Heat Transfer
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Lecture – 05
Resistances in Composite Wall case

Alright.

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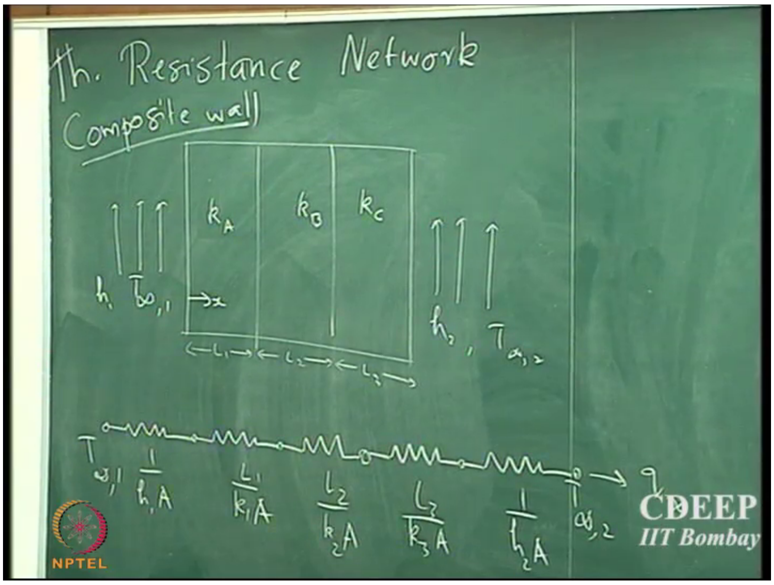

So we stopped by looking at the Resistance Network qualified by strength thermal Resistance Network for composite walls. So, that is where we stopped in the last lecture. So, supposing there are multiple walls which are associated; so if the length is l 1, l 2 and l 3 and we have fluid which is flowing on either side of the Composite wall.

So, that is the Composite wall. And if the conductivities are k A, k B and k C. So, we said that we could construct the network which has essentially 5 resistances, that is this x direction. And if the heat transport coefficient is h 1 on this side and the temperature is T infinity 1 and this is h 2 and t infinity 2. So, we said that it is t infinity 1 and t infinity 2 and the resistances are 1 by h 1 A, l 1 by k 1 A, l 2 by k 2 A and l 3 by k 3 A and 1 by h 2 A. So, one could define, we know what are the total resistances.

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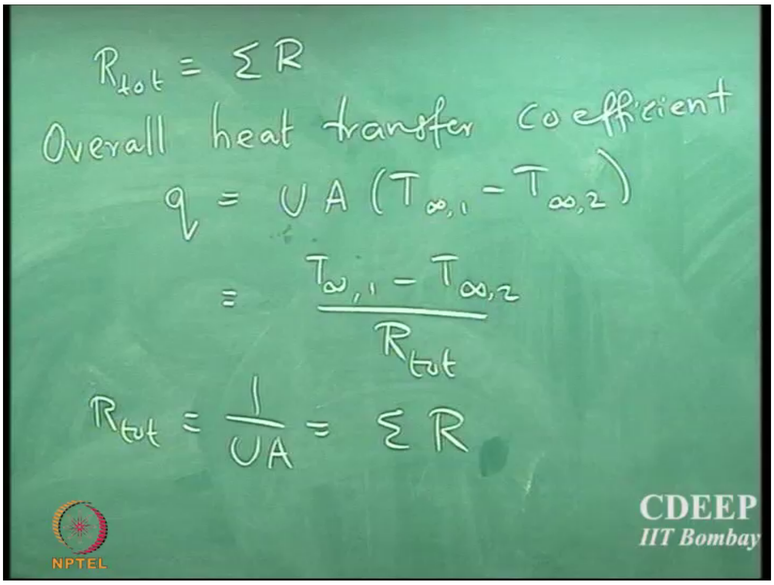

So, the total resistance, R total, will simply be sum of all the resistances. Sum of all the individual resistances, which is essentially sum of these.

Now, what we are going to start with in today lecture is, we are going to look at what is called the Overall heat transport coefficient. So, one could define overall heat transfer coefficient. And the reason for doing that is ultimately from measurement point of view. In fact is what I would measure is the temperature here and temperature here. So, I need to know what is the total amount of heat that is transferred from let us say, hot fluid on this side. Or I use 1, so I can put 1 2 and 3 and cold fluid on this side.

