Heat Transfer
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Lecture - 34
Flow through pipes VI - Correlations for laminar & turbulent conditions

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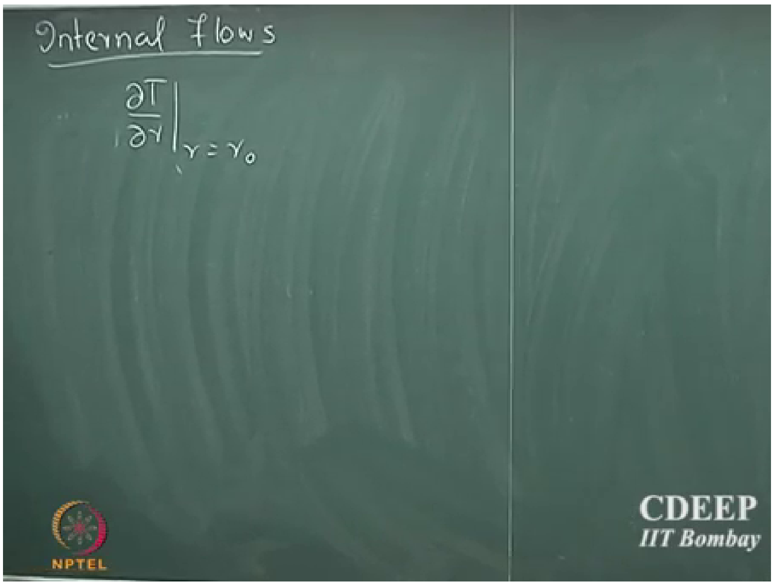

We need to find dT by dR at r equal to r naught right, that is what tells you what is the transport coefficient. So, what we want to do is in order to find this, we need to solve the model equation, all we have done. So, far is we have only looked at what are the properties? What are the insights that we can get without solving the equation? So, now, we are going to solve the actual equation and we are going to find out, what is the actual heat transport coefficient value right.

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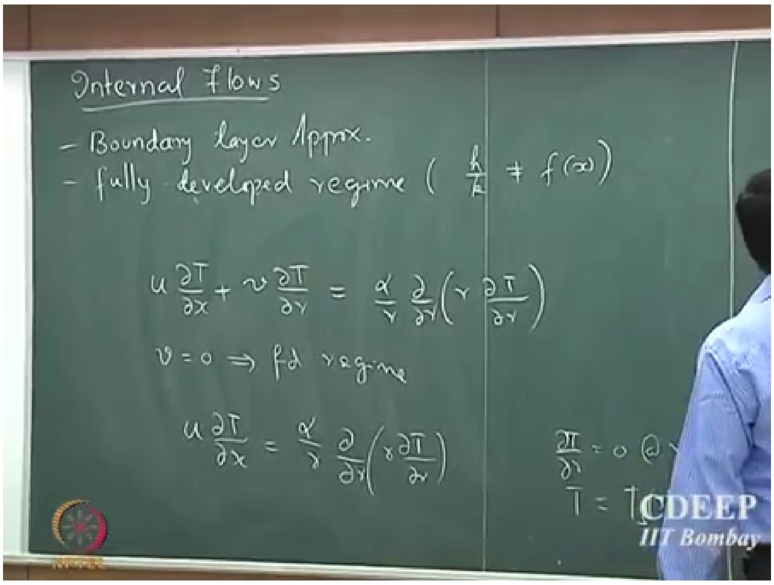

Internal Flows

- Boundary layer Appx.
- fully developed regime ($\frac{h}{k} \neq f(x)$)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} = \frac{\alpha}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)$$

$v = 0 \Rightarrow$ fd regime

$$u \frac{\partial T}{\partial x} = \frac{\alpha}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)$$

$\frac{\partial T}{\partial r} = 0$ @

$T = T$

So, suppose, we write the ; suppose, we assume that the boundary layer approximations are valid, we assume that the boundary layer approximations are valid for the fluid, which is flowing through the tube and we assume that it is in the fully developed regime, what is the nature of heat transport coefficient in the fully developed regime

Student: (Refer Time: 01:26).

