say I have a surface A 2 and I have let us say another surface A 1 ok, I have two surfaces and I need to find F 1 2, which is the view factor for emission, which is emitted by surface A 1 and what how is that with you by surface A 2 ok. So, we need to find the view factor or the geometric factor all right.

So, let us say that this is the centre of the sphere, and if I assume a differential element here. So, this is dA 2 and this differential element is dA 1 ok. So, if the sphere is the radius of the sphere is R then; obviously, the center distance from the center to any location the sphere is R; and if the linear distance between the 2 is S. So, that is the linear distance between the two surfaces or two differential elements ok.

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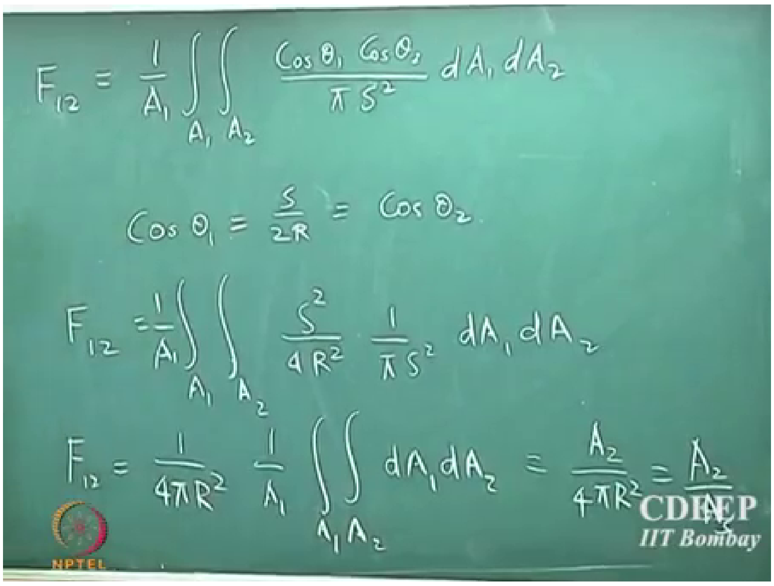

So, the objective is to find F 1 2 and the formula for F 1 2, the integral is 1 by A 1 integral A 1 integral over A 2 cos theta 1 cos theta 2. So, if the angle here is theta 1 and theta 2 and the angle here is theta 1. So, then I will have F 1 2 is 1 by A 1 integral A over A 1 integral over A 2 cos theta 1 by cos theta 2 pi S square into dA 1 dA 2 ok. So, that is the view factor.

What is cos theta 1? Cos theta 1 is let us say I drop a perpendicular line. So, this distance is S by 2 S by 2. So, cos theta is S by 2 R and that is also equal to cos theta 2 ok, because it is a isosceles triangle and so the angles are seen and that is equal to S by 2 R. So, now, if I plug this in, A 2 S square by 4 R square into 1 by pi S square dA 1 dA 2 ok.

So, we cancel out S 2. So, that will be 1 by 4 pi R square. So, note that R square the radius of the sphere is independent of what is the area of A 1 and A 2, and it is independent of the location of these surface areas. So, that will be and will be 1 by A 1 integral A 2 dA 1 dA 2 ok. And that is nothing, but A 2 by 4 pi R square.

What is 4 pi R square? It is the area of the whole sphere right. So, that is nothing, but A 2 by the surface area of the sphere. So, that is an interesting formula. So, note that the view factor F 1 2 is; now completely independent of where these two are located, it is independent of where these two are located along the surface and not just that it depends only on the receiving surface area.

So, it is independent of the size of the surface area, which is size of the surface, which is emitting and it depends only on the surface area of the receiving surface. So, this is a really useful and very very important formula and in fact, we will see several examples mostly in the next lecture or the one after and we look at several different problem, we will see that this formula actually comes in very handy to calculate view factors for various three dimensional geometries.