So, I want to know, what is the effective heat transport, that is occurred from the hot fluid to the cold fluid. And that is simply because I may not be able to measure the temperatures in between. So, one could define the total amount of heat that is transported, similar to the way we defined Newton's law of cooling as the constitutive equations. So, we could write, we could define an overall heat transport coefficient U, multiplied by A, multiplied by the net temperature difference of the observable temperatures or measurable properties. And that should obviously be equal to, that should be equal to what?

Yeah, by R total; so that's right. So, that should be equal to T infinity, 1 minus T infinity, 2 divided by R total simply by the definition of the resistances. So, from here we can clearly read out that R total equal to 1 by U A and that should be equal to this sum of the individual resistances; sum of all the resistances that is involved in the system that we are considering. So, this is a ubiquitous property of a definition of any heat transport system. So, in principle one could define a overall heat transport coefficient for any heat transport system.

And what we are doing essentially is, we are lumping all the properties, all the transport processes which are occurring inside, everything is lumped into these one quantity called universal or overall heat coefficient. And we will see many different variations across the overall heat transport coefficient, depending upon the system that we are considering; that we are going to see in today's lecture and several lectures in future.

So, it is very important to understand the definition of overall heat transport coefficient. And note that it is a fictitious quantity. It can be detected based on the individual properties of the system that we are considering. However, overall heat transport coefficient is mainly for convenience purposes and for calculation purposes. It really helps in defining such kind of a quantity.

So, there is 1 small aspect about resistances.

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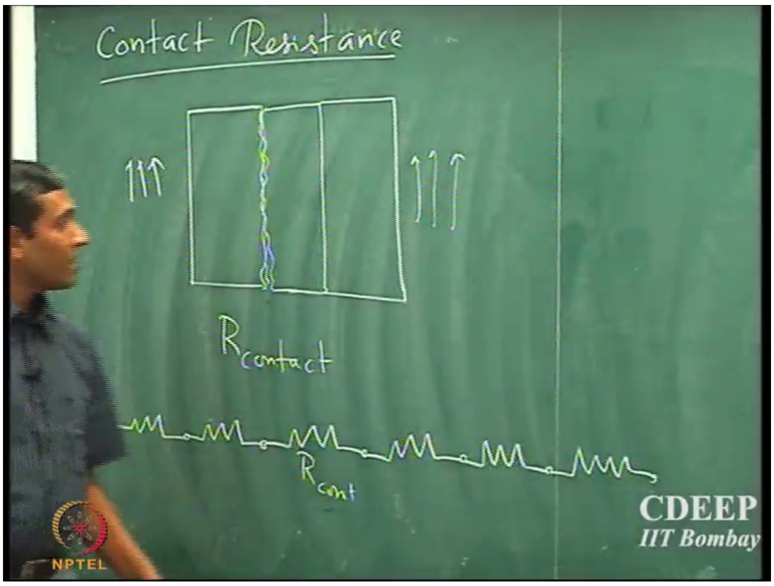

There is something called Contact Resistance. So, so far and all the that couple of examples that we described in the last lecture and start of today's lecture, the we assume that the contact between the slabs is supposed to be a smooth contact; however, in reality that need not necessarily be the case. For example, if you have, when you say smooth, it is smooth all the way up to the microscopic level right. So, it is not this smoothness that you observe in your eyes; it is the smoothness that you would observe when you see under a microscope. That you will not expect.

There is no reason why you should expect that the wall that you are having has a smooth surface all the way up to the microscopic level. So therefore, there will always be certain resistance which is offered by the non smooth contact between the 2 surfaces of the composite wall. So, as a result, one could define something called a Contact Resistance. And it depends upon the smoothness properties. So, there is no clean way to estimate what should be the value of this contact. There are some correlations. We will not go into those correlations in this course, but as and when it is required particularly from the problem solving or exam point of view, these kinds of numbers will be provided to you. So, give me a sec.