It does not vary. So, it is the h by k is not a function of the actual position. So, the model equation for energy balance is u d u by d x plus d dT by dr that is equal to alpha by r ok. So, that is after you introduce the boundary layer approximation, what is v in this? Fully developed regime; v 0, because if flow is only in the x direction. So, v is 0 in the fully developed regime. So, these equation essentially boils down to udT by dx equal to alpha by r d by dr dT by dr.

So, now how do we solve this equation boundary conditions are easy the dT by dr is, is 0 at r equal to 0 and T is, let us say, if it is constant temperature, it is (Refer Time: 02:56) Ts. So, let us say, let us say, it is not constant, it is a general boundary condition T equal to Ts at Ts of x at r equal to r naught ok. So, that is the boundary condition (. How do we solve this? Note that u u is now a function of the radial position, what is u? u is 2 times into 1 minus r by r naught the whole square.

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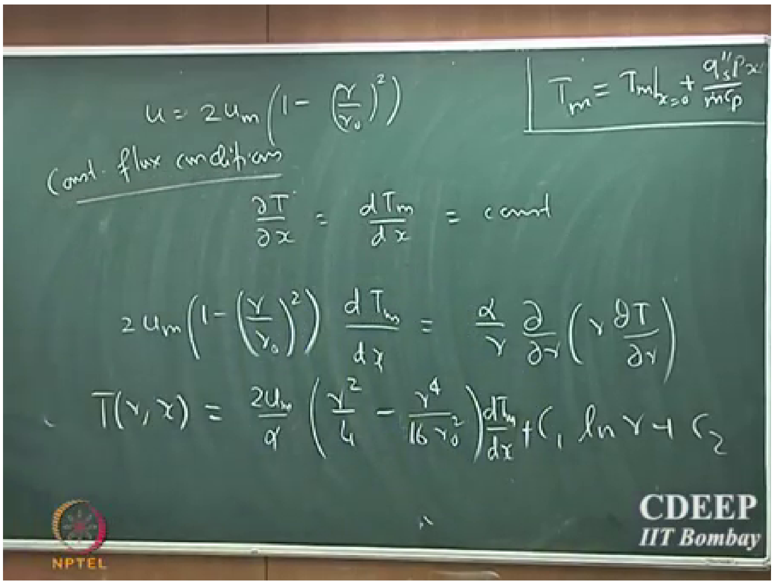

$$T_m = T_m\big|_{x=0} + \frac{q_s'' P x}{\dot{m} c_p}$$

$$u = 2u_m\left(1 - \left(\frac{r}{r_0}\right)^2\right)$$

Const. flux condition

$$\frac{\partial T}{\partial x} = \frac{dT_m}{dx} = \text{const}$$

$$2u_m\left(1 - \left(\frac{r}{r_0}\right)^2\right)\frac{dT_m}{dx} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$T(r,x) = \frac{2u_m}{\alpha}\left(\frac{r^2}{4} - \frac{r^4}{16 r_0^2}\right)\frac{dT_m}{dx} + C_1 \ln r + C_2$$

So, separation of variable, it will not work, simple separation of variable, it will not work. So, how do we solve this ok. So, we have to now tap on some of the properties that, we actually derive without solving the equation. So suppose, I start with a constant flux condition. So, constant flux condition, we know that dT by dx dTm by dx that is the constant ok. Why is this? Because we said that the temperature profile in the fully developed regime is similar at any x location and therefore, dTm by dx and therefore, the local temperature gradient is x direction remains the constant.

So, now, if we impose that condition here, we can rewrite this equation as 2 u m 1 minus r by r naught square into dTm by dx, which is a constant flux equal to alpha by r d by dr dT by dx, simply by using the properties. Now, it is become a solvable equation it is now only an equation in r direction right, we can solve this equation. Now, there will be T of r comma x will be 2 u m by alpha multiplied by r square by 4 minus r r power 4 by 16 r naught square plus C 1 lawn r plus C.

