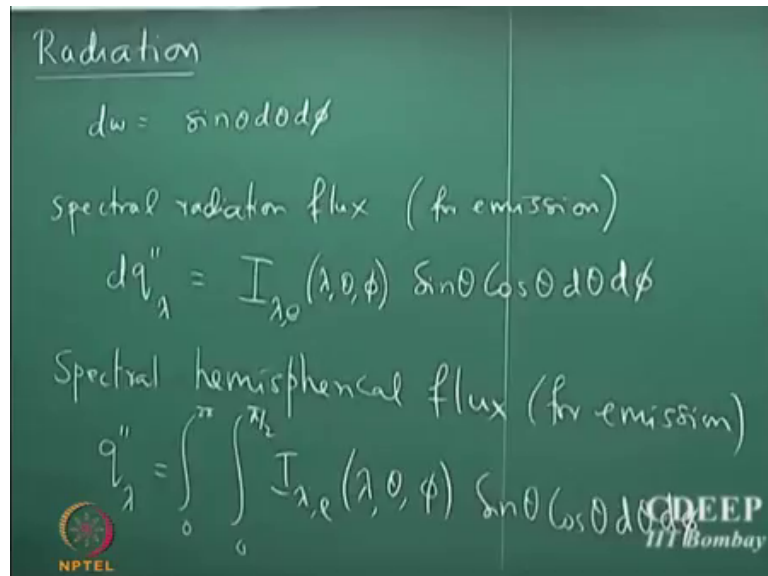


**Heat Transfer**  
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**Lecture – 45**  
**Radiation: Spectral properties, Blackbody**

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Let us continue with our discussion on radiation. We are going to see the first half we are going to see some more definitions, and then we are going to move on to characterizing blackbody radiation, that is what we are going to do in today's lecture. So, just to recap of what we did before, the solid angle is given by  $\sin \theta d\theta d\phi$  that is the solid angle, this is the solid angle of an object with respect to the object which is the emitting radiation.

And then we said that, the Spectral radiation flux, which is  $dq_{\lambda}$ , that is the spectral radiation flux at a certain wavelength, and that is given by  $I$ , which is the radiation intensity, and the subscript  $\lambda$  stands for the corresponding wavelength, and if I put a subscript  $e$ , it stands for emission, and that is the function of wavelength and position, in principle it is a function of temperature, but we will see that in a short while, that will be  $\sin \theta$ ,  $\cos \theta$ ,  $d\theta$ ,  $d\phi$ . So, that is the definition of the spectral radiation flux, I can qualify it by saying for emission for emission from a surface.

Now, based on this one could define, what is called a Spectral hemispherical, one could define a Spectral hemispherical flux. Now what it means is in, remember that in conduction we said, when you average over a certain dimension, it is called lumping right. So, here theta and phi are basically the angular directions in spherical coordinates.

And so, Spectral hemispherical flux is essentially, averaging over the 2 dimensions. So, this basically tells you, what is the total emission, that comes out of that surfaces, summed over all directions, at a given wavelength. So, that is nothing but, so if I say  $q_{\lambda}$ , that will be integral over 0 to  $2\pi$  on  $\phi$  direction, and 0 to  $\pi/2$  on theta direction, it will be 0 to  $2\pi$ , 0 to  $\pi/2$ ,  $I_{\lambda,e} \sin\theta \cos\theta d\theta d\phi$ . So, that is the, that is the Spectral hemispherical flux.

So, now we need a introduce a new definition.

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Diffuse Emitter

$$I_{\lambda,e}(\lambda, \theta, \phi) = f(\theta, \phi)$$

$$q''_{\lambda} = I_{\lambda,e}(\lambda) \int_0^{2\pi} \int_0^{\pi/2} \sin\theta \cos\theta d\theta d\phi$$

