Heat Transfer
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Lecture – 29
Flow through pipes l

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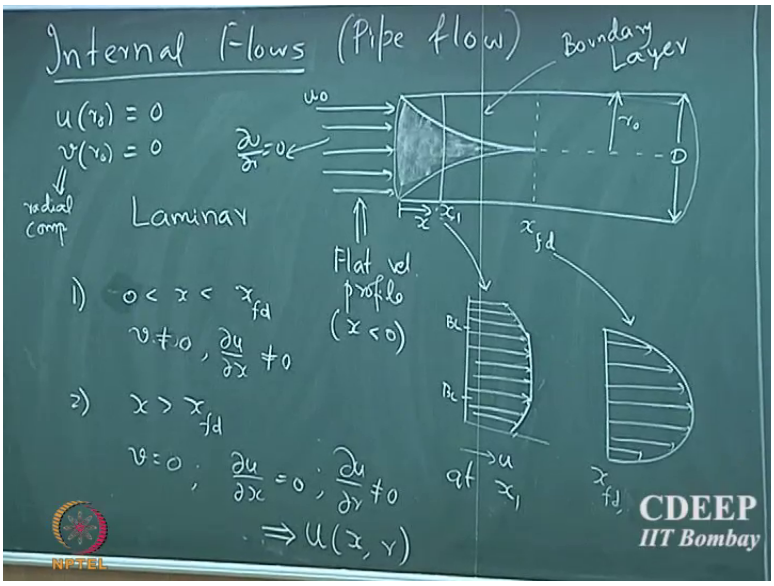

So, we will switch gears today and will move into the next chapter is internal flows. So, so it is really like it is domently look at pipe flow, look at heat transport mass transport during pipe flow.

So, suppose let us consider a tube and we have a fluid which is flowing through this tube let say that u naught is the velocity with which the fluid is entering the tube. So, let us assume that it is a flat velocity profile before it enters the tube and if I put coordinate. So, I call this as x direction. So, this is x less than 0, it is a flat velocity profile and if I mark the midpoint if the radios of the tube is r naught and the diameter of the tube D.

So, now, suppose if we let say the fluid flows in to the tube. So, similar to flat plate the fluid is note that it is a circulate tube. So, it is a circular cross section, circular cross section. So, as soon as the fluid enters the tube it experiences the wall which is present and location r naught right. So, similar to the flat plate the velocity of the fluid will come to 0 at the wall right. So, u at r naught that is equal to 0.

Now, suppose if I say v is the radial component velocity. So, u is the x component velocity and v is the radial component radial velocity component suppose v at r naught is also 0. So, note that it is a flat velocity profile which means before it enters the tube the fluid is primarily moving in only one direction, but as soon as it touches the wall as soon as the experience the wall its suddenly the fluid comes to rest and so the viscous effect will start playing role and therefore, the fluid will start moving in both direction both radial and the actual direction and therefore, it results in the formation of boundary layer. So, that is the boundary layer.

So, what you see here the location in between. So, note that it is a circulate tube. So, the boundary layer will be present in every location in the rim of the tube and they will slowly merge in convert together at a certain location. Now, in this regime here the fluid will continued to be flowing with a as an invisible fluid because the viscous effects due to the presence of the wall will be experience slowly and gradually as the fluid goes inside the tube.

So, at some location, some location where these boundary layers merge and after certain point you get what is called the fully developed, fully developed regime. So, fully developed regime is a location where every location in the every location on inside the tube the fluid experience is the presence of the wall. So, the viscous effect to the start playing the role at every fluid particle in any cross section after the fully developed regime, so there is no invisible flow regime after the fully developed location all the way to the end of the tube.

Now, suppose if I draw the velocity profile let say at position x 1, at position x 1 if a draw the velocity profile. So, this is the location of the boundary layer. So, if I draw the velocity profile it will be maximum, the maximum will be somewhere at the centre. So, will have profile which look like this as to be symmetric. So, it is a symmetric velocity profile. So, it should be flat velocity. So, between the boundary layers it will still have same velocity profile as the fluid as their as that of the fluid which is entering the circulate tube. So, it will have a flat velocity profile between the boundary layers and then the fluid will be retarded in the boundary layer and. So, you will have a velocity profile which looks like the this in the x direction

So, this is the x component velocity profile at x 1 location. So, now, if I look at the profile, this is the profile at x 1 location, now if I look at the velocity profile after of at the fully developed regime what I will have is the parabolic velocity profile. So, this is x fully developed regime, anywhere after the fully developed regime the velocity profile will continue to remind the same as long as you go inside and as long as the regime changes. So, you could always go from laminar if it is laminar, laminar regime or turbulent regime the profiles are different. So, let us assume that we are looking at laminar profile we have laminar regime then you will see that as soon as you cross the fully developed regime the velocity profile does not changed anymore. So, now, simply based on this intuition you can actually detect lot of interesting properties.

