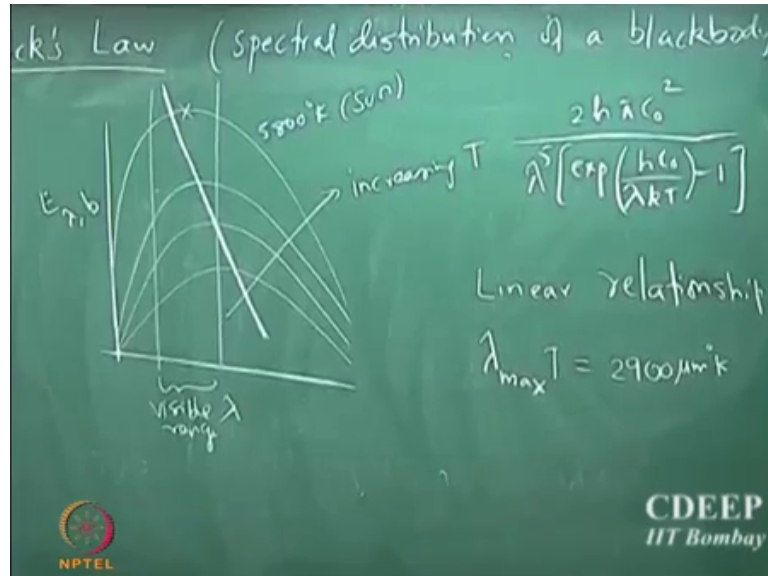


Heat Transfer
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Lecture – 46
Properties of a Blackbody

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If I plot lambda versus $E_{\lambda,b}$ essentially I am plotting $\frac{2 h \pi C_0^2}{\lambda^5 \left[\exp\left(\frac{h C_0}{\lambda k T}\right) - 1 \right]}$ So, that is what I am plotting.

So, what I find is for different temperatures, that is the kind of behaviour that you will get. This is as increasing temperature and this has been plotted for it is very easy to plot this just put a different temperature and plot it as a function of the wavelength and. So, people have done this for the temperature of sun is approximately 5800 Kelvin and it turns out that the maxima of that curve it actually falls in the visible range, you know where you can see, visible range is the wavelength at which we can see things.

In fact, the reason why we are able to get so much light from sun is because the maximum radiation that is emitted by sun which is close to a blackbody (Refer Time: 01:55) supposed to be close to a blackbody. So, the maximum emission that is radiated by sun it falls in the visible range that is why we are able to get so much light from the closest star which is sun.

So, one can actually join these so, it turns out that by join it with a thicker lines, If I join all the maxima together at different temperatures, they all fall into a certain pipe certain pattern. They all fall into a linear pattern that is the lambda max which is the maximum intensity wavelength multiplied by temperature so, that is at constant in fact, I will show you how to derive this; it is about 2900 micro meter Kelvin. So, that is the sort of inverse linear relationship that actually one would find that the maximum wavelength as a function of temperature it seems to remain a constant.

In fact, this short very difficult to show this how do I find maxima of each of these curve? How do I find the maxima? You take the first derivative and set that to 0 and I have done that for you.

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The image shows a chalkboard with the following handwritten text and equations:

Radiation - Blackbody

$$E_{b,\lambda} = \frac{C_1 \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right) - 1}$$

$$C_1 = 2\pi h c_0^2$$

$$C_2 = \frac{h c_0}{k}$$

$$\frac{d}{d\lambda} [E_{b,\lambda}] = 0$$

$$\lambda_{\max}$$

$$\Rightarrow \exp\left[\frac{C_2}{\lambda T_m}\right] \left[C_1 C_2 \frac{\lambda_m^{-7}}{T_m} - 5 C_1 \lambda_m^{-6} \right] + 5 C_1 \lambda_m^{-6} = 0$$

$$\Rightarrow \left[\frac{C_2}{5 \lambda T_m} - 1 \right] + 5 \exp\left(-\frac{C_2}{\lambda T_m}\right) = 0$$

Logos for NPTEL and CDEEP IIT Bombay are visible at the bottom of the chalkboard.

So, d by d lambda, the way to see this is to express E b as in this form c 2 by lambda t minus 1, where C1 is 2 pi h C naught square and C2 is h C naught by k all we have done is just clubbed all the known constant into a single constant and then define d by d lambda E b lambda evaluated at lambda max equal to 0.

