Heat Transfer
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Lecture – 38
Heated plate in a quiescent fluid – II

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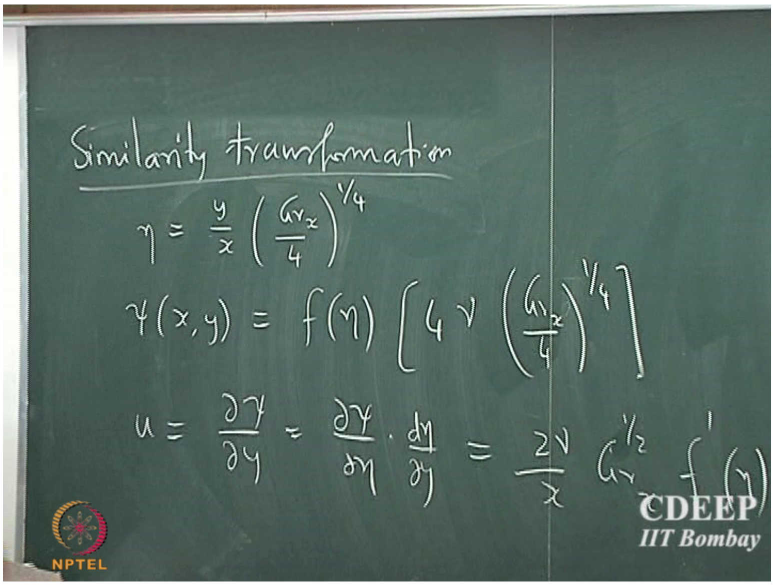

So, we introduce a similarity transformation or that we are not trying to get analytical solution because it is a coupled equation. There is no clean way to solve it analytically. So, what we going to do is we are going to introduce similarity transformation it just helps in solving the problem numerically and also to get the gradients in an easy fashion. So, what we are going to do is we going to introduce similarity variable which is y by x into the local grash of number by 4 to the power of 1 by 4.

So, it is not the magic that these expressions have come to people have actually done careful detail analysis of the modeled equation and scaling in order to get these things it just out of the scope of this course and you will not go into the details of how to get these similarity, but there is a formal method to do this and then you define psi which is essentially this stream function the modified stream function.

So, now we rewrite all the model equations in terms of these similarity solution quantities. So, you write mu will be d psi by d y which will be d psi by d eta into d eta by d y and that will be 2 nu by x into Grashof number to the power of one by 2 into f prime of eta; so, using these definitions. So, you can rewrite the model equations as f triple.

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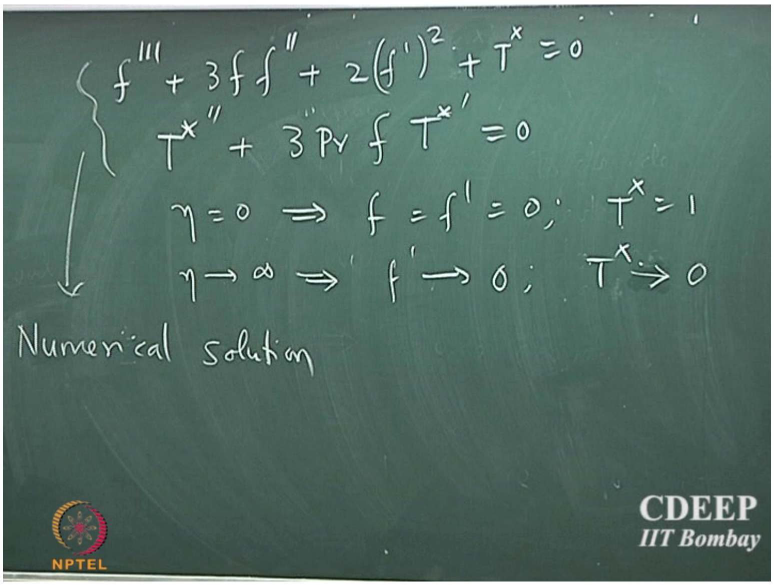

Prime plus 3 f into f double prime plus 2 f prime square plus t star equal to 0 and t star double prime plus 3 times prandtl number into f into t star prime equal to 0 and the boundary conditions would be eta is 0 f equal to f prime equals to 0 and t star equal to one and as eta goes to infinity.

You have f prime goes to 0 and t star goes to 0. So, that is the model equation and. So, one can find numerical solution for this there is no analytical way to solve this equation. So, one has to find this solution is the numerical methods to solve this equation and. So, the numerical solution would look something like this.

