Heat Transfer
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Lecture - 35
Example problems: Forced Convection

What we going to do in today's lecture. So, the first half of today's lecture we are going to look at some of the example problems for a internal flows and little bit about external flows, and then will march on to the next topic of the course.

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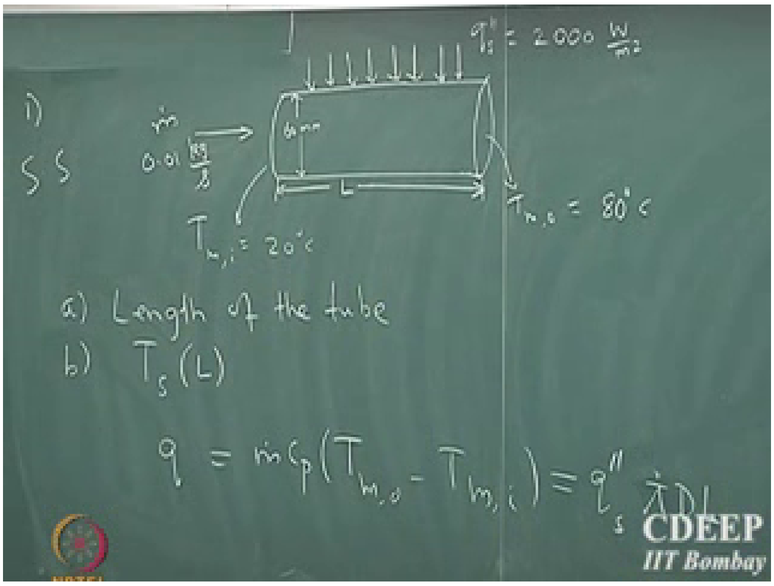

1)
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a) Length of the tube
b) $T_s(L)$

$$q = \dot{m} C_p (T_{m,o} - T_{m,i}) = q_s'' \pi D L$$

So, the first problem is a virtue which a fluid is flowing at a mass flow rate at 0.01 k gs per second, then the external surface is maintained at a constant flux of 2000 watt per meter square, then the diameter is 60 millimeter the maxin cup temperature at the inlet is 20 degree C to yes 8 degree C. Oops 80 maxin cup temperature at the exit of the tube is 80 degree C. So; obviously, you could imagine that the fluid is being heated up as it goes through the tube and it is taking up heat from the fluid which is present around it.

It maintains a constant flux of 2000 watt per meter square. Now question is, what is the length of the tube, and the second question is, what is the surface temperature at L. So, L is the length of the tube, what is the temperature of the curved surface at L. So, these are the two question. So, how do we find this?

Student: (Refer Time: 02:36).

So the net amount of heat that is taken up by the fluid to increase it is capacity. So, that is given by m dot C p into T m o minus T m i. So, that is the net amount of heat that is taken up by the fluid under steady state condition that is also equal to the amount of heat to s prime into the curves of it is area right. So, that is given by pi d L right. So, that is pi into d into L. We have all the information m dot C p is given. So, I can put the numbers here

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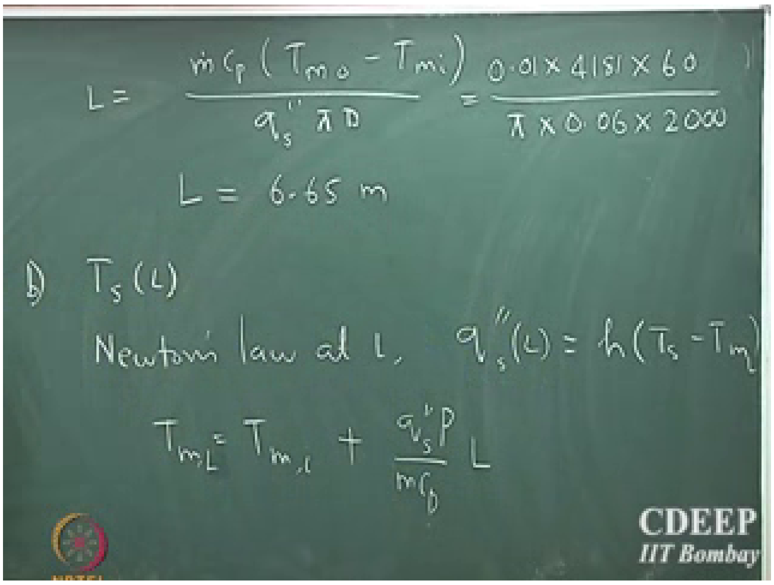

$$L = \frac{\dot{m} C_p (T_{mo} - T_{mi})}{q_s'' \pi D} = \frac{0.01 \times 4181 \times 60}{\pi \times 0.06 \times 2000}$$

$$L = 6.65 \text{ m}$$

b) $T_s(L)$

Newton's law at L, $q_s''(L) = h(T_s - T_m$

$$T_{mL} = T_{m,i} + \frac{q_s'' P}{\dot{m} C_p} L$$

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So, the length will be m dot C p T m o minus T m i divided by pi into d and that will be 0.01 into 4181 into 60 divided by pi 0.06 into 2000.

