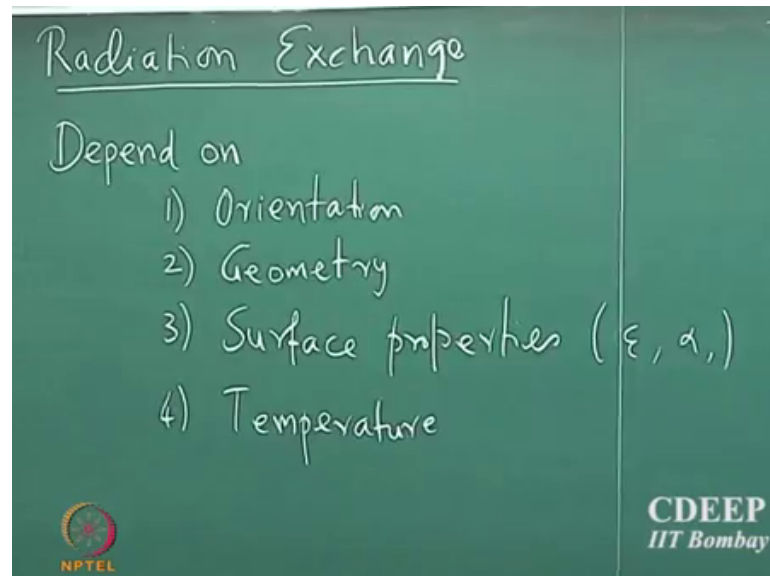


**Heat transfer**  
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**Lecture – 49**  
**Radiation exchange: View factor**

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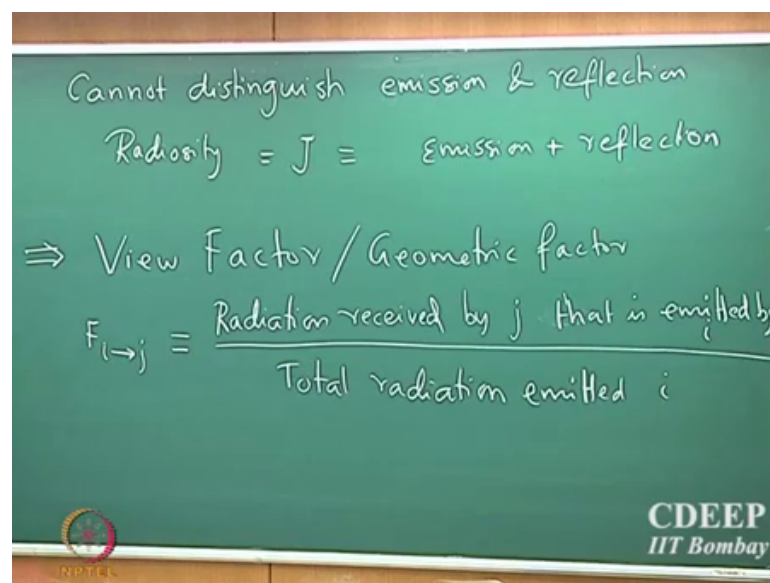
So, so far we have been discussing about individual aspects of radiation. So, now, we are going to from today's lecture for the next three or four lectures, we are going to look at radiation exchange between different surfaces. So, the radiation exchange would depend on several factors. So, the first one is obviously, the geometric orientation depending upon the orientation of the objects the radiation exchange is going to be different, depending upon the orientation. For example, you might have an object which is facing perpendicular to each other, or you might have objects which is facing parallel to each other, so depending upon the orientation between each other, the radiation exchange is going to be different.

And it is of course, going to be a function of the geometry of the material which is exchanging radiation. And it is going to be a function of these surface properties. For example, emissivity, absorptivity, etcetera. And it is going to be a function of temperature. So, these are the four primary factors on which the radiation exchange would depend on. And what we are going to see in the next few lectures including

today's is that how to characterize radiation exchange between two surfaces and in fact the net radiation which is received by a surface is essentially what is the net amount after the exchange has taken place between the two objects.

