Heat Transfer
Prof. Ganesh Viswanathan
Department of Chemical Engineering
Indian Institute of Technology, Bombay

Lecture – 37
Heated plate in a quiescent fluid – I

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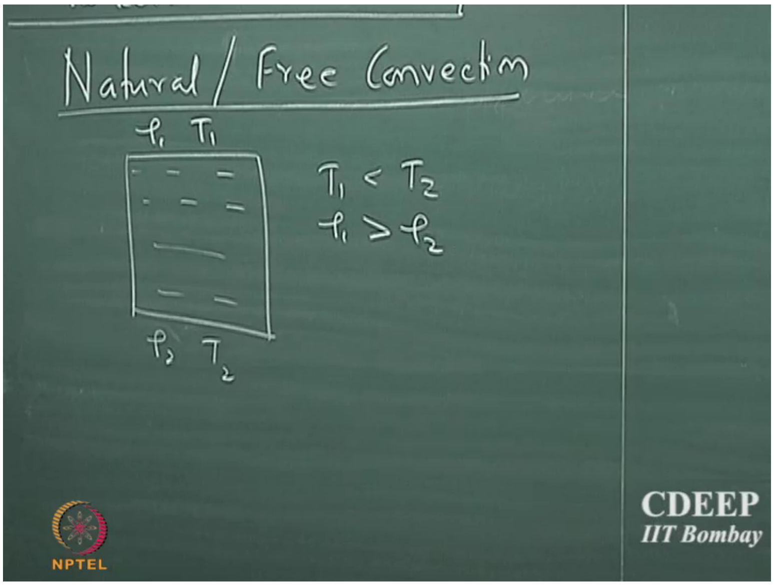

Here, we started discussion on natural or free convection. So, we started by looking at the example of a fluid which is placed in a box, and we looked at the differences in the temperature which leads to density difference and that leads to a convection. So, supposing if T 1 is less than T 2, we said that rho 1 is greater than rho 2 and therefore, the heavier fluid is sitting on top of the lighter fluid which is an unstable situation.

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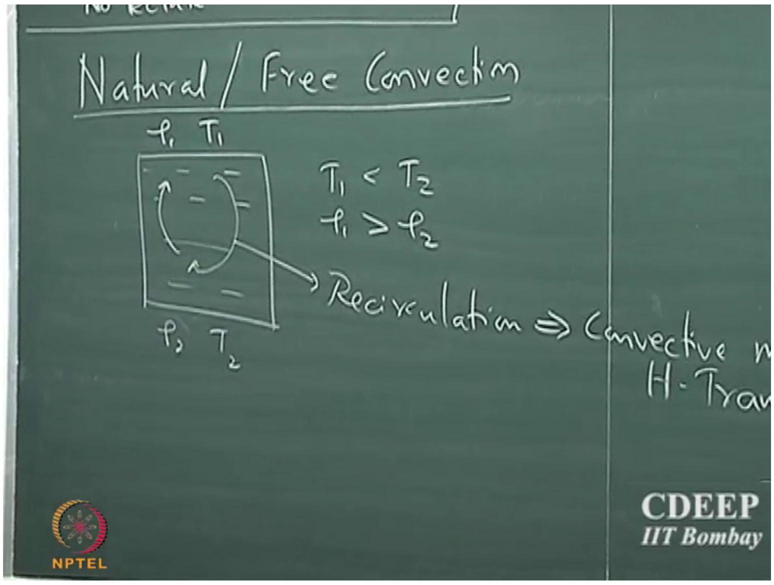

And so that is going to set up a circulation leading to convection leading to convective mode of heat transport in the horizontal direction.

This re circulation. So, its.

Student: Sir, (Refer Time: 01:35).

That is correct where the driving force is gravity here now the gradient in the horizontal direction, because of the re circulation not every location in the horizontal direction is going to be maintained at same temperature. So, there will be heat transport in the horizontal direction too, but the primary driving force for convection is the vertical direction because of gravity, gravity is a driving force ok.