So, that is the solution. So, all I have done is, I have integrated the expressions with respect to r. I have integrated it twice ok. So, you take r on this side that gives you r square by 2 and then you have, you get r r square r square by r to the power of 4 by 64 you will get, then you divide by r again, it will be r by 2 and then you integrate it again, it will be r square by 4. So, every integral in the first term will give you a 1 by 2 and every integral here will give you a 1 by 4

Student: (Refer Time: 06:10).

DT sorry, that is a dT m by dx thanks right.

So, that be here a, a dTm by dx I forgot to copy this here.

Student: (Refer Time: 06:20).

Which one?

DTm is already a cup mixing temperature.

Student: (Refer Time: 06:24).

It is already there, if we not put in, put in the expression for dTm by dx. Now, that is the function of x the lower right. So, Tm is already integrated over r. So, Tm is not a function of radial position anymore.

Student: (Refer Time: 06:37).

Yeah.

Student: (Refer Time: 06:40).

No.

DTm by Tm is a function of x, but dTm by dx is constant and. So, which means Tm is linear yes, right.

So, Tm is linear and this is nothing, but qs prime into P by m dot CP that is the constant

Student: (Refer Time: 06:58).

No, this is a gradient not Tm be very careful. This is dTm by dx, it is not Tm, Tm goes as Tm at x equal to 0 plus qs double prime, T into x by m dot (Refer Time: 07:19) that is the expression for Tm, but dTm by dx is a constant.

Student: (Refer Time: 07:24).

dT by dx.

Student: (Refer Time: 07:29).

Yeah.

Student: (Refer Time: 07:31).

From here dT by dx, which will be,

Student: (Refer Time: 07:37).

Yz.

Student: (Refer Time: 07:38).

Dtm by dx, at what r location

Student: (Refer Time: 07:52).

So, let us first put all these constants, we will see that this constant is now going to play a role, we will see that.

So, now, if I substitute these boundary conditions T of r comma x that should be.

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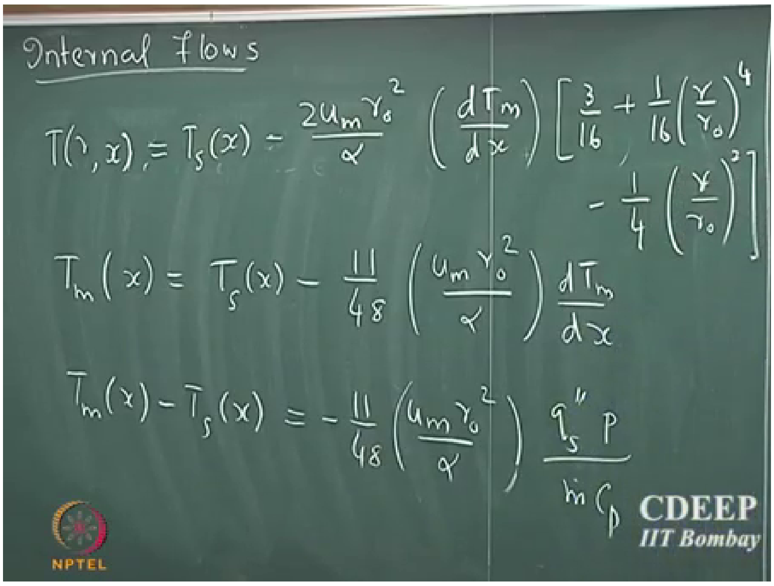