So, there has to be a because one object emits radiation there is absorbed or reflected or transmitted if it is not an opaque object by the other object; and the other object is going to emit and the reflected radiation is also going to be received by the source object. So, there is a constant exchange of radiation between them.

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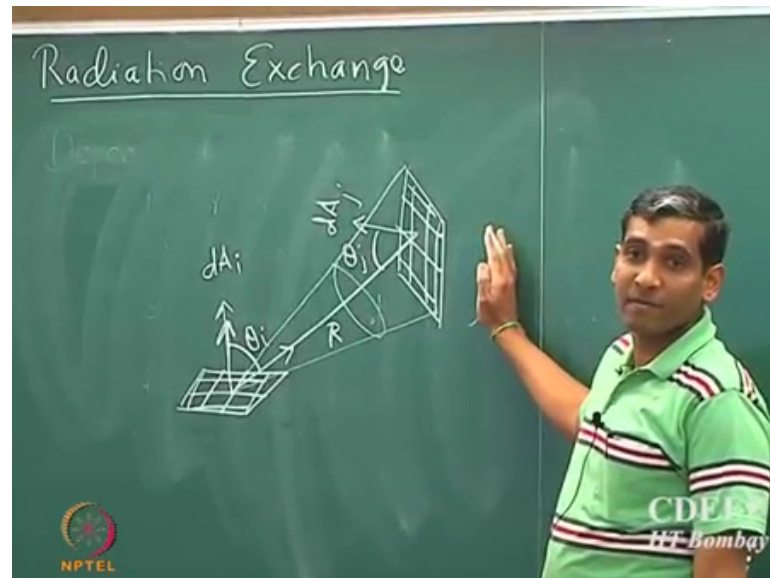


So, therefore, in order to characterize the radiation exchange we cannot distinguish between so whatever net radiation that leaves the surface is essentially sum of what is emitted plus what is reflected. So, the symbol that we will use is called  $J$  which stands for radiosity and this is nothing but emission plus reflection, emission plus reflection is what is called as radiosity. And one could certainly define the spectral hemispherical total etcetera dot dot dot all the (Refer Time: 04:21) integrals you can define for radiosity as well.

So, with this definition, we the first aspect we are going to look at in today's lecture is how to characterize orientation geometry. So, we need to define what is called a view factor or it is also sometimes called a geometric factor, also called a geometric factor. So, this view factor essentially characterizes or it takes into account the orientation and geometry of the two surfaces which is or  $n$  surfaces which is exchanging radiation. And

it is defined as the ratio of radiation or received by let us say object j that is emitted by object i divided by total radiation emitted by object i. So, this is the view factor. So, this term capital F is this symbol that we will use for view factor. So, this defines the view factor as what is the fraction of radiation which is emitted by surface i and is received by surface j.

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So, suppose I have a point surface. And if the area of this surface is  $dA_1$ , so that is a differential surface. And let us say I have another surface here and the differential area is let us say  $dA_i$  and  $dA_j$ ,  $dA_i$  and  $dA_j$ . So, if that is the line that connects the midpoint of the differential element, and if the distance is  $R$ , so the view factor is nothing but whatever is the total emission that comes out of this surface i and what fraction of that is received by surface j.

Now, it is the geometry which actually defines what is going to be refraction. The reason is that not all radiation that comes in all directions of this surface is intercepted by this object; so it is only a fraction which is intercepted and therefore, it cannot take all the radiation that comes out of this surface can only be a fraction and that is what is defined as view factor all right.

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$$dq_{i,j} = I_{i, \epsilon + \rho} \cos \theta_i dA_i d\omega_{j \rightarrow i}$$
$$d\omega_{j \rightarrow i} = \frac{dA_j \cos \theta_j}{R^2}$$
$$q_{i,j} = \iint_{A_i, A_j} dq_{i,j} = \iint_{A_i, A_j} \frac{I_{i, \epsilon + \rho} \cos \theta_i \cos \theta_j}{R^2} dA_i dA_j$$

So, supposing if this angle is theta i and this angle is theta j what is the radiation that is emitted by surface i and received by surface j, what is the radiation that is emitted by surface i and received by surface j. How do we find this? So, the first thing is, so this is you should remember the definition of the radiation intensity that we discussed at the start of radiation, so that is the supposing if I is the intensity of radiation that comes out of this surface, and that is i comma emission plus reflection right. So, note that when we have to deal with radiation exchange, we need to look at both reflection plus emission together right multiplied by cos theta i.

