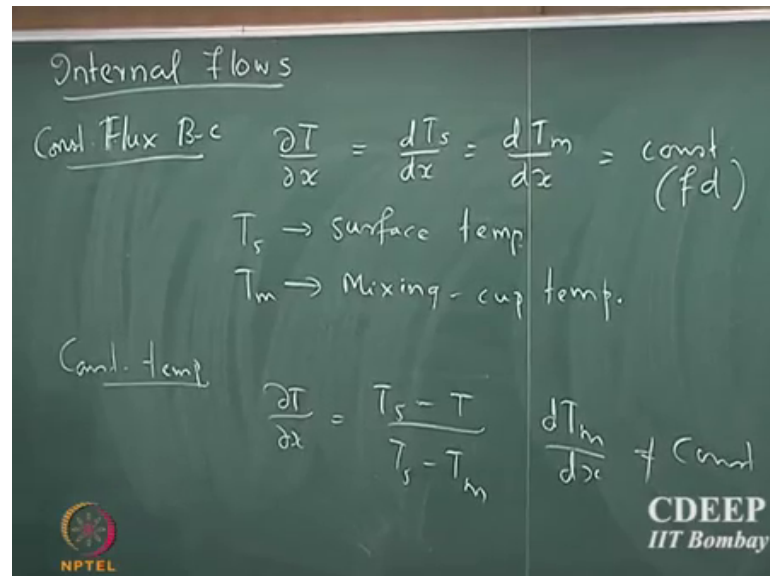


**Heat Transfer**  
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**Lecture – 33**  
**Flow through pipes V: Log mean temperature difference**

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By observing some of the interesting features of heat transport, when there is a fluid which is flowing through a pipe. So, we are going to develop further on that. So, just to recap we said if it is a constant flux boundary condition. So, note that without solving equations in the fully developed region, we said that  $dT$  by  $dx$  that is equal to  $dT_s$  by  $dx$  that is equal to  $dT_m$  by  $dx$  and that is equal to constant ok.

So, not just that for  $T_s$  is the surface temperature, and again this is in the fully developed region and  $T_m$  is the average or the mixing cup temperature. So, similarly we offer for that for constant temperature boundary condition. So, you said that  $dT$  by  $dx$  is  $T_s$  minus  $T$  by  $T_s$  minus  $T_m$  into  $dT_m$  by  $dx$  so; obviously, this is not a constant, you can clearly see that it is not a constant ok.

So, we are going to develop further from here, and see what we can do from these useful insights, and also today we are going to actually solve the full modal equation, and then look at how these kinds of insights actually are going to help to solving the full modal equation all right.

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$$\frac{dT_m}{dx} = \frac{q_s'' P}{m c_p} \Rightarrow \text{valid from } x=0 \text{ to } x=L \quad q_s'' \neq \text{const}$$
$$q_s'' = h(T_s - T_m)$$
$$\frac{dT_m}{dx} = \frac{h P}{m c_p} (T_s - T_m)$$
$$\Delta T = T_s - T_m$$
$$-\frac{d\Delta T}{dx} = \frac{h P}{m c_p} \Delta T$$

So, let us look at the constant temperature case. So, we said that  $dT_m$  by  $dx$ . So, we wrote an overall energy balance for the mixing cup temperature, and we found that that should be equal to  $q_s$  prime  $p$  by  $m$  dot  $c_p$ . So, note that the minus sign will depend upon the direction of the heat transport from the fluid to the surrounding cause surrounding to the fluid.

So, now if it is a constant temperature case; obviously, the flux is not constant,  $q_s$  double prime is not constant, in the last lecture we showed what is the mixing cup temperature profile when the flux is constant or the constant flux case which. So, this equation is the an equation for every location in the pipe. So, this is a general balance which we wrote from  $x$  equal to 0 to  $x$  equal to  $L$  ok.

So, this is valid for  $x$  equal to  $L$ . So, now, so, the flux is not constant; however, from Newton's law of cooling, we know that flux is  $h$  into  $T_s$  minus  $T_m$  so, that by definition right. So, now, we can use this expression here equal to  $hP$  by  $m$  dot  $C_p$   $T_s$  minus  $T_m$ . So, now, if we integrate this expression, we should be able to find the profile of the cup mixing temperature right. So, we will have to do something slightly different. So, now, I am going to define a variable called  $\Delta T$ , a  $\Delta T$  is the temperature difference. So,  $\Delta T$  is the  $s$  minus  $T_m$ .