So, that is what will find you the relationship between lambda max in the corresponding temperature and so, that will be exponential of C2 by lambda T m into C1 C2 lambda power minus 7 divided by T m minus 5 C1 lambda power minus 6, that should be lambda max plus 5 C1 lambda max to the power of minus 6 equal to 0. So, if you take the derivative that is what it will turn out to be and after a little bit of algebra you can

actually reduce the equation to this form C_2 by $5 \lambda_m T_m$ minus 1 plus 5 exponential of minus C_2 by $\lambda_m T_m$ equal to 0. You can rewrite after little bit of algebra you can actually reduce it to this form.

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Wien's Law (spectral distribution of a blackbody)

$$x = \frac{C_2}{\lambda_m T_m}$$

$$\left[\frac{x}{5} - 1 \right] + 5 \exp(-x) = 0$$

$$\frac{C_2}{\lambda_m T_m} = 4.965$$

$$\Rightarrow \lambda_m T_m \approx 2900 \mu\text{m} \cdot \text{K}$$

Wien's Displacement Law

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And now, you set x to C_2 by $\lambda_m T_m$ and T_m as T_m corresponds to the corresponding temperature and so, you can solve the equation x by 5 minus 1 plus 5 times exponential of minus x equal to 0. So, this has to be solved numerically so, you solve this what you will find is that C_2 by $\lambda_m T_m$ is 4.965 and if you plug in the value of C_2 you will find that $\lambda_m T_m$ is about 2900 approximately.

So, you should actually solve this equation and try to find out the correct answer for that particular expression and this is what is called Wien's displacement law which relates the temperature at which the maximum intensity is achieved by a given blackbody, once again a little bit of definition here something called band emission.

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Radiation - Blackbody

Band Emission

$F(0 \rightarrow \lambda) = \frac{\int_0^\lambda E_{b,\lambda}(\lambda, T) d\lambda}{\int_0^\infty E_{b,\lambda}(\lambda, T) d\lambda}$

$= \frac{1}{\sigma T^4} \int_0^\lambda E_{b,\lambda}(\lambda, T) d\lambda$

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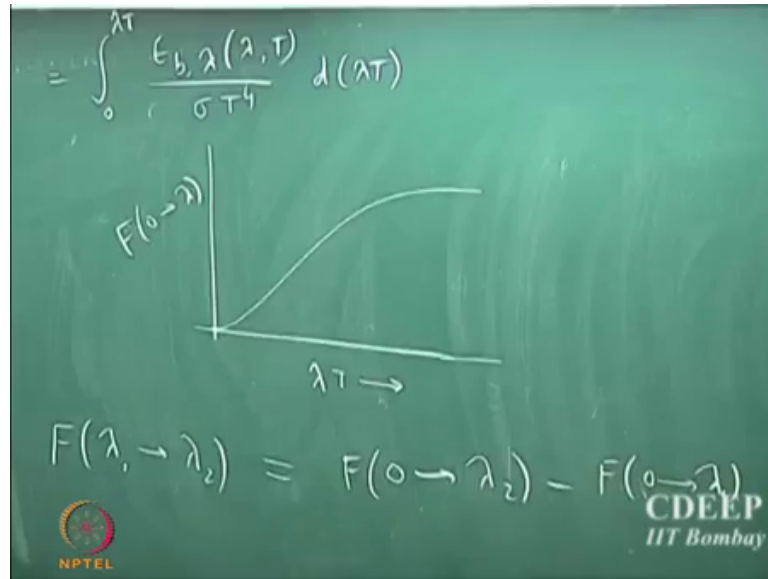
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So this is useful particularly when you are solving problems, otherwise my opinion I do not see much use for this band emission concept, but you should know this because it is useful in solving some problems in the exam etcetera.

So, what it tries to answer is you define a quantity called F which is the fraction of emission so, supposing F 0 to λ is basically the fraction of total emission from a black body so, the total emission from black body is nothing but integral over all wavelengths and F 0 to λ will tell you what is the fraction of the total that is emitted in that range. So, simply from the definition 0 to λ E_b λ divided by integral 0 to infinity d λ , what is the denominator? Integral E_b λ d λ 0 to infinity that is your Stefan Boltzmann law σT^4 and so, that is nothing but 1 by σT^4 integral 0 to λ d λ .

So, we can rewrite this expression in a slightly more convenient form.

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And that will be, we change the variables and that will be E_b divided by σT^4 into $d(\lambda T)$. So, we just introduce a change of variables so, people have calculated this expression and you have these standard tables with x axis having λT and the y axis having 0 to λ . So, it turns out that this is the kind of behaviour so; it is particularly useful when you are doing certain calculations on blackbody, if you want to know what is the emission at a given range of λT then you can simply find out using this curve here by subtracting the corresponding integrals. So, if you want to know what is the emission $F(\lambda_1 \rightarrow \lambda_2)$ it is nothing but what is this?

