Heat Transfer
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Module - 09
Lecture - 44
Spectral Intensity

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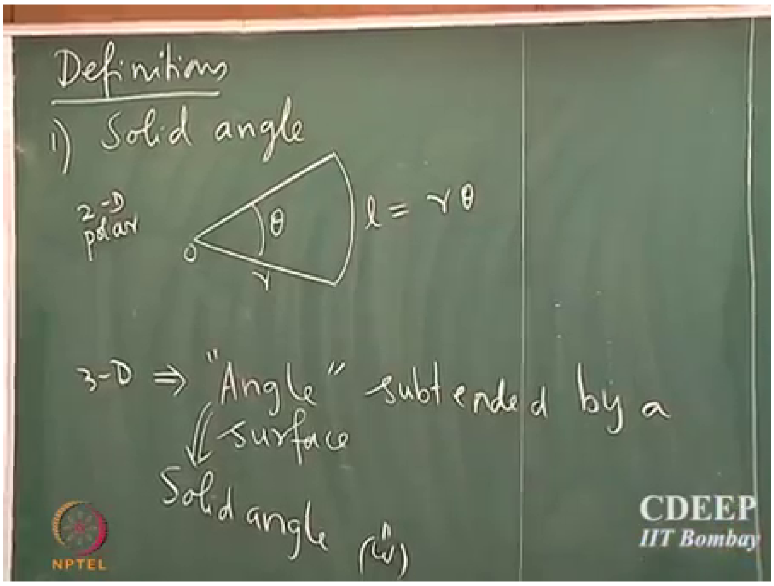

So, let us look at few definitions now, the first on the list is solid angle ok. So typically in normal geometry that you have done so far so supposing if you have a 2 dimensional of system and if this is theta, and this is r, you know what is going to be the length of that arc right. What is the length of arc?

Student: (Refer Time: 00:44).

R theta right, R theta is the length of the arc. Now if you want to know this theta then meaning of this angle is the what is the angle that is subtended to by this arc to that particular point to the center right.

If I draw circle if I had complete this circle this angle tells you what is the angle that that this arc subtends to the center of that particular circle. Now we need to extend this so this is very good a in 2 dimensional polar coordinate. So, this definition is for 2 dimensional system and polar coordinate.

Now we need to extend to a 3 dimensional system so note the radiation is a 3 dimensional process so you need to extend this concept of the angle subtended by a given surface. So, here it is the angle subtended by this arc which is 1 dimensional system arc is a 1 dimensional system. So, what is the angle that is subtended by this 1 dimensional system to the center of that circle.

We need to define what is the angle subtended by a surface, we need to define what is the angle that is subtended by a given surface to a particular point ok. So supposing, supposing here is point object by the way this angle is what is called as the solid angle.

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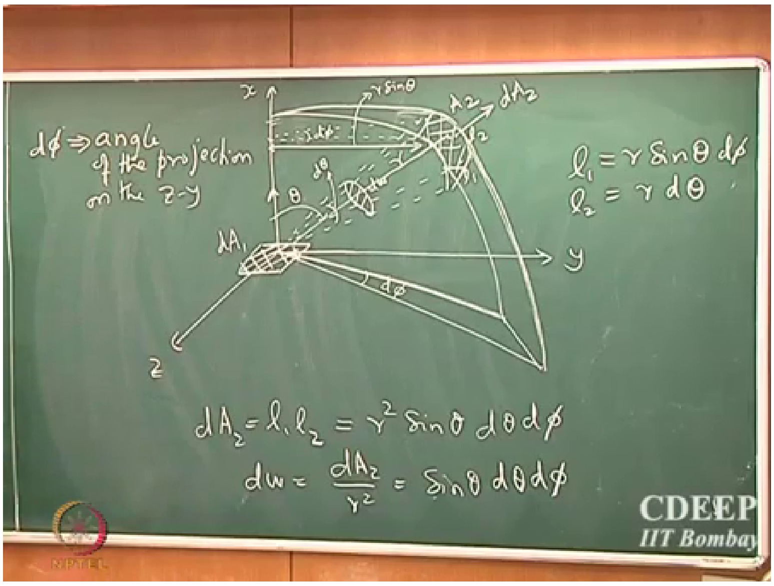

And the symbol that is given is omega; omega is the symbol that is given for solid angle. So, supposing if there is an object here and you had surface this is a surface ok. So, if a call the object as let us say d A 1 is the surface area of that object and the vector points in the normal direction to that surface. So, note that surface area can actually be quantified as vector. So, it is a vector which is pointing in the normal direction and let us says this is A 2, surface A 2 and this is the vector so depending on the direction you can say it is pointing inwards a outside.

