Heat Transfer
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Lecture – 30
Flow through pipes II

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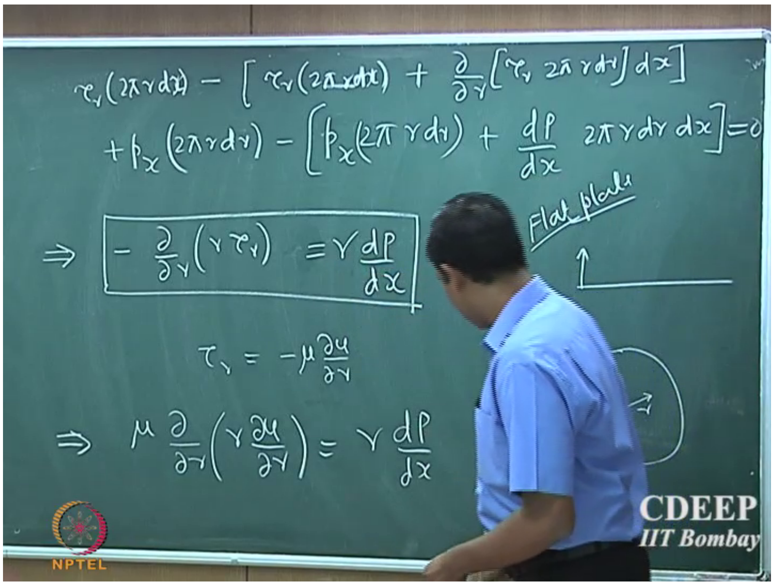

These, I can write it as tau r, so I can rewrite the balance as minus tau r 2 pi r d r plus d by d r, tau r 2 pi r d r into d x, so I get it right, yeah 2 pi r d x, sorry 2 pi r d x. So, I can open it up and write it in Taylor series expansion.

So, the variation of radius, remember that it is a varying cross sectional area, so it has to be preserved inside the differential. Plus p x, 2 pi r d r minus 2 pi r d r plus d p by d x 2 pi r d r into d x, that should be equal to 0. So now, there will be minus d by d r, r into tau r plus that should be equal to d p by d x, so that is the force balance. What is tau r? Should be very careful about the sign that you have, when you are in you actually using cylindrical coordinates, it is not the same as what you have in Cartesian coordinates.

So, remember that in Cartesian coordinates in flat plate case, if you look at the flat plate, the wall is actually the affect of the wall is actually in the positive y direction, but when you take a tube the effect of the wall is actually in the negative r direction, remember that if you take a, so compare that with a tube. If this is the centre, so the positive r direction is going outside from the centre. So, the wall is now actually located at r equal to r naught. Therefore the effect of the wall is actually in the negative r direction that is why you have a minus r. To be very careful when you go from Cartesian to cylindrical coordinates or spherical coordinates, we should always be very careful about the sign that you use.

So, therefore, if you plug that in, it will be mu into d by d r or d u by d r equal to oh, I have forgotten r here, r d p by d x. So, r does not cancel out because, it is presents inside the differential, should be r d p by d x please correct it.

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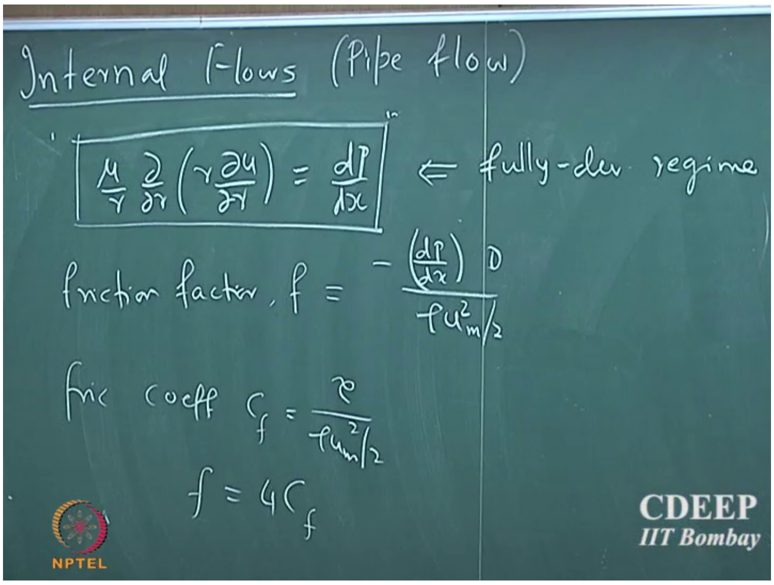