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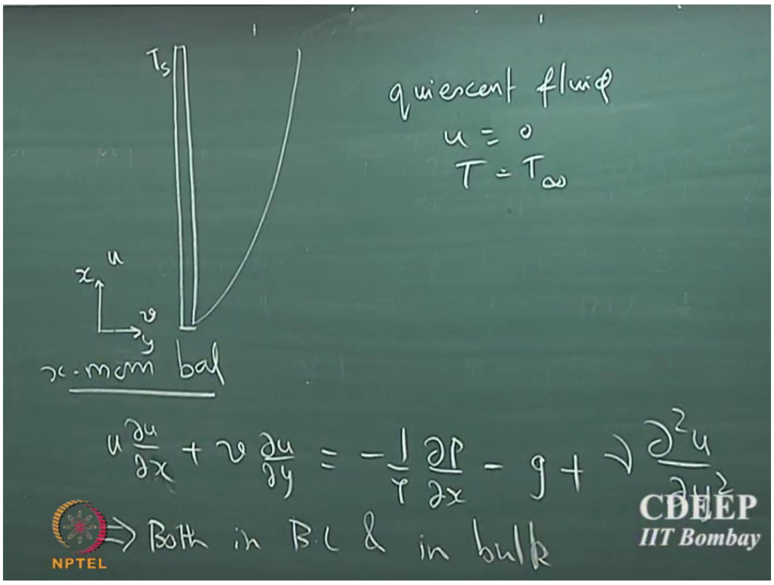

So, now we said that suppose if we have a plate which is maintained at a certain temperature, and then it is certainly drop into a quiescent fluid, which means u is 0 and T is some T infinity and then it results in a certain boundary layer, and we looked at the x momentum balance in the boundary for this problem, which will be u supposing if this is y direction and this is x direction and u and v are the x and the y component velocities. So, we said that u d u by d y plus v u du by d x plus v d u by d y that is equal to minus 1 by rho d p by d x minus g d y square. So, that is the x momentum balance.

Now, this equation is valid at every location in the in this problem and. So, it is valid in the quiescent medium as well in the quiescent fluid as well. So, this equation is valid both in boundary layer and in bulk. So, now, if I look at the equation in the bulk we know that u and v are 0, velocities are 0 and not just that all the gradients are 0 in the bulk stream.

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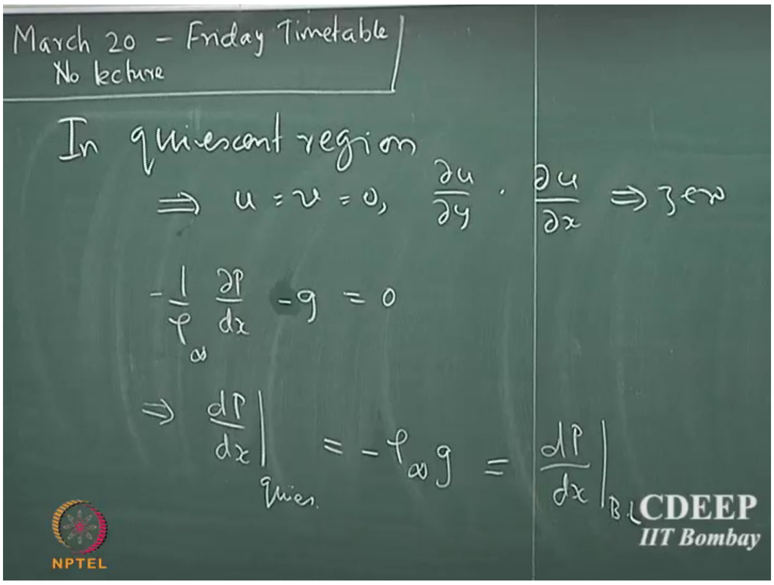

In the quiescent region u and v are 0, because the fluid is not moving and also d u by d y d u by d x they are also 0 all the gradients are negligible in the quiescent region.