So, the question is what is the angle that is subtended by this surface A 2 on to that point object that is what we need to find so, the solid angle is so that is the solid angle. If I call this as d omega so I can extend this concept of finding the angle in 2 dimensional into a 3 dimensional system ok. And the way I do that is supposing the linear distance between center of these 2 is r ok. So, d omega is essentially defined as d A 2 by r square.

So this is the definition of solid angle. So, this is the area of the object divided by the distance between the center of the objects yeah cleared everyone. We are going to know find out how to find this solid angle for any arbitrary 3 dimensional distance. So, supposing I now define this my y direction, this is x and this is let us say z direction ok.

Now I can draw a hemisphere if I can draw section of the hemisphere, so this is the section of hemisphere ok. Let us assume that there is a hemisphere which is present in 3 dimensions and if I assume that so this is the angle if I call this as d phi. So, this is the angle that is actually subtended by this particular area on the x, z, y plain. So, d phi the angle of the projection on the z y plain so d phi is the angle of the projection on the z y plain and if I call this angle theta so that is the angle of the of the line that joints the center of these two object would be x axis so that is angle theta.

So, I have now clearly defined 2 coordinate systems. So, this is my theta and this is my phi, phi is the angle that actually comes out of the bold and theta is the angle which actually goes in the x, y plain ok. Now I can define this small angle here this is d theta this is the differential angle that is subtended in the x, y plain of that surface ok. So, keep it in mind that there are 2 different angles in 2 different plains. So, d theta is angle that is subtended by this particular surface if you project it on the x and y plain ok.

So, the objective is now is to find the area d A 2 in terms of these angles that is what we to do ok. So, supposing I call this length as l 1, and this length as l 2. So, this the linear length scale of the 2 side, so d A 2 is nothing, but l 1 into l 2. So, if I know what l 1 and l 2 are done I can find the solid angle ok. What is l 1? How do I find l 1?

Student: (Refer Time: 07:39).

From, from d phi how do I find l 1 from d phi ok. So, I will help you so supposing this angle is d phi. What is this side?

Student: (Refer Time: 08:05).

What is that distance? So, this is r what is this distance r sin theta ok. So, l 1, l 1 is r sin theta and what is l 2 oops sorry l 1 is r sin theta d phi. So, this length; this length is r sin theta and therefore, l 1 is r sin theta d phi. What is l 2?

Student: (Refer Time: 08:44).

Whatever it is it does not matter it is a differential element, whatever is length of the arc it is not a linear line, it is an arc, it is does not matter.

All I am saying is what is length of the arc? That we should be able to find, does not matter whatever the shape of the object. So, all I am saying is just uh differential element now if you integrated over the whole surface whatever surface you have you should be able to make it. What is l 2?

Student: (Refer Time: 09:21).

R. So, note that this angle is d theta and so the projection is this cone here. So, we see this triangle that is the projection right so l 2 is nothing, but r d theta. So, this is the radius and the angle between them is d theta. So, the length of the arc l 2 is r d theta and therefore, the area is given by r square sin theta d theta d phi ok.

So, from here I can read out that d omega which is the solid angle which is subtended by that differential surface on d A 1 is given by d A 2 by r square which is sin theta d theta d phi that cleared everyone yes understand this very clearly ok.

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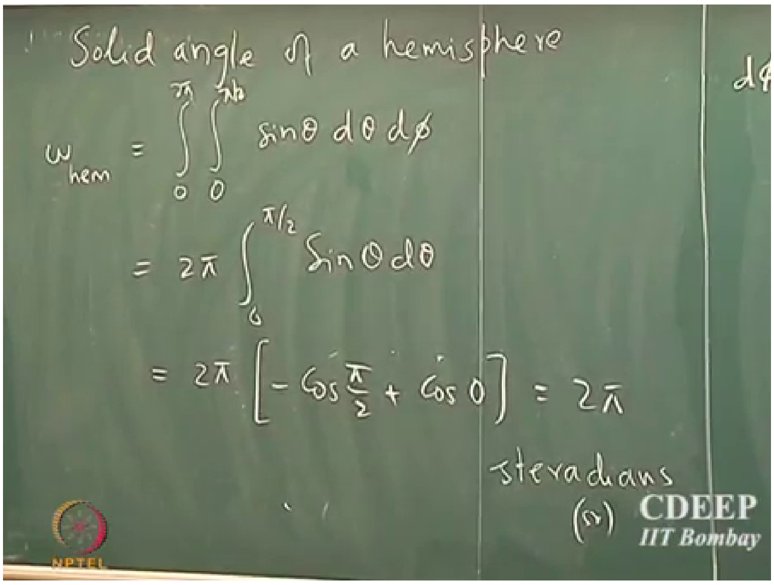

Supposing if I want to find the solid angle of the hemisphere how do I do this solid angle of hemisphere I want to find the solid angle of hemisphere. How I find the solid angle of hemisphere yeah.

