Heat Transfer
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Lecture – 32
Flow through pipes IV: Mixing-cup temperature

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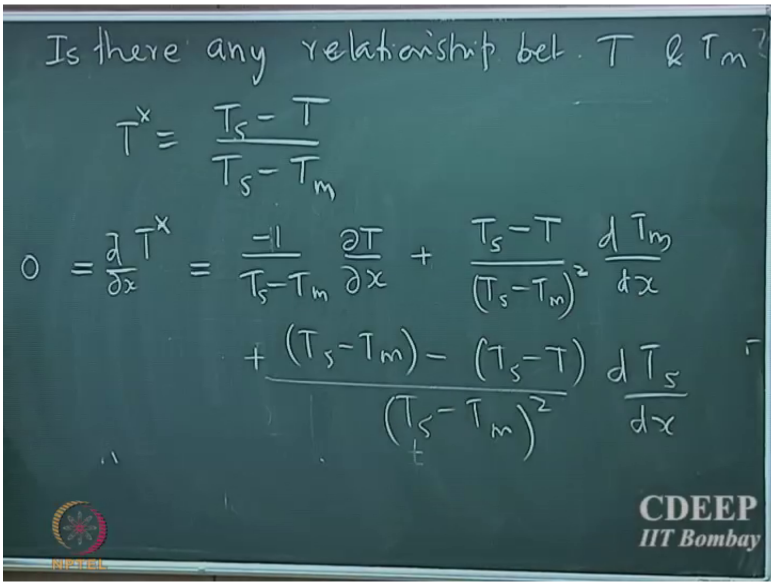

Suppose I take constant flux condition constant flux conditions.

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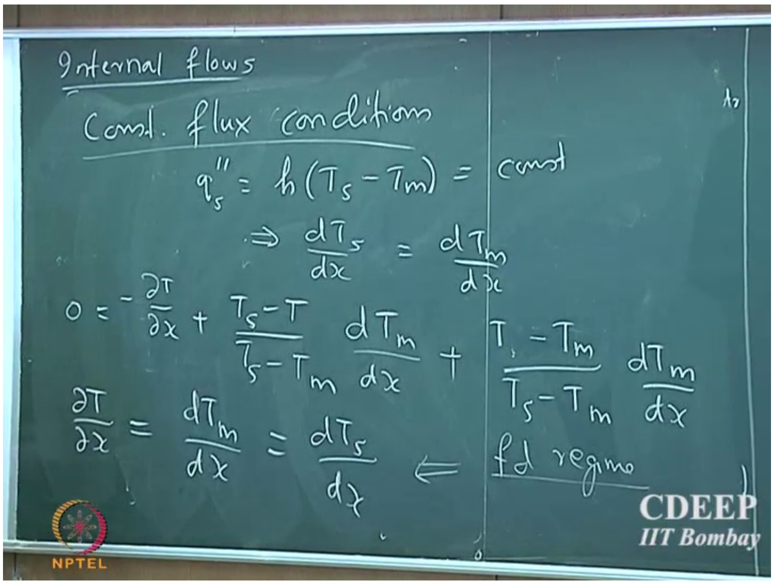

So, the local flux qs is h into Ts minus Tm and that is constant right. So, which means that dTs by dx should be equal to dTm by dx right, but it is constant all I have done is I have taken the first derivative with respect to x position. So, the gradients of these two have to be equal.

Now, if I plugging that condition where we will see that 0 equal to minus dT by dx plus Ts minus T divided by Ts minus Tm into dTm by dx plus. So, that will be. So, Tm minus T divided by T minus m divided by Ts minus Tm into dTs by ds. So, for I can replace dTs by dx with the gradient of the average temperature so that will be dTm by dx, then if I joint these two together you will find that dT by dx that is equal to dx and that is also equal to dTs by dx n. So, all I have done is. So, I have just club these 2. So, the local temperature term will cancel out. So, what you will have is Ts minus Tm and. So, you divide by Ts minus Tn that goes away.

So, what you have is that the local temperature gradient in the x position should be equal to the local temperature gradient of the mixing cup average, and that should also be equal to the temperature gradient of this surface. So, that is an important inside. So, we have got two important things one is that the heat transport coefficient remains constant in the fully developed region, and not just that the gradients are actually equal. Again this is in the fully developed region. So, the gradients are equal. So, this is an important in (Refer Time: 02:42) of insight from experimental point of view.

