Heat Transfer
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Lecture - 31
Flow through pipes III

We have been looking at internal flows.

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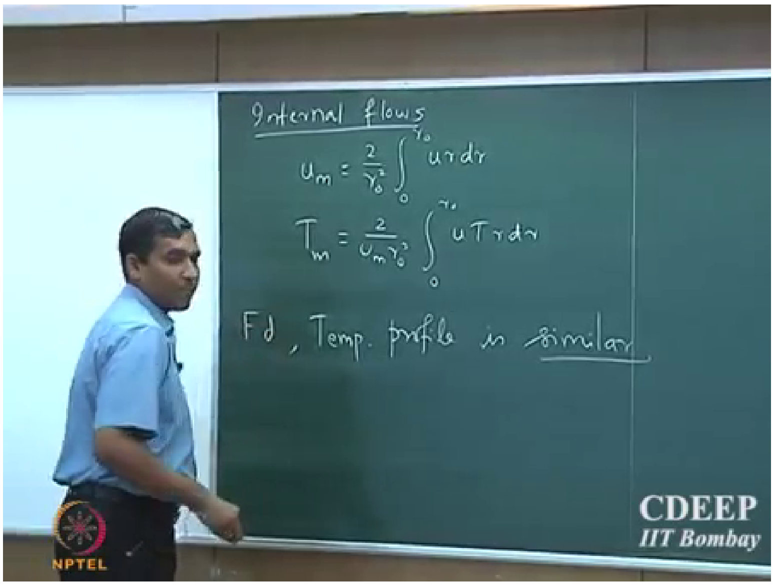

So, we observed it that, there is no reference velocity and there is no reference temperature. So, we defined the mean or cup mixing temperature is given by 2 by r naught square integral 0 to r naught, u r d r. And similarly we defined cup mixing temperature which is anyone remember 2 by 2 by U m r naught square integral 0 to r naught u times T time r d r.

So, that is the cup mixing temperature or the cross sectional average temperature and that is be cup mixing or the cross sectional average velocity. Now we said that there is a the temperature profile is similar. So, we going to show today what is meant by similar and how to see it how to see that the profiles or the temperature profile the fully developed regime is similar. So, we said fully developed regime temperature profile is similar ok.

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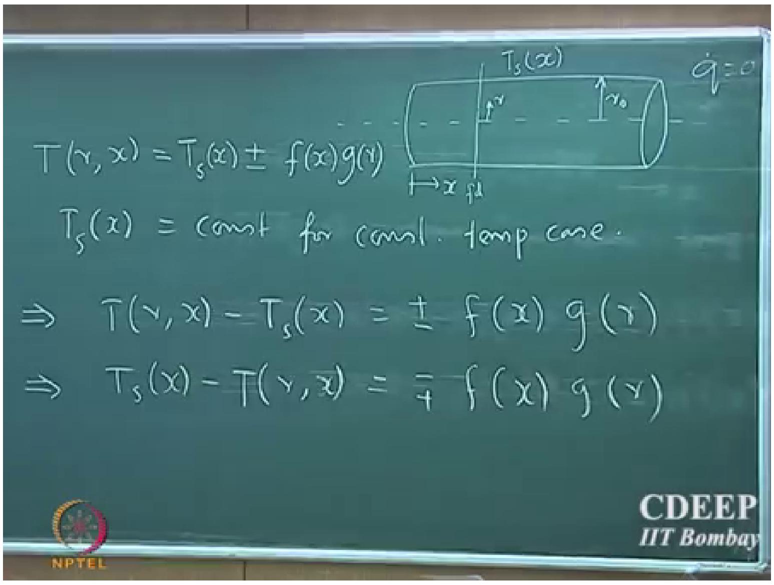

So, suppose here is a tube. So, for both the cases by we can make a general representation that the temperature of the exterior surface of the of the tube is T s of x, where T s is the surface temperature as if it is constant flux when the surface temperature can in principle vary with the x position this is x direction and if it is constant temperature when it does not change with position right. So, now, if I want to find the temperature distribution I want to find the temperature distribution inside the tube let us say in the fully developed regime.