Internal flows

$$T(r,x) = T_s(x) - \frac{2u_m r_0^2}{\alpha}\left(\frac{dT_m}{dx}\right)\left[\frac{3}{16} + \frac{1}{16}\left(\frac{r}{r_0}\right)^4 - \frac{1}{4}\left(\frac{r}{r_0}\right)^2\right]$$

$$T_m(x) = T_s(x) - \frac{11}{48}\left(\frac{u_m r_0^2}{\alpha}\right)\frac{dT_m}{dx}$$

$$T_m(x) - T_s(x) = -\frac{11}{48}\left(\frac{u_m r_0^2}{\alpha}\right)\frac{q_s'' P}{\dot{m} C_p}$$

Equal to Ts at any x location minus 2 u m r naught square by alpha into dTm by dx multiplied by 3 by 16 plus 1 by 16 r by r naught to the power of 4 minus 1 by 4 r by r naught to the power of x ok. So, that is the [ex] expression. Now, if I find out, what is Tm from here ok. So, that is by integrating over the cross section. So, that will be Ts that remains constant minus, it will be 11 by 48 into u m r naught square by alpha into dTm by dx ok.

So, that is the expression for Tm and. So, now, we know that Tm dTm by dx is qs double prime T by m dot dT by substitute that here. So, Tmx minus s that is equal to minus 11 by 48 into, into r naught square by alpha into qs double prime P by m dot CP. All right. So, now, we know qs prime, qs prime is defined as, is defined as the heat transport coefficient h into to Tm Ts minus Tm right.

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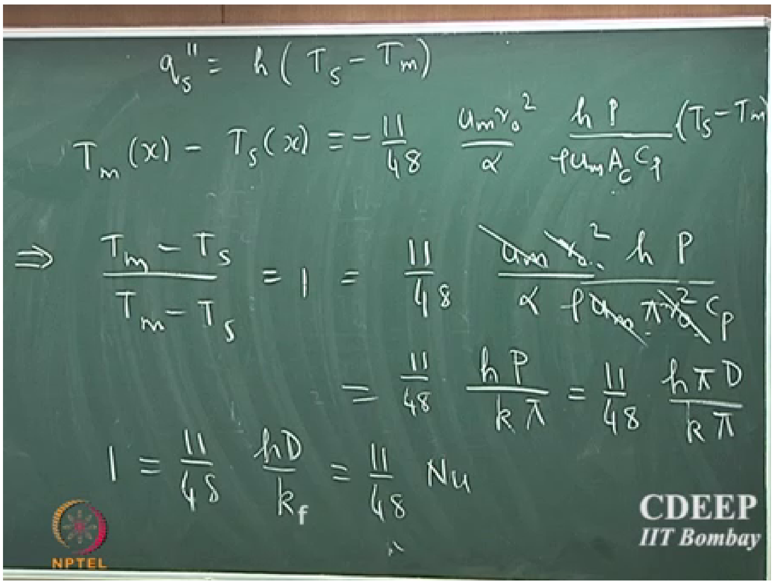

$$q_s'' = h(T_s - T_m)$$

$$T_m(x) - T_s(x) = -\frac{11}{48}\frac{u_m r_0^2}{\alpha}\frac{hP}{\rho u_m A_c c_p}(T_s - T_m)$$

$$\Rightarrow \frac{T_m - T_s}{T_m - T_s} = 1 = \frac{11}{48}\frac{\cancel{u_m r_0^2}\, hP}{\alpha\, \rho \cancel{u_m} \pi \cancel{r_0^2} c_p}$$

$$= \frac{11}{48}\frac{hP}{k\pi} = \frac{11}{48}\frac{h\pi D}{k\pi}$$

$$1 = \frac{11}{48}\frac{hD}{k_f} = \frac{11}{48} Nu$$

So, that is the heat transport coefficient ok. So, we can rewrite this as Ts Tmx minus Ts that is equal to 11 by 48 into r naught square by alpha into h into the perimeter divided by what is m dot it is rho into u m into the cross sectional area a s sorry cross sectional area multiplied by CP into Ts minus Tm. So, now, I can this is minus one. So, that is Tm minus Ts, which is equal to 1 that is equal to 11 by 48 r naught square divided by alpha into h into P divided by rho u m by d square by 4 or phi r naught square into CP.