So, that turns out to be 6.65 meters. So, I wanted to stare at the numbers, by the way this is why realistic set of number. So, look at that the diameter of the tube is 60 millimeters, 60 millimeter is how much, it is this much right 6 centimeter in the diameter and then the length of the tube is 6.65 meters that is how much, that is almost the length of this dais right 6 meter we have to divide yeah.

So, this, if you actually remember your visit to r c f, if you see the tube lengths that you have very large in industry, the reason is, it takes the lot of effort to heat the fluid or cool the fluid, and therefore, 6 meter is really not that much.

When the diameter may be very small in fact, most of the tube that you saw for heating up probably not bigger than this, you know this is the pretty reasonable size dais of the tube, but then 6 meter is literally long. So, the idea of heat transport is actually done along with the process, when you do this process design and process flow sheet courses in the future semesters, you will see that the length of the tubing actually is the very important factor in designing how a plant has to be made.

So, you want to line your tube in such a way that you are not, your actually maximize, your minimizing the equipment; that is the tube size of the tube, but at the same time you maximize the amount of the heat transport for a given C p. You can imagine that in a plant, there will be many different fluid which is actually flowing in different directions to different equipments, some have to be heated some have to be cooled. So, heat transport is a very important challenge, how to maintain the heat transport and. So, that you cut down the net energy cost. So, that is an important challenge.

It is important to understand, and if you get an alarming number of 6 meters. You should really not to be worried, because it is not of it is quite fair to have that longer tube and that thinner tube. Of course, if you do a reacted design, if you get the reactor size of 6 meter you have to worry about it, because that is an impossible things to do, it is very difficult to build a 6 meter tall reactor, and there are reactor which are that tall, but really not, it is not a very standard thing to have a reactor which is that tall ok.

So, you have to have some feel for the numbers. So, if you have a piping system 6 meters is reasonable, but we have a reactor, you cannot have 6 meter tall reactor, it is very difficult to maintain it. So, you must have some feel for numbers while you go through some of these problem, and most of the problem in the text are fairly realistic, these are industrially related problem.

So, it gives a good feel of the different kinds of numbers that you will encounter, when you go to industry in the future. So, your second problem is how to find the temperature of the temperature of the surface at L, how do we do this, how do we find the temperature. So, it is a constant flux condition

Student: (Refer Time: 07:29).

Yeah.

Student: (Refer Time: 07:31).

It transport coefficient that is not there right, we do not know that yet.

Student: (Refer Time: 07:38).

So, we know that supposing I write in Newton’s law of cooling at L to where q s at L will be sum h into T s minus T m at L right. So, that is the Newton’s law of cooling at L. Now if I know what is the, what is the heat transport coefficient and what is m, I am done right, but I know much more than that. So, it is a constant flux condition, I know what is the temperature profile of the mixing cup temperature right. So, I know that T m is given by T m i plus.

Student: (Refer Time: 08:34).

Yeah

Student: (Refer Time: 08:37).

Q s double prime.

Student: (Refer Time: 08:40).

Into T divided by.

Student: (Refer Time: 08:45).

M dot C p into x. So, T m at L is given by T m i plus q double prime P divided by m dot C p into L right, we know this. So, we can use this information and then we can also find, I am not given other numbers here. So, we can also find out, what is nusselt number.