And you know I can replace the whole expression in terms of  $d\Delta T$  in terms of  $\Delta T$  this is specific reason why I call it  $\Delta T$ . In fact, you will see in a short while what is

the rationale for that and how you can connect with some of the experiments that you are actually performed in the labs. Now I can integrate this expression here 1 that is equal to minus  $h P$  by  $m \dot{C}_p$ .

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Internal flows

$$\ln\left(\frac{\Delta T_o}{\Delta T_i}\right) = -\frac{P}{m\dot{C}_p} \int_0^L h \, dx$$

$$\Delta T_i \Rightarrow \Delta T(x=0) = T_s - T_m(x=0)$$

$$\Delta T_o \Rightarrow \Delta T(x=L) = T_s - T_m(x=L)$$

$$\ln\left(\frac{\Delta T_o}{\Delta T_i}\right) = -\frac{PL \bar{h}_c}{m\dot{C}_p}$$

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So,  $\Delta T_i$  is nothing, but  $\Delta T$  at  $x$  equal to 0 and  $\Delta T$  at  $x$  equal to  $L$ . So, this is the  $\Delta T_i$  is nothing, but  $T_s$  minus  $T_m$  at  $x$  equal to 0 and that will be  $T_s$  at  $x$  equal to  $L$  minus  $T_m$  at  $x$  equal to  $L$  ok.

But we said it is a constant temperature condition. So, this is nothing, but  $T_s$  minus  $T_m$ .  $T_s$  remains constant among the surface. So, its  $T_s$  minus  $T_m$  at  $x$  equal to 0 and  $x$  equal to  $L$ . So, this is nothing, but the temperature difference of the average fluid temperature and the surface temperature and this is the difference at the exit of the tube. So, these two are actually measurable quantities. I can now find out what is the average temperature at the inlet, I can find out what is the average temperature at the outlet of the tube and these two are measurable quantities and. So,  $\Delta T_i$  stands for yeah  $i$  stands for input and  $o$  stands for output. So, these two are measurable quantity if experimentally measurable quantity.

So, therefore,  $\ln(\Delta T_o / \Delta T_i)$  that is equal to what is indicate  $0$  to  $L$   $h \, dx$  yeah.

Student: (Refer Time: 07:11)

Yeah where is.

Student: (Refer Time: 07:12)

Where is what  $\bar{h}$  into  $L$  right. So, this is  $\bar{h}$  into  $L$  into  $\bar{h}$  defined based on the length of the circulate you, that is the average heat transport coefficient remember that we are really not very interested in the local heat transport coefficient what is more interesting and important is the average heat transport coefficient right and that is an important design for (Refer Time: 07:39)  $\dot{m} C_p$ . Now if I look at the total amount of heat that is lost.

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$$\begin{aligned} q_{conv} &= \dot{m} C_p (T_m|_{x=0} - T_m|_{x=L}) \\ \ln\left(\frac{\Delta T_o}{\Delta T_i}\right) &= \frac{-PL \bar{h} L}{q_{conv}} (T_m|_{x=0} - T_m|_{x=L}) \\ &= \frac{-PL \bar{h} L}{q_{conv}} ((T_s - T_m|_{x=L}) - (T_s - T_m|_{x=0})) \\ &= \frac{-PL \bar{h} L}{q_{conv}} (\Delta T_o - \Delta T_i) \end{aligned}$$