So, therefore, the x momentum balance simply reduces to minus 1 by if rho infinity is the density of the fluid in the quiescent region. So, that it will d p by d x equal minus g equal to 0. So, from here we can say that d p by d x in the quiescent region, that should be equal to minus rho infinity into g, but there is no external forcing which is actually there is no force convection. So, the fluid is not being force to move at any location.

So, therefore, the net pressure gradient that the fluid experiences in the boundary layer should be equal to the net pressure gradient that the fluid experiences in the quiescent region because the fluid is not being force too at all its actually at rest and. So, the net pressure gradient in the boundary layer should be equal to the net pressure gradient in the quiescent region.

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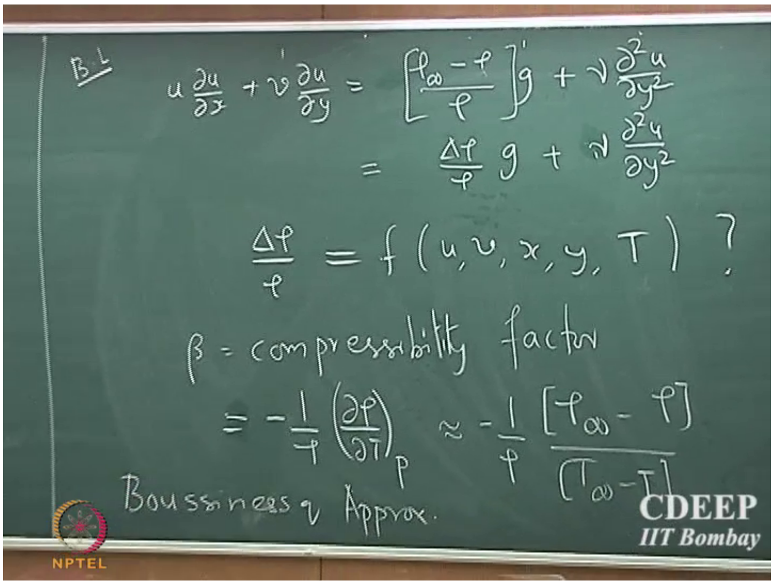

So, now, if we incorporate that into our momentum balance so, what we will see we incorporate that into the momentum balance into u d u by d x, plus v d u by d y and now I am writing the momentum balance in the boundary layer. So, that should be equal to minus rho infinity minus rho plus nu into d square u by d y square ok.

So, this is nothing, but the difference in the density delta rho, which is the difference in the densities in the bulk and any location in the boundary layer, divided by the density in the that location in the boundary layer plus nu into d square u by d y square. So, now, if we know what is delta rho by dou rho we are done, and we should be able to solve the equation and. So, what is the relationship between the density gradient and the other system variable?

For example velocity or temperature or concentration whatever, here we are not looking at mass transport let us not worry about concentration for now. So, what is the relationship between delta rho by rho and u, v, x, y temperature, etcetera? So, that is the new variable or new body force term that we did not see so far and all the convection topic that you start seeing natural convection. So, any thoughts on how do we find this.

Student: (Refer Time: 07:17).

Yeah compressibility. So, we said that the driving force. So, note that in natural free convection the density gradient is the function of temperature right.

We said that the temperature difference is causing the density gradients and therefore, they have to be related in some way. The so far and all the cases that we looked at the momentum boundary layer equation was not did not have any temperature dependence, it was independent of the temperature concentration. So, now, we will start seeing dependence of temperature on the momentum boundary layer equations. So, the way to that is what is called the compressibility factor compressibility factor.