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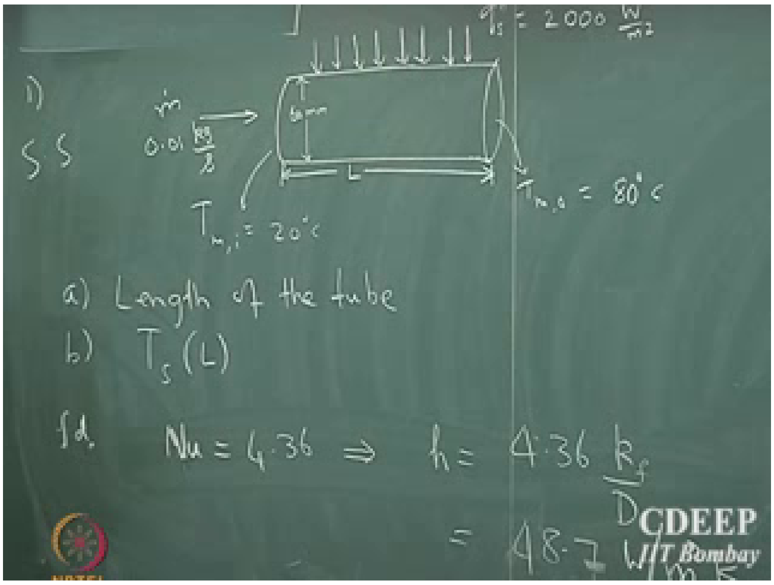

The constant flux case if I assume that fully developed, what is the Nusselt number.

Student: (Refer Time: 09:21).

Its constant right, the constant and it is 4.36. So, from in here, I can find out what is the heat transport coefficient 4.36 into k by D. So, that is the conductivity of the fluid k f. So, conductivity of the fluid divided by D, from here I should be able to find out what is the heat transport coefficient right. So, if I throw in this numbers that will be 48.7 watt per meter square Kelvin.

Now, how do we know whether it is really fully developed or not x by d in the fully developed regime.

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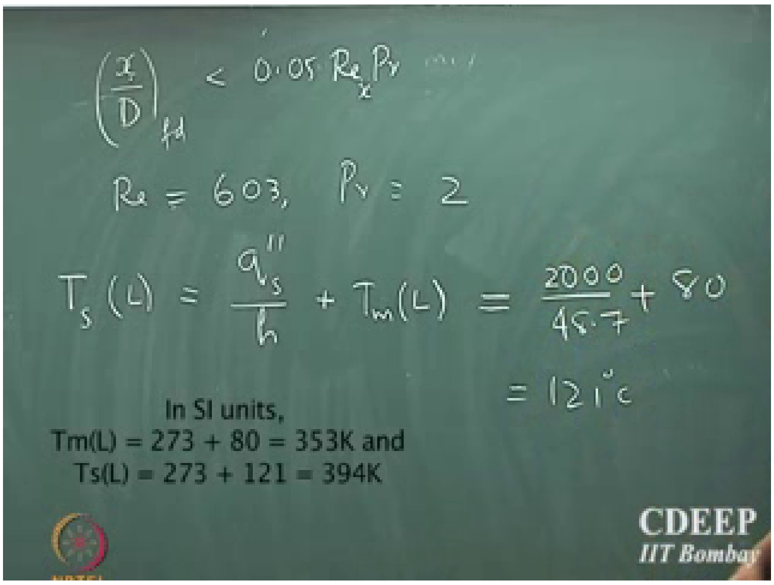

$$\left(\frac{x}{D}\right)_{fd} < 0.05\, Re_x Pr$$

$$Re \approx 603, \quad Pr = 2$$

$$T_s(L) = \frac{q''_s}{h} + T_m(L) = \frac{2000}{48.7} + 80$$

$$= 121^{\circ}C$$

In SI units,
Tm(L) = 273 + 80 = 353K and
Ts(L) = 273 + 121 = 394K

If this is 0.05 times Reynolds number times Prandtl number. So, that is if this is satisfied, then you can assume that it is a fully developed system. So, we can calculate the Reynolds number and Prandtl number. So, Reynolds number is about 603, and Prandtl number is about 2 or something like that. So, it will be much higher than the x by d. So, when this condition is satisfied. In fact, that is one of the things; I discussed when we started the discussion on convection and internal flows.

So, this condition is satisfied then you can assume that it is a fully developed system and. So, therefore, the T s at L, that is given by q s prime divided by h plus T m at L. So, that is given by 2000 divided by 48.7 plus 80, it is gets about 120 degree C. So, that is the. So, that is the temperature of the surface at x equal to L ok.

Here suppose if it is water, I dint tell you which fluid is going inside. Suppose if it is water it is a reasonable temperature; no, why it is above the boiling point of water, supposing if a fluid that is going inside is water ok.