So, supposing if you have another wall here, now if I want to construct resistance network including the contact resistance, then what I would do is; supposing I have fluid which is flowing here, then you will have. If there is no Contact Resistance, then there total of 5 resistances, but because the contact position is not smooth, it is going to offer a certain resistance to heat transport and therefore, you will have an additional network which is basically the Contact Resistance which will come in series. So, the other 5 are the same thing what we saw a short while ago. So, we need to include a Contact Resistance which is present in between. Supposing we want to include a Contact Resistance between the second and the third wall, we could do that. We could include another resistance here. Any questions?

Student: Why should because of there is a lack of.

Lecturer: Correct.

Student: (Refer Time: 08:32)

Lecturer: Yeah. Supposing if they are not in contact with each other; a very simple example is, supposing you are holding a coffee cup, there are 2 ways of holding it: I could hold it like this with all my fingers on the curved surface of the tumbler or I could hold it just at the top. So, if you look at the workers who are like drinking coffee and tea, the hot tea, they usually hold the tip or they hold a cup like this. And the reason is that, you do not want to have a direct contact with the surface which is hot.

Now taking that parallel here, if this surface is not smooth, then the contact, the transport of heat from one slab to the other depends upon the overall effective surface area which is available for transport. Now if the contact is not very smooth, then the total surface area which is available for heat transport from one slab to the other slab is not as much as the overall surface area which is available. And therefore the total amount of heat that is transport is not exactly the amount of heat that comes at this end and therefore this offers a resistance. Yes, you had a question.

Student: (Refer Time: 09:46).

Yes, there will be dissipation because of that. See, what is the resistance? Note that the resistance is basically characterizes the total amount, the ability of the system to transport heat, what is the resistance that it offers to transportation of heat from one location to the other. Because the contact point is not significantly good, there is going to be some dissipation. And therefore that that offers a certain resistances and that is what is captured by the Contact Resistance. Any other question?

Alright. So, next what we are going to see is, we assumed so far in all the cases that we considered that the area of heat transport is constant.

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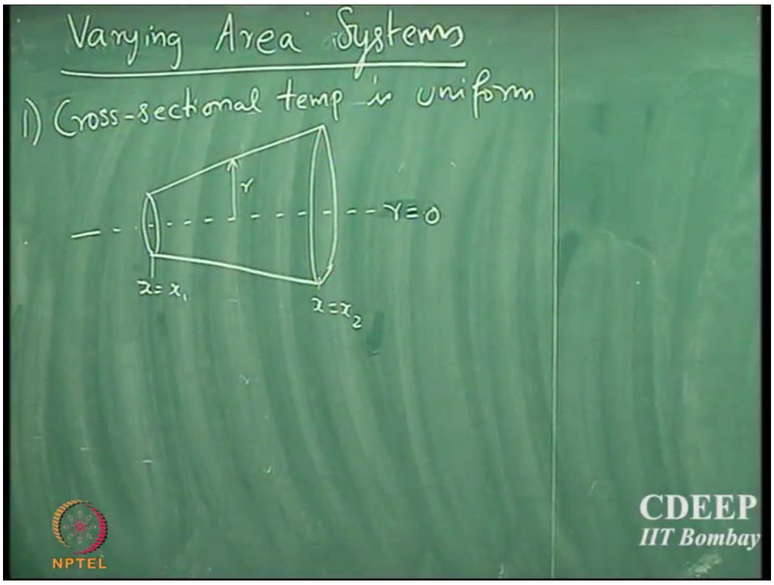

So, today we are going to look at the Varying Area Systems. So, how to characterize and quantify conduction process through a system where the surface area or cross sectional area for heat transport is constantly changing. A very simple example would be that, supposing I have a truncated cone, I have a truncated cone and let us say I am looking at. So, this is let us say at equal to 0.

So, I want to know what is the heat that is being transported from let us say, let me call this as x direction, x equal to x 1, x 2. I wanted to call it radius. So, that is the center of radius is 0. So I want to know what is the amount of heat, that is transported from x 1 to x 2. And I am going to make an assumption; because I am looking at 1D systems, I am making an assumption that the Cross-sectional temperature; that means, that at every cross section I assume that the temperature is uniform in the cross section and the gradients are 0.

So now, so this is my radius at any location and so I could write my balance.