So, r naught square goes off and then. So, that will become u m goes off ok. So, what is alpha rho into CP, what is alpha into rho into CP? Is nothing by k right, conductivity. So, that will be 11 by 48 into h into P divided by k. What is the perimeter phi into d right into phi sorry. So, perimeter is 11 by 48 into h into phi into d divided by k into phi ok. So, now, from here you will see that 1 equal to 11 by 48 into h D by k what is h D by k? Is Nusselt number, right. So, that will be 11 by 48 into Nusselt number ok.

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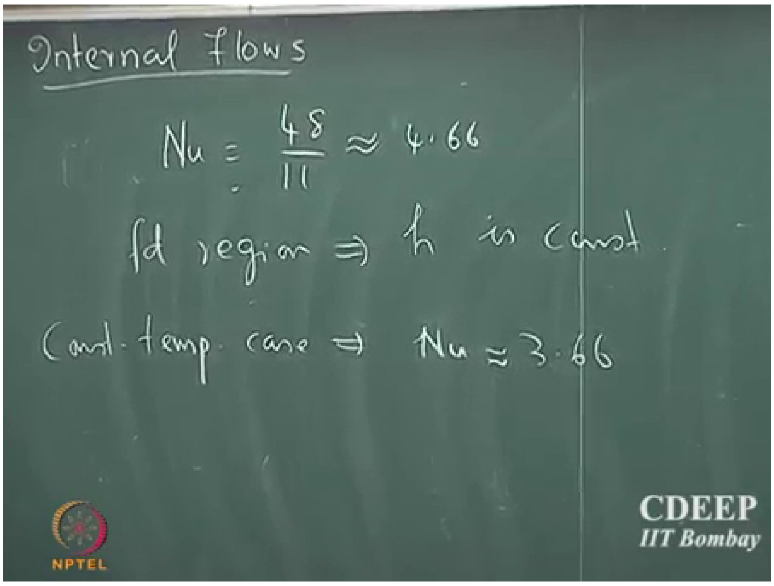

Internal flows

$$Nu = \frac{48}{11} \approx 4.66$$

fd region ⇒ h is const.

Const. temp. case ⇒ $Nu \approx 3.66$

So, from here we find that Nusselt number is 48 by 11. So, Nusselt number is 48 by 11 is approximately 4.66 ok. Why are we getting it constant? Because the heat transport coefficient is constant right. So, in a fully developed regime. So, we find that Nusselt number is also a, a constant in that regime and. So, similarly one should do exercise for the constant temperature case and one would find that Nusselt number will be about 3.66. So, same exercise, you solve the equation, put all the characteristics of the local temperature gradient and you will find that the Nusselt number is about 3.66. So, it is consistent with the properties and the characteristics that we observed without solving the equation.

So, remember I always told you all through this course that, before you solve the equation, you try to get maximum information out of this and you attempt to get insights of this problem only, that is going to help you to verify whether the solution that you have got is right, otherwise you have no way to compare the solution that you got with the actual insight with the problem. So, it is always useful to get the insights first and intuitively predict what is going to be the nature of the behavior and then you go and get the actual solution. So, that way, you have a benchmark to compare this ok, all right. So, now, this is all the story about fully developed regime, but when it comes to like entry region, where the boundary layer is definitely (Refer Time: 14:44) it turns out that the entry region, there is no analytical solution possible.