So, what will be the general functional form? So, remember the model equation. So, what are the process is it are supposing here is my fully developed regime. So, what are the processes which are acting here? You have diffusion and you have connection right is it linear with respect to temperature we assumed that there is no heat generation heat generation term is 0. So, it is linear with respect to temperature right. So, what will be the functional form of the temperature as a function of r and x? This is r direction yeah what will be the functional form? I am not solving the equations, what will be the functional form which will get the solution.

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Some function of r and some function of x right. So, it is by separation of variable you should be able to get a 2 functions, it will be a product of 2 functions will each of these functions will be 1 will be function of the radial direction 1 will be function of the axial direction. So, therefore, the general form of the solution with T s of x if you assume that is heat that is been supplied from the external region to the fluid, or from fluid to the surface.

So, the general solution will be T s plus or minus some function of x and some function of the radius right. So, that is the general form of the temperature distribution right. The upper limit is T s is the heat is being supplied from the surface to the fluid and the lower limit is T s if it is vice versa. So, that is a function of x position if it is a constant flux case and T s of x is constant for constant temperature case. So, this is the general functional form, not that we are not solved anything we have just observed what is the general functional form of the temperature profile. So, from here

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Sure, you can because if there is no heat generation, if you look at the structure of the equation is linear in temperature. So, you should be able to separate. So, therefore, we can rewrite the functional form as T of r comma x minus T s equal to plus or minus f of x into g of r. So, now I want to rewrite this as I just want to rewrite it as, T s minus T equal to minus plus f of x into g of r. So, all I have done is I have taken T on the other side and the function on the left hand side. So, once I do this, I can introduce the averaging property here.

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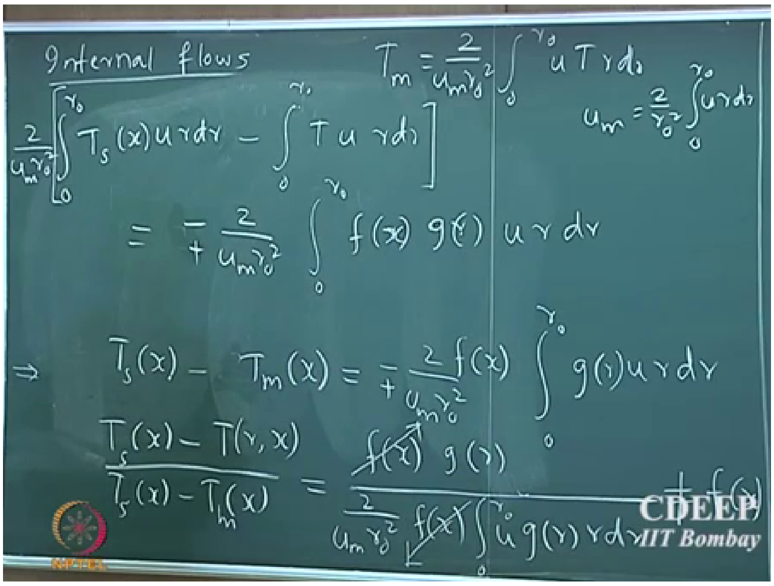

So, now, if I we know that Tm which is the average temperature is 2 by r naught square integral 0 to r naught u T r d r ok.

So, the objective is I want to find the mean temperature, remember I told you that the mean temperature or the average temperature, and the local temperature there is some similarity between them and therefore, we can actually see that the temperature profiles are similar. We are going to see that in little bit more rigorous way as to how it is similar. So, suppose I want to find out T m from this expression.

So, I can integrate 0 to r naught 2 by r naught square T s of x u into r d r minus integral 0 to r naught T into u into r d r that is equal to minus plus 2 by r naught square integral 0 to r f of r naught f of r f of x into g of r into u into r d r. So, that is the functional form. So, some here. So, T s is not a function of the radial position, it is the wall temperature right. So, I can pull this out from the integral is nothing, but itself is 2 by r naught square is already there in. So, this is T s of x minus T m of x. Remember that is 2 by r naught square into integral 0 to r naught u r d r. So, that goes away and the right hand side you will have minus plus 2 by r naught square f of x is not a function of the radial position. So, that will be integral r naught g of r u into r d r ok.