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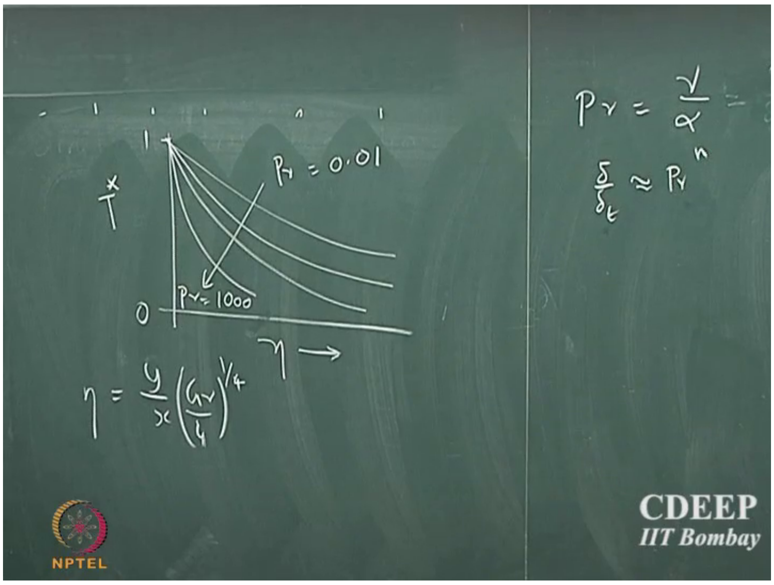

So, the numerical solution of for the temperature profile the star going from 0 to 1 and this is eta. So, the solution would look. So, this is increasing prandtl number. So, in increase prandtl number will have a faster fall and temperature profile why is that. So, let say this is point 0 one pr equal to let say thousand what happens in increase prandtl number what is prandtl number its nu by alpha right. So, what happens in increase prandtl number momentum boundary layer is more important this is the ratio of delta over yeah good delta over delta T yes.

So, this is other I should take delta over delta t the approximately scales as prandtl number to the power of n right. So, so when prandtl number is very large the thermal boundary layer thickness is very small. So, remember look at your scaling variable theta theta is y by x into Grashof number by 4 to the power of one by. So, when boundary layer thickness is very small so; obviously, eta is very small. So, all the temperature profile has to be captured in a very small value of eta. So, that is the reason why it falls very quickly for a large prandtl number.

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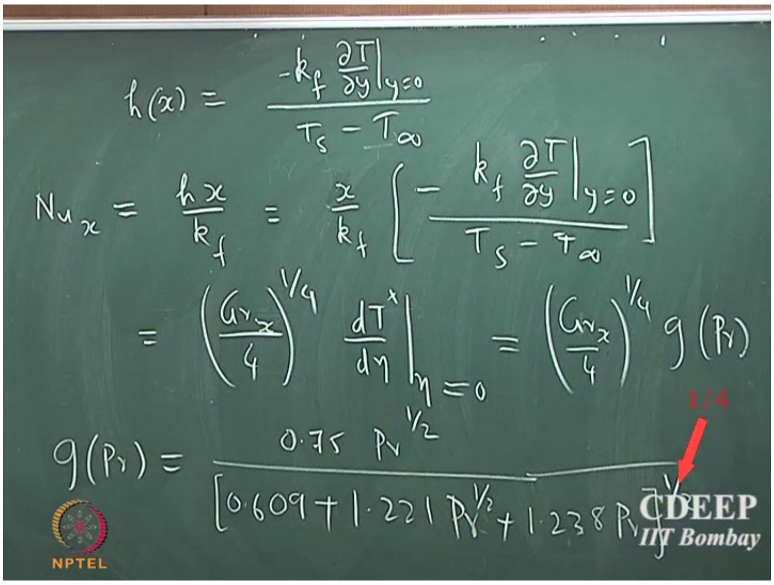

Heat transport coefficient is defined as minus k f is the conductivity of the fluid into d t by d y at y equal to 0 divided by t s minus t. So, that is the definition of t transport coefficient and. So, from using this expression using this definition you can define the local Nusselt number k f that will be x by k f into minus d y divided by t s minus t infinity. So, in terms of the dimensionless quantity, it will turn out to be Grashof number by 4 to the power of 1 by 4 d t star by d eta; eta equal to 0 and that turns out that it is a function of prandtl number and is as and that will be 0.75 into prandtl to the power of half divided by 0.609 plus 1.221 prandtl number to the power of half plus 1.238 prandtl number to the power of half.