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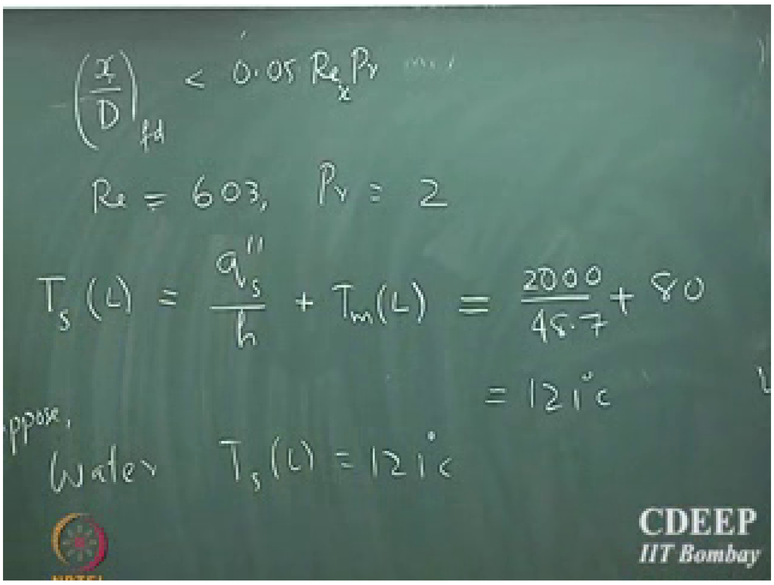

$$\left(\frac{x}{D}\right)_{fd} < 0.05\, Re_x Pr$$

$$Re = 603, \quad Pr = 2$$

$$T_s(L) = \frac{q''_s}{h} + T_m(L) = \frac{2000}{45.7} + 80$$

$$= 121^\circ C$$

ppose,

Water $T_s(L) = 121^\circ c$

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Suppose it is water, then T s of all equal to 121 is quite unreasonable. So, you would expect that water would, I actually boiled at that time, you will see wafers. So, all your calculation will fall apart. So, you have to pay attention to the temperature that you get.

If you assume that it is a single phase, not that we never looked at multiphase system. As of now we assume that there is no phase change and the fluid acid goes through the tube. So, you have to pay attention to these numbers. You have to make sure that these numbers that you get are not about the boiling point of the fluid, that you are actually looking at. So, if this has to be correct, and it has to, it cannot be water, it has to be some other fluid. So, it is important to pay attentions. The point is that you have to pay attention to the numbers that you get, it is very important.

All right let us go to the next problem we have.

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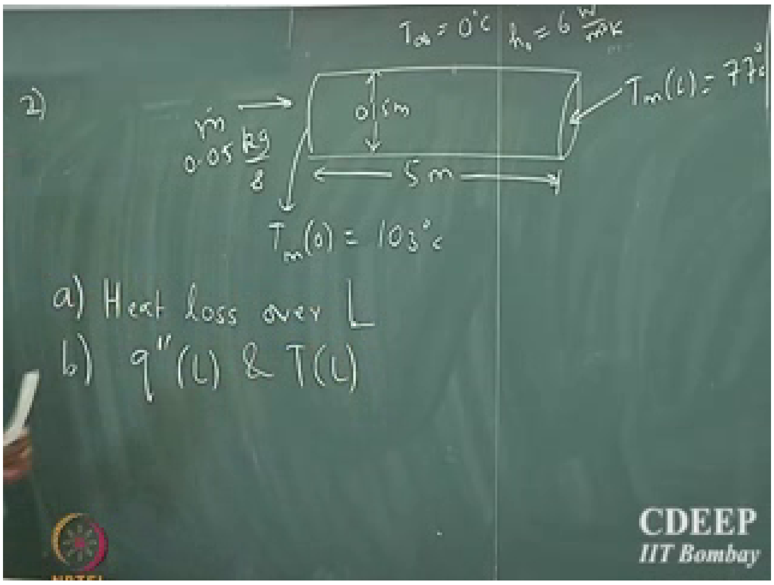

A tube and we have a fluid which is flowing through it 0.05 k gs per second and the diameter is 0.15 meters, and let us say there is a fluid which is flowing past the outer curved surface, and at a zero degrees C and the external heat transport coefficient is 6 watt per meter square of Kelvin. I will then the T n at L is 77 degrees C, and T m 0 is 103 degree C and the length is 5 meters. So, the question is, what is the heat loss over L.

So, the question is, see supposing I look at different x values at which the fully developed regime is going to appear. So, I just want to make sure that for most of the tube is, it is fully developed that is all. So, the entry length is very small. So, the Nusselts number, if you look at the Nusselts number that you got for the correlations that you got when you included entry length is.