So, now if I divide these two expressions, you see there is an expression here which is T s minus T that is the difference between the surface temperature and the local temperature and here is a difference between the surface temperature and the cup mixing temperature. So, now, if I divide these 2 expressions minus T divided by T s of x minus T m of x. So, this is both function of r and x right. Now if I divide these 2 expressions what do you see is something very interesting you see f of x, g of r divided by 2 by r naught square into f of x into integral 0 to r naught r d r.

So, all I have done is I have just divided these 2 expressions and so, interestingly the functional form will actually cancel out the x dependence will actually cancel out in this ratio and therefore, this ratio is not a function of the x position. So, let me explain again look at the right hand side carefully f of x, g of r divided by 2 by r naught square f of x comes out and its integral of gr multiplied by u. So, that is the cross sectional average of the function g.

So, f of x will cancel out. So, you will see that this ratio T s of x, minus T of rx where the local temperature is now a function of both radial and the axial position, and not just that the surface temperature and even the cup mixing temperature both of them can be a function of the axial position. But irrespective of that that is an important result irrespective of that this ratio is not going to be a function of the axial position.

So, that is what one means by a similar profile. What it means to say similar profile is that T s minus T the profile that you will get for T s minus T and that for T s minus T m would be exactly the same in the x direction. So, the profile that you will get for the difference in the surface temperature and the local temperature and the difference in the surface temperature and the mixing cup temperature that difference will be the same for at any location the fully developed regime. So, as a result this ratio is not a function of the axial position. So, that is an important result, it is very similar to what we got in the momentum boundary layer where we said that the velocity profile does not change.

So, here the correct statement is that if I define a new variable new dimension less temperature, which is T s minus T by T s minus the cup mixing temperature. So, that dimensionless temperature is now independent of the axial position. So, this is a fundamental difference between the momentum boundary layer and the thermal boundary layer, in the in internal flow case.

So, it is not the temperature profile which is not changing its the dimensionless temperature profile, it does not change to the axial position. So, this observation actually can lead to some really interesting and intuitive results, which is what you will see in a short while.

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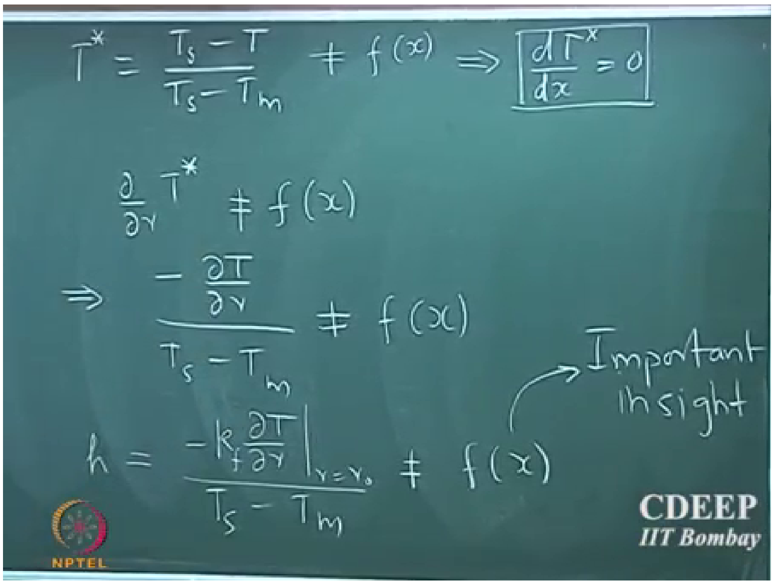

So, this means your T star Ts minus T by T s minus T m, I am just removing the functional, but we should always assume that T s is the function of position if it is a constant flux condition, and T s is constant if it is a constant temperature condition and. So, that is not a function of the axial position which means that dT star by dx is 0. So, that is the first observation.

So, it is the gradient of the dimensionless temperature is 0, and not the gradient of the actual temperature and in fact, the definitions that we have used in all or to this course we have used a certain form for non dimensionalizing temperature, and that really comes from this idea. That using such a type of dimensionless quantity for temperature were actually helps in understanding the problem better and it gives much better insights.