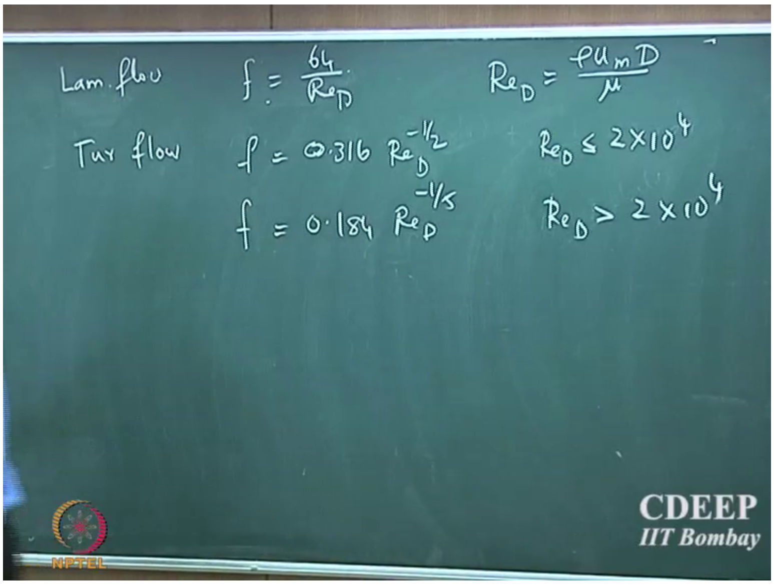
So, mu by r, d by d r into r d u by d r equal to d p by d x, so that gives you balance for the velocity in the fully developed regime, so this is not true in the developing or the entry regime its true only in the fully developed regime, where d p by d x the pressure gradient is not a function of the radial position. Therefore, you can find out the velocity profile in the fully developed regime. How do you find the pressure gradient? Is it a measurable quantity?

Student: Yes.

Yes, how? You want pressure gradient in the fully developed regime. So, how do you find the pressure gradient? How many of you heard friction factor? So, if you know the friction factor, so that actually defined us f. If you know friction factor, friction factor is actually related to the pressure gradient in the fully developed regime. So, if you know the friction factor then you should actually be able to estimate the pressure gradient. So, that is given as minus d p by d x into D divided by rho, so that is the friction factor. How is the friction factor and friction coefficient defined? What is friction coefficient?

So, friction coefficient that c f that is tau divided by rho u n square by 2. How are these two related? Yeah, they not same, is definitely not same, but how are they related. So, this is, so for laminar flow conditions it actually turns out that it is 4 times C f, it is only true for laminar flow.

Now, if you are looking at turbulent flow in other condition, there are some correlations which is available for the friction factor, which is in terms of the Reynolds number. (Refer Slide Time: 06:15)



So, that is given by, so for laminar flow, so this 4 times C f approximately translates to 64 by the Reynolds number. Reynolds number is defined based on the diameter of the tube not the radios. So R e D is defined as rho u m D by mu, so which means that the Reynolds number does not change with the location. So, it is defined based on the mixing cup velocity and for turbulent flow is if given by 0.316 into R e D power minus half and this validity is when it is less than or equal to 2 into 10 power 4. What is the transition Reynolds number for pipe flow 2000?

Student: (Refer Time: 07:18).

2100 (Refer Time: 07:22) and, fine and this is valid when Reynolds number is greater than 2 into 10 to the power 4. So, that is the correlation for turbulent conditions and if you know the friction factor you should be able to estimate the pressure gradient. So, if we know the pressure gradient you should be able to integrate the expression and find the velocity profile and that is what we are going to do now.