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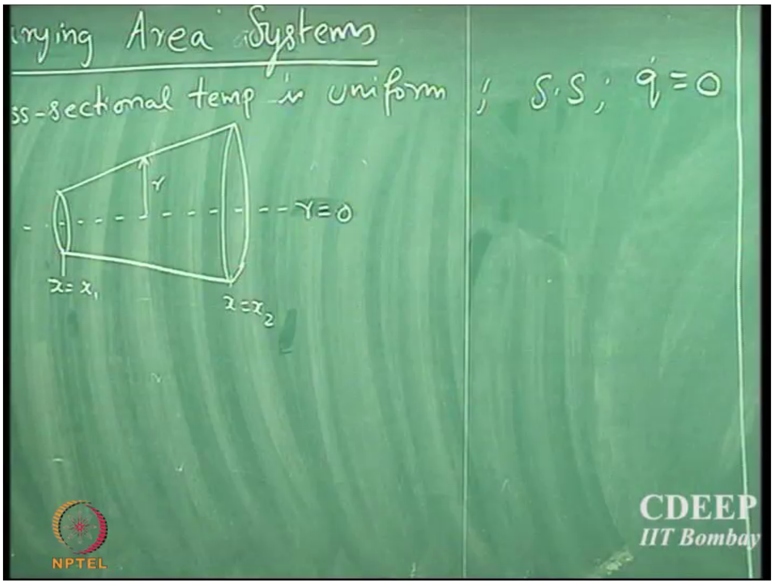

And if I continue to assume that it is a Steady State system and heat generation is 0, same assumptions as what we made before.

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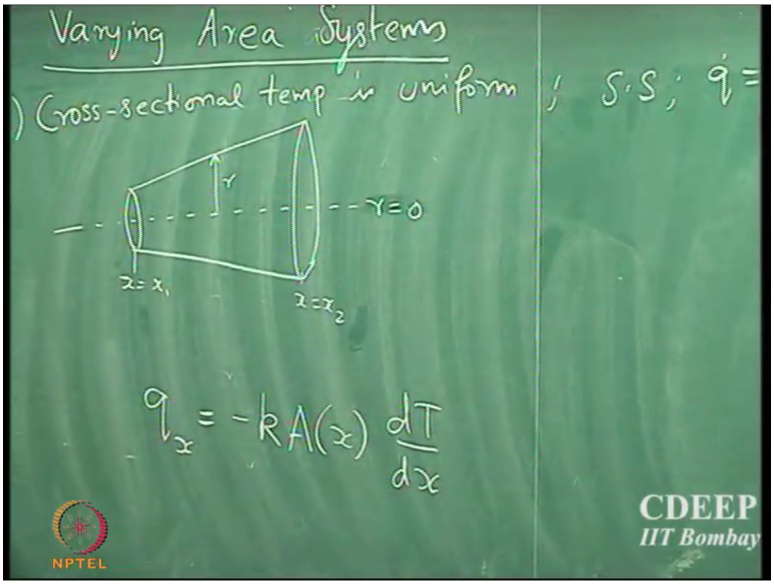

So, I could simply write the total amount of heat that heat transfer rate q x is given by minus k which is the conductivity of that material multiplied by the cross sectional area of heat transport. So, note that now the cross sectional area is a position of the function is a function of the position, excuse me; multiplied by d T by d x. So, what is the objective? We need to find the temperature profile that is the objective of the problem right? So, we said that if we know the temperature distribution, if we know the temperature profile, we are done. We have quantified the system. So, that is what we need to find. So, let us say we integrate this equation.

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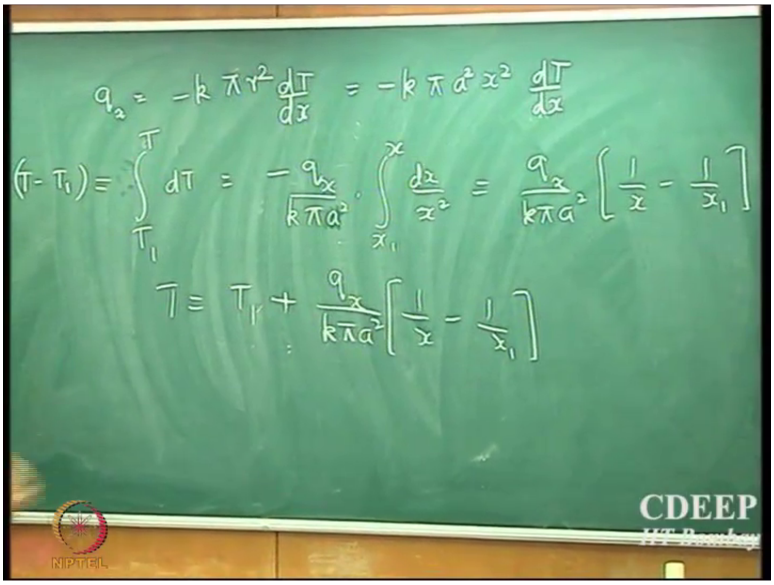