So, the typical symbol that is used is beta now the definition of compressibility factor is minus 1 by rho d rho by d T at constant pressure. So, that is the definition of compressibility factor. Now what do you expect the density gradients to be is it expected to be very large compared to the density the local density or not. So, it is not because, it is a free convection problem there is no force convection and. So, we expect even the boundary layer thickness to be significantly smaller and. So, we expect that the density gradient also to be very very small.

So, therefore, we can approximate this as minus 1 by rho. So, this approximation is what is called as boussinesq approximation, after the person boussinesq approximating the gradient the density d rho by d T as simply the differences in the bulk and the local density divided by the difference in the bulk temperature and the local temperature that is what is called boussinesq approximation yes haransh.

Student: (Refer Time: 09:48).

That comes from the relationship between the pressure gradient and the quiescent region.

So, if you write the momentum balance in the quiescent region, where there is no velocity velocity is 0 because of fluid is at rest and the velocity gradient is 0. So, therefore, you will see that the x momentum balance will simply reduce to the pressure gradient in the x direction should be equal to minus rho infinity times gravity. And because there is no external forcing the gradients have to be equal in the boundary layer and the quiescent region and therefore, you replace the d p by d x with minus rho infinity into g and that is how you get this expression.

Student: (Refer Time: 10:27).

Yeah, but there is no flow flow of fluid right. So, the pressure gradient is going to be very very insignificant.

Student: (Refer Time: 10:39).

Yeah.

Student: (Refer Time: 10:41).

Right, but that is already take into account rho into g is already take into account, the body force accounts for the force that the fluid is experiencing because of gravity that is already accounted for in the momentum of balance.

Student: (Refer Time: 10:54).

See we look at only steady state case here and these course there is very little dynamics we look at the only place, where we looked at some dynamics was in conduction where we looked at the semi finite slap we really going to look at a transient cases in the these kinds of problems.

So, this approximation is what is called as boussinesq approximation, a just to get a feel of what this compressibility factor really how it plays an important role is that the compressibility factor; obviously, it is a function of temperature compressibility factor itself is a function of temperature suppose if we say it is an ideal gas.

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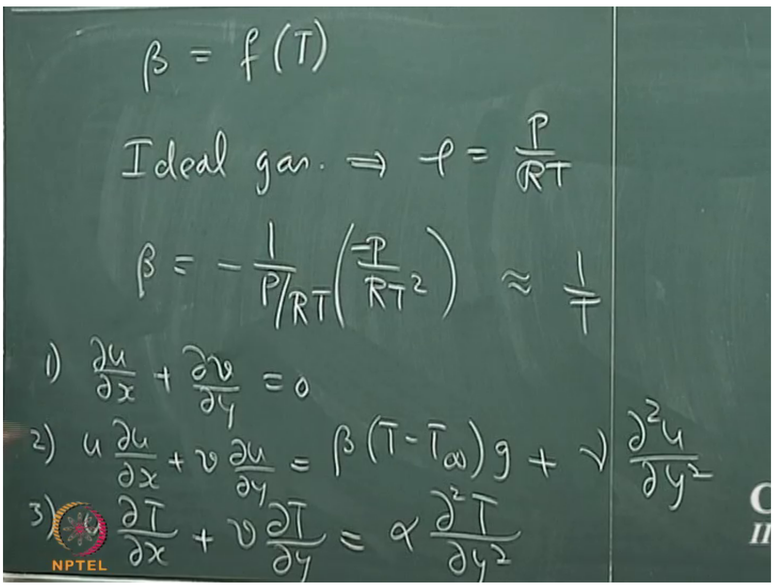

Rho is given by P by R T for an ideal gas. So, now, from here we can write that beta is minus 1 by P by R T into d rho by d T at constant pressure.

So, that will be P by RT square with a minus sign. So, that will really scale as sort of 1 over temperature. So, a compressibility approximate these scales as 1 over temperature, but let us for the moment assume that beta is some measurable property and it is not varying significant in the temperature range that we are looking at. So, it scales as 1 over T do you must understand this difference it scale as 1 over T, but just for sake of getting insight as to what is happening in this problem.