Student: (Refer Time: 11:01).

180 degree no.

Student: (Refer Time: 11:04).

hm Theta goes from 0 to phi by 2 that is right and then note that there are 2 angles, hemisphere is a 3 dimensional object so it is defined by 2 angles. So, omega of a hemisphere is given by integral 0 to 2 pi on phi, so phi so note that phi goes all the way around and z, y plain. So, and this will be 0 to pi by 2 sin theta d theta phi that is it. What is it is integral not hard 2 pi integral 0 to pi by 2 sin theta d theta what is that integral 0 to pi by 2 sin theta d theta (Refer Time: 12:03).

Student: (Refer Time: 12:05).

One is not that hard minus cos pi by 2 plus cos 0 that is equal to 2 pi. So, the solid angle that is subtended by a hemisphere is 2 pi not 180, and what will be that is for sphere v 4 pi. So, the units that is used here is called the steradians and it is typically represent as s r so that is the unit for solid angle called steradians any questions on this. So, far you must understand this geometry we will be using we will not I will not be drawing this picture again and again.

But you must clearly understand how this geometry is constructed and how we find the surface area of the how we find the area of this arbitrary surface and how we are able to calculate the solid angle. So, the whole concept of quantification of radiation which hinges upon this picture, the ability to calculate the solid angle is very very important when we are doing any radiation calculation.

And all through the radiation topic that we will see I will not use the word solid angle, but the solid angle is actually inbuilt in some of the expression that I am going to derive shortly the today's and next lecture. So, the sin theta d theta d phi is something that you will see that will appear in all most all the expressions that you will see for radiations it is very very important to understand how you get the solid angle any questions on this ok.

So, l 1 is essentially the length of this arc l 2 is the length of the this arc right, so this is a differential element of arbitrary surface. So, l 1 is nothing, but this length multiplied by d phi and this length is r into sin theta. So, this is theta and so sin will be opposite the hypotenuses sin theta and so the length of this particular you know linear distance is r sin theta multiplied by d phi will give you l 1 and this length is in the x y plain and so that is radius multiplied by the angle in that plain which is d theta. So, r d theta will be l 2 and r sin theta d phi will be l 1, any other questions yes.

Student: (Refer Time: 14:51).

It is a projection it does not matter where you write the projection I have just for convenience sake I have written it here does not matter. The angle remains the same wherever you write the projection whichever plain you write the projection whichever x whichever x value you write the z y projection the angle reminds the same it just the length will be different.

So, if I write my projection here I can always do that right, I can always do that. So, this distance is actually equals to this distance on the projection I have just represented this projection so that is easy to understand that the length of arc is r sin theta d d phi. So, all I have done is I have just taken this projection and represented to here that is all

Student: (Refer Time: 15:43).

No it is not we will see that so; obviously, the distance at which the object is placed is going to make a difference, we will see that some of the definition we will see any other questions all right. We are going to move to the next definition.

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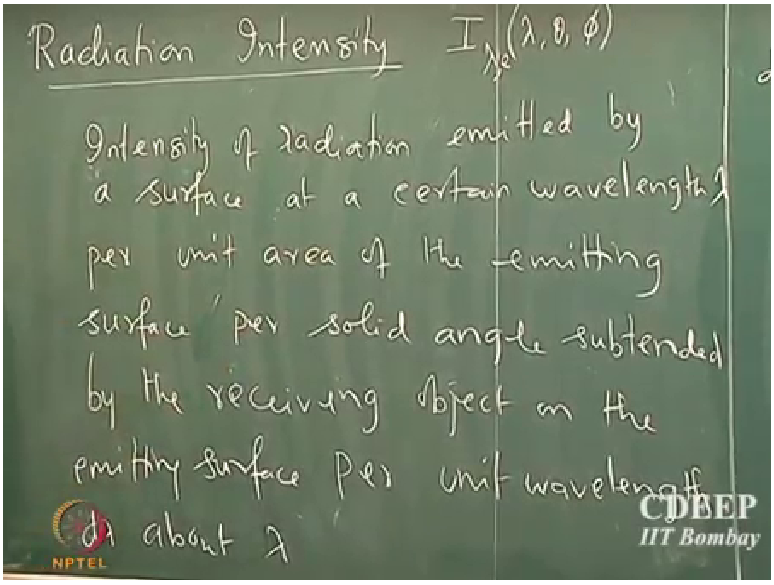