So, we say that q x is minus k. What is the area? It is pi, cross sectional area is pi into radius square. Pi r square, multiplied by d T by d x right. So, supposing I say that r goes as a linear function of the position, if I say that the radius of the local radius goes as the linear function of the axial position, then I could simply write this as minus k pi a square x square into d T by d x, into d t by d x. And now what will be q x? Will it be constant or it will change; rate of heat transfer that will be constant. Why will it be constant? Because of the energy balance, you see that there is no heat that is being generated.

So, whatever comes in it has to go out here at steady state condition. Note that steady state is very important. Yes. So, the question is if you dissipation of heat, how will it be constant, but when we say that there is no generation or less of heat which means that the dissipation is 0. We will come to that. So, there are ways to consider that. We are actually consider, when we are doing a 2 dimensional system, you can actually look at dissipation from the outside walls and we will actually see it in one of the examples in the future lectures. Alright. So, because q x is constant, we should be able to integrate this expression to find the temperature profile.

So, supposing I integrate between T 1 and T. T 1 is the temperature at the boundaries of this system. T 1 is the temperature at the boundaries of this system. I integrate between T 1 and T, that is equal to q x minus q x by k pi r square or k pi a square into d x by x square going from x 1 to x. It is a pretty simple integration. So, it is q x by k pi a square into 1 by x minus 1 by x 1.

So, note that because it is 1 by x square, the minus sign will go away because of the integration. And this is T minus T 1. So, therefore, T is T 1 plus q x by k pi a square into 1 by x minus 1 by x 1. Is it a complete description? We do not know the q x value right. So, it is not a complete description yet. So, how do we find q x? So, we know that the temperature on the other boundaries is T 2. So, we can use that property to find out what is q x. So, we do not know what q x is.

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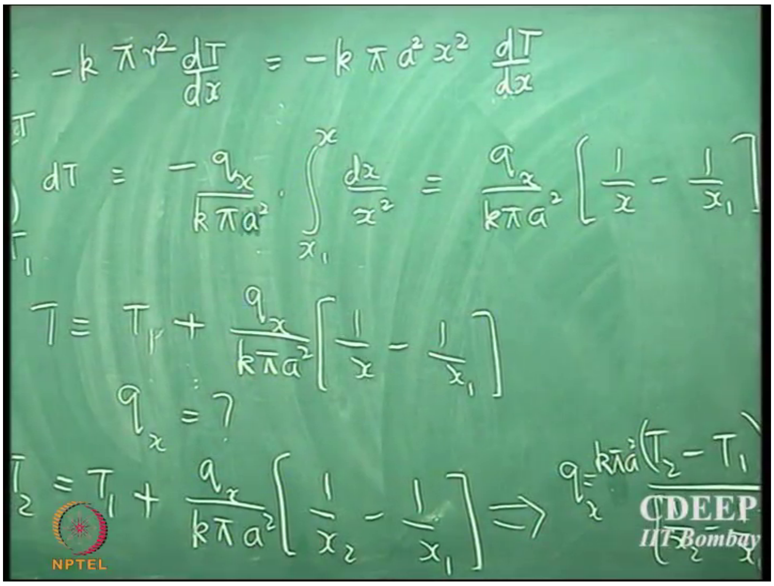

So, how we are going to do that, we are going say T 2 is T 1 plus q x by k pi a square multiplied by 1 by x 2 minus 1 by x 1 ok.

So, from here q x is given by T 2 minus T 1 divided by 1 by x 2 minus 1 by x 1 multiplied by k pi a square, multiplied by k pi a square. And so now we can plug this into our solution. We can plug this into our solution.