Particularly, because you do not have to calculate or you do not have to construct the three dimensional geometry anymore, if you are able to use this formula straight; you do not have to construct the three dimensional geometry and do the integration. So, it becomes very handy, if you really understand what this formula means and how to use this that is very important, how to use this formula for solving or finding new factors for various kinds of problem that you will be looking at in the future.

All it assumes is that it is a diffuse emission and what you account for is radiosity. So, that accounts for both emission and reflection everything is accounted. The only assumption, which is important here is that; we assume that the emission is diffused now if it is not diffused, then you will have to integrate over the whole angle all the angles in all the directions.

So, if the emission plus reflection the radiosity is the diffuse radiosity, then this is a golden formula that will be very very handy to solve different kinds of problems. So, it sounds very counterintuitive, but this is where this is one place where actually intuition does not work. So, if you actually plug in all the formula, it does not matter where you place the surface area it really does not matter.

So, that is a very very stocking result that you will see it is completely counter intuitive, but it is very very useful. You can actually construct a whole three dimensional geometry and integrate with it exactly the same result. So, we have not done any magic here, it just comes out from the formula, which we had derived rigorously in the last lecture. And we have just applied the formula with the normal angles that we already know.

So, all that matters is that the geometric factor, if the areas are placed inside the periphery of the sphere this is very important; if the surfaces are placed in the periphery of this of the sphere or the surface of the sphere irrespective of where the emitting surface is located it only the view factor only depends upon the receiving surface area and the total sphere ok. So, that is a very very useful formula.

Any questions on this; it is counter intuitive and the reason why it is counter intuitive is that; you would always think that, if I have to if I have to place the surface here, then how come that it depends only on the surface area of the receiving surface, but note that the area of the sphere is still here. So, it is accounted in some sense the area of the emitting surfaces, so in some sense accounted in the total area of the sphere.

But it just turns out that it is relevant. It is relevant, what is the size of the area and where it is located is completely irrelevant. And similarly where the second surface is located is also completely irrelevant, when you want to calculate the new factor, it just turns out that it is completely irrelevant, it is counterintuitive and it is we have done it we have not done any magic here it is completely rigorous all right.

So, this is a very very useful formula that you must all remember. So, recall that when we initiated discussion on new factor in the last lecture, the objective was to characterize radiation exchange between surfaces. So, once we know how to calculate new factor, we can now attempt to find out what is the net radiation exchange between different surfaces. So, that is what we are going to do now.

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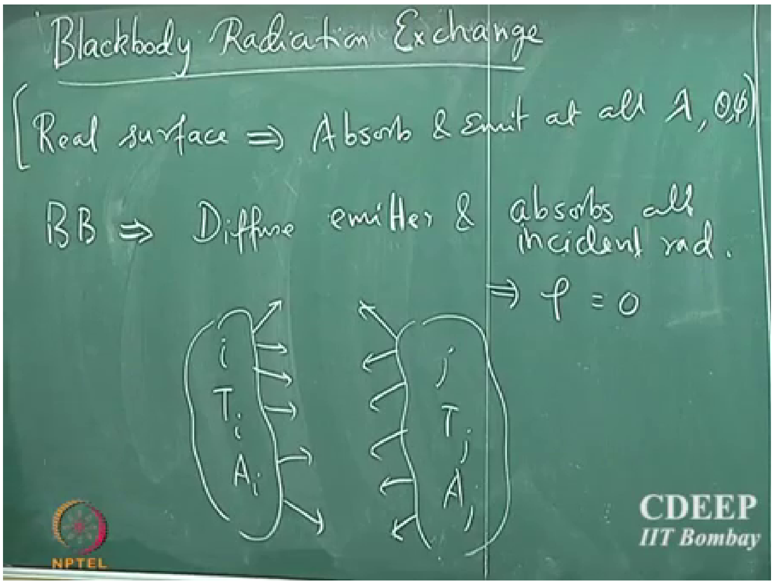

So, supposing we take real surface. So, we know that real surfaces absorb and emit at all wavelengths all lambda and all theta and phi ok, but with blackbody, we have a special property blackbody has a special property, what is that property? What is that property? What is the special property of blackbody? It absorbs all the incident radiation right. So, blackbody is a diffuse emitter and absorbs all incident radiation; all incident radiation, which means that reflection is 0 right; so this simply means that reflectivity is 0 for a blackbody ok.