So, if I know how to measure the local temperature of the surface, I am done I can find the temperature gradient of the fluid inside the tube. So, typically the entry region is very small. So, by and large is going to be a fully developed region in the tube and. So, I should be able to estimate the temperature gradient of the fluid inside the tube, just by looking at the surface temperature. So, (Refer Time: 03:10) excellent method to measure experimentally. So, such kind of an insight is very difficult to get from experiment guys you looking at simple analysis of the nature of the solution, we are able to estimate an important property of the system that the temperature local temperature gradient in the x direction is same as the temperature gradient of the surface alright.

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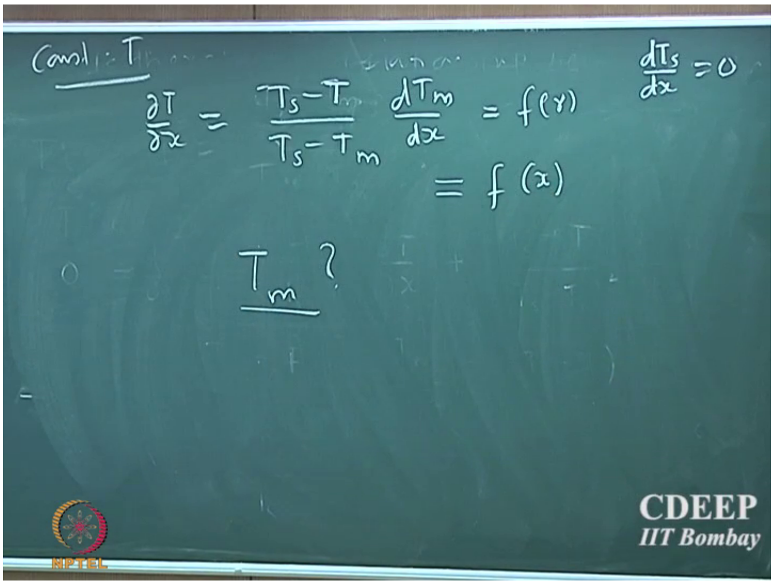

One could do the same exercise for constant temperature. So, you will see that dT by dx that will be Ts minus Tm or Ts minus T divided by Ts minus Tm and dTm by dx. So, this is for the constant temperature case, where dTs by dx is 0. So, if it is constant temperature then the temperature gradient in the wall is 0 and. So, we will see that dT by dx will Ts minus T by Ts minus Tm into dTm by dx. This is the function of radial position of course, yes this is the function of axial position yes or no is it or no; it is not this is dimensionless temperature.

Student: (Refer Time: 04:52).

DTm by?

Student: (Refer Time: 04:56).

Yeah.

Student: (Refer Time: 04:57).

So.

Student: (Refer Time: 04:58).

Good. So, this actually can be a function of axial position because although you have a dimensionless temperature here, the gradient dTm by dx we do not know anything about dTm by dx. So, dTm by dx can in principle be function of x. So, now, the real exercise once we know this the real exercise is to find out what is the cup mixing temperature. So, can we find the cup mixing temperature? So, if we find the cup mixing temperature profile with respect to x direction and we have done; we get a lot more piece of information without solving all the gory equations ok.

So, what we are going to do next exercise is to find out what is the method to estimate the cup exchange temperature. So, what we are going to write is we going to write a balance energy balance or estimating cup mixing temperature Tm.