Student: $F(0 \rightarrow \lambda_2)$.

$F(0 \rightarrow \lambda_2)$ minus $F(0 \rightarrow \lambda_1)$ so, that will tell you what is the net emission in the range λ_1 and λ_2 . So, it is just a useful tool for solving some problems that you may get in your exam later, otherwise I do not see much use for the band emission concept, but I think it is just for it is a useful tool as what you must understand. Any questions on this so far? All right.

So, we have few more definitions before we really march into quantification of radiation process.

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Radiation - Emissivity

$$\epsilon_{\lambda}(\lambda, \theta, \phi, T) = \frac{I_{\lambda, o}(\lambda, \phi, \theta, T)}{I_{\lambda, b}(\lambda, T)} = \frac{\text{Emission from real surface}}{\text{Emission from BB at same T}}$$

Spectral hemispherical emissivity

$$\epsilon_{\lambda}(\lambda, T) = \frac{E_{\lambda, e}(\lambda, T)}{E_{\lambda, b}(\lambda, T)} = \frac{1}{\pi} \int \int I_{\lambda, o} \cos \theta \sin \theta \, d\theta \, d\phi$$

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Now the reason for such a definition is often you may not know; what are the properties of a real surface. So, remember when we define blackbody we said that blackbody is considered to be a standard so, you want to define all the properties of a real surface using the properties of the standard which is the blackbody radiation. So, therefore, it is useful to define a quantity called emissivity which is represented as epsilon and again it is a function of lambda and temperature, lambda theta phi and temperature and it is essentially defined as the ratio of the intensity of emission from the real surface which is again a function of phi theta and temperature with the corresponding emission from blackbody.

So, the corresponding emission from black body which is not a function of the angular position so, this is essentially ratio of radiation emission or emission from real surface divided by emission from blackbody at same temperature, that is very important it has to be at the same temperature, it is a ratio of the emission from the real surface with that of the emission from a black body which is maintained at the same temperature.

So, similarly one could define a spectral hemispherical emissivity so, the definition is a bit tricky. It is very different from what you would have seen a short while ago. So, the spectral hemispherical emissivity is now a function of lambda and temperature so, that is the ratio of the spectral hemispherical emissivity of the real surface versus the spectral hemispherical emissive flux from the corresponding blackbody which is maintained at

the same temperature. So, it is not just the integral of the emissivity here so, the definition is emissivity is always defined as a ratio of the corresponding quantities. So, here it is the ratio of these spectral hemispherical emissive flux from the real surface and these spectral hemispherical emissive flux from the blackbody which is maintained at the same temperature.

So, now we know what is the expression for E_λ what is it? It is integral of the numerator term there so, that will be I_λ I can rewrite this as.

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$$\begin{aligned} \epsilon_\lambda(\lambda, T) &= \frac{\int_0^\pi \int_0^{2\pi} I_{\lambda, e}(\lambda, \theta, \phi, T) \sin\theta \cos\theta d\theta d\phi}{\pi I_{\lambda, b}(\lambda, T)} \\ &= \frac{1}{\pi} \int_0^\pi \int_0^{2\pi} \frac{I_{\lambda, e}(\lambda, \theta, \phi, T)}{I_{\lambda, b}(\lambda, T)} \sin\theta \cos\theta d\theta d\phi \\ &= \frac{1}{\pi} \int_0^\pi \int_0^{2\pi} \epsilon_\lambda(\lambda, \theta, \phi, T) \sin\theta \cos\theta d\theta d\phi \end{aligned}$$

Epsilon lambda will be integral 0 to 2 pi integral 0 to pi by 2 I lambda E lambda comma theta comma phi comma T all the sin theta cos theta d theta d phi divided by what is E lambda b? Same so, it is nothing but pi I lambda b of lambda T.

So, we know that black body is a diffuse emitter therefore, I can simply integrate the sin theta cos theta and that will lead to there will be pi into I lambda comma p. So, that is 1 by pi integral 0 to 2 pi 0 to pi by 2 I lambda E I could take this inside the integral because it is a diffuse emitter and so, it will be sin theta cos theta d theta d phi and so, that is 1 by pi by 2 this quantity this ratio here is nothing, but the emissivity. So, there will be epsilon lambda comma lambda.