So, supposing now I take d by dr of T star d by dr of T star is that a function of the x position? No the star is not a function of x position therefore; this is also not a function of the x position. So, now, dT star by dr is nothing, but minus dT by dr divided by T s minus Tm. So, all I have done is I have differentiated the dimensionless quantity. So, T s is not a function of radial position and T s and T m both are not function of radial position. So, dT by d T star by dr is nothing, but minus dT by dr divided by T s minus Tm.

So, that is not a function of the axial position, what is this form, what is this ratio minus dT by dr by T s minus T m? It is the form that we have all seen a heat transport coefficient has a similar functional form right. So, what is the definition of heat transport coefficient? Heat transport coefficient is minus k dT by dr at r equal to r naught divided by T s minus T m right. So, that is the heat transport coefficient by definition right.

So, now, because this ratio is not a function of the axial position, this ratio is also not a function of the axial position. So, this is also not a function of the axial position as long as the properties are constant if k is constant. So, you put a subscript f 50 conductivity of the fluid. So, if kf is constant, then the heat transport coefficient is not a function of the axial position the fully developed regime. So, that is an important result.

So, we get an important insight. So, that is an important insight into the problem. So, not that we have not solved the equations, we have only looked at the structure of the equations and we have made we have looked at what is going to be the profile of the dimensionless quantity and from that we are able to look derive an interesting insight that the heat transport coefficient in the fully developed regime. So, is not going to be a function of the axial position.

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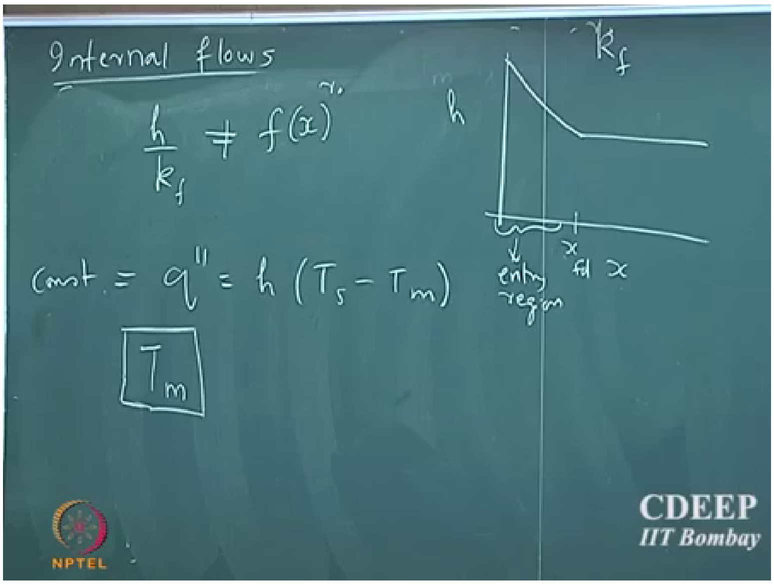

So, which means that h by k is not a function of the axial position. So, this provides an excellent framework oh k f this provides an excellent framework for actually comparing the transport properties for different fluids. The conductivity of different fluids are different.

So, this provides a very nice framework for comparing the properties of heat transport for different fluid which will have different conductivity. As a result if I draw a plot of x versus the heat transport coefficient for a fixed kf. So, supposing if this is my fully developed regime fully developed regime starts at that location.

So, the heat transport coefficient is constant the product function of the axial position what about this region? It will be higher or lower, what would you guess? We will see will see the little bit more rigorous way what will be the nature of the heat transport coefficient in that location, but what would you guess it will be higher or lower? So, I give you a hint.

So, q the flux of heat transport the local flux of heat transport defined as h into T s minus T m right. So, the surface temperature minus the reference temperature i; so, that is the definition now can you guess. So, let us take constant flux condition yeah it will be.

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Higher why?

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So the temperature gained will be smaller in the entry region. So, this is what us called the entry region. So, the temperature gradient is going to be smaller the entry region and therefore, in order to maintain constant flux, the heat transport coefficient has to be higher. So, that is the profile of heat transport coefficient with respect to the axial position with you, we can see this in a much more rigorous way.

So, suppose I want to find out. So, the exercise is now to really find out what is the mean temperature. So, if I know the mean temperature, the lots of thing that I can actually estimate. So, before we go and find out what is this mean temperature, let us see if you can find the relationship between the mean temperature and the actual local temperature.