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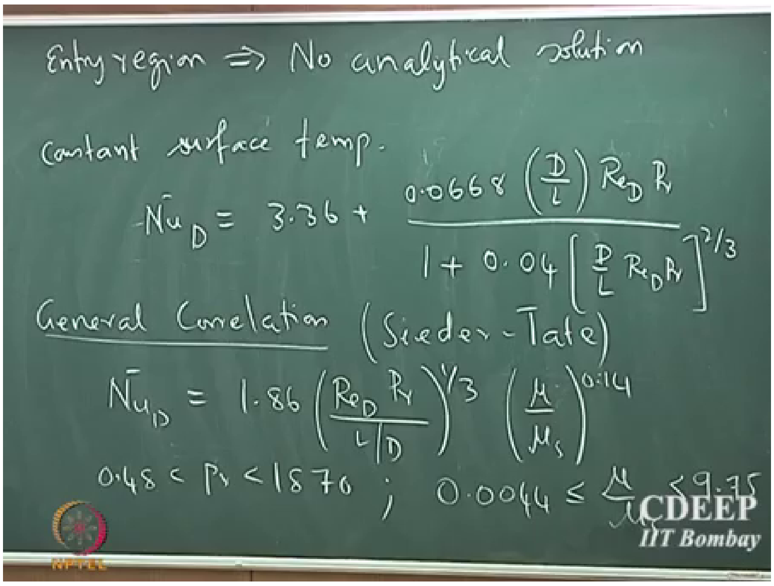

Entry region ⇒ No analytical solution

Constant surface temp.

$$\bar{Nu}_D = 3.36 + \frac{0.0668\left(\frac{D}{L}\right) Re_D Pr}{1 + 0.04\left[\frac{D}{L} Re_D Pr\right]^{2/3}}$$

General Correlation (Sieder-Tate)

$$\bar{Nu}_D = 1.86\left(\frac{Re_D Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$0.48 < Pr < 1870$; $0.0044 \le \frac{\mu}{\mu_s} \le 9.75$

No analytical solution is possible and therefore, we have to rely upon something, some of these correlations and. So, if it is a constant surface temperature, if it is a constant surface temperature. So, the correlations to find out the Nusselt number, again this is average Nusselt number, it is being found, based on experiments, the empirical correlations and. So, that will be 3.36 plus 0.6, it is 0668, Prandtl number divided by 1 plus 0.04 into D by L into R e D Prandtl number to the power of 2 by 3. Now, I do not expect you to remember the correlations for your exams, if it is, if these correlations have to be used, they will be given to you ok, you do not have to memorize any of them, what would be useful is to understand how to use these correlations, rather than trying to memorize these correlations ok. So, if at all these correlations are required, it will be given ok.

Now, there is a general correlation, which is available, called the Sieder Tate. Some of you may have already heard this name Sieder Tate equation. So, Nusselt number is given by 1.86 Reynolds number, Prandtl number 1 by 3 one fourth and. So, here, this correlation is valid when Prandtl number is between 0.48 and 1870 and the ratio of mu to mu s, what is mu? So, viscosity is; obviously, a function of temperature right. So, mu s is the viscosity of the fluid evaluated at the surface temperature and mu is the viscosity evaluated at intermediate temperature right.

So, mu by. So, this is valid when mu s is less than 9.75. So, that is a pretty large validity. So, now, you are going to have a heat transport problem, where the ratio of viscosity is really going to change more than 9.75 that is too much ok. So, that is a pretty good equation and. So, the properties are evaluated. So, the properties are evaluated at mean temperature.

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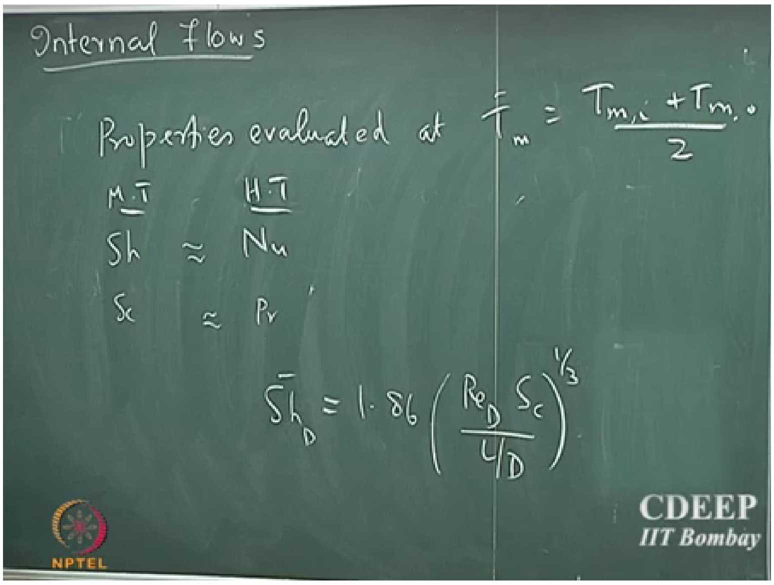