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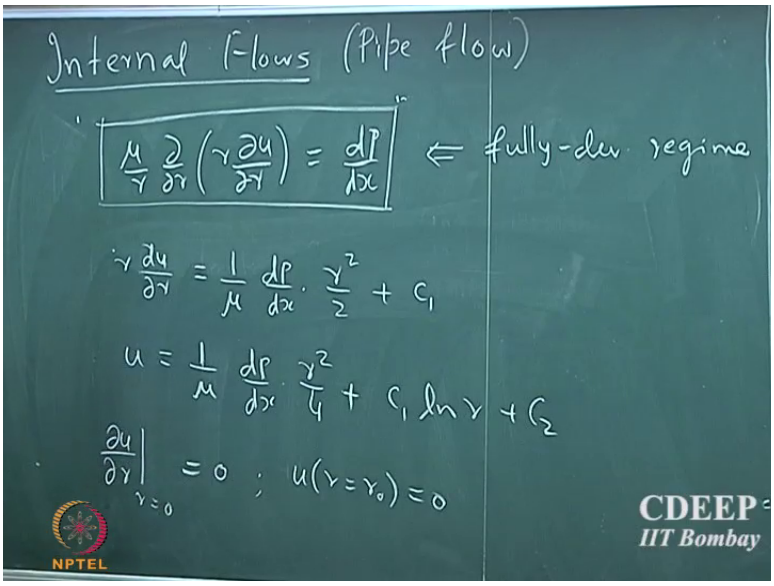

So, we can integrate this expression here, so the integral will be I can do it here. So, I can integrate this expression, so it will be r d u by d r, that is equal to 1 by mu d p by d x into r square by 2 plus c 1 and u is 1 by mu d p by d x, r square by 4 plus c 1 lon r plus c 2, so that is the integral. What are the boundary conditions? R equal to 0, so d u by d r at r equal to 0, that is equal to the 0 because of the symmetry and what about the other boundary conditions, u at r equal to r naught is 0.

So, now if I put the first boundary condition which constantly disappear, c 1 will disappear right, so we can say that it would be u by d r at r equal to 0, so that means, c 1 is 0, so that constant will go to 0.

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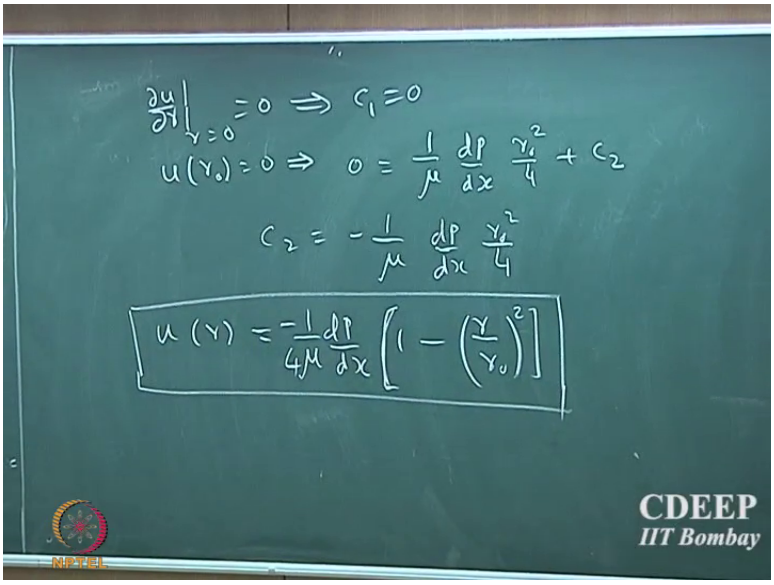

Now, if I put the second constant u at r naught equal to 0, so that would be 0 equal to 1 by mu d p by d x into r naught square by 4 plus c 2, because c 2 will be minus 1 by mu, d p by d x into r naught square by 4. So, from this I can find the velocity profile that will be 1 by mu, d p by d x, I can pull 4 out, 1 minus r by r naught the whole square. You note that I have pulled a minus sign outside; r by r naught is greater than 1 or less than 1? It is less than 1, why is there a minus sign because d p by d x is negative, right.