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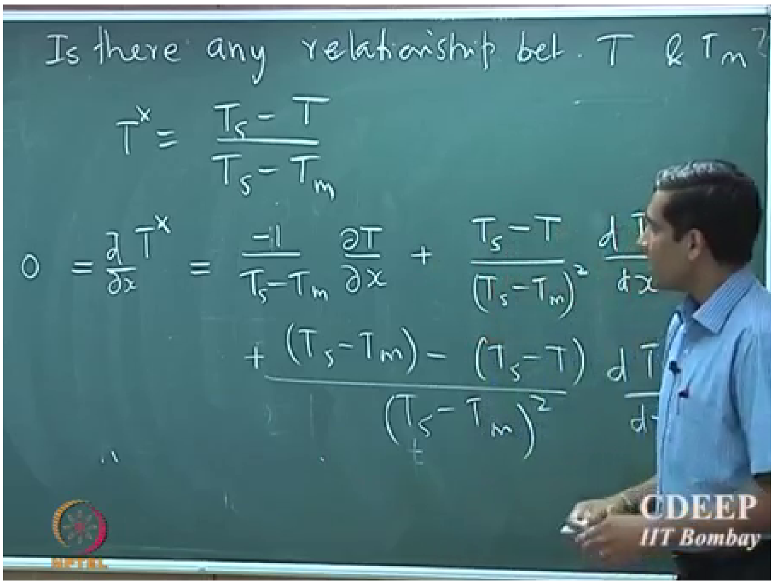

Can we is there any relationship between T and T m. Because we say that T m provides a way to scale the temperature which leads to a one important insight that the non dimensional temperature profile does not change with the axial position in the fully developed regime.

So, which means that there must be relationship between the two; so, can we find out what that relationship? So, T star is T s minus T by T s minus Tm. So, remember that we are not solving the equation you not even attempting to solve the equation as if now. Without solving what is the maximum piece of insight that we get.

So, now, if I take the derivative of this with respective x, what is that? That should be that should be 0 right T star is not a function of the axial position in fully developed regime and so, that should be equal to 0. So, that will be T s is a function of position T m is a function of position and T is a function of position and so, we can now write this as 1 by T s minus T m minus 1 into dT by dx.

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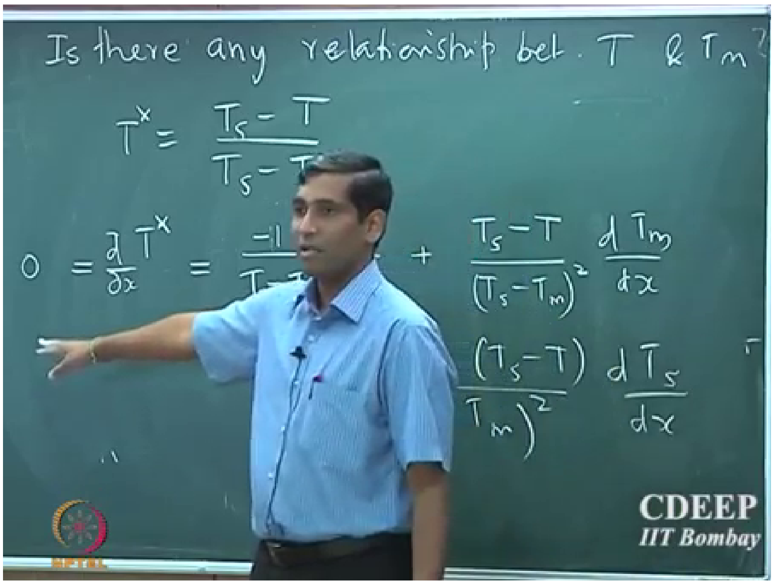

So, that is the x derivative with respect to local temperature plus we can take a derivative with respect to the temperature T s plus the whole square into d T m by dx plus T s minus T m minus T s minus T divided by T s minus T m the whole square into d T s by dx. And that should be equal to 0 because the dimensionless temperature is now a independent of the x position the fully developed regime and therefore, this derivative is 0 and therefore. So, that this gives you an expression which relates the gradient of the cup mixing temperature and the gradient of the surface temperature, with respect to the gradient of the local temperature.