So, the first thing is that, between x 0 and x fully developed regime. So, the velocity in the radial direction is not slow and not just that du by dx it also not 0 in all location. Remember that before it enters the du by dx is 0 because there is no change in the velocity and it flows at a flat velocity profile. So, then if x is greater than fd at is if you reach the fully developed regime. So, suddenly you will see that the y component velocity goes to 0 and then will have du by dx also will be 0, the velocity profile does not change the fluid will continued to move in that direction, but it will have a definite parabolic profile. So, here if you see before it the fluid enter in this regime you will see that du by dr is also equal to 0 because you start with a flat velocity profile.

But what has been introduced because of the presence of the wall and because of the retardation of the fluid because of the viscous effects to du by dr is not equal to 0. So, this immediately means. So, this immediately implies that the velocity x component velocity is now a function of both x and r. So, that is an immediate implication now going to be a function of both x and r. So, with this let see what to do. So, in the flat plate case we always had a reference velocity in a flat plate case.

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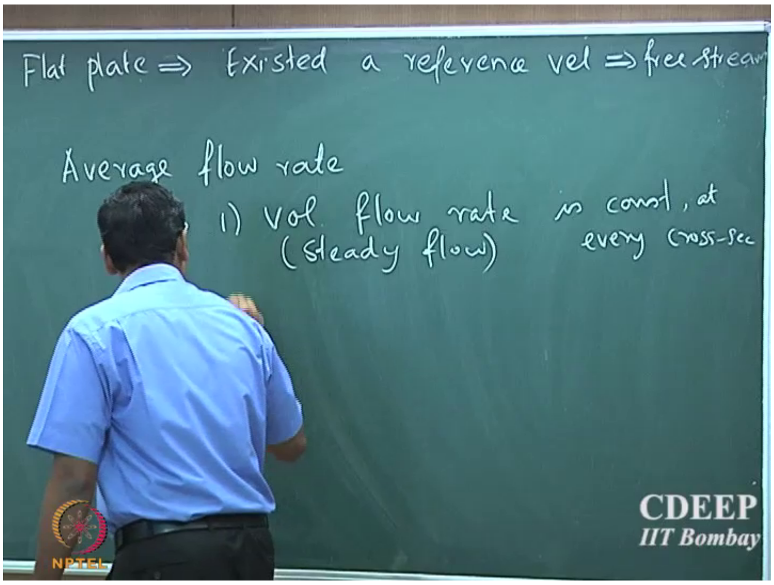

There was always a there existed always a reference velocity which is basically the free stream velocity, but in a pipe flow there is nothing called a free stream velocity right. In a flat plate you had whatever fluid flowing well pass the boundary layer which that is a free stream velocity and we use that for all scaling properties, but in an internal flow when fluid is flowing through a pipe there is no referring velocity. So, we have to start from defining the reference velocity. So, what is the most appropriate reference velocity? Yeah.

Student: (Refer Time: 10:24).

Yes, any suggestion.

Student: (Refer Time: 10:31).

U naught no that is before the pipe right. So, what is the reference (Refer Time: 10:36) inside the tube? Maximum at the center that is one possibility what else; average. Why average?

Student: (Refer Time: 10:48).

Because at every cross section. So, we can define an average flow rate there are two reasons why it is a good measure, one is volumetric flow rate is constant if it a steady flow right. So, again if it is a steady flow, the steady flow is constant at every cross section that is very important, but more important than that much more important is that if I say I put a measuring devise a flow measuring devise at the end of the pipe. Let us say I know how I put a rotameter at the end of the pipe or I put some other meter at the end of the pipe or eventually meter for example. So, now, what I can measure or what I would measure is actually the average flow rate.

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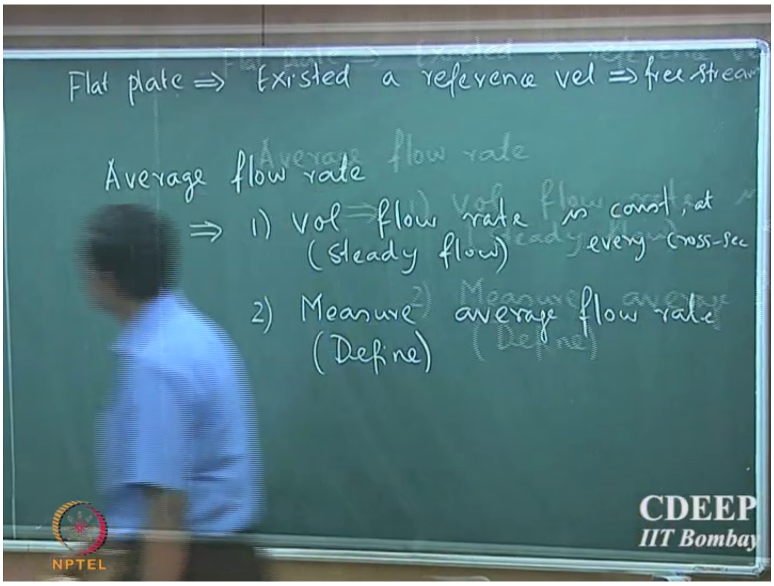