I do not except it will remember any of these expression that it just important to realize the these expression 2 exist. So, it is required it will always be given to you in your exam.

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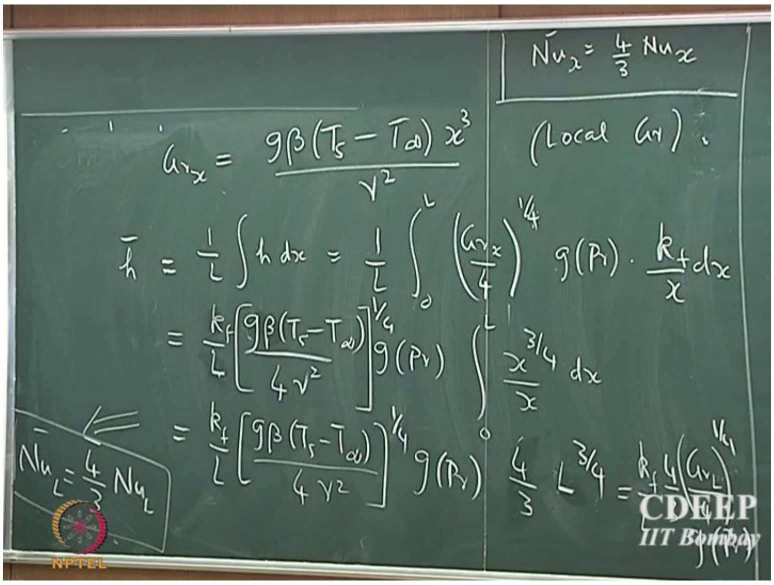

So the local Grashof number recently defined as g beta t s minus t infinity into x cube by nu square. So, it is simply the same expression that we had report multiplied by the Reynolds number square. So, we get x cube by nu square which is the local Grashof number. So, this is the local grashof number.

So, the next exercise is to find out the average. So, we had 2 objectives one is we find the local heat transport coefficient we want to find the average heat transport coefficient and that is simply given by one by l h into d x. So, that will be one by l integral of. So, from Nusselt number we can actually use that expression. So, we can find out what is the local heat transport coefficient. So, that will be Grashof number by 4 to the power of 1 by 4 into g p r into k f by x into d r. So, that is the local heat transport coefficient and that will be 1 by 4 g times eta t s minus t infinity by nu square. So, all these are not function of x. So, we can pull them out g p r is not a function of x we can pull them out pull it out k f is not a function of x that can be pulled out. So, that will be integral of that to the power of 1 by 4.

So, that would be x to the power of 3 by 4 divided by x into d x and that will be k f by l g beta t s minus t infinity 4 nu square one by 4 what is this integral 0 to l 4 by 3 come on its 4 by 3 into x to the power of 3 by 4 l to the power of 3 by four. So, this is nothing, but integral of x to the power of minus one by 4 into d x. So, that you integrate data 4 by 3 which is which comes out because of integration l to the power of 3 by four.

So, that turns out to be. So, that turns out to be k f by l 4 by 3 into Grashof number based on the length to the power of 1 by 4 multiplied by the function of prandtl number. So, from here what we observe is that the average Nusselt number is based on the length should be equal to 4 by 3 times the Nusselt number based on the the local Nusselt number based on the length ok.

It is very similar to what we got in the flat plate case where the average Nusselt number is some function of the or some constant modulo constant of the local Nusselt number itself. So, therefore, we can actually say that the average Nusselt number at any locations which signifies the net amount of heat that is transported till that location in the flat plate that should be equal to 4 by 3 times the local Nusselt number. So, that is an important observation. So, if I can measure the properties at the exit or at the end of the flat plate, then I should be able to find out what is the local Nusselt number at that location.

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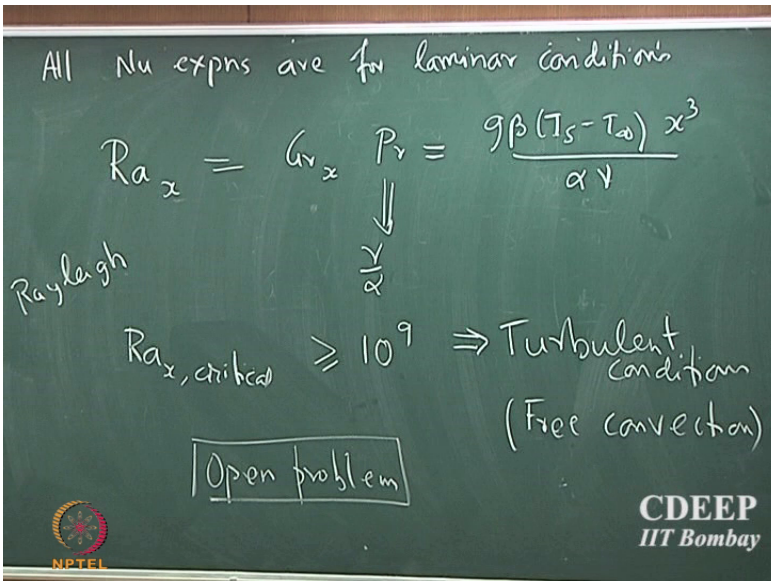