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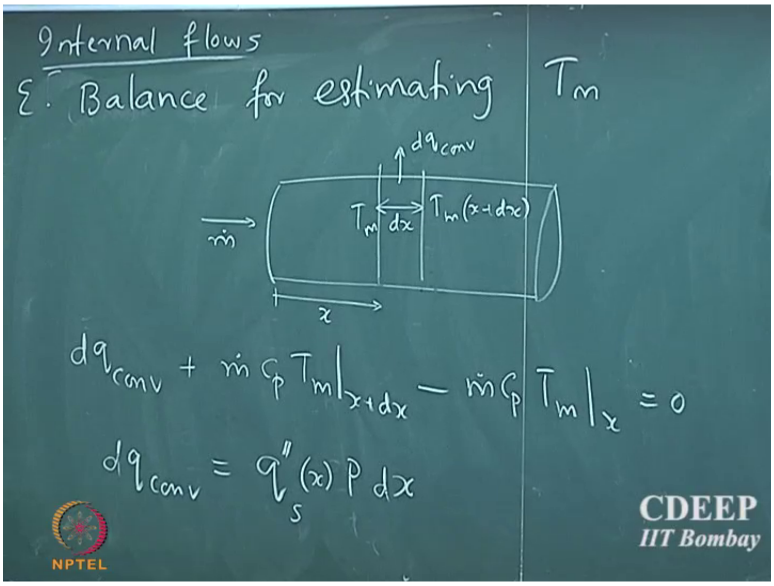

So, this is very similar to the lumping I talked about during conduction fine. So, if you have a full temperature profile, we can actually integrate in the radial direction and you would actually get a same energy balance and it is a good exercise to tried out ok.

So, suppose I take a tube, then in dx and if the mixing cup temperature at that location is Tm, and this is Tm x plus dx this is x, and the mass flowrate is m dot, and if the amount of peak that is last is dq convection, that is the (Refer Time: 06:56) that lasts from the surface. So, what is the heat balance? Whatever heat that is coming into this shell at this location, should be equal to what leaves plus whatever is leaving from the other end of the shell right. So, that should be dq convection plus what is the amount of energy that is actually leaving this place is location yeah m dot.

Student: (Refer Time: 07:30).

So m dot Cp into Tm right at x plus dx minus m dot Cp Tm fx equal to 0 right. So, that is the total amount of heat that is actually entering this and leaving this place. So, from here it is a sine right.

Student: (Refer Time: 08:08).

It is ok.

Student: (Refer Time: 08:18).

What about the.

Student: (Refer Time: 08:19).

What about the.

Student: (Refer Time: 08:20).

Yeah.

Student: (Refer Time: 08:22).

There will be conduction there is a overall balance right the conduction is accounted here.

Student: (Refer Time: 08:28).

This is no.

Student: (Refer Time: 08:31).

Be very careful this is the mixing cup temperature right. Now when you want to write conduction we have to write the local temperature. So, here we have lumped everything.

Student: I think conduction (Refer Time: 08:44).

it has it takes place in the z direction.

Student: (Refer Time: 08:48).

Right, but whatever heat comes in leaves the everything is accounted in the total amount of accumulation, this is the amount of heat that is carried net amount of heat carried by the fluid. Now this can be split into two parts what is the diffusive yeah what is the conduction term and what is the convection term. It could be split into two parts, but this is just a generic balance. So, from here; so what is dq convection? So, if the flux of heat transport is q double dash we do not know whether it is constant flux or not let us say in general flux is qs, it could be a position of x multiplied by the perimeter right into dx right. So, that is the convection and.

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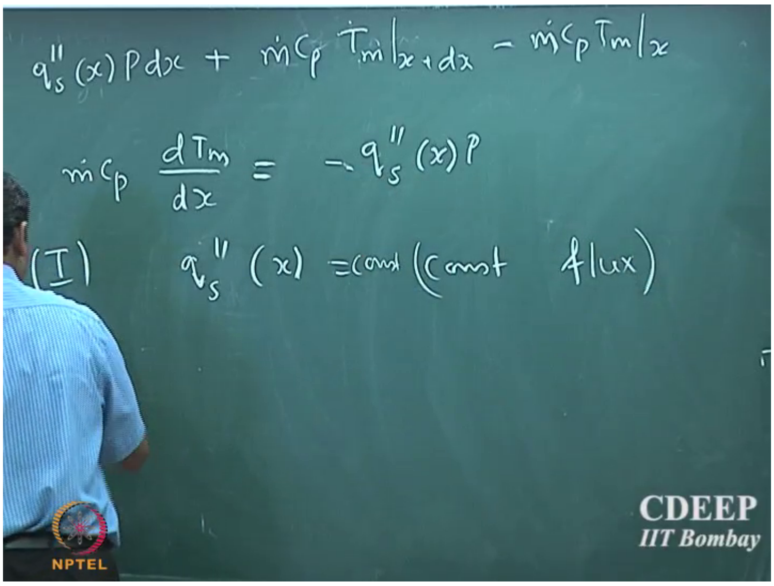

So, therefore, you plug this in we will see that Qs double prime we are right. So, I can rewrite this as m dot Cp into dTm by dx that should be equal to minus qs into p. So, how do I define qs what is qs? If it is a constant flux then this is constant for first case, it is a constant flux then this is constant, if it is constant temperature qs is given by some heat transport coefficient multiplied by Ts minus Tm right.