So, we are going to define a quantity called radiation intensity which the radiation intensity and the typical symbol that is used is I ok. And a subscript lambda means that radiation intensity at a particular wave length lambda and that is going to be function of lambda theta and phi it is going to be a spectral property. So it is going to be dependent on which lambda it is going to be depended on what angle in theta direction, and what angle in phi direction ok. So, it is defined as the intensity of variation I can also put e here which stands for emission. So give me a second.

Intensity of radiation emitted by a surface at a certain wave length lambda per unit area of the emitting surface per unit area of the emitting surface, per solid angle subtended by the receiving object on the emitting surface per unit wave length lambda so that is the definition of radiation.

So, intensity of radiation emitted by a surface at a certain wave length lambda per unit area of the emitting surface if d A 1 is the emitting surface per unit area of the emitting surface per solid angle that is subtended by the receiving object. So, note that intensity now depends upon both the receiving object. What is the amount of intensity that is emitted by the surface on that direction?

So, it has to; obviously, be a function of the receiving object its area and a solid angle per unit wave length d lambda about that particular wave length. So, this is the definition of the radiation intensity.

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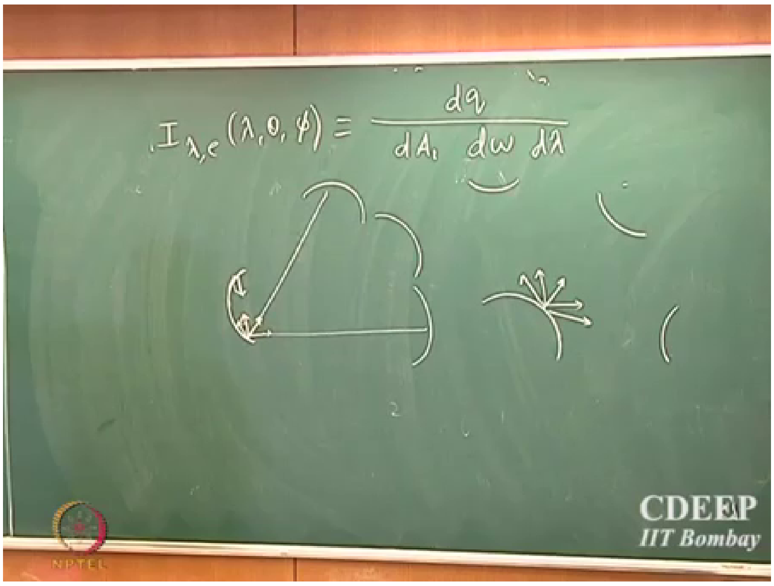

And so what we are going to see is how to define heat transfer rate based on the so need translate this definition into to a mathematical form lambda comma theta comma phi so that will be given d q which is the differential amount of heat transport rate, the rate at which the heat is being emitted by that surface divided by d A 1 ok. So, that tells you the flux divided by the solid angle d omega divided by d lambda. If this is my surface which is emitting radiation ok, now if matters where it is being placed where the receiving object is being placed.

Because the it does not matter, even if you keep the distance constant, even if you keep the distance constant ok. So, supposing if you have a concave surface whatever is emitted by a concave surface is not received by itself ok. Whatever is emitted by a concave surface always leave, and go out of the concave surface.

Now so supposing I have a convex surface, supposing if I have a convex surface then whatever is emitted by this object in this location is now going to be intercepted by the same object, so the orientation plays a role. So, for an example see here supposing I take the surface here it is emitting radiation in all direction right.

So, the radiation that is emitted is going to be intercepted by an object which is paced here. Radiation which is emitted here is going to be intercepted by an object placed here. There is no reason why I should assume that all the radiation is constant in all directions. So, it does matter where the object is placed even though the distance between the object is the same it does matter where it is placed any other question ok.