So, this is the velocity profile with respect to r. So, here clearly we show that it is not a function of the x position because the velocity profile does not change and so d p by d x has to be constant and d p by d x has to be constant for a steady flow and therefore, it has to be a function of only the radial position in the fully developed regime. It is still do not know what happened in the entry regime we will come to that later.

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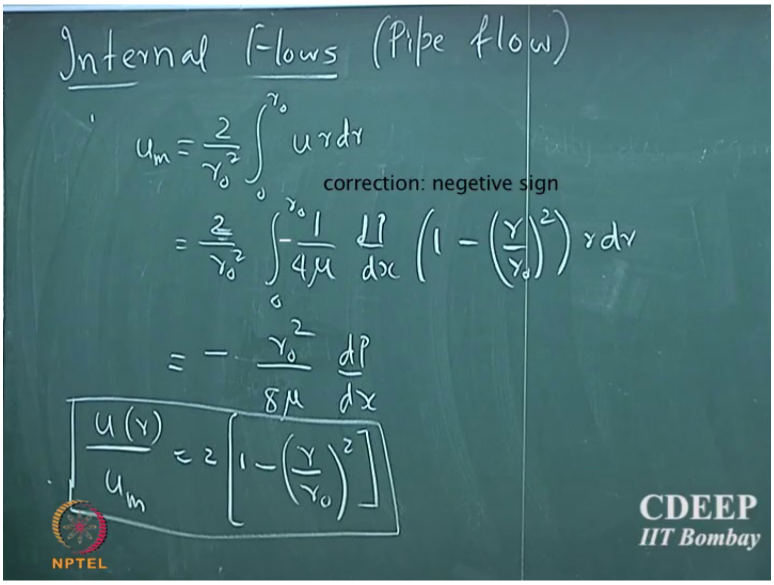

So, from here now we should be able to find out what is the average profile. U m which is the mixing cup or the average velocity profile is 2 by r naught square, integral 0 to r naught u into r d r, right. So, I can substitute now the velocity profile, so that will be 2 by r naught square or one by mu, 4 mu d p by d x. I am now going to the integration here, this give you the final expression. So, the integral will be minus 1 by 4 mu d p by, so there will be r naught square by 8 mu in to d p by d x, so that will be the average velocity.

So, now I can use that as a scaling, remember we defined average velocity for scaling purposes, right. If I know d p by d x, I am done. I know my cup mixing velocity and so I will, now know the scaling velocity so now I can use that to scale my actually velocity profile. So, therefore, u by u m is now given by 2 into 1 minus r by r naught the whole square. So, that is the velocity profile in the fully developed regime. Is that cleared to everyone, any questions on this so far? So, now, the purpose of all this exercise, remember that in this course the purpose is to find the heat transport coefficient and the mass transport coefficient. So, what we are really interested in, is the thermal boundary layer and the concentration boundary layer.

But the momentum boundary layer they have a strong effect on the thermal and the concentration boundary layer. So, once we know the velocity profile, now we can proceed and find out what does it effects on the thermal boundary layer. So, what we are going to see next is the thermal boundary layer.

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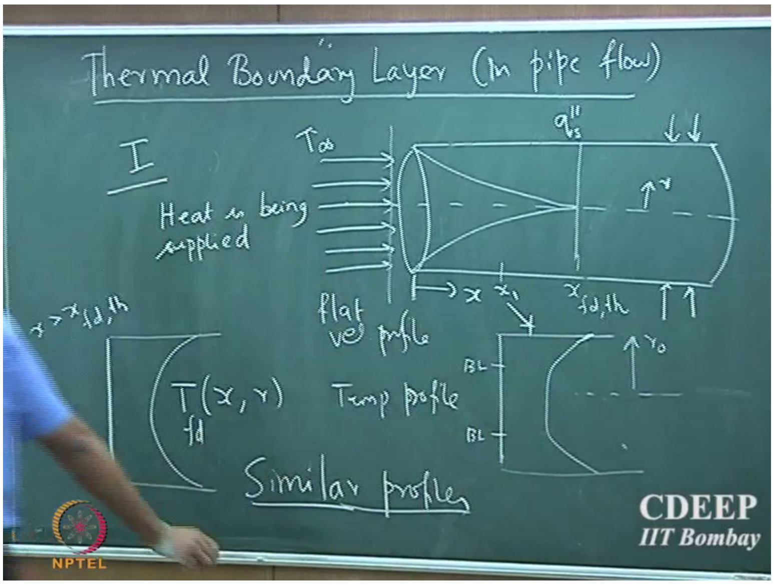