So, when I put a flow meter at the end of the pipe. So, note that the profile does not change after the fully developed regime. So, I can measure that is a very important point you can measure the average flow rate. Remember that the whole course so far always said that anything you ask u is as reference has to be something that should be a measurable quantity otherwise it does not make much sense to use it as a reference.

So, average flow rate is actually one of the good measurable quantity. In fact, we will seen heat transport case that the average temperature is also very good measurable quantity alright. We can now we have to define this quantity, all we do not know what it is from a theoretical point. Yes.

Student: (Refer Time: 12:59).

D u by dx is a 0.

Student: (Refer Time: 13:08).

U is, well actually at after this location you correct in observing that u is not x and r we are going to show that rigorously that it is not. So, right now we do not know anything about where this x fd is there before this point you are correct in the observing that before this point u is a function of both x and r an after this point it is only a function of the radial position you will see that in short while. Any other question?

So, now, we need to define what is this average velocity and we have to use that as a scaling parameter for all the modeling equations where we going to right in order to define the average flow rate we are going to use the first observation that the volumetric flow rate is the conserved property. So, now, we can define m dot which is the volumetric flow rate at any cross section is basically average over the whole area density into the local velocity, local velocity into d A c right that is the deferential cross sectional area. So, that is the density of the fluid that is the local velocity multiplied by d A c integrated over the whole cross section will tell you what is the mass flow rate at any location x.

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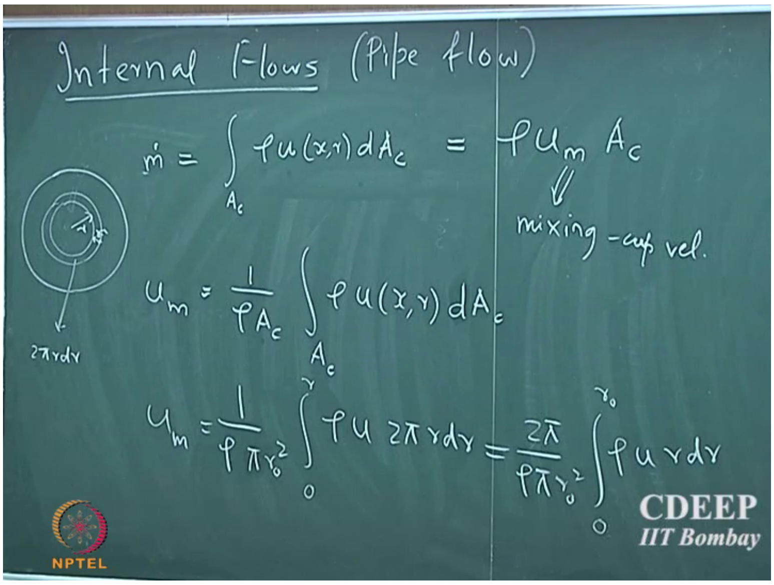

So, now we can define this as rho into. So, that is the definition u m is the average velocity and in fact, I put m here because in classical literature it also called as mixing cup velocity called cup mixing or mixing cup either way it is used. Mixing cup is basically area average or cross sectional average were quantity is called mixing cup quantity that multiplied by the cross sectional area. So, that is a definition. So, from here we should be able to find out what is u m. So, u m is 1 by rho A c integral over the cross section rho u into d A c what is d A c, what is the differential area and a cross section, come on.

Student: (Refer Time: 15:46).

2 pi r e r, it is not hard. If I draw a cross section an if I take a small angular element inside and if this is location r and this thickness is dr the area of the shallot 2 pi rdr. So, I can plug that in here u m will be 1 by rho area of cross sectional area is pi r naught square because r naught is the radius of the tube integral 0 to r rho u 2 pi rdr. So, that will be 2 pi by rho pi r naught square integral r naught rdr. So, now, if I assume that it is an incompressible fluid. So, suppose if I assume that it is an incompressible fluid, an incompressible fluