Heat Transfer
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Lecture – 04
1D Steady – state conduction: Resistance concept

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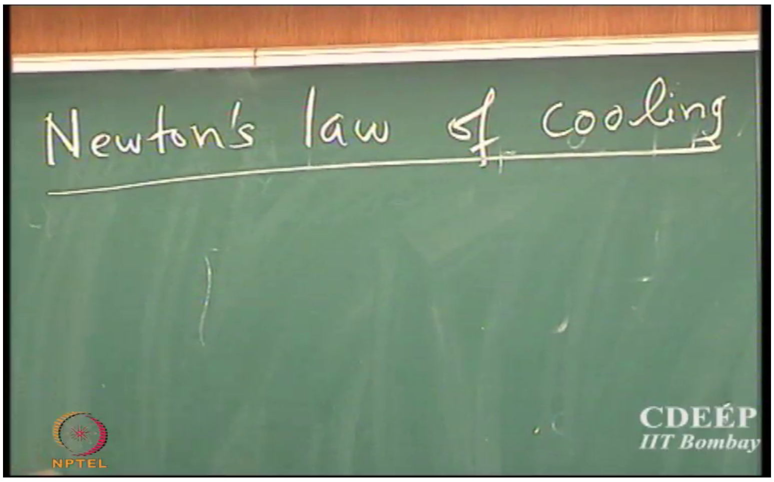

Similar to Furriers law which is the constitutive relationship between the flux due to conduction and the local temperature gradient Newton's law of cooling is the constitutive relationship for heat transport from a solid system where conduction is the mode of heat transport inside and there is a conduction outside the solvent.

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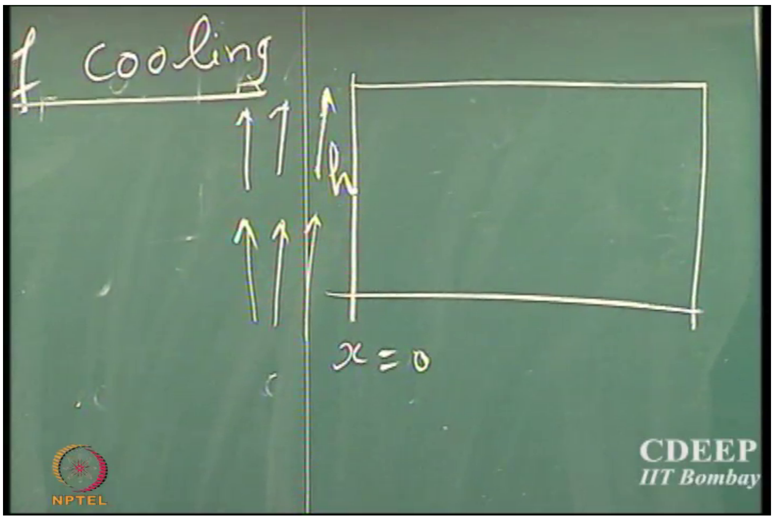

The way to describe it supposing if we have a solid here and there is some fluid which is flowing past this solid, this is x is equal to 0 will be flowing at a certain velocity V we will see a lot more about it when we discuss convection topic may be about 10 lectures 10 15 lectures from now and there could be a transport coefficient h, we have to now define a heat transport coefficient and the reason for defining such a coefficient is that note that here is heat transport across different phases.

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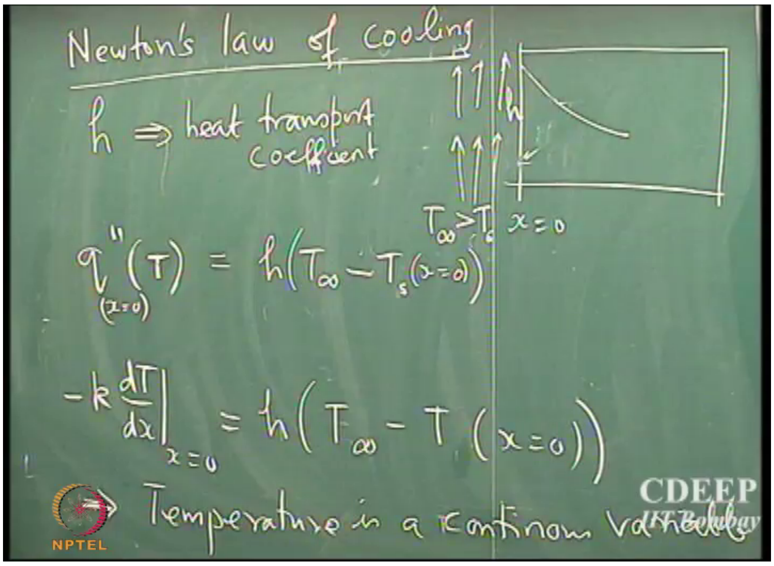

Here this is a solid and this is a liquid, you have transport across different phases and we need to define some part of a coefficient that is a fictitious coefficient and we will actually see how to estimate these coefficients it is for convenience purposes such a coefficient has been described and Newton's law cooling simply says that q prime at x equal to 0 that the flux at x equal to 0 should be equal to the heat transport coefficient h and supposing I assume that the temperature here is T infinity and if the temperature inside if this temperature is higher than the temperature at the boundary.

This is the surface temperature, if I assume that the fluid temperature is higher than that of the solid temperature which means that the heat is transported from the fluid to the solid, then one could define the flux as T infinity minus T s which is basically at x is equal to 0. That is the Newton's law of cooling and if I flux this into my third boundary condition and get minus k d T by d x at x equal to 0 equal to heat transfer coefficient into T infinity minus T s or you can say T at x is equal to 0. This gradient will take care of that, note that when T infinity is less than T s then you are going to have a profile which is different.

Supposing if T infinity is greater than T x you are going to have a profile which looks like this alright because there is heat that is going from the fluid into the solid. Now supposing if T infinity is less than T s we are going to have a different profile, the gradient will care of the sign whether it is T infinity is greater or lesser T s. This is a general form irrespective of what is the sign of T infinity minus T s because the gradient is what is going to dictate what is going to be the sign and also you should note it is very important to note here that this gradient describes the flux inside the solid and this formula here represents the flux just outside the solid.

This is a balance between flux of the heat transport just inside the solid and flux of heat transport just outside the solid. This balance really comes about because we assume that the temperature is continuous is that continuous variable because it is a continuous variable there has to be a gradual change in the temperature and if the way to account it mathematically is that the gradients have to be equal that is how you ensure that the temperature is a continuous variable that is it changes continuously without any jump at the boundary point. This is very important to understand how this balance comes about you will see very often just in this course in several other courses in reaction generating etcetera in the future you will see that these kinds of flux balances at the boundary will come into picture.

Let us take a specific example, let us say we start with 1 Dimensional steady state conduction process.

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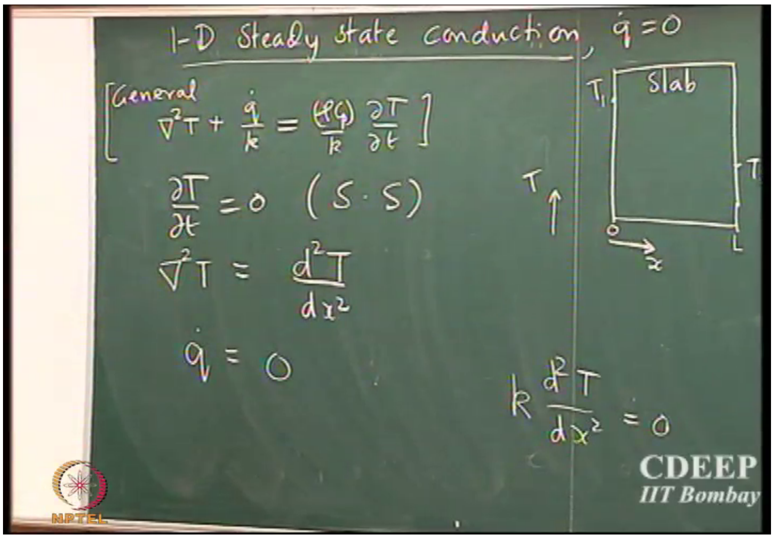

We start with a simple 1 Dimensional conduction, let us say we have a slab, that is a slab and we assume that it is one dimensional. Let us say that the conduction process is in the x direction and if you assume that the, this is the temperature axis. If the temperature here is T 2 and let us say temperature here is say T 1 and T 2, but x equal to 0, the temperature is T 1 and x is equal to l the temperature is T 2. So, what is the energy balance for this, keep in mind that the general energy balance we should always try to sort of look at the general framework with del square T plus q dot by k equal to rho c p by k into d T by dt.

So, you can always start with the general energy balance, we said that it is a steady state system with time derivative goes to 0 to d T by d t is 0 because it is a steady state system we are considering a steady state problem and then we said it is one dimensional system, which means that the Laplacian is simply in the x direction d square T by d x square and supposing if you also assume that q dot is 0 that is the there is no energy generation or loss inside the system no matter what is ever and, q dot is also equal to 0.

These are the immediate translation of some of the assumptions that we have made and based on this we can write a simple model it will be k into d square T by d x square equal 0 that is the simple model equation for the system that we have considered where it is a 1 D steady state conduction with no heat generation slash heat loss.

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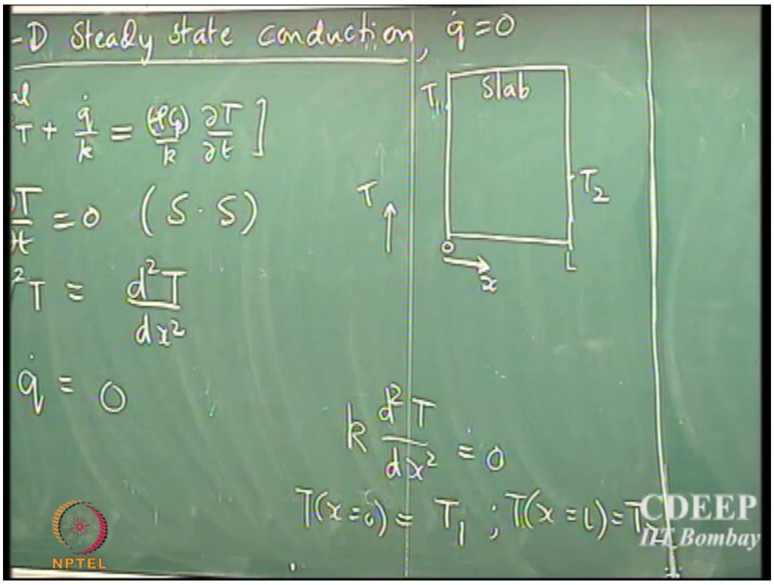

What are the boundary conditions at x equal to 0 equal to T 1 and T of x equal to L equal to T 2, can I solve this equation of course, we have done this it is not a big deal.

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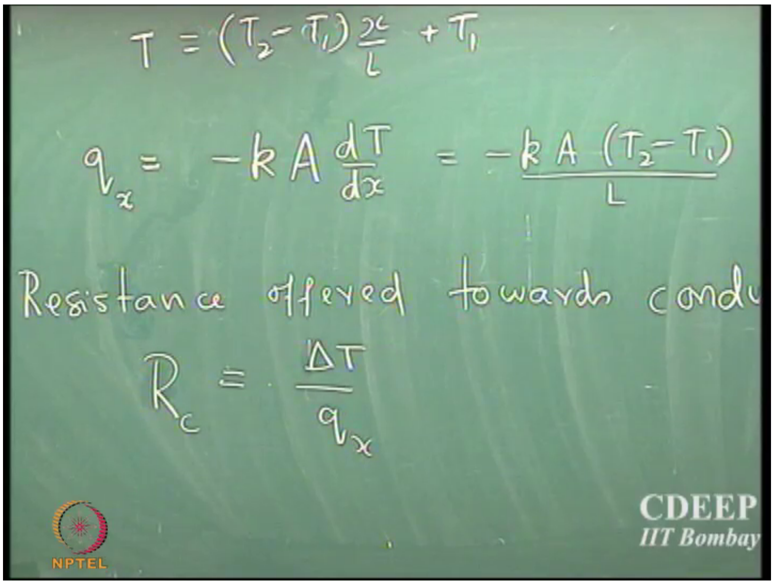

Temperature is given by T 2 minus T1 into x by L plus T 1, that is the temperature T 2 minus T 1 into x by L plus T 1 that is the solution very trivial if you did not understand how this comes about you should go home and try to work it out and get the solution very easy to get this.

We want to find out what is the heat transfer rate we want to find out what is the rate at which heat is being transferred for this system, remember in the last lecture I am mentioned that heat transfer rate is the quantifying description of the transport of heat via conduction and in general it is the quantifying description for any heat transport process.

So, you need to find out what is the heat transfer rate and also you should keep in mind that it is very very important from application point of view. So, when you are looking at certain system what somebody wants to know is how much heat is being transported, what is the amount of heat that the system would generate at one wall of or one end, then what is the amount of heat that I am supposed to take from that that is that end of the wall.

This is an important question you will always have when you from the application point of view, but finding the energy distribution is really leading towards finding the heat transfer rate and the way to find out is very simple we know that minus k area of heat transport multiplied by the corresponding gradient it is very easy, what is the gradient, d T by d x it is T 2 minus T 1 by L. There will be minus k A into T 2 minus T 1 by L, with this framework we could introduce what has called the resistance that is offered by the system.

We should reflect that one of the points that we discussed a short while ago what is meant by thermal diffusion is that there are 2 competing process these which are occurring one is the ability of the system to store heat and the other one is the ability of the system to transport heat because of conduction. One way to look at is the resistance that is often a clean and elegant way to capture the one of the competing processing is to calculate the resistance that is offered by that particular system for that process. It is similar to what you would have seen in your electrical engineering or some of the electricity things if you have studied Pre J E.

So, we can find out what is the resistance offered to towards conduction towards conduction and the way it is defined is.

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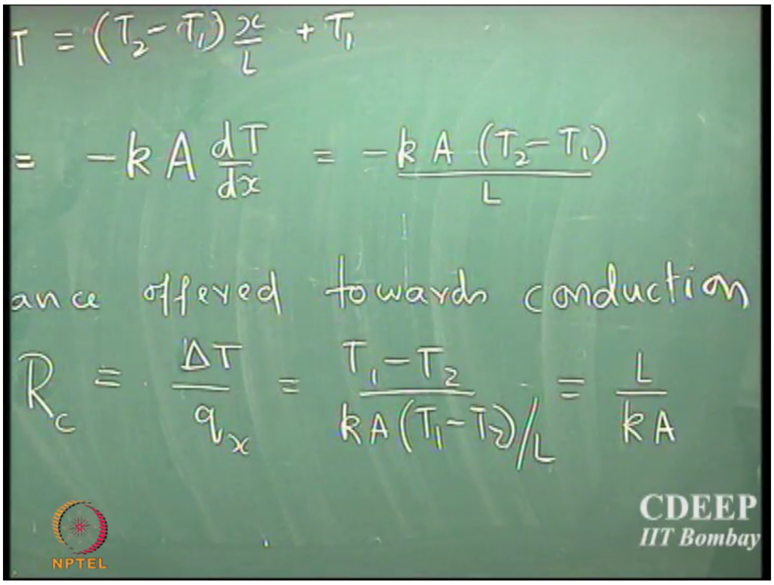

So, R is generally is a symbol I will use for resistance, R for conduction sometimes I use a subscript sometimes I do not, but generally R means the resistance which is involved. So, resistance offered is simply given by delta T divided by the rate at which heat is transported in that direction and that is given by T 1 minus T 2 divided by k into A divided by L. I have just taken the minus sign inside and that is nothing, but L by k into A.

The resistance that is offered by the system for thermal diffusion is given by L by k A, note that it is not just conductivity, but also the length and the area of heat transport plays an important role here and that you would sort of intuitively guess if the length is longer then you have a longer span for diffusion to occur if the length is smaller you have a shorter span so; obviously, you would intuitive that length and the cross sectional area is going to play a role and that sort of clearly comes out from the analysis of the moral that we have written so far.

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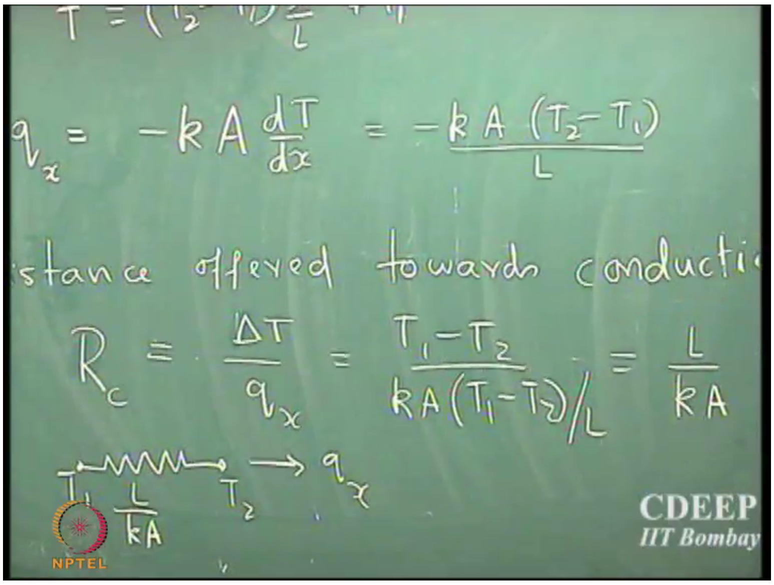

Therefore we can easily translate this into a simple resistance network, we can easily translate it into a resistance network and this sort of network you are familiar with from your P J E days where we can say that there is a network and it is connected by the temperature of the 2 surfaces of your system and the resistance of the network is given by L by k A.

We can now add a small complication to it or rather small addition to it, let us say that there is some fluid which is flowing on both side.

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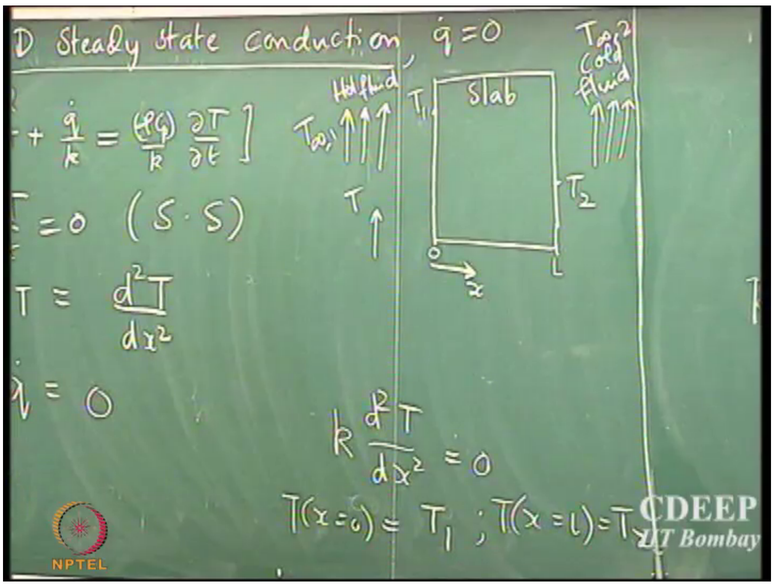

Let us say I call this as the hot fluid and I call this is the cold fluid and if the temperature of the hot fluid is T infinity 1 temperature of the cold fluid is T infinity 2, what will be the resistance offered for heat transport from the hot fluid to the surface, how do we find that, we need to find the resistance which is offered for heat transport from the hot fluid to the let us say surface at x is equal to 0 how do we find that is described by Newton's law of cooling.

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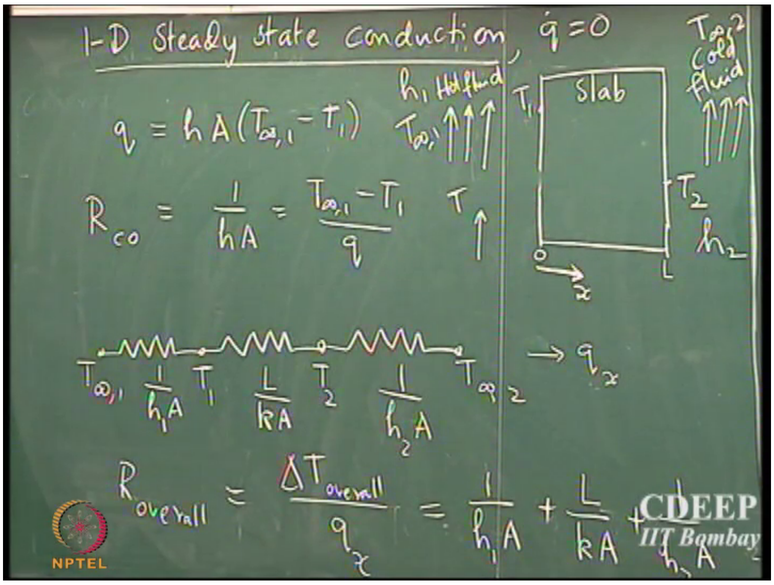

So, q which is the rate of heat transport at this location is given by h into A which is the heat transport coefficient multiplied by the cross sectional area which is outside the board into T infinity,1 minus T at x equal to 0 which is T 1. That is the rate at which the heat is being transported from the hot fluid to the slab and we can use the resistance network on step and we can say this is the convection resistance and that is simply given by 1 h A.

It is very simple to see this is nothing, but T infinity,1 minus T 1 divided by q very easy to see this it is a same formula I have used delta T divided by q, h depends on many things like flow velocity properties of the fluid we will see all that we will see how to find h it is not very easy to explain in the next 5 minutes. There are about almost like 12 lectures we are going to have in this course which describes what is the method by which you can estimate h. So, it really requires that much time, we will actually look at it sometime around the mid sem and after that.

So, now, therefore, we can now extend this resistance network into 3 resistances, supposing if the heat transport coefficient from the hot fluid to the slab is let us say h 1 and from the slab to the cold fluid that is h 2 if that is the heat transport coefficient. Then we can say that the temperature of the hot fluid is T infinity 1 and the resistance offered for heat transport is one by h 1 A and the temperature at the boundary is T 1 and the resistance offered by the slab for heat transport inside the slab is given by l by k A and the temperature on the boundary is T 2 and similarly the resistance here is given by h 2 into A.

So, we can almost read out what the resistances are so, that is the power of resistance method that you have learnt in your electricity things. So, you can always read out the properties and present it in terms of the network on there is something even more cute you can do. So, if this is the total amount of rate at which the heat is transport from the overall system from the hot fluid to the cold fluid then one could represent q x as the total resistance or maybe we should write. So, one could say that the overall resistance is simply given by the overall temperature difference divided by the total amount of heat that is being transferred right and very easy.

What is the overall resistance it is in series, just some of the resistances very trivial. So, it is 1 by h 1 A plus l by k A plus 1 by x 2 into A alright. So, that is the overall resistance that is offered by the whole system together. This gives you an elegant opportunity. So, if you know what are the measurable quantity. So, this is the first time I am using measurable quantity, when you are doing any modeling work when you are representing any system using mathematical representation or model equations you must always understand what are the observable quantities what is it that that you can measure experimentally, what is it that you can measure in physical system and actual system.

So, whenever you have if you take the example of the geyser you know, all that you can measure is the temperature of the fluid that comes in and goes out. So, can use can we write our model equations and can we write our mathematical expression in such a way that we are able to use the observable parameters for observable quantities in order to estimate the properties of the system for example, it could be I want to know what is the temperature T 2, but I all I do all I know is I can only measure the temperatures of the outside fluid. So, can I use the resistance network? So, that is what these kind of resistance network method provides you an elegant opportunity to use the measurable quantity and find some of the quantities that is not known to you via normal experimental measurement. So, we are going to see some examples of these mostly in the next lecture. So, we will just take a little bit more complicated case of this resistance network and we will finish with that for today's lecture.

So, let us take the next example of Composite wall.

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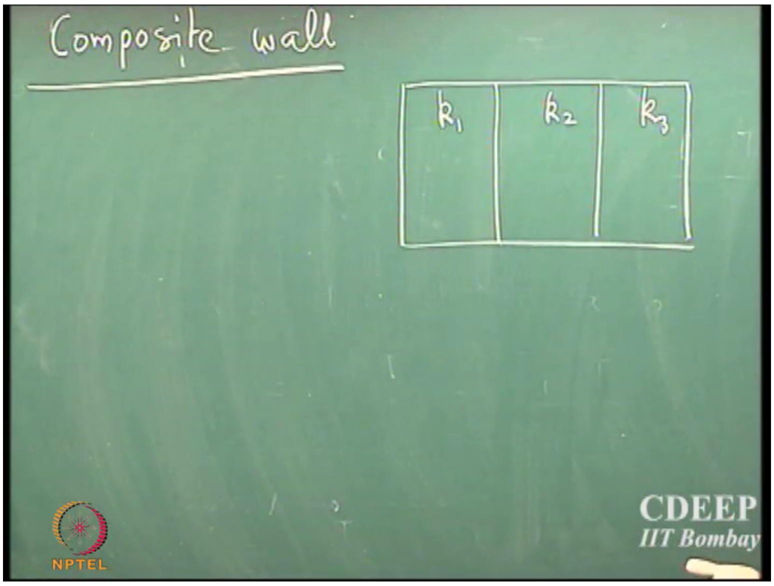

Composite wall, supposing instead of one wall I have multiple walls which are stacked with next to each other and this is the very very common thing that you would observe in a insulating system. So, supposing if you have an insulator usually it is stacked with different walls and each of them have different conductivities it is very very often very common to design system with multiple wall which actually have different properties. So, you can have k 1, k 2 and k 3 each of these could have different conductivities. So, you could imagine that you might want a certain material which is exposed to the hot fluid, but you do not want the second material which has better conductivity to be exposed to the hot fluid because there could be some reactivity with the fluid.

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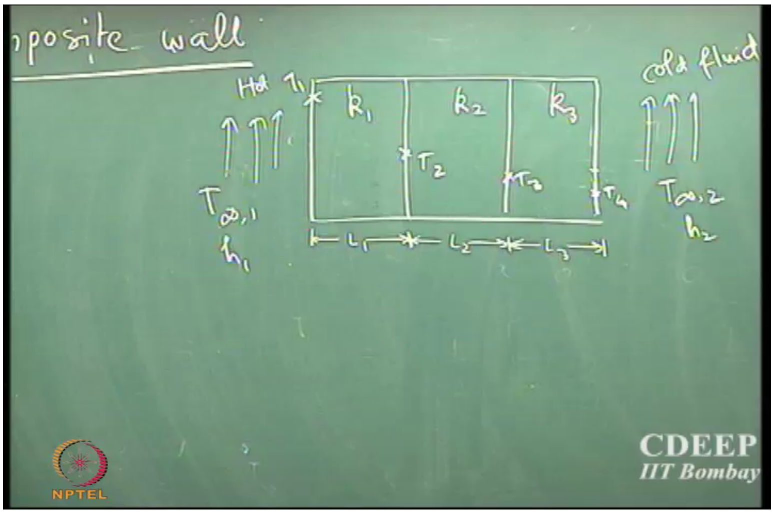

So, in order to avoid that it is a very common practice to design these kinds of composite walls and if I assume that the length of this first wall is L 1, length of the second wall is L 2, length of the third wall is L 3 and if I have let us say hot fluid which is flowing before the first wall and there is a cold fluid which is flowing after the third wall and if the temperature of the hot fluid is seen T infinity 1 and if the heat transport coefficient for Newton's law of cooling is h 1 here and if it is T infinity 2 and h 2 and I can now specify temperatures T 1,T 2, T 3 and T 4. So, I can easily read out and draw the resistance network with minimal effort.

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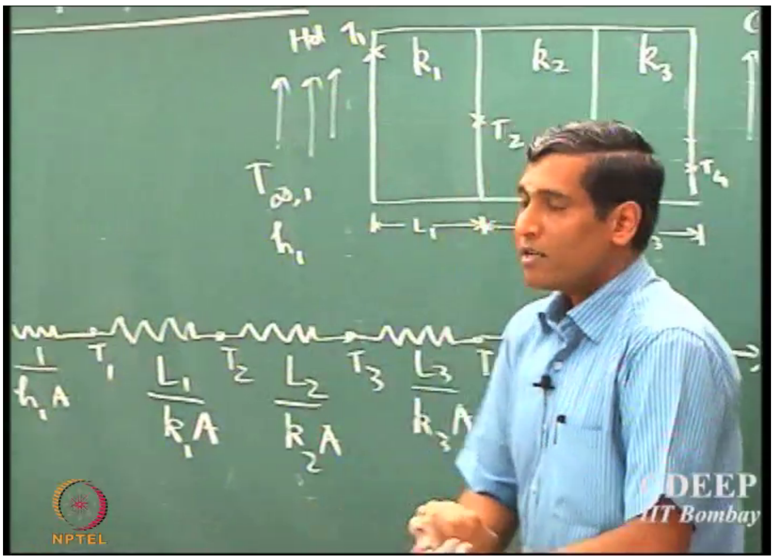

So, how many resistances are there how many 5 right. So, we have 5 resistances the first one being the resistance offered for transport from a transport of energy from the hot fluid to the first slab. So, that is 1 by h 1 into A, T 1, T 2 and this is L 1 by k 1A T 3 L2 by k 2 A T 4 L 3 by k 3 A and this is T infinity, 2 and that is the amount of heat that is being transported from the system from hot fluid to the cold fluid and that is q x I think the good point to stop.

So, in the next lecture what we are going to see is we said we assume that the cross sectional area is constant. So, we are going to start with a specific case where the varying cross sectional area of system and we are going to see how the equations are going to change and how the temperature distribution is going to change, by the way one important point before we close what will be the temperature distribution in this slab is linear, very obvious from the expression.

So, it is linear temperature profile and what will be the temperature profile if there is a varying cross sectional area and we look at other geometries we looked only at Cartesian coordinates here we will start looking at radial systems and other coordinates.