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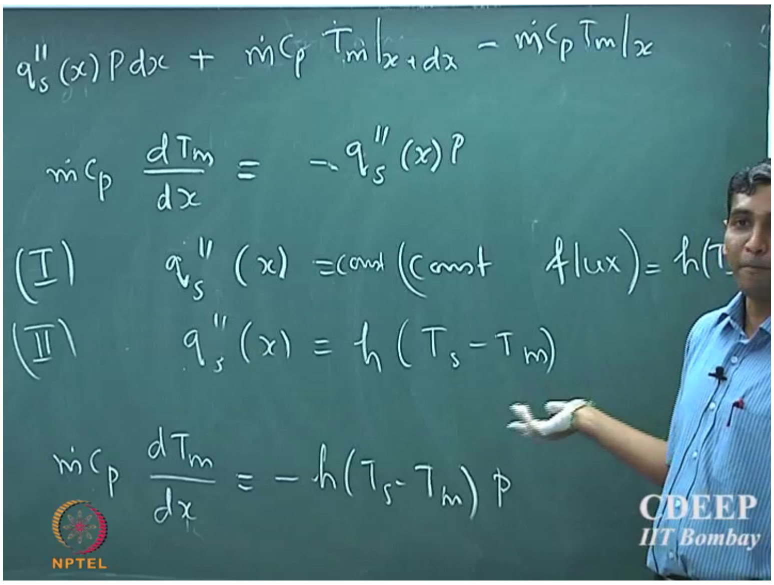

So, I can plug that in. So, that will be m dot Cp into dT m by dx, that is equal to h want to Ts minus Tm into p. Yeah you need a temperature gradient right flux what is the newtons law of cooling flux will be equal to heat transport coefficient multiplied by the temperature of the surface minus whatever is the reference temperature.

So, a reference temperature for our problem is the cup mixing temperature right. So, that is where reference temperature.

Student: (Refer Time: 12:10).

Yeah, it is cross sectional average Tm is the cross sectional average.

Student: (Refer Time: 12:19).

Correct. So, that is a weighted cross sectional average, it still a cross sectional average. We have to take the velocity into account, because the u can be different at different location right. So, you have to take if you want.

Student: (Refer Time: 12:34).

Yeah that at a given cross section you have a velocity profile, it is not a flat velocity profile you have a parabolic velocity profile. So, therefore, you have to take into account the effect of the velocity on the amount of energy that is carried, and that is why Tm will account for the local velocity, and also the local temperature in the fluid at any cross section. It is a cross sectional average weighted with the velocity term weighted with the profile of the velocity.

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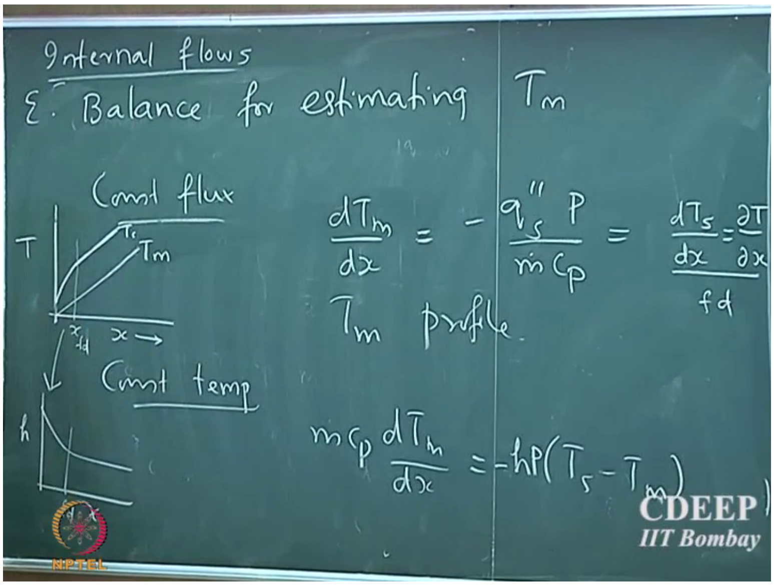