$$= 2\pi I_{\lambda,e}(\lambda) \frac{1}{2} \left[ -\cos(\pi) + \cos 0 \right]$$

$$q''_{\lambda} = \pi I_{\lambda,e}(\lambda)$$

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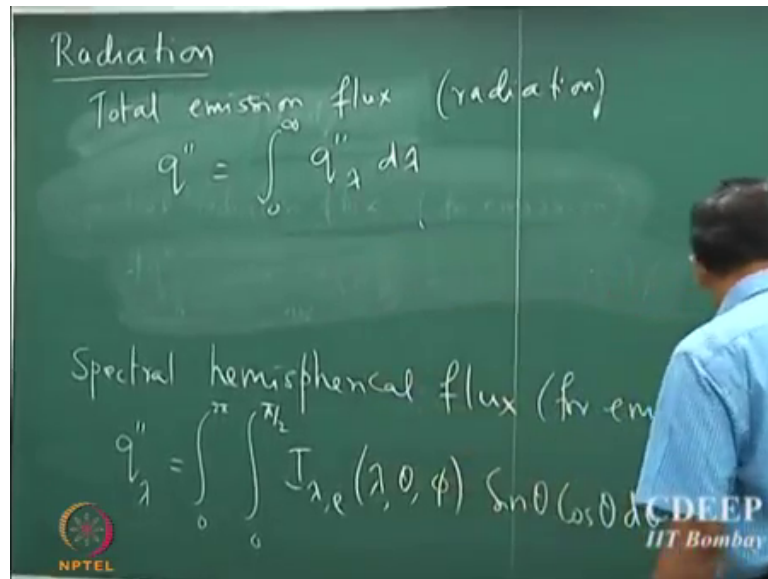
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So supposing, if the surface is a Diffuse Emitter so, Diffuse Emitter it means that, the intensity of emission is not a function of the angular direction, which means that, in all directions the emission by the surface is going to be uniform, which means that the intensity of emission is independent of the angular position. So now, if we plug in that aspect on this expression here, so will get  $q''_{\lambda}$ , and this is for Diffuse Emitter, excuse me,  $I_{\lambda,e}$ , that is only a function of  $\lambda$ , and of course, temperature.

What is this integral? With respect to  $d\phi$ , it is  $2\pi$ . What about  $\sin\theta \cos\theta d\theta$  over  $0$  on  $\pi$  by  $2$ ?  $2\pi$ ,  $\lambda e$ . How do you integrate  $\sin\theta \cos\theta$ ? Half of  $\sin 2\theta$ , so that will basically be,  $1$  by  $2$  minus  $\cos\theta$  by  $2$  plus  $\cos 0$  divided by  $2$ , minus  $\cos\pi$ , minus  $\cos\pi$  plus  $\cos\theta$  divided,  $\cos 0$  divided by  $2$ , so that will nothing be,  $\pi I \lambda e$ . So, that is nothing, but that is equal to  $\pi$ .

So, similarly one could define, what is called the total radiation.

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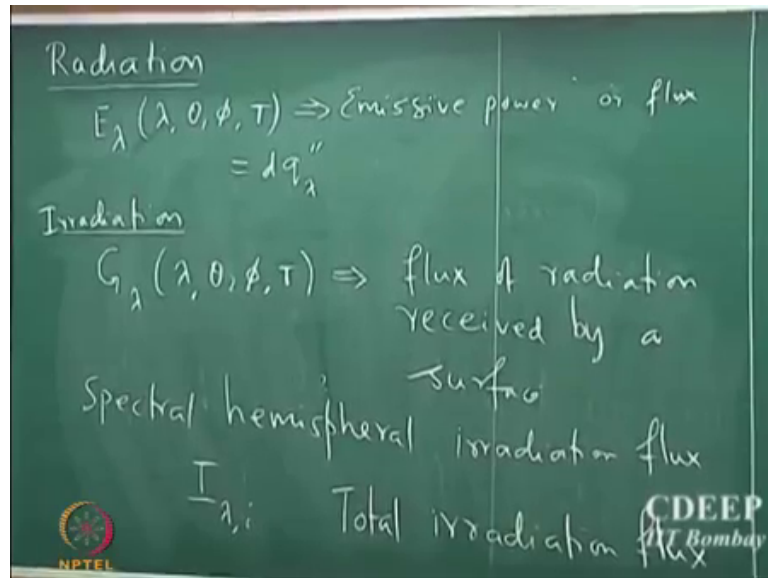


Flux or Total emission flux, Total emission flux or total radiation flux in general, so that, that is simply  $q$ , which will be integral over all  $\lambda$ , all the whole range of the wavelengths, that you one can think off into  $d\lambda$ . So, that is given by, so that is essentially the integral of this quantity here, over all possible wavelengths. So, that will give you the, total radiation flux from a given surface at all wavelengths, and emitted in all directions.