So, how do we characterize thermal boundary layer and there is a fluid which is flowing through a pipe. So, I have a pipe here and this is r this is x. So, now when it comes to heat transport problem, so there are two specific cases that one can sort of think of right, so one is you can maintain a constant flux of heat that is actually removed from the surface.

So, first case would be, case one would be I have a constant surface heat flux, so I can have a situation, where the flux of heat that is either removed or given to the cylinder or given to the tube is maintained constant. So, if it is at a higher, is the fluid is at higher temperature then there is a constant flux with at which the heat is removed and if the fluid is at a lower temperature where you want to heat the fluid, then it could be that there is the constant flux of heat that is supply to the tube.

So, if I say that the fluid is now flowing inside the tube and if I assume that the it is a flat velocity profile, before it enters the tube and if I assume that the temperature is T infinity, so that is the temperature of the fluid before it enters the tube. So, now, similar to momentum boundary layer I will also have a thermal boundary layer, where the effect of the surface is felt by the fluid as soon as the fluid enters the tube and up to a certain location, only if a fraction of the fluid in any cross section is experiencing the presence of the wall or the heat transport from the wall and up to a certain location and a location called fully developed thermal fully developed regime.

So, at that location every fluid particle in the cross section will now experience the presence of heat transport from the wall to the fluid or vice versa if the temperature of the fluid is hotter than the surroundings. So, now, similar to momentum boundary layer we can define a thermal boundary layer and also a regime for fully developed thermal boundary layer conditions.

So, now what will be the temperature profile, suppose I take x 1, I want to draw the temperature profile. So, let us say that this is the boundary layer location and the temperature if I assume that the heat is being supplied to the fluid. So, now, what will be the gradient, what will be the flux at the boundary, will it be positive or negative. So, heat is being supplied from the surrounding to the fluid negative flux.

So, now, so that is the temperature profile. So, note that this is the temperature profile and there is a constant flux at the boundary. So, this is an, this is the center x r equal to r naught. Now supposing if I take a second case, where I maintain the boundary at a constant temperature I can always do that right. So, what happens in the fully developed regime? What will be the temperature profile?

Student: (Refer Time: 18:03).

Yeah. So, this is at x 1, what about fully developed regime. So, supposing I have a x greater than the fully developed location, is no boundary layer because fluid particle at every location in the cross section is now experiencing the presence of heat transport from the wall or to the wall. So, which means that, so there will be a temperature profile which looks like this, is this is the function of factual position, you remember in velocity boundary layer we said moment it touches the fully developed regime the velocity profile does not change. What about in the case of thermal boundary layer? Yeah, same, no. Even here, you will have heat that is being supplied to the fluid right. So, how can you have a same temperature profile?

So, the temperature profile; the local temperature profile is now going to be a function of both x and r in the fully developed regime, unlike what you saw in the momentum boundary layer. So, this is a significant difference from as compare to what you saw in the flat plate case, flow past a flat plate case, where all 3 boundary layers have a similar profile, but in case of a pipe flow the local temperature profile is now going to the function of both x and r, but then what is interesting is are you can still do something. Yeah, it is not necessarily linear.

Student: (Refer Time: 19:55).

Not necessarily it is linear, will actually show it rigorously that it need not be linear. So, what you see is we will see what is called the profiles will be similar. Will actually mostly in tomorrow's lecture, I will actually show you how to see that the temperature profiles are the similar. It is not very obvious, if you look at the profiles; here it is not very obvious, what you mean by similar profile.

So, what you will have is similar profiles, so in any location after the fully developed regime in the thermal boundary layer case, is that you will have similar temperature profiles. So, note that they are not same, it is only similar. We are going to define, what is mean by similar profile? Not in today's mostly in tomorrow's lectures.