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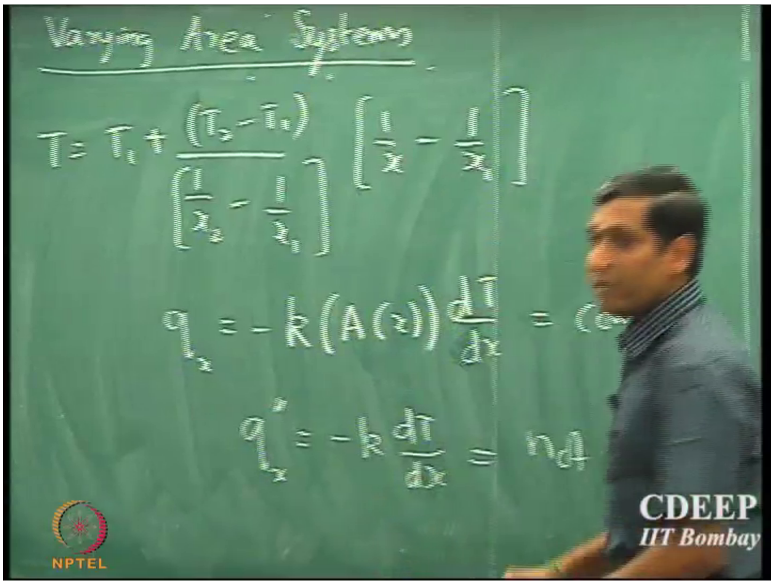

So, T equal to T 1 plus k pi plus T 2 minus T 1 divided by 1 by x 2 minus 1 by x 1 multiplied by 1 by x minus 1 by x 1. So, that is the distribution, temperature distribution in the system with the varying cross sectional area. So, an important message of this example is that what you need, what is preserved here or what remains constant in this system; because there is no heat generation or dissipation is the heat transfer rate and not the flux. So, you have to make a distinction here. So, what we said is the heat transfer rate, these remains a constant. However, the flux which is given by q prime which is minus k d T by d x, this is not a constant.

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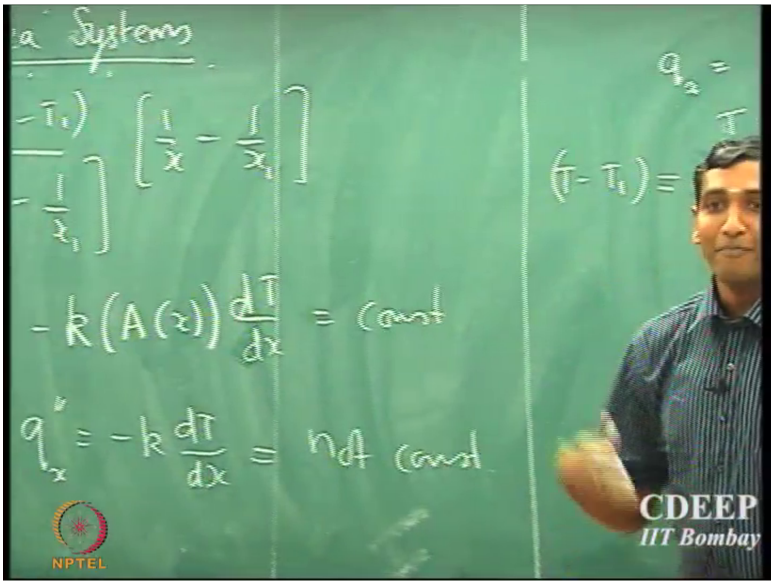

So, this is an important observation which you would not have made in the simple 1 D system where the cross sectional area is constant. The cross sectional area of heat transport is constant, where you would not be able to make such a distinction between the 2. So, when you have varying cross sectional area, what is really conserved and what is really preserved is the heat transfer rate and in fact that is the reason why you write a rate balance and not a flux balance.

So, this is very important to understand this distinction. It is important to write transfer rate balance because that is the final quantity that is importance and the flux need not necessarily remain constant even in a small element. So, it is very important to understand this distinction. And we are going to next see how these things are going to play a role when you are looking at radial systems where this becomes extremely important.