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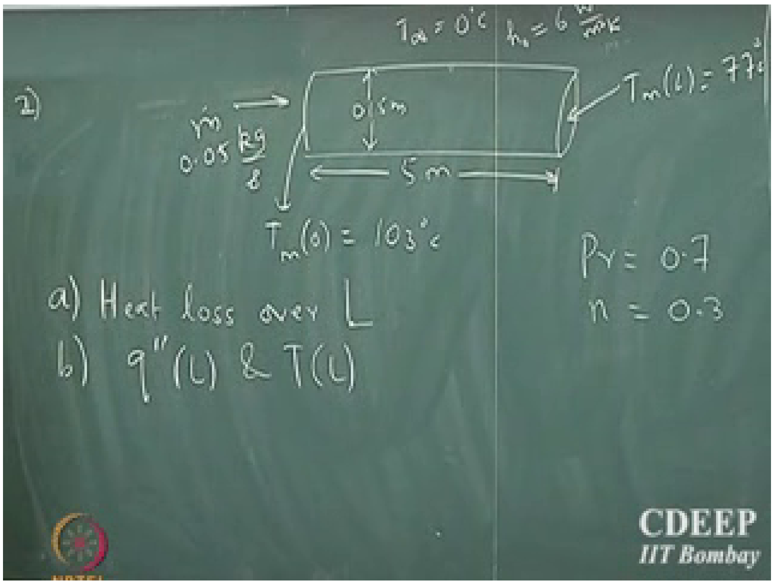

Always the Nusselts number will be, Nusselts number for fully developed plus some contribution.

Student: (Refer Time: 14:44).

But is 7 meters.

Student: (Refer Time: 14:47).

The total length is 7 meters is very small right. So, if you notice the Nusselts number correlation, you will always see that it is Nusselts number fully developed plus some contribution, because of entry length, if this contribution is insignificant. So, you assume that it is almost constant that is all. So, here the question is, what is the heat loss over whole length of the tube, and what is the flux and what is the temperature distribution, temperature at L and some numbers are given here. So, Prandtl number is 0.7 n is 0.3 ok.

So, how do we go for calculating this? So, first one is easy, it is m C p delta T. So, q s m.

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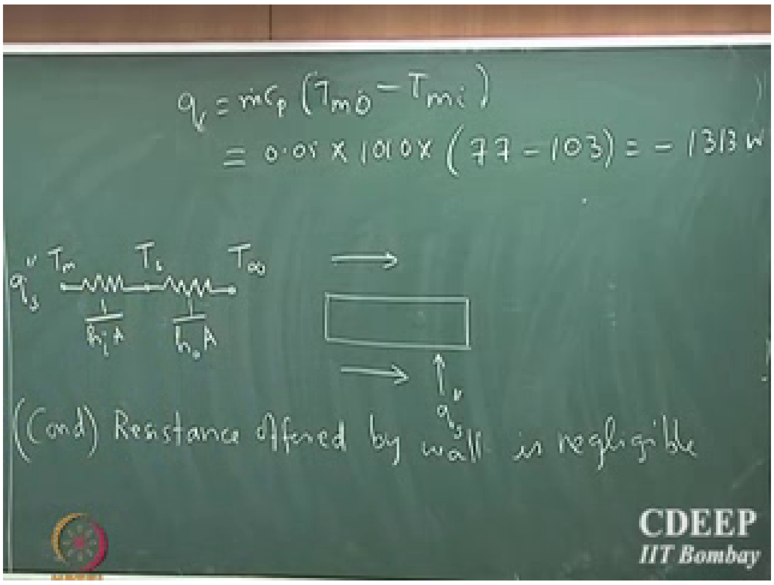

$$q = \dot{m} c_p (T_{mo} - T_{mi})$$
$$= 0.05 \times 1010 \times (77 - 103) = -1313\ W$$

(Cond) Resistance offered by wall is negligible

Dot C p into T m T m i minus T m o minus T m i, and that is equal to the numbers 0.05 into 1010 into 77 minus 103. So, that is minus 1313 watt. Why it is negative, because fluid is now loosing the heat to the surroundings right. So, that is the sign convention that we have used ok.

Then what about how do we find q double prime flux of heat that goes from the fluid inside the tube to the fluid outside the tube, I will give you a hint, so you can look at in a slightly different way. So, remember what we have learnt in your conduction topics. So, if you have a wall here yeah.

Student: (Refer Time: 16:46).

So, the way to see that is suppose you have a square. Now there is one fluid which is flowing here a another fluid which is present on this side. So, now, the flux of heat transport, whatever flux of heat transport is essentially what is transport to the wall and from the wall it is transport to the fluid outside. So, if I assume that the resistance flow offered by the wall is negligible to assume that the resistance offered by wall for conduction. I should also mention conduction resistance. it is not a very bad assumption to make, because if the thickness of the tube will be typically very small and also the conductivity of the solid is significantly higher than the conductivity of the gas flowing, that we have or the fluid stream.