So, suppose I have two black bodies I call it i and j and let us say that the temperatures are T i and T j and the surface area are A i and A j ok. So, now, the emission is going to occur at all locations in all directions, because it is diffused it is going to be equal in all directions right the diffuse emitter. So, now, the question is, what is the net rate of heat exchange between these two surfaces that is what we want to find ok?

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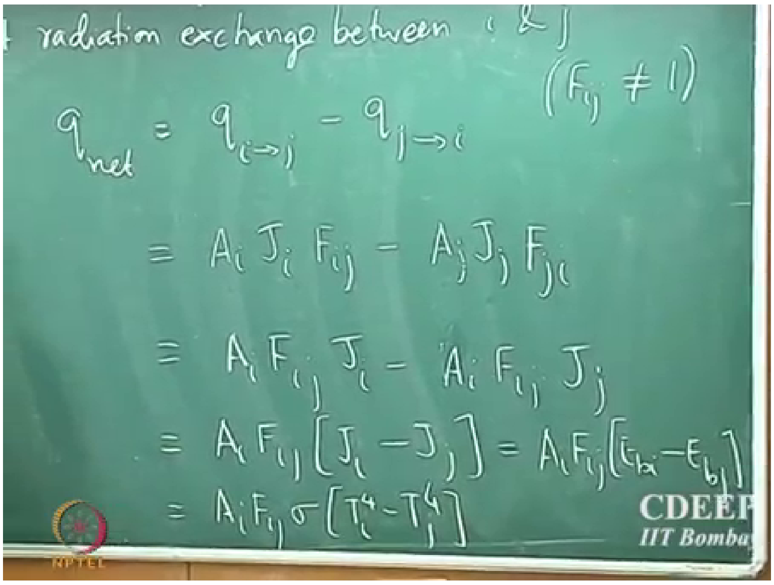

So, what is the net radiation exchange between i and j that is what we want to find ok. So, q net will be; so all of you must have heard that sigma Ti power 4 minus Tj power 4 will tell you the net radiation extreme, what is missing; that is not that is not the complete story, what is the key assumption that you make?

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That is forget all the conduction properties. Let us say there is only radiation mode of heat exchanging, what is the key assumption that you make when you use that formula.

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That is fine it is a blackbody. So, it has to be a diffuse emitter yes.

Student: (Refer Time: 12:59).

E equal to; it is a blackbody. So, it is a blackbody radiation.

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View factor you see. So, that is the first thing primary assumption that you make you assume that view factor is 1. So, most of the radiation calculations that you would have done so far you assume that there the view factor the geometric factor they are all geometrically aligned and that that the geometric factor does not contribute anything to the radiation extreme.

So, remember that in last lecture, we said that one of the crucial property that you need to find out if you want to calculate the radiation extent is the view factor you cannot assume that view factor is one between any two surfaces. This is the fundamental underlying assumption that went into all the calculations you have done so far ok. So, the first observation is that F i j is not equal to 1.

So, therefore, the net radiation is simply the difference between, what is emitted by i and received by j minus, what is emitted by j and receive by I ok. So, this q is given by Ai, Ji. So, that is the net radiation rate of radiation emitted by surface i multiplied by the corresponding new fact all right. So, that is what tells you what is that fraction, which is received by the surface j minus q j i is given by Aj Jj F ji ok. So, this A j into J j that is the rate at which the radiation is emitted by surface j and what fraction of that is intercepted by surface i is given by product of that with the view factor ok.

So, from reciprocity relationship we know that A i Fi j and A j F j i r related right. So, that will be A i F ij J i minus Ai F ij into Jj ok. So, that is equal to Ai F ij into Ji minus Jj. What is Ji for black body. So, keep in mind that the properties of black body are such that; it is a diffuse emitter and it absorbs everything which means reflection is 0 right.