So, suddenly because of the definition you will see that there is 1 by pi that is come in because of the definition, unlike what we saw in emissive flux, spectral hemispherical

emissive flux and the total flux here you will suddenly see a 1 by π that comes because of the definition of this ratio.

Student: (Refer Time: 07:20).

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$$\epsilon(T) = \frac{E(T)}{\epsilon_b(T)} = \frac{\int_0^\infty \epsilon_\lambda(\lambda, T) d\lambda}{\int_0^\infty \epsilon_{\lambda b}(\lambda, T) d\lambda}$$

$$\epsilon(T) = \frac{1}{\sigma T^4} \int_0^\infty \epsilon_\lambda(\lambda, T) d\lambda$$

So, one could define what is called a total emissivity that is epsilon which is only a function of temperature so, that will be E of the real surface is a function of temperature and E of the blackbody which is only a function of temperature. So, there will be integral 0 to infinity E lambda comma real surface into d lambda divided by what is the denominator term? σT power four. So, epsilon E is 1 by σT power 4 into integral 0 to infinity E lambda T d lambda.

So, the purpose of defining these quantities is if I know the emissivity of a surface there are ways to measure this so, if I know the emissivity of a surface I am done because I know how to characterize the blackbody radiation we are the Planck's law which is an exact equation. So, from that I will know what is the emission flux from a given surface similarly, if I know the spectral hemispherical emissivity I am done. So, if I know the local emissivity I should be able to calculate these spectral hemispherical emissivity because that is given by 1 by π double integral the local emissivity which is lambda E into \cos theta \sin theta d theta.

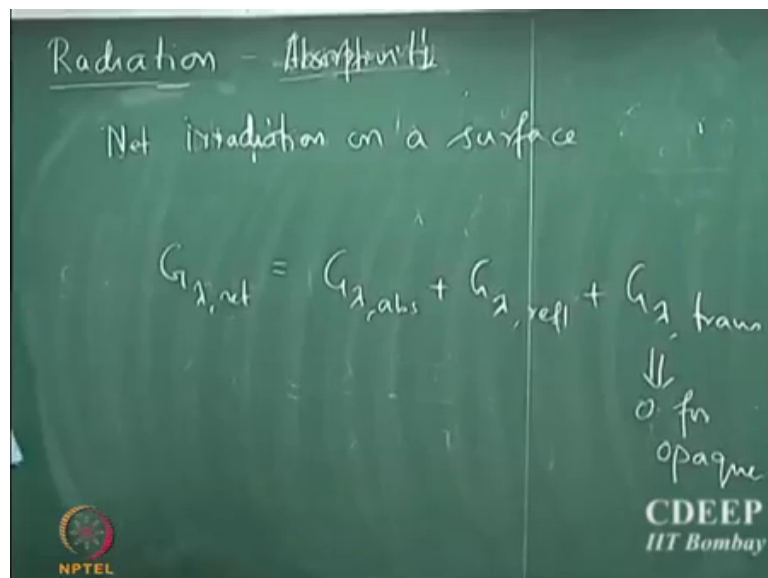
So, if I know the local emissivity I should be able to calculate the local emissive flux, spectral hemispherical emissive flux and also the total emissive flux. So, most of the real surfaces look at almost all the real surfaces the properties that you will know is the emissivity of the surface and in fact, it is an intrinsic property. We will see a little bit about this in one of the future lectures why it is an intrinsic property is here. All right.

So, one could again define a similar quantity called absorptivity so, how do we define absorptivity? Note that we cannot use black body as a standard here.

Student: (Refer Time: 20:12).

Right it absorbs everything so, what do we do?

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Student: (Refer Time: 20:17).

What is the net irradiation to the surface? And so, the absorptivity is now defined based on the net irradiation that is received by a surface. So, if we know how to quantify that then we are done. So, whatever fraction is received is what is called as absorptivity and whatever is not received can either be transmitted or reflected.

So, we are going to next class what we are going to do is we are going to introduce saying that the net irradiation that is received is equal to the net irradiation that is absorbed plus reflected plus transmitted. So, transmitted is in some cases supposing if

you have a translucent glass so, there some of the radiation which is incident to the surface can actually be transmitted to the other end of the surface or if you have a liquid for example, the radiation which is received on one surface of the container which is containing the liquid could actually be transmitted through the liquid to go into the other end, but if it is opaque solid then the transmittivity is 0, the amount of radiation that is transmitted is 0. So, if it is opaque object this is 0.

So, what we are going to start in the next lecture is by looking at a couple of more definitions very quickly we look at this definition and then we will start looking at how to quantify some of the aspects of radiation exchange and the emission from a given surface.