Therefore the conduction resistance that is offered may not be that significant compared to the convection yes.

Student: (Refer Time: 18:08).

Will be the same it just that it just make the calculation easy that is all you can always include conduction. It is not a big deal; it is very easy just adding one sticking, one more extra resistance inside that is all. So, what I can simply do is, now I can construct a resistance network and what is this resistance. This is the temperature inside the tube, and this is the wall temperature and this is the temperature of the fluid outside, and this is 1 by h inside into a is 1 by h outside into a that is the resistance.

So, if we know the heat transport coefficient I am done. So, I know the mixing cup temperature. So, the representative temperature here is the mixing cup temperature. So, if I know the temperatures, if I know the heat transport coefficient I am done. So, now, the question is to calculate the inside heat transport coefficient, outside heat transport coefficient is given. We just have to calculate the inside heat transport. So, that is what we are going to do now. So, will say we are going to calculate the.

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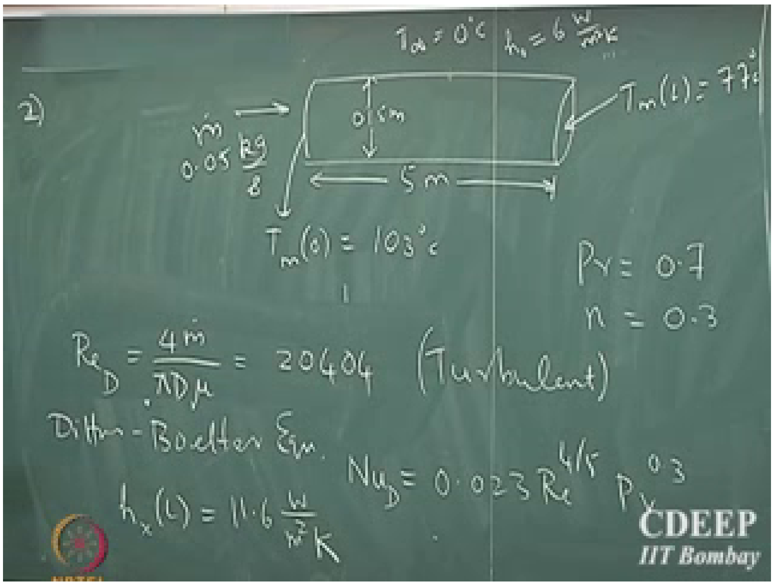

Reynolds number is 4 m dot pi d mu. So, that will be about 20404. So, clearly it is a turbulent flow turbulent condition. So, we have to use the diffuse Boelter equation, reduce the diffuse Boelter equation to find the Nusselt number.

So, n u based on the, it is 0.023 Reynolds number to the power of 4 by 5 Prendtl to the power of 0.3. So, from here we find that will be 11.6 watt per minute square Kelvin. So, that is the heat transport coefficient at L. So, now, using the resistance network

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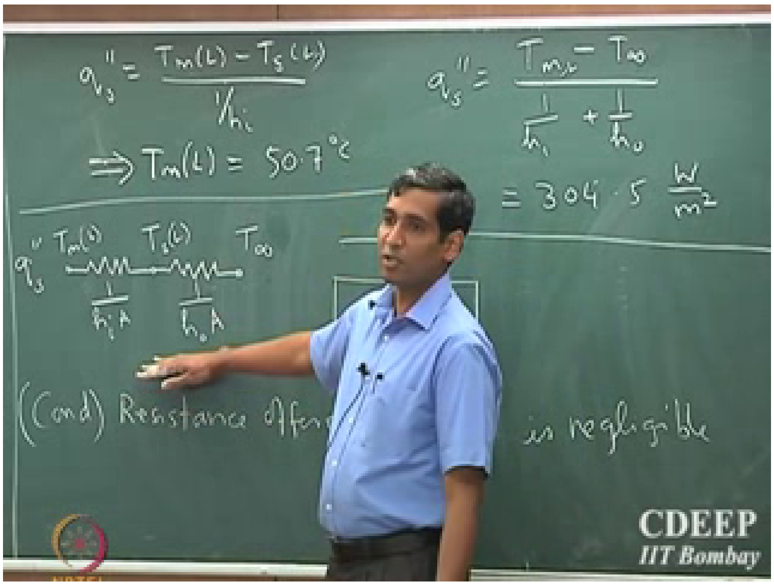

q s double prime is nothing, but the overall temperature difference divided by the resistance right. So, that will be T m minus T infinity at L divided by 1 by h i plus 1 by h naught. Remember I am calculating flux, you have removed the area. So, if I know h and h 1, I am done. I know what is T m L. I know what is T infinity. I can just plug the numbers and this will turn out to be 300 and 4.5 watt per meter square. So, that is the flux. How do I find. The next question is, I need to find the temperature at the boundary.