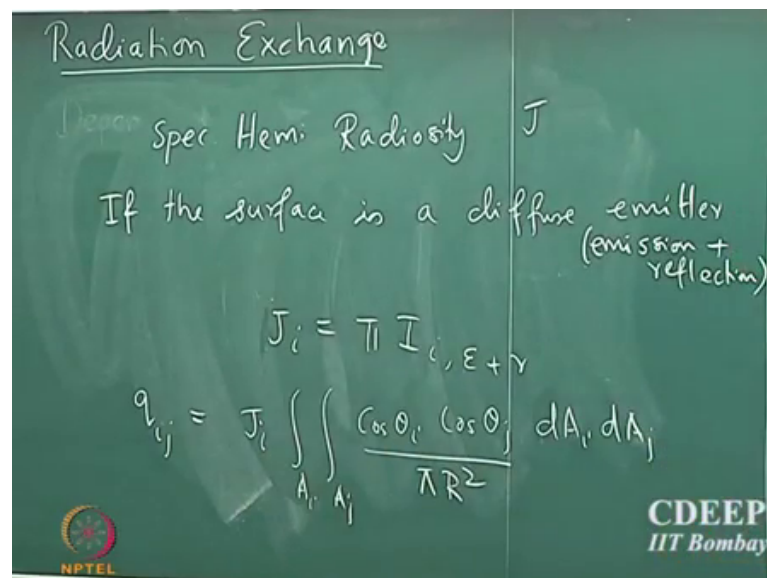
So, if the radiation intensity vector is pointing in the outward perpendicular direction, so I times cos theta will be the intensity in the direction towards the second object, and that multiplied by dA i will give you the rate. Note that this is flux right multiplied by dA i will give you the rate at which the radiation is leaving from this surface in the direction in which it is pointing to the second surface multiplied by the corresponding solid angle. Multiplied by the corresponding solid angle will tell you what is the amount of radiation that is intercepted by this surface.

So, we need to find the solid angle that is subtended by this surface on that point object and that solid angle multiplied by the rate at which the radiation is leaving that surface will tell you what is the differential radiation that is emitted by this surface; and this is received by the second surface alright.

What is  $d\Omega_{ji}$ , what is the differential solid angle, what is the definition of solid angle  $d$ . So, note that  $dA_j$  is pointing in the normal direction,  $dA_j \cos \theta_j$  right. So, the area which is pointing towards the emitting surface is  $dA_j \cos \theta_j$  right. So, the solid angle is  $dA_j \cos \theta_j$  divided by  $R^2$ , so that is the differential radiation that is emitted by a surface  $i$  and is received by surface  $j$ .

So, therefore,  $d q_{ij}$ ,  $d q_{ij}$  is  $i$  plus  $r \cos \theta_i \cos \theta_j$  into  $dA_i dA_j$  divided by  $R^2$ . Now, if you want to find the total radiation that is received by the second surface and that emitted by the first surface, we simply have to integrate this  $q_{ij}$  over  $A_j$ , we integrate this differential element over the area of both the surfaces is that cleared everyone alright.

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Now, the spectral hemispherical radiosity right spectral hemispherical radiosity is given by so the symbol I use this  $J$ . So, supposing if the intensity where to be diffuse. So, it is a diffuse emitter. If the surface is a diffuse emitter, so when I say emitter here I mean it is diffused with respect to both the emission from its surface plus the reflection emission plus reflection all right. So,  $J$  will be, how are  $J$  and  $I$  related it is diffuse it is  $\pi$  into  $I$  epsilon plus  $\rho$ . So, from here we can substitute the expression.

So, you will get  $q_{ij}$  will be. So, note that it is diffused which means its not a function of the angular position and so it will be  $J_i \int_{A_i} \int_{A_j} \cos \theta_i \cos \theta_j dA_i dA_j$  divided by  $\pi R^2$ .