Let us assume that beta remains almost constant to the temperature range that we are looking at. So, now, we can write now the all the balance equations in the boundary layer and. So, the first one would be continuity equation that is the continuity equation. When we have the momentum boundary layer equation boundary layer momentum balance. So, that will be u d u by d x plus v d u by d y that is equal to rho infinity minus rho by rho so that we can replace using the compressibility factor so that we can rewrite as beta into T minus T infinity into gravity plus nu into d square u by d y square.

So, all I have done is I have just replace delta rho by rho with the compressibility factor expression using boussinesq approximation and then we have the temperature balance. So, we assume that there is no energy generation etcetera. So, this is the energy balance we have the momentum balance and we have the continuity equation. So, we need to find a way to solve these boussinesq approximation in order to find the heat transport coefficient remember that the objective is to find the local heat transport coefficient that is the objective and of course, the average heat transport coefficient.

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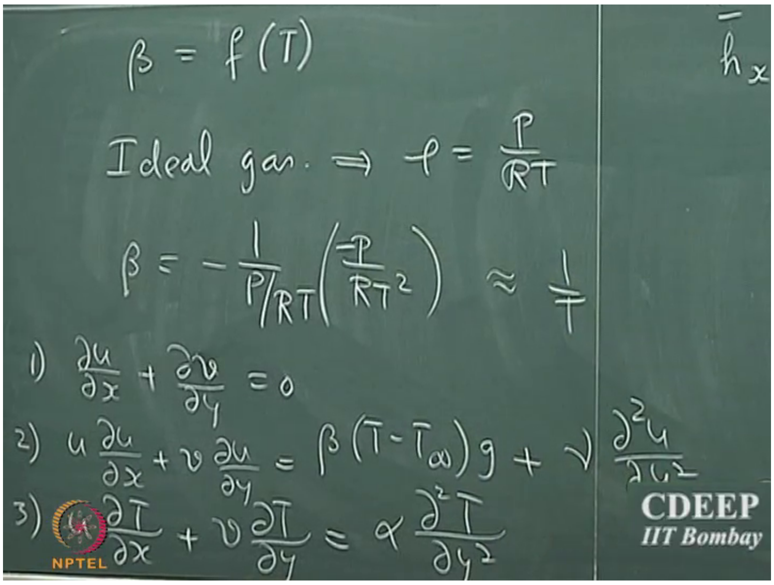

If there are any length or based on the length of the full play that is the objective.

So, we have to solve some of these equations in order to find the heat transport coefficient. Remember that if we know the gradient at the wall we are done right the heat transport coefficient if because the temperature of the plate is constant and the bulk temperature is constant. So, if we know the gradient at the wall we are done. So, the objective is really define the temperature gradient at the inter phase between the plate and the quiescent fluid.

So, I mentioned in the last lecture that we are going to look at the reference velocities for this kind of a problem.

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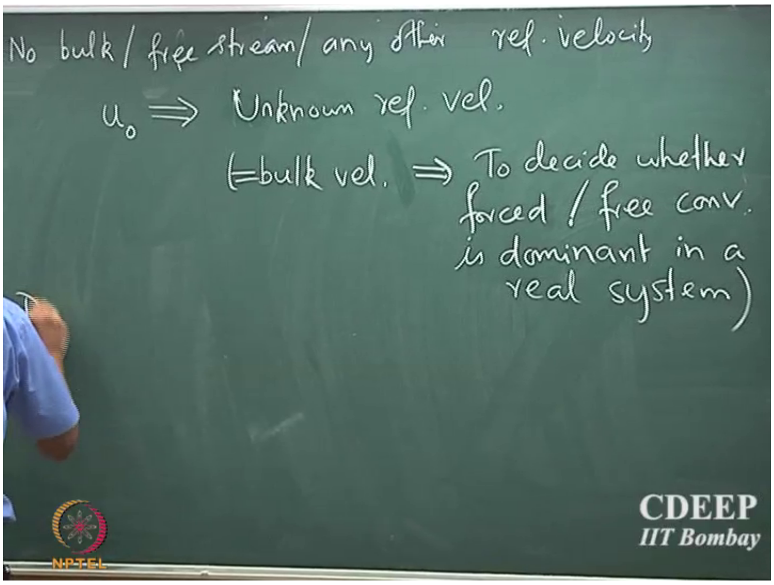