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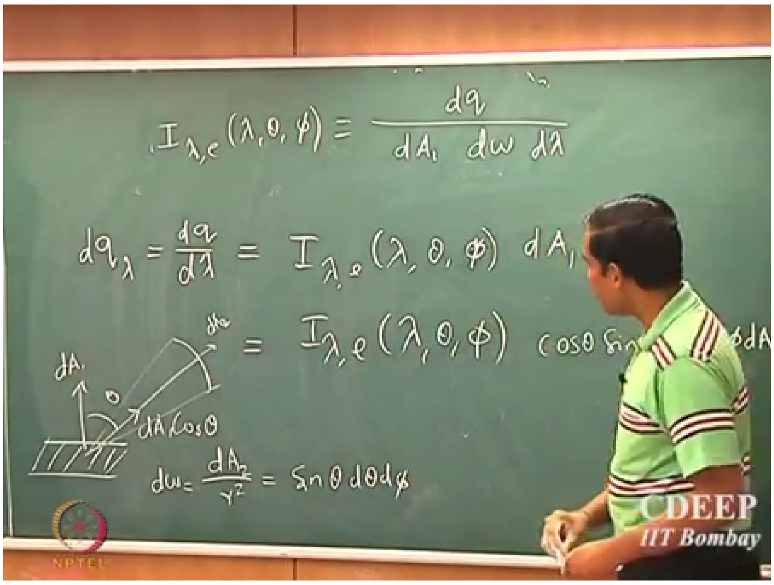

So, now, d q by d lambda ok, so this is the the so this is called as generally the terminology that is used in your text book is d q lambda which is basically the heat transfer rate per unit wave length and so that is given by I lambda e lambda theta comma d phi multiplied by d A 1 into d omega ok.

What is the d A 1? What is the area? So, keep in mind that they are draw the picture again so this is the area vector of that object, and if this is cos theta, this is theta then the area that is actually in the direction of the receiving object is d A 1 into cos theta right.

So, that is the so the area vector which is pointing in the direction of the receiver is d A 1 into cos theta ok. And what is a solid angle d A 1 by d A 2 by r square. So, note that the solid angle depends upon the area of the receiver not the one that is emitting. So, keep in mind that if this is A 2, if let us say that d A 2 is the area of the receiving surface then d omega is defined by d A 2 by r square.

So, this is the area component in the direction of the line that joint the center of these 2 objects and so d omega is d A 2 by r square and that is given by sin theta d theta d phi. So, that is the solid angle that is subtended by this surface on the emitting surface d A 1 ok. So, if I put that that will be I lambda e, phi cos theta, sin theta, d theta d phi d A1 so that is the that is the heat transfer rate per unit wave length.

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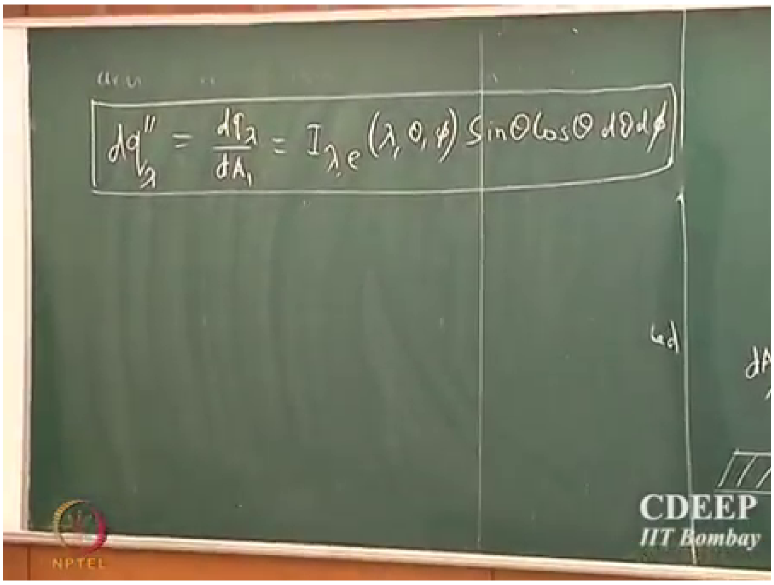

So if I can define flux d q lambda note that double prime when I use its flux so that will be d q lambda by d A 1 which is the fluxes at which the emission is occurring that surface so that will be I lambda e, lambda comma theta comma phi, sin theta, cos theta, d theta, d phi that is the definition of flux of radiation from emitted by a given surface.

So, I would you know encourage all of you actually go over this before you come to the next class you must completely understand how this expression is derived. So, everything that we discuss in radiation completely hinges upon how this equation is derived and how the 3 dimensional geometry, is constructed and how this solid angle is actually evaluated for a given system ok.