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Total emission from surface  $i = J_i A_i$

$$F_{i \rightarrow j} = \frac{J_i \iint_{A_i, A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i dA_j}{A_i J_i}$$

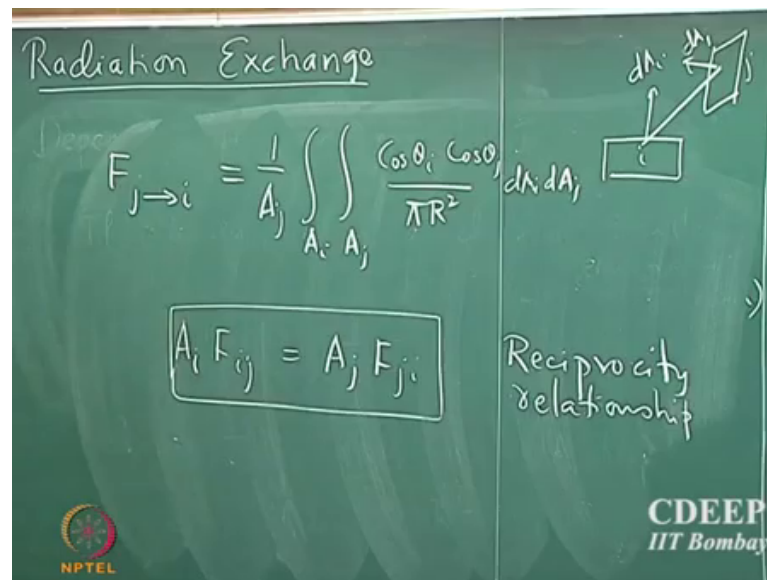
$$F_{i \rightarrow j} = \frac{1}{A_i} \iint_{A_i, A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i dA_j$$

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What is the total emission from the surface  $i$ , what is the total emission from surface  $i$ ? It is so this is diffused  $j$  is a diffuse emitter. So, it will simply be, what will it be what would be the total emission, it is diffused. So, it is constant in all directions. So, it will be nothing but  $J_i$  is the flux of emission plus reflection multiplied by the surface right, so that will give you the total rate of emission from the surface  $i$ . With this, we can now define the view factor as  $J_i \int A_j \cos \theta_i \cos \theta_j \pi R^2 dA_i dA_j$  divided by  $A_i J_i$ . So, note that this is valid only when it is a diffuse emitter; otherwise, this  $j$  will go inside the integral and you will have to integrate over the whole area.

So, note that if it is diffused then the area scales out like this; otherwise you will have to integrate over the whole area right, because the differential amount of emission plus reflection is not going to be dependent on the position. So, now I can cancel out the  $J_i$ 's then so it will be  $F_{i \rightarrow j}$ , so it will be  $\frac{1}{A_i} \int A_j \cos \theta_i \cos \theta_j \pi R^2 dA_j$ . So, this is the general formula for view factor if the emitting surface is diffused. It is very important. This is valid only when the surface is a diffuse emitter. Diffuse emitter is where the emission is now independent of the angular position.