So, the first and important issue is that there is no bulk or free stream or any other reference velocity. There is no reference velocity for this problems because the fluid is active at rest, we do not know any velocity to (Refer Time: 15:26) the boundary layer. So, what we are going to do is we going to assume that there is some reference velocity we are going to show later that we will not be using this reference velocity for any of the calculation purposes in fact, to even define any of the domains.

So, the only place where we will be using this reference velocity. So, it is an unknown reference velocity. Now the only place where the reference velocity plays an important role is when you want to make an important decision whether the free convection or a force convection is playing a dominant role in a real problems. So, here we have assumed that the problems is only free convection problem, but in principle even when you are going to drop a plate inside there is going to be some disturbance of the bulk fluid and. So, you would expect that there will be some velocity.

So, the question is; how do I decide whether for a given problem is the free convection or the force convection which is the dominating mode of heat transport. So, it is at that situation where this unknown reference velocity will play an important role so. In fact, in that situation this unknown reference velocity be equal to the bulk velocity itself, to decide whether force or free convection is dominant. We will see that mostly before the end of today's lecture as to how to decide which one is dominant, and how to decide particularly which correlations we use ok.

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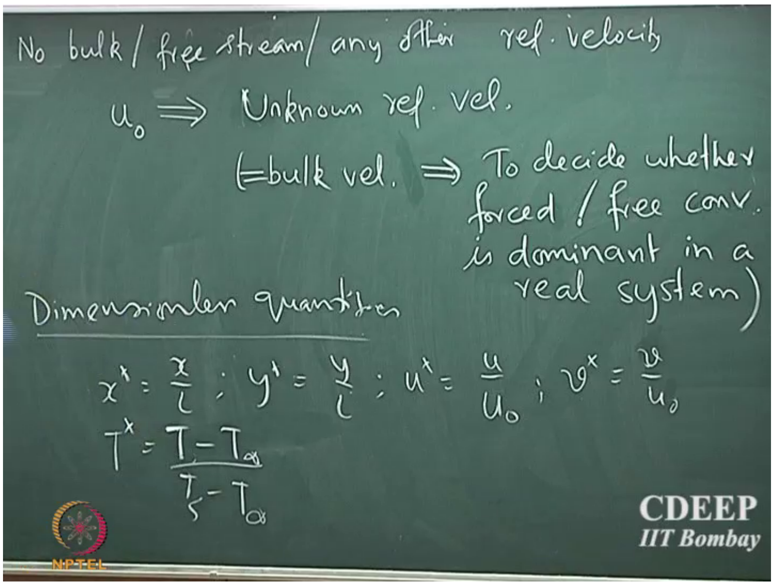

So, suppose we use this as the reference velocity and then we introduce the dimensionless quantities introduce the dimensionless quantities as x star as x by l y star as y by l u star as u by u naught note that we do not know what u naught is it still an unknown reference velocity and v star is v by u naught. So, there will be T minus T infinity by T s minus T infinity.

So, that is my definition of all the dimensionless quantities, then I can convert all these model equations into the dimensionless form and. So, that will be something like this. So, that will be u star d u star by d x star plus v star d u star by.