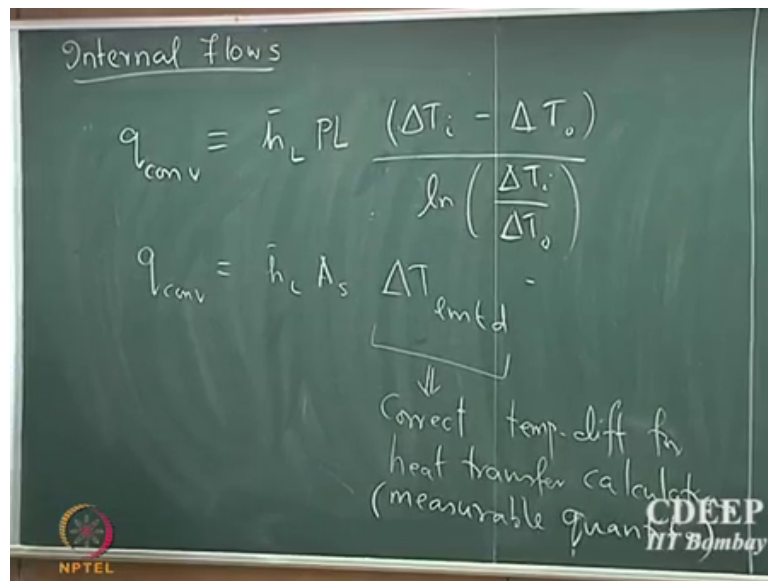
By the fluid then the net amount of heat that is actually lost by the fluid you do convection. So, that is given by  $\dot{m} C_p$  into the mixing cup temperature at the outlet minus the mixing cup temperature at the inlet.

So, that is the net amount of heat that is lost by the fluid by simply looking at the difference in the capacity of fluid at the inlet and at the outlet. So, how I can rewrite I can use this expression and substitute for  $\dot{m} C_p$ , will have one  $\Delta T$  naught by  $\Delta T_i$  that is equal to  $t$  into minus  $PL \bar{h} L$  divided by  $q_{conv}$  into  $T_m|_{x=0}$  minus. But we know that  $\Delta T$  naught and  $\Delta T_i$  are the difference in the surface temperature and the corresponding cup mixing temperature right. So, I can rewrite this as

minus  $PL \bar{h}$  [laufter],  $q$  convection I can add and subtract a surface temperature term here. So, I will get  $T_s$  minus  $T_m$  equal to  $L$  minus  $T_s$  minus  $T_m$  equal to 0.

So, all I have done this I have just added and subtracted  $T_s$  and so, this is nothing, but this is  $\Delta T_o$  right is the output. So, this is the minus  $PL \bar{h} L$  by  $m \dot{C}_p$ ; so,  $q$  convection into  $\Delta T_o$  minus  $\Delta T_i$ . So, this is the temperature difference at the outlet this is the temperature difference at the inlet.

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So, from here I can rewrite estimate what is the net amount of heat that is transferred. So, that is given by  $\bar{h} L P$  into  $L \Delta T_i$  minus  $\Delta T_o$  divided by  $\ln \Delta T_i$  by  $\Delta T_o$ .

What is this expression? This is the log mean in temperature difference. So, so  $q$  convection what is  $P$  into  $L$ ? It is the curved surface area right. So, its  $\bar{h} L$  into the curved surface area into  $\Delta T_{lmtd}$ . So, it is not just a magic that you use log mean temperature difference in your calculations, which actually rigorous analysis that shows that that is the correct temperature difference that if it actually used for your heat transport calculation.

So, log mean temperature difference is what most of you who have done this  $\lambda$  then turbulent flow experiments, we have already encountered this term called  $\Delta T_{lmtd}$  now. In fact, that comes from a rigorous analysis of the cube  $x$  (Refer Time: 11:32)

cube heat transfer from a fluid which is flowing towards you and. In fact, one could shows similar exercise for the fluid which is going outside the tube as well ok.

So,  $\Delta T_{\text{lmtd}}$  is actually the correct temperature difference for heat transfer calculations. So, what we learn from here is that the log mean temperature difference is the correct temperature difference that you have to use for heat transfer calculations not always, when you are using the measurable properties. Reason why this is the correct measure is that you cannot measure the local temperature profile. So, only a measurable properties you know is the input and the outlet inlet and the outlet temperatures. So, if you know the inlet and outlet temperatures then  $\Delta T_{\text{log mean temperature}}$  is the correct temperature difference that we have to use for finding the total amount of heat that is transferred.

Student: (Refer Time: 13:04).

Yeah.