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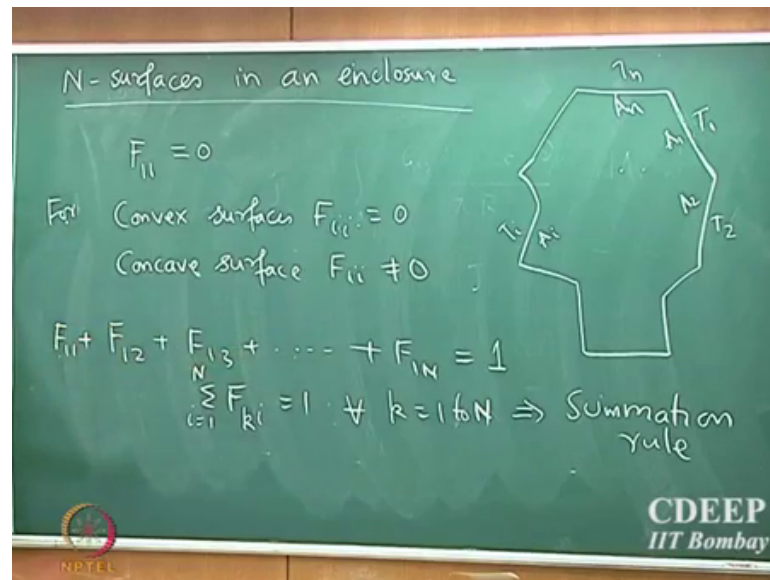
What will be  $F_{j \rightarrow i}$ ? What will be  $F_{i \rightarrow j}$ , so that is the view factor for emission which is leaving surface to maybe I will put the cartoon here. So, here is surface  $j$  and here is surface  $i$ . So,  $F_{i \rightarrow j}$  tells you what is the view factor for emission which is coming out of the surface  $i$  and is received by surface  $j$ .  $F_{j \rightarrow i}$  is emission coming out of surface  $j$  and that fraction which is received by surface  $i$  fine. So, what will that be based on this expression, it will be  $\frac{1}{A_j} \int_{A_i} \int_{A_j} \cos \theta_i \cos \theta_j \pi R^2 dA_i dA_j$ , once again this assumed that it is a diffuse emitter.

So, from this you can clearly see that  $A_i F_{i \rightarrow j}$  will be equal to  $A_j F_{j \rightarrow i}$ , because this geometry which is seeing the other geometry, and this is essentially the relationship between the solid angles. What you have essentially connected is the solid angle which is subtended by this object on  $j$ , and solid angle which is subtended by  $j$  on  $i$ . So, we have just taken care of that and this relationship is what is called as reciprocity relationship. So, it is very important to find these relationship you are going to find a couple of other relationships in a short while.

So, the first exercise whenever you get a radiation problem to be solved, the first exercise is to find the view factor that is the first step when you want to calculate radiation exchange between any surface. So, such conservation relationship helps you in finding various view factors with almost no effort. For example, if you know one view factor the other one comes for free. So, as long as you know the area you should be able to find the

other view factor. So, such kind of conservation relationship helps you in finding as many view factors as possible without doing the query algebra. So, if you know this, you do not have to integrate the expression again all right.

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Supposing I have n surfaces, supposing I have n surfaces in an enclosure and it is exchanging radiation. So, we have let us say they are maintained at  $T_1, T_2$  etcetera up to  $T_n$ , there are n surfaces, and each of these have a surface area of  $A_1, A_2, A_i$  and  $A_n$  in general. So, from here we can easily read supposing if I want to calculate  $F_{11}$  what is  $f_{11}$ . So, supposing if it is a planar surface what is the  $F_{11}$ .

So, whatever emission which is actually coming out of this surface which is planar, so I said it is a planar surface. So, whatever emission that comes out of this planar surface is not going to be intercepted by itself because of the orientation right. So,  $F_{11}$  is actually 0 or in general one could say that convex surfaces for all convex surfaces for all convex surfaces the view factor of itself is 0. So, if I is the fact that I give to the surface then the view factor to itself is 0; and if it is a concave surface,  $F_{ii}$  is not equal to 0, so that is an important observation.

What about  $f_{\text{sum of } F_{11}, F_{13}, \dots, F_{1N}}$  what will be the sum of all the view factors it will be 1 right because by definition view factor is the fraction of radiation that is emitted by that surface by a surface and as received by another surface. So, this should be equal to 1 plus if it is a closed enclosure note that this is valid only for a closed enclosure if it is not



a closed enclosure then you will have to identify or intuit has to what should be the correct view factor summation that you have to use.

We will see some example problems where you will get a much more clear picture as to what you should do when you have a open enclosure all right. So, it is essentially  $\sum_{i=1}^N F_{1i}$ ,  $i$  going from 1 to  $N$  that is equal to 1. So, this rule is valid for all the surfaces right. So, in general, I can write this as  $\sum_{k=1}^N F_{ki}$ , for all  $k$  going from 1 to  $N$ . So, this rule is called a summation rule, this is called the summation rule. So, if you know  $N$  minus 1 view factor, the  $N$ th one comes for free simply by using this summation rule.