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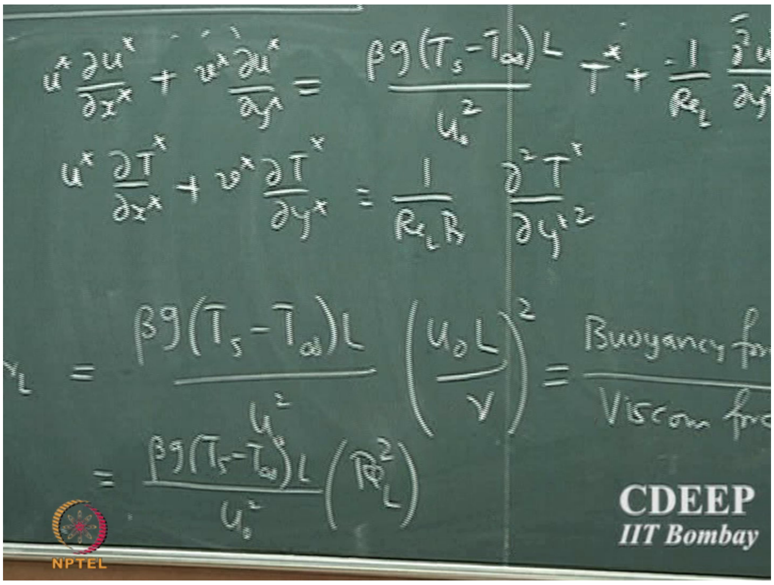

D y star that is equal to beta gravity, that will be T s minus T infinity into L by u naught square into T star plus 1 by Reynolds number into based on the length into d square u star by d y star square. And the temperature balance will be u star d T star by d x star plus v star into d T star by d y star that is equal to 1 by r e l into prandtl number into d square T star by d y star square.

They are not very different from what we saw in force convection expect that your path is new dependence of the velocity on the local temperature, that is because of the density gradients and that is because of the body force that we have included in the model equation. So, now, they are coupled now the earlier case the momentum balance and the concentration and temperature equation were de coupled. So, we could solve the velocity profile independently with respect to the temperature and you are able to get the gradient we cannot do that here. So, still you have you can still solve this problems by some transformation you going to see that in the short while.

So, before we do that let us look at this expression here, in which is a coefficient to the local temperature.

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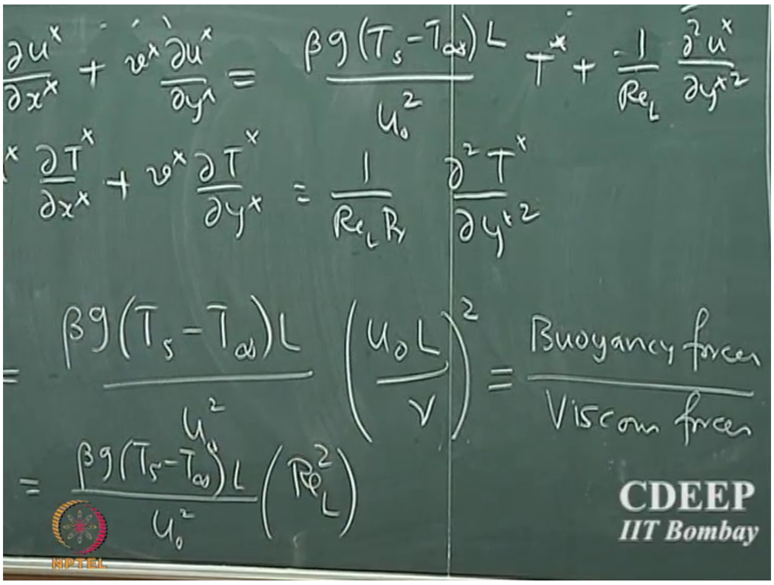

So, that is beta time g into T s T infinity into L by u naught square. So, we still do not know what u naught square is remember that it is a is an unknown reference velocity. So, therefore, we define a new number called Grashof number which is not simply the coefficient. So, in all the earlier cases we attempt to define a dimensionless quantity for the coefficient and here we distinguish the effect of different course here and similarly we are going to distinguish the effect of different forces and the way to do that is you multiply it by the square of Reynolds number. So, we remember that u naught L by new is nothing, but Reynolds number based on the length of the plate.