So,. So, suppose I find for the constant flux case. So, dTm by dx will be equal to minus qs prime p by m dot Cp. If I know this and if it is a constant flux the flux is a measurable quantity. So, I know what qs is, I know what p is, I know what m dot and Cp is. So, I should be able to find the temperature profile right. So, I can find out what is the; obviously, linear you can see that. Now it is a constant temperature case I could stress this all this equation. So, it will be m dot Cp into dTm by dx equal to heat transport coefficient into Ts minus Tm ok.

So, now here that mixing cup temperature is now going to be a function of the heat transport coefficient, what will be the temp mixing cup temperature profile?

Student: (Refer Time: 14:18).

It will be exponential function right minus sign into T that should be a p right. So, that will be a function of the heat transport coefficient. Also we can clearly see that when you have constant flux the temperature profile is going to very linearly in the fully developed region, while in the case of constant temperature it is going to be an exponential profile. So, one can actually draw the temperature profile for a constant flux case. So, you will have. So, remember we said that for constant flux case dTm by dx is also equal to dTs by dx that is also equal to dT by dx in the fully developed region right.

So, remember the derivatives we took a short while ago. So, there we clearly showed that for a constant flux case, the gradient of the mixing cup temperature is equal to the gradient of the surface temperature and the local temperature in the fully developed region. If this is the temperature profile of the mixing cup temperature, and this will be the temperature of the vcdt surface temperature. So, that will be the surface temperature. So, the surface temperature gradient, this is the fully developed region. So, the surface temperature gradient and the mixing cup temperature gradient are same in the fully developed region, while they are not same in the entry region ok.

So in fact, this is the reason why the heat transport coefficient is actually significantly higher in the entry region remember we do this plot of heat transport coefficient (Refer Time: 16:08) x were you have a constant heat transport coefficient in the fully developed region and that is because the gradients are equal and also the mixing cup temperature is linear. So, therefore, the gradient is constant when you have a constant flux condition, and therefore, the heat transport coefficient has to remain constant for a given set of properties tike conductivity of the fluid, while in the entry region the gradient is very small and therefore, the heat transport coefficient has to be very large.

Student: (Refer Time: 16:41).

No we can always have the reverse if you have. So, we started the discussion by assuming that there is heat transport from the outside to the inside of the fluid, you can always do the reverse it does not matter the profiles will be different.

Student: (Refer Time: 16:56).

Yeah sure because this balance we wrote this is for the whole tube going from 0 to arrow; this is the independent of whether you have a fully developed region or an entry region, this balance is valid everywhere in the tube. So, it is linear everywhere, but the surface temperature is not linear everywhere.

Student: (Refer Time: 17:22).

Why does the.

Student: (Refer Time: 17:25).

Right because the fluid is now. So, the mixing cup temperature is now catching up with the temperature of the wall. So, wall temperature is higher, it is going to catch up with the.

Student: (Refer Time: 17:37).

Right.

Student: (Refer Time: 17:38).

No, but this is constant flux, keep in mind this is constant flux not constant temperature right. You already removing constant amount of heat. So, the temperature of the wall is the one which is going be vary with the action position right you are supplying or leaving. Whatever it does not matter whether Ts is higher than Tm or Tm is higher than Ts you will have exactly a similar behavior except that the profiles have going to be slightly. So, Tm is going to be higher than Ts if you are actually removing heat from the fluid if you are actually supplying heat when this profile will going to be above the Tm that is all that is the only difference. Any question so for?

Remember that we still have not solve the full profile. So, without solving the full profile of the temperature, we are able to get lot of insights into what is the nature of the heat transport coefficient, what is the nature of some of the observable quantities like the surface temperature and in fact, we are able to predict some of the properties of the actual temperature profile without even solving the equation. We will come to solving the equations in the next lecture.