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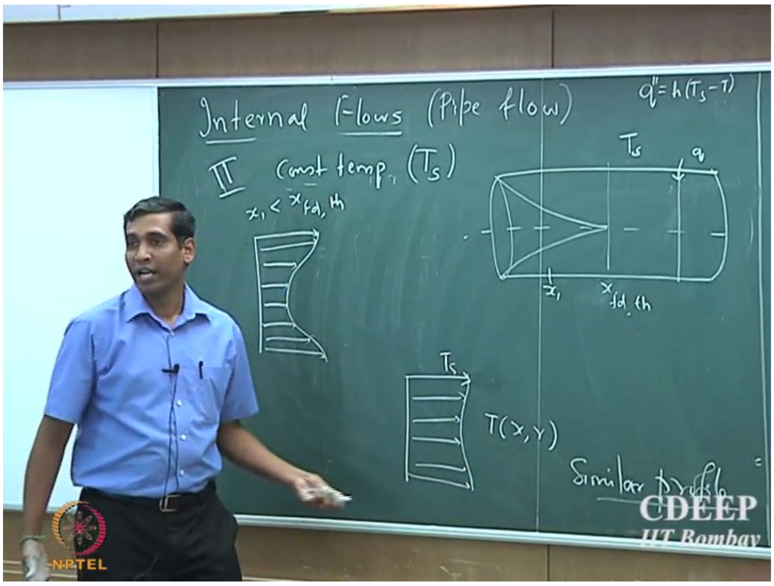

So, now, once we know this, we can go to the next case where you have a constant temperature. So, supposing I maintain the surface temperature at a certain constant value. So, now, I have a tube here, once again I have a boundary layer; thermal boundary layer and I maintain the surface temperature as constant. So, now, I can draw a profile, so let us say at some location x 1, so that is a constant temperature. So, this is that position x 1 which is less than, the fully developed regime and if I look at the profile at the fully developed regime, once again that is a temperature of the surface and as you go fast into the pipe there is continue, there will be continuous transport of heat from the wall to the fluid and so there will be a change in the temperature profile, once again this will be a function of x and r.

But what you will see is that you will get what is called as similar profile and we actually going to capitalize on this aspect that the profile is similar and we are going to attempt to solve the problem and get the heat transport and mass transport coefficient. So, before that, before we even understand what is a similar profile, we need to have a reference, right. Yes.

Student: (Refer Time: 22:44).

Because even if you have constant temperature, the you have a flux of heat transport, right. So, until the fluid temperature equilibrates, I because of Newton's law of cooling. What is Newton law of cooling, at the local point the flux is given by some heat transport coefficient into the temperature different right. So, till the fluid equilibrates with the wall temperature, there is going to be constant addition of heat from the wall to the fluid. So, therefore, there is addition of capacity to the fluid and so you will see that the temperature is going to change with x location in the fully developed regime. This is fundamentally different from what you saw in the momentum boundary layer, but we will see that because of the similarity or the profile is similar, you will see that there is some intuitive way by which you can actually solve this problem.

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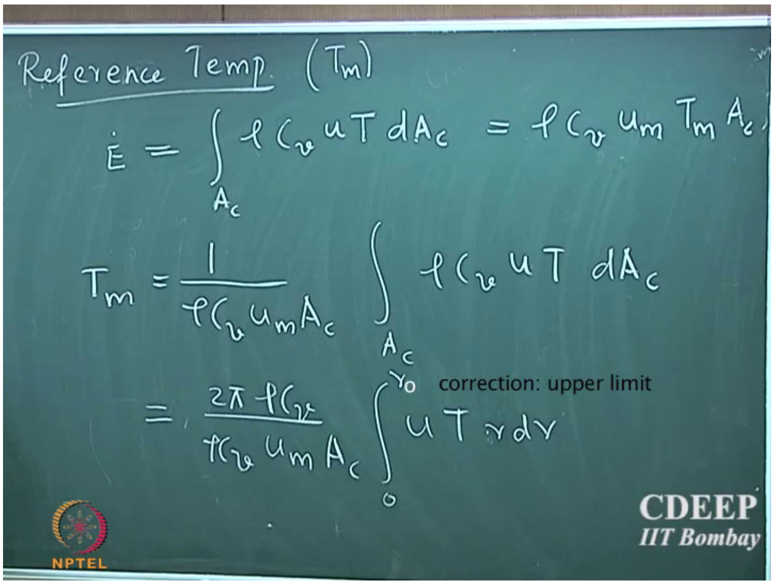