Internal Flows

Properties evaluated at $\bar{T}_m = \frac{T_{m,i} + T_{m,o}}{2}$

M.T H.T

Sh ≈ Nu

Sc ≈ Pr

$$\overline{Sh}_D = 1.86\left(\frac{Re_D\, Sc}{L/D}\right)^{1/3}$$

At mean temperature, which is Tmi o by 2 plus Tmo by 2 plus T mean temperature at the inlet and the outlet ok. Now, one could actually extend this to a mass transport problem ok. How do we extend this by boundary layer analogy right. So, we now, that we have boundary layer analogy. So, Sherwood number is equivalent for Nusselt number in heat transport.

So, we have mass transport and heat transport. So, Sherwood number and Nusselt number are equivalent to each other, Schmidt number and Prandtl number are equal right. So, you replace Nusselt number with Sherwood and Prandtl with Schmidt, you will get the corresponding correlations for mass transport. So, just for completeness, we will write them here, 1.6 Schmidt number divided by L by D 1 by 3, all right. So,. So, the last piece of the puzzle, for intern flows is the heat transport coefficient, during turbulent condition ok.

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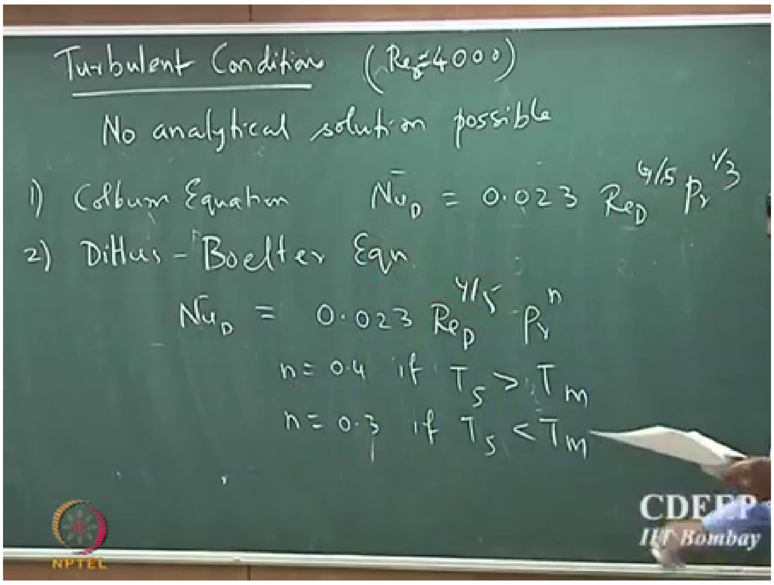

Turbulent Condition ($Re \geq 4000$)

No analytical solution possible

1) Colburn Equation $\overline{Nu}_D = 0.023\, Re_D^{4/5}\, Pr^{1/3}$

2) Dittus - Boelter Eqn

$$Nu_D = 0.023\, Re_D^{4/5}\, Pr^{n}$$

$n = 0.4$ if $T_s > T_m$

$n = 0.3$ if $T_s < T_m$

So, we never talked about Turbulent condition. What is the Reynolds number transition?

Student: 2100 (Refer Time: 20:00).

2100 for laminar to.

Student: (Refer Time: 20:07).