So, you multiple it by the Reynolds number. So, you get an expression which essentially gives you the ratio of any L guess ratio of beta times g what does it signify gravity. So, it is the buoyancy forces or force that the fluid is experienced in because of gravity. So, u naught square will cancel out what does nu signify viscous force that is all. So, it is very easy. So, Grashof number which is essentially beta g T s minus T infinity into l by u naught square into R e L square. So, that is the ratio of buoyancy forces to the viscous forces yeah, but this just some functions of viscous force.

So, its say just square, you will have square of the viscous forces that is because of the scaling that you will require for the buoyancy force. So, that gives us I mean an interesting framework to answer the question that it force a short while ago how we decide whether the free convection or the force convection is the dominating mode of heat transport.

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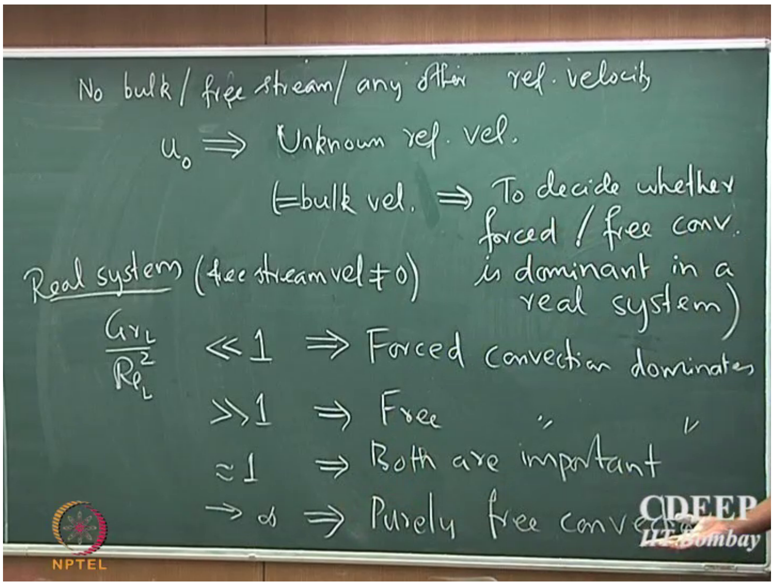

So, Grashof number by R e L square. So, that is provides the framework for understanding which one is the dominating mode of heat transport ok.

So, supposing if you have in a real system. So, real system the bulk velocity or the free stream velocity is really not 0. So, if we use the free stream velocity for a real system as the scaling velocity. So, Grashof number by square of Reynolds number gives you a dimensionless quantity, which can be used to decide which one is a dominating force. So, supposing if it is this much smaller than 1 what would you infer, which 1 is important force or free convection? Re forced convection which is important forced convection dominates and if it is much larger than 1 free convection dominates, and if it almost equal to 1 then you would expect that both are important and what if it tends to infinity.

So, look at the ratio u naught tends to 0. So, that is the purely free convection problem right. So, that is a purely free convection problem. So, this when it goes to infinity simply means that the bulk Reynolds number goes to 0 which means that the bulk velocity is 0, which is the definition of free convection. So, we started by assuming that it is a quiescent fluid. So, when this tends to infinity means that the Reynolds number is 0 and. So, therefore, the bulk fluid is at rest and so, it is a purely free convection problems.

So, it just provides a framework to decide what there in a real system you never going to have a completely free convection problem, there will always be some disturbance even in the (Refer Time: 25:16) of example we saw there will always be some disturbances you going to have air flow you are going to have a fan so, all that is going to disturb the fluid. So, therefore, it gives you a nice framework to decide which 1 is the dominating mode of heat transport.