I need to find T s. I need to find T s at L. How do we do that?

Student: (Refer Time: 21:23).

Yeah. So, same flux. So, q s double prime is also equal to T m L minus T s L divided by 1 by h i. So, from this I can calculate T m of L that is equal to 550.7 degree. So, that is the temperature of the fluid at the location, where the fluid is leaving the tube at L. So, remember that the resist tube concept that we have actually developed and discussed in conduction topic actually comes very handy, anyhow simultaneous conduction convection process used, if you do not have any heat generation in the wall which is not the bad assumption to make.

You can see that this resistance concept, you will start seeing that it is coming in again and again many different scenario. So, that is the very important tool to learn the resistance concept which was also used in radiation. Those are aspect of using resistance is to calculate different properties, is an important and very useful practical tool all right. So, let us look at an another problem.

So, once again we have a tube. Then now instead of having constant flux we have a variable flux, but a known variability. So, we say that it is a sum maxima multiplied by sin pi x by L. So, L is the length of the tube, x is the actual direction and m naught m dot is the rate at which we mass is flowing through the tube.

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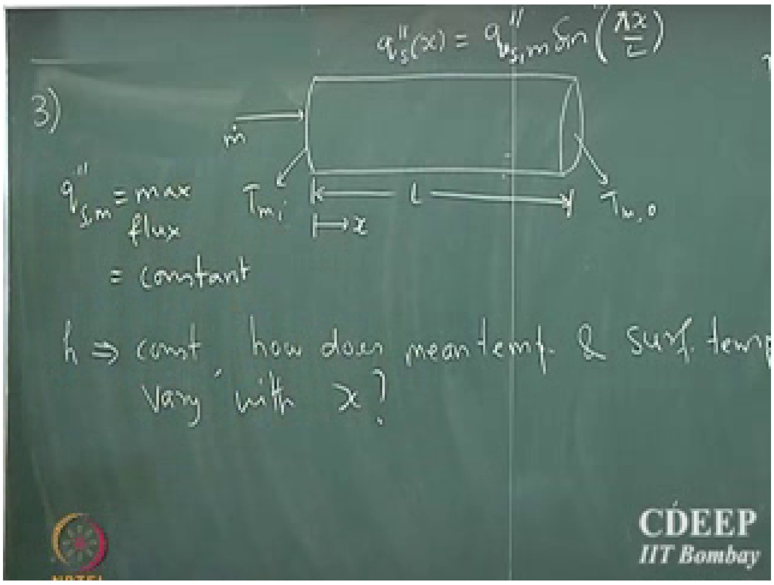

And T m i is the temperature mixing cup temperature of the inlet, and T m o is the mixing cup temperature of the outlet. And if you assume that q s m which is the maximum flux and that remains constant, we assume that the maximum flux, maximum flux is present somewhere in between, and we assume that maxima remains (Refer Time:23:54) and. So, we have a known function for the flux. So, a question is. So, if heat transport coefficient is constant, which means that you are really looking at fully developed regime. So, the heat transport coefficient is constant. How does mean temperature and surface temperature vary with x ok.

We said that d T m by d x; that is nothing, but q s prime into perimeter by m dot C p.

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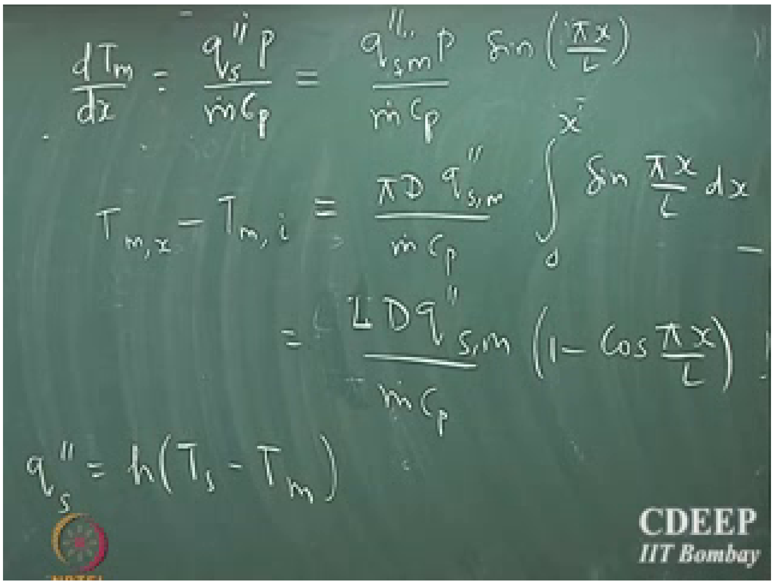