To, to the transition region and typically 4000 of course, it varies a lot depending upon the nature of the fluid and the surface of this few. So, approximately 4000, you would consider that the fluid is actually in a turbulent regime and. So, in those condition also there is no analytical solution available, analytical solution possible. There is no analytical solution possible as of today and. So, the first correlation is called Colbum equation, where Nusselt number is given by 0.023 Prandtl number to the power of 1 by 3 4 by 5.

So, remember that you start seeing this product of Renault to Prandtl, because Reynolds number dependence comes from the flow and the Prandtl number actually comes from the temperature gradient, that the boundary or the interface between the fluid and the solid and a general correlation as called the Ditters Bolter equation. So, this is actually something which is very-very widely used, Ditters Bolter voltage equation is very widely used and. In fact, you must have seen this in your turbulent flow experiment. Those who have already performed and the equation is average. So, that will be 0.023 Reg to the power of 4 by 5 into Prandtl to the power of 10 n

So, the difference between colbum and the Ditters Bolter Equation is that depending upon, whether the heat is being lost from the fluid or whether, the heat is gained by the fluid, the exponent n is slight visible ok. So, n is 0.4, if Ts is greater than T, mixing cup temperature at any cross section. So, which means that there is heat, that is met heat, that is gained by the fluid and it is 0.3, if Ts is less than Tm ok.

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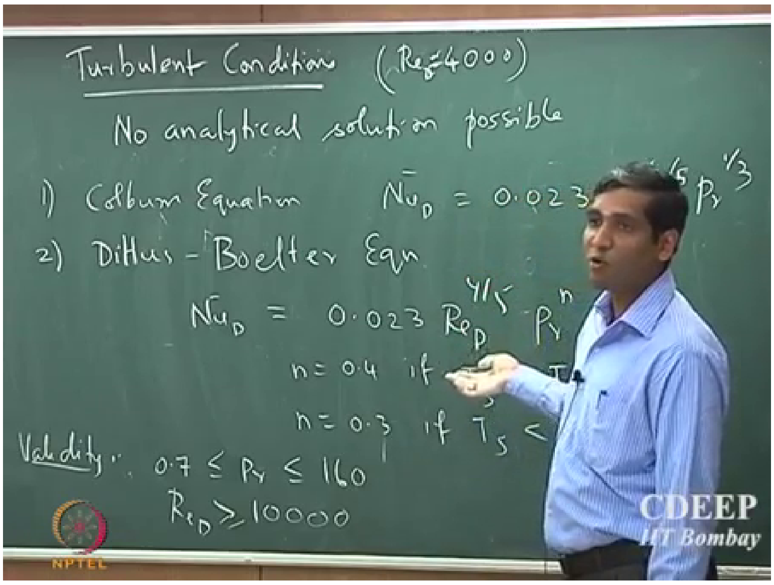

And of Course there is some range, validity range. So, the validity is, it is 0.7 less than or equal to Prandtl number 160 and 10000 ok. Those of you who have already performed the turbulent flow experiments, you been told that the Reynolds number should be about 10000, the reason why that is the case is the validity of the equation, the correlation that is used is only when the Reynolds number is sufficiently large. So, if you actually have some of, you have actually done this experiment and have not calculated the Reynolds number, you should have, if you have not calculated and checked, whether the Renault number is greater than 10000, you will see that there will be a variation in your prediction, the straight line fit that you are trying to fit your experimental data, it will not fit your experimental data very well and thus your Reynolds number is sufficiently large, where the correlation is valid ok. So, that is very important.

So, these validity regions, they actually play a very important role and just like in any statistics, you cannot apply a fit to the range, which is outside the validity of the fit right. So, if it is a, if it is a linear regression fit, you cannot apply the fit to any range, which is outside the validity of that fit.

So, therefore, pay attention to these validity regimes depending upon the Reynolds number and depending upon the properties, you may have to different correlations, once again of course, I can translate the same thing to mass transport problem by replacing the Nusselt number with Sherwood and Prandtl number to Schmidt number.