So, now; so, all what we have done is basically for laminar conditions; although I should mentioned it earlier. So, all that have been done so far; all equations and all Nusselt number expression are for laminar conditions there is a particular reason why I did not mention laminar. So, remember that there is no reference velocity. So, the question comes in as to how would I define what is laminar and what is turbulent. So, a new dimensionless quantity called Rayleigh number it actually used for distinguishing the laminar and turbulent conditions for these kind of problem. So, we define Rayleigh number based on the length which is the product of Grashof number based on the length multiplied by prandtl number and that is given by g beta t s minus t infinity x cube by alpha into nu.

So, remember prandtl number is nu by alpha. So, I multiplied it by grashof number. So, the square cancels of and we get an alpha. So, the turbulent and laminar transition is actually defined by what is called the critical Rayleigh number. So, the critical Rayleigh number exists 10 power 9, then the flow conditions inside the boundary layer remember that if the flow conditions inside the boundary layer the fluid outside is still at rest is a quiescent medium is at rest. So, the critical Rayleigh number excuse me, Rayleigh, r a y l e i; Rayleigh number is 10 greater than 10 power 9, then it is considered to be turbulent conditions.

This is something which is very rare and would never encounter a turbulent conditions. In fact, the people had not looked at this problem it is still an open question as to how we transport ducker under turbulent conditions free convection. So, there is people have not done experiments. So, there is really no cold nation and no analytical expression to find out what is the heat transport in these conditions. So, it is not something that has been studied. So, it is still an open question it is an open problem an open problem.

So, when would Rayleigh number become very large? So, look at this expression gravity is fixed there is a fixed bound for compressibility factor the temperature gradients is also fairly fixed, it is not that you can have a significant temperature gradients length scale is also fairly. So, it is the viscosity and diffusivity which has to be tells you when the Rayleigh number is going to be significantly large. So, it is very difficult to encounter a situation where the fluids viscosity and the thermal diffusivity is in such a way that the Rayleigh number is significantly large is about 10 power 9.

So, that is not something it has been encounter, but maybe there is a situation we do not know that all right. So, we will switch to another geometry for free convection. So, that will finish today's lecture. So, this is natural convection around cylinders.

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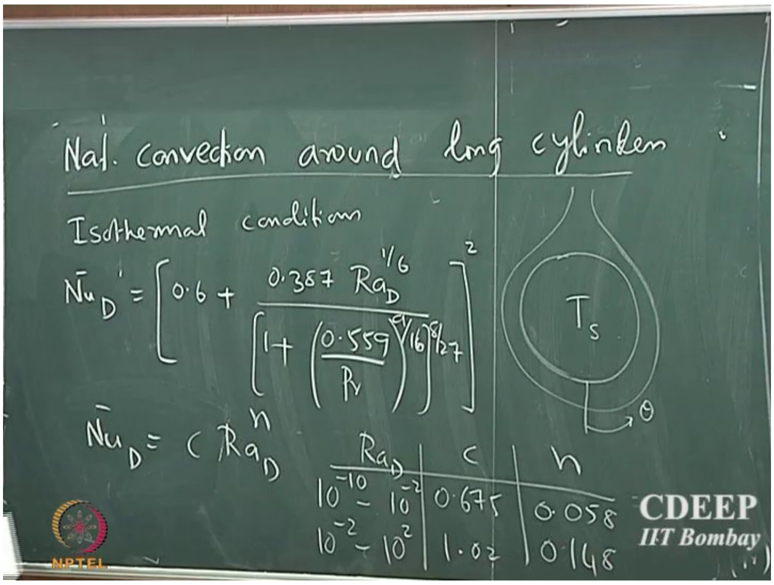

So, the analysis and the boundary layer formation is very similar to what we actually discussed in the external flow case where we have a long cylinder we have a long cylinder which is actually going inside and outside the board and. So, that let say that this cylinder is suddenly drop inside a squeeze in fluid and now you will have a boundary layer which is formed around it.