So, in order to define similar profile, we need to define a .Reference temperature. Just like how we defined mixing cup velocity, we can also define what is called the mixing cup or the average cross sectional average temperature. So, how do we define this, what is the conserved property. So, remember in the velocity boundary layer we use the mass flow rate as the conserved property, so what we do here.

So, you use what is called the internal energy in a given cross section, so use the property of internal energy at given cross section to define the cross sectional average temperature. To the internal energy at any location is given by integral over the cross section, rho times the capacity of the fluid; internal capacity of the fluid, which is rho C v multiplied by u into T into d A c, right. So, that is the internal energy that is actually being transported.

So, now we can define this, as rho into c v into in terms of the mixing cup velocity, multiplied by the average temperature into the cross sectional area. So, note that it is a definition, we defined the average amount of heat that is actually scored in the fluid at any cross section, is given by the capacity multiplied by the mixing cup velocity, multiplied by the mixing cup temperature, multiplied by the cross sectional area.

So now, so from here, we find T m is 1 by rho C v, u m, A c, integral over the cross section, T into d A c. So, I can, d A c is nothing but 2 pi r d r. So, that will be 2 pi into rho c v producing incompressible fluid, u m A c, u into T into r d r, 0 to r.

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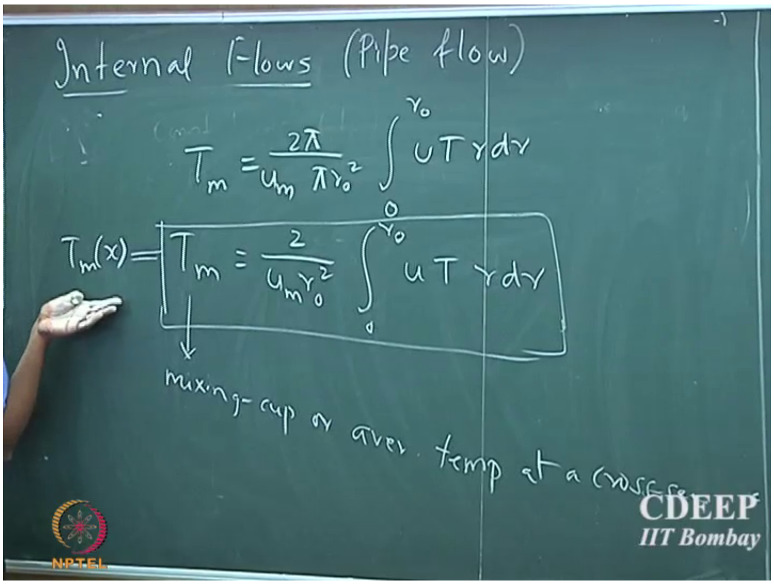

So, from here, we can deduct and T m, you can reduce that T m is 2 pi divided by u m into cross sectional area is pi r naught square into integral u T r d r, 0 to r naught. So, that will be 2 by u m r naught square, r d r. So, that is the definition of mixing cup or average temperature profile at any cross section, it is called the mixing cup or average temperature. This is the function of x position? No, why?

Student: (Refer Time: 27:30).

Does not matter, whether it is constant flux or constant temperature, this is constant with respect to x position, depends whether T is a function of x and r right, so this integral is only a function of r. So, this obviously, has to be a function of x position. Unlike what we saw in the velocity boundary layer, the mixing cup temperature is now a function of the x position.

So, what we will do in the next lecturer is, we will start by defining what is called as a similar profile and that definition, we will capitalize on the fact that the mixing cup temperature is also a function of the axial position. We will show that the temperature profiles in the fully developed regime, is very similar and with that we will be able to solve the equations and we will be able to find the heat transport coefficient.