So, this is very important, particularly when we talk about blackbody radiation, you will start seeing some special applications of these expressions. Now at this point I will pause for a moment, and remind you that, these hemispherical quantities, this integral, and the Total emission integral, you will see in many different places, in all over radiation concepts. So in this course, and also if you happen to read advance radiation topics, you will see a lot about these hemispherical and total, because all the definitions and all the quantitation of radiation, they are all depend upon these 2 quantities.

So, it is very important to understand, what is meant by the Spectral hemispherical flux, and what is meant by the Total emission flux, all right, so general symbol that I will use, which is consistent with your texts.

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For emission, I will use symbol E. So, that is a function of theta, phi, and temperature. So, this is the Emissive power, or flux we could call either of them. So, and that is nothing but, that is  $d q_\lambda''$ . So, that is the definition of Emission power.

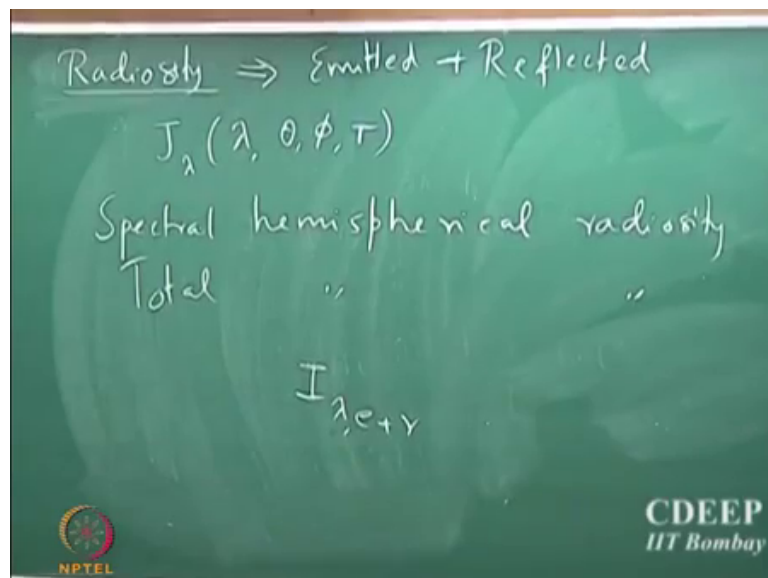
Similarly, one could define Irradiation; explain in a minute; what is the Irradiation. So, this is the flux of radiation received by a surface, the flux of radiation received by a surface, so far we talked about, what is the net emission from a surface. So, remember that, when we first discuss the mechanisms of radiation, we said that, anybody, any form of matter is going to emit and absorb radiation. So, if you have to characterize the radiation process, you will have to characterize the emission, you will have to characterize the absorption, and you also have to characterize the reflection.

So, similarly one could define, just like how we define these Spectral hemispherical quantities, you can have Spectral hemispherical, I am not going to write the formula, but it is just important to realize that, one could define a Spectral hemispherical irradiation flux. So, it is just the integral, etcetera, etcetera.

Now, the symbol that is used in your text, and this is what I also will use is, so  $\lambda_i$ , that stands for irradiation, that is intensity of radiation that is absorbed by a given surface.

And so, similarly one could also define, so like Spectral hemispherical, one could also define a Total irradiation flux, all these are definition, we just have to replace  $\lambda_e$  with  $\lambda_i$ , in all the integrals that we wrote. It is just boring to write the integrals too many times, so, but you understand what I mean right. So, instead of  $\lambda_e$ ,  $\lambda_i$  in your earlier expression, where it is the intensity of radiation, that is emitted, you replace it with the intensity of radiation, that is actually absorbed, and similarly one could define, what is called Radiosity.