So, Ji is just the emission from the blackbody and that is given by Stephen Boltzmann law right. So, that is where this sigma t power 4 comes in. So, that is equal to Ai F ij E b i minus E b j. So, that is the blackbody radiation. So, that is equal to ai F i j sigma Ti minus Tj to the power of 4 ok, because there is no reflection. So, note that this is because reflectivity is 0 that is why j I is equal to eb I there is no reflection from any surface it is all the emission that from that surface.

So, the net radiation exchange between the surfaces this quantity. Now, why is this important? Why is this important? Why do we need to know what is the net radiation exchange? Supposing, if I want to maintain the temperatures ok. So, let me put it in a different way supposing, if I want to maintain the temperatures of these two surface this is the amount of heat that either I have to provide to a body or remove depending upon the temperatures.

So, q net can be positive or negative depending upon what is q i j and q j i we do not know that; you have to put the numbers and find out whether q net is positive or negative. So, for a given body if q net is positive right which means that there is a net radiation that it is actually taking because of radiation exchange.

Therefore there has to be a heat sink inside that surface in order to maintain a constant temperature, if you want to maintain an isothermal temperature; that that is the amount of heat that has to be removed from that system if q net is positive. Now supposing if it is negative then that is the amount of heat that you have to supply. So, this is a very very important property remember. That in conduction you always said you always have to find out what is the net amount of heat that you have to supply or remove from the system.

So, that is the whole purpose of calculating the radiation exchange off course blackbody is an idealization, we are going to see how to do this for a gray surface in a short while, which is a real system ok. So, black body just gives you an advantage to an idealized system to understand; how the system is behaving because all the properties of all the radiation properties are scaled with respect to that of the corresponding black body all right.

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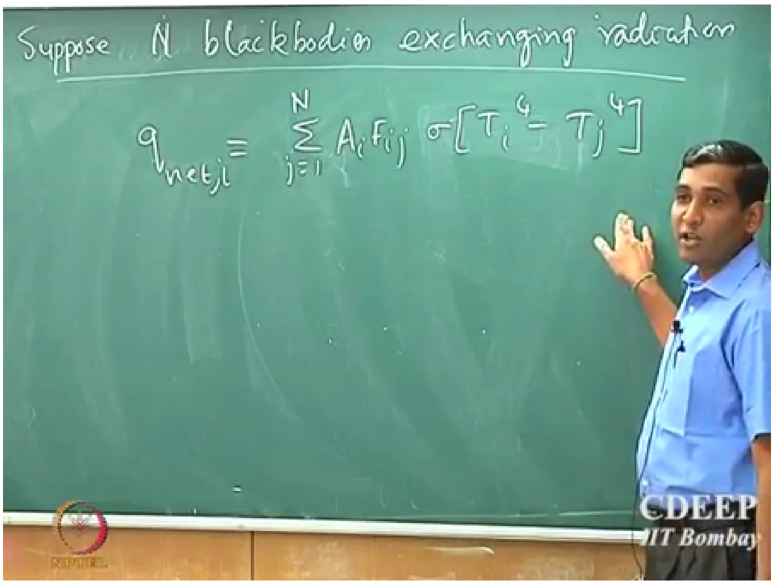

Supposing I have N surfaces like this, suppose I have N N blackbody exchanging radiation; suppose I have N blackbody then what will be the net radiation exchange;

what will be the net radiation exchange; it will be sum over all I right. So, it will be sum Ai F ij, so there will be sigma Ti to the power of 4 minus Tj to the power of 4.

So, this is the net radiation exchange of the I th surface and it is summed over 1 to all n ok. So, this is without loss of generality we have written this formula. You do not have to assume that there is no radiation exchange between itself. So, that is also countered for here because J equal to i would be Fi i. So, that accounts for radiation exchange with itself. So, if it is a convex surface, then Fi I will be 0, but is the concave surface it is not 0. So, this is a general representation of the net radiation exchange with N surfaces.