$$\frac{dT_m}{dx} = \frac{q''_s P}{\dot{m} C_p} = \frac{q''_{s,m} P \sin\left(\frac{\pi x}{L}\right)}{\dot{m} C_p}$$

$$T_{m,x} - T_{m,i} = \frac{\pi D q''_{s,m}}{\dot{m} C_p} \int_0^x \sin\frac{\pi x}{L}\, dx$$

$$= \frac{L D q''_{s,m}}{\dot{m} C_p}\left(1 - \cos\frac{\pi x}{L}\right)$$

$$q''_s = h(T_s - T_m)$$

So, that is the balance for the mean temperature or the cup mixing temperature and. So, we know what q s m. We substitute q s m P into sin n pi. Oops fine pi x by l divided by m dot into we integrate this we get it very easy. So, we find that t m at x minus t m at the inlet that is equal to pi d which is a perimeter prime divided by m dot C p integral zero to x sin Pi x by L d x.

So, that gives you the expression for the mean temperature as a function of x location. How do we find the surface temperature?

Student: Q s equal to h into T s.

So, easy. Again we know that q s prime is h into T s minus T m. So, you just plug in the expression for T m x here, will be able to find it. it is very easy. So, really if you start from really basic first principle and try to understand what is been asked and how to go about finding it. Most of these problems will actually fall out if you do it systematically. They all fall out very easily all right. So, I think we will finish the internal flows topic with this example, and (Refer Time: 26:23) after a quick summary will go to the next topic.

So, what we are looked at so far is first we started with the boundary layer analogies.

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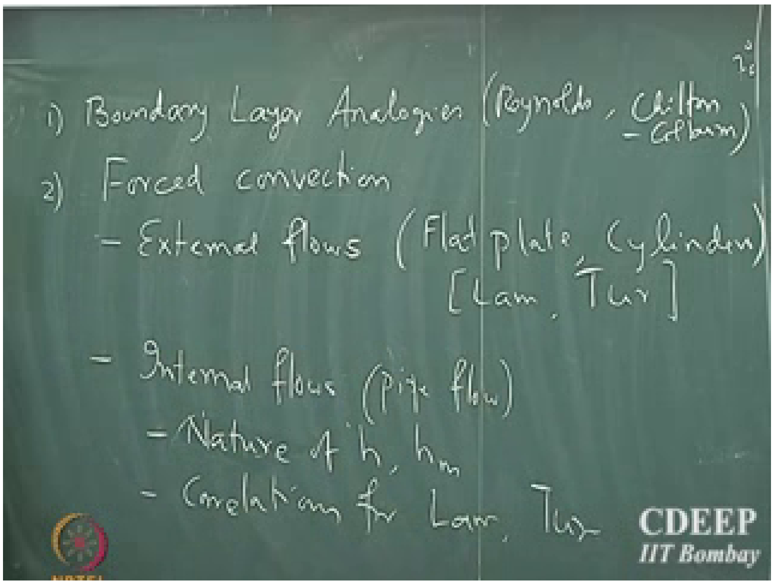

1) Boundary Layer Analogies (Reynolds, Chilton - Colburn)

2) Forced convection
   - External flows (Flat plate, Cylinder) [Lam, Tur]
   - Internal flows (pipe flow)
     - Nature of h, h_m
     - Correlation for Lam, Tur

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We looked at boundary layer analogies and we looked at Reynolds and Chilton Coburn analogy. This is also called as modified Reynolds analogy. With that we moved on to forced convection. We looked at heat transport and mass transport, when there is x fluid which is being pumped by a pump, or it is a pressure driven flow. And in this case we looked at external flows and we looked at flat plate. We looked at cylinders and we also looked at laminar turbulent in both these case. And then we moved on to internal flows really pipe flow, is what we looked at.

We looked at pipe flow and we found the characteristics or I should say nature of heat transport coefficient, nature of h and h m we found that, and then we looked at correlations for laminar and turbulent. So, that is all we have covered. So, next what we are going to move on, is the next topic which is the free convection or natural convection.