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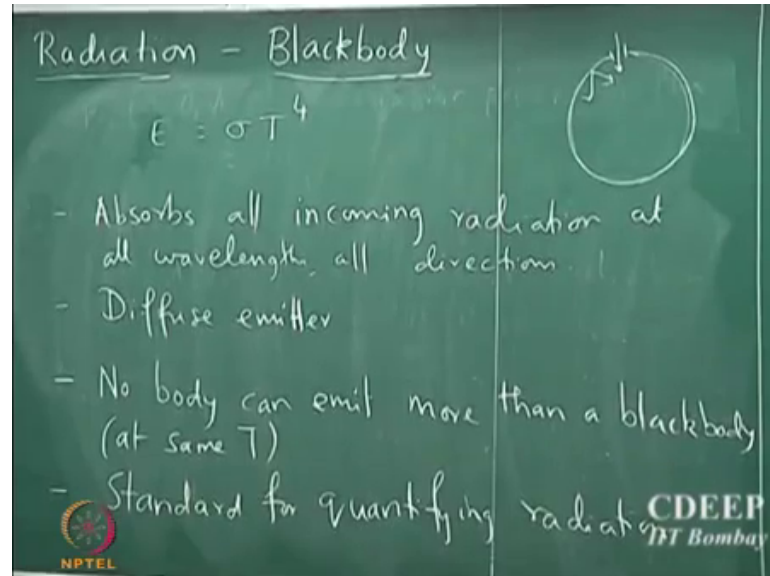


So, Radiosity is essentially the sum of, what is Emitted plus the Reflected radiation. It is useful to define Radiosity, because the net radiation that comes out of a surface, is essentially sum of, what is Emitted plus what is Reflected right. So, that is the net emission. And the symbol that is given as,  $J$ ,  $J$  is a symbol that is given, and  $J$  at a specific  $\lambda$ , again it going to be a function of,  $\lambda$ ,  $\theta$ ,  $\phi$ , and temperature. And once again, one could define quantities like, Spectral hemispherical radiosity and Total hemispherical radiosity.

One could define similar quantities, and the symbol for intensity representation is,  $e$  plus  $r$ . So, that is again the notation that is given in your text.

And we will follow the same notation here. So, it is the, intensity of radiation that is emitted plus the intensity of radiation that is reflected at a given wavelength.

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So, that is it. And essentially, you can get these integrals by replacing,  $I_{\lambda}$ , with these a new intensities that we have just defined, following?

So, that we are going to move into the next aspect of Radiation. Next aspect is, we are going to discuss, what is Blackbody radiation. Now, most of you, probably all of you would have, heard about Stefan boltzmann constant right. So, you say that radiation, whatever the intensity, Emission intensity, is  $\sigma T$  to the power of 4, etcetera etcetera. Why is it  $T$  to the power of 4, does anyone know?

See in conduction we said that, the driving forces temperature gradient. Why is it here that, it is  $T$  to the power of 4, why not temperature gradient? So, let us start with Blackbody radiation, so what is a Blackbody? they are, again, Blackbody there is nothing called a Blackbody. So, Blackbody is an idealization, and it is used for standardization purposes, in fact all the concept of Diffuse, Emitter etcetera is essentially defined for characterizing Blackbody. And therefore, we can introduce a, certain standard for quantifying the radiation process.

So, there are certain properties, we will come back to why it is  $T$  power 4 in a short while. We quantify Blackbody Radiation.

So, it Absorbs all incoming radiation, at all wavelengths, and all angles, all directions, so, that is the first property. Second property is that, it is the Diffuse emitter so, that is the second property of Blackbody. So, No can emit more than a Blackbody, again it is an idealization, there are some experimental, you know, geometries, that I have been close to Black body, but it is not the area. So, it is just an idealization, no body can emit more than the radiation, that is emitted by a Black body at the same temperature, that is very important, same temperature.

So, therefore, is the Diffuse emitter, it absorbs all incoming radiations, at all wavelengths, and all angles, and so it is generally considered as a, Standard for, for quantifying radiation process, for quantifying radiation process. So, it is believed that, if there is a cavity, spherical cavity so, it has to be a sphere, and if there is a small pinch hole, which is present at the cavity, and if the inside of the sphere is coated with black, thing, then that is considered to be a Black body. And the reason behind is that, whatever is emitted in all directions, is a, it is absorbed by itself. So, there is only a very small cavity, and whatever radiation that comes inside, they are internally reflected, and absorbed completely, at all wavelengths. So, that is why it is considered as, close to blackbody radiation, all right.

So, just like in conduction, we said that, if we want to quantify the transport process, we need constitutive law, like Fourier's law and newton's law.