So, this is nineteen at a certain temperature t s and let say this is angle theta. So, one has. So, one can work out what is the; what are the correlations for the Nusselt number. So, suppose if the cylinder is nineteen under isothermal conditions let say we know how to maintain the cylinder. So, the Nusselt number based on the diameter is given by 0.6, 0.387; this is empirical correlation remember that Rayleigh number to the power of 1 by 6; 1 plus 0.559 divided by prandtl number power of 9 by 16 power of 8 by 27 and the square of this. So, all this funny numbers you can see immediately tells you that it some sort of correlation this much is not completely analytical ok.

So, this is a general correlation for this kinds of problems and there is also an alternative correlation which is given by some c times to the power of n and there is a table which sort of tells you what is the what are these constants for various values of Rayleigh number. So, there will be 10 power minus 10 minus 2.675 and 0.058 and 10 power minus 2 to 10 square 101.02 and 0.148. So, that is the Rayleigh number that is the constant free and n for different ranges of Rayleigh number.

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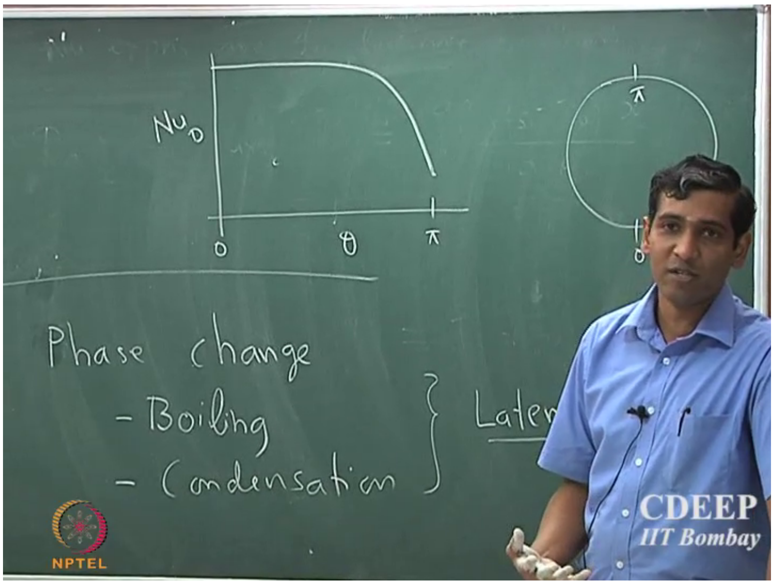

So, similar to what we saw in the cylinders. So, we Nusselt number verses theta remember we saw that the Nusselt number is increases and then it suddenly falls and it is in a similar fashion you would except that the Nusselt number would actually in 0 and pi it simple fall; what is the reason for this fall remember your cylinder with external flows what is the reason this going to be a separation point right. So, when this fluid is flowing around the fluid is now no more in contact with the surface and so, you expect that the Nusselt number that you would calculate using these correlation to would actually fall as you go close to the other end of the cylinder remember that theta 0 to pi actually goes from this is 0 and pi is here ok.

So, that is the kind of profile for Nusselt number that you would expect and once again I do not expect you to remember these expressions and. In fact, you would not be expected to remember these expressions even in future when you are actually have to use these expressions in real systems. So, these expression are always catalog in several books and its always available to you buts it is important to know and realize how to use these expression and what is the values behind it.

So, what you will see in the next lecture is we will start a new topic. So, we look at some sort of an application. So, so far we never looked at phase change we never looked at phase change. So, we always assume that the fluid which is flowing is always in a same phase. So, what we going to see in next in next classes we are going to start discussion on boiling what happens to a system when there is phase change of the fluid that is it goes from liquid to the vapors state and a few lecture down the line we going to see the rivers what happens when there is condensation. So, these 2 they play an important role from industry point of view because there is always steam which is flowing in different locations and different pipelines in a process industry and. So, you will always see that at some location a fluid is being boiled because of heat transport and in some location the heat that is carried by this steam is actually transported to another fluid through heat exchanger where the steam is actually condensed.

So, it; so, that the nature of the heat that is actually left or taken up by the fluid is the latent heat; so, we are going to start looking at when there is latent heat which is actually playing an important role in the transport proceeds how do these analysis change and what is the nature of these correlation we going to get and what the nature of solutions that we going to get. So, that is what we going to start the next lecture.