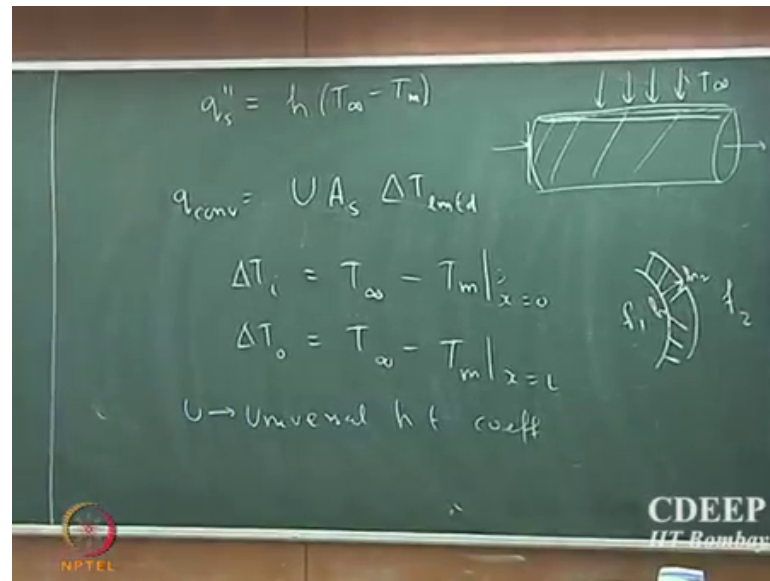
Student: (Refer Time13:06).

This expression is valid for both. So, I will show you in a short while. Actually when you have let us say if you have a fluid which is flowing outside the tube which is at a certain temperature. So, the, this can actually be extended to varying temperatures also we have.

Used a constant temperature, but you can also use the same expression same  $\Delta T_{\text{log mean temperature}}$  for the varying expression which are varying temperature which we will see when we are actually going to do the heat transfer equipment design there we will see that this is a general expression which is valid for whether you have constant or a varying temperature.

So, we will see that when we actually look at the heat transport equipment design. So, suppose we say that there is a fluid which is flowing ok.

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So, this is there is the fluid which is flowing through the tube, and there is another fluid which is flowing around this around the curved surface and let us say that this at a temperature  $T_\infty$ . So, now one could define the local flux of heat transport can be defined as  $h$  into  $T_\infty$  minus  $T_m$ . So, what we know is only the temperature of the fluid which is flowing fastest. So, this tube and so, we know the net flux of heat transport at any location as heat transport coefficient multiplied by the temperature difference of the fluid which is outside and the mixing cup temperature at any cross section. So, that is the definition by the way.

So, now, if I incorporate that definition here, I will have  $q$  convection will be equal to some heat transport coefficient, I write this as  $U$  and I will explain in a short while why I used  $U$  here  $U$  into  $A_s$  into  $\Delta T_{lmt}$  where  $\Delta T_i$  is defined as minus. So, the  $U$  is what is called as a universal heat transport coefficient. So,  $U$  is the universal heat transport coefficient and the reason why it is universal is, the wall of the tube is a certain definite thickness right. So, there is going to be a resistance to conduction in the wall of the circular tube and there is also be resistance for heat transport between the external surface and the fluid which is actually flowing fastest ok.

So, universal heat transport coefficient essentially it accounts for what is the heat transport coefficient for transport from the fluid to the wall. So, if I zoom this location. So, if this is the wall of the tube. So, there is fluid one which is flowing here and fluid

two which is flowing here. So, you could have a heat transport coefficient at the internal location for heat transport from fluid one to the wall of the circular tube, and then there will be heat transport from the inner wall to the outer wall of the tube and there will be heat transport coefficient for transport of heat from energy from the wall to the outside fluid. So, this  $U$  universal heat transport coefficient it essentially accounts for all three quantities will see a lot more of these when we are actually looking at the design of heat exchanges.

So, will see what is the definition of universal heat transport coefficient, we have seen very briefly when the discuss conduction, but we will see in a lot more detail as to what is this universal heat transport coefficient, what are the different factors that are accounted in that universal heat transport coefficient. So, this is what this  $U$  is what most of you would have used if you did your laminar flow and turbulent flow experiments, its universal heat transport coefficient concept is what you would have used.

To estimate the heat transport coefficient for that experiment we will see a lot more details about, what are the different factors accounted for, what are the different quantity and; what are the different pieces of the universal heat transport coefficient that you can neglect under different conditions so that we will see we reserve to the heat transport equipment design all right. So, all the thing that we have studied about heat transport through internal flows is so far early I upon the characteristics and properties of the heat transport coefficient we never found what is the actual number right. So, that is what we are interested in.