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Planck's Law (spectral distribution of a blackbody)

$$I_{\lambda,b}(\lambda, T) = I_{\lambda,b}(T, \theta, \phi) = \frac{2hc^2}{\lambda^5 \left[ \exp\left(\frac{hc}{\lambda kT}\right) - 1 \right]}$$

$$h = 6.6256 \times 10^{-34} \text{ J s}$$

$$k = 1.3805 \times 10^{-23} \text{ J/k}$$

$$c = 2.99 \times 10^8 \text{ m/s}$$

$$E_{\lambda,b}(\lambda, T) = \int_0^{\infty} I_{\lambda,b}(\lambda, T) \sin\theta \cos\theta d\theta d\phi$$

$$= \pi I_{\lambda,b}(\lambda, T)$$

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The law that is used for, quantifying radiation is called the Planck's Law that is the Planck's Law. And the law is, the radiation intensity  $I$ , which is a function of; obviously, it is not a function of  $\theta$  and  $\phi$ , we are going to see that in a short while, and that is equal to  $2 \text{ times } h c \text{ naught square by, } \lambda \text{ to the power of } \phi$ , into exponential of  $h$  into  $h c \text{ naught by } \lambda k T \text{ minus } 1$ . So, that is the law, which relates the intensity of radiation emitted by Blackbody, I should qualify it with a  $b$ , saying it is for Blackbody. And there are some constants that I should write here. So, the constant  $h$ , and note that, it is a Diffuse emitter, therefore, it is only a function of  $\lambda$  and temperature.

So, the Planck's constant is  $6.6256 \text{ into } 10 \text{ power minus } 34 \text{ joule second}$ , and Boltzmann constant so,  $k$  is Boltzmann constant, that is  $1.3805 \text{ into } 10 \text{ power minus } 23 \text{ joule per kelvin}$ , and  $c$  naught is this speed of light.

So, this actually comes from the electromagnetic wave theory. So, this is called the Spectral distribution. So, what it captures is the, Spectral distribution of intensity of radiation that is emitted by a Black body. So, because it is a Diffuser editor, emitter, excuse me. So,  $E$  is the symbol that we will use for emissive power. So, the emissive power of Black body, which is only a function of  $\lambda$ , and temperature, is given by, if I know that,  $I$  is the intensity of emission, what is the emissive power?

So, that will be  $\int_0^{2\pi} \int_0^{\pi/2} I, \lambda \text{ comma } b, \sin \theta \cos \theta \text{ d } \theta \text{ d } \phi$ , but we said that, it is a diffuse emitter right, so, which means that, the intensity is not a function of the angular position. So, we can integrate so, this will be  $\pi$  into  $I \lambda \text{ comma } b$ . So, that integral simply boils down to  $\pi$ . So, integral of  $0$  to  $2\pi$   $\text{d } \phi$  is  $2\pi$ , integral  $0$  to  $\pi/2$   $\cos \theta \sin \theta \text{ d } \theta$  is  $1/2$ .

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Where?

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Yeah, it is understood. It understood. So, the integral is simply  $\pi$ .

So, the emissive power of, the Spectral emissive power. So, we only integrated over all the wavelength, all the angular directions. So, the spectral emissive power of a Black body is given by,  $\pi \text{ times } I \lambda$ .



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Radiation - Blackbody

Total emissive power

$$E_b(T) = \int_0^\infty E_{\lambda,b} d\lambda = \int_0^\infty \frac{2h\pi c^2}{\lambda^5 [\exp(\frac{hc}{\lambda kT}) - 1]} d\lambda$$
$$= \sigma T^4$$
$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$$

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Now let us find, what is the Total emissive power, Total emissive power of a Black body. So, that is given by E, which is only a function of temperature, because we are going to now, integrate over all wavelengths so, that will be a function of temperature. I put a subscript b, for Black body, 0 to infinity to d lambda. So, when you integrate this expression so, that is integral 0 to infinity,  $2 h \pi C$  naught square, lambda power 5 exponential,  $h c$  naught, by lambda  $k T$  minus 1 d lambda.

So, note that, this pi comes from the, averaging over all the angular directions, and so, now, if we integrate over whole lambda, what you get is this, magical formula called sigma T power four. So, the way you have to integrate is, you have to use, we you have to use the convolution integral so, you do a integration by parts 4 times, and what you get is sigma t to the power of 4. And the value of sigma is 5.67 something. So, that is 5.67 into 10 to the power of minus 8 watt per meter square kelvin 4. So, this magical formula, that all of you have seen so far, actually comes from a rigorous analysis, by using the quantification of Blackbody radiation. So, this T power, sigma T to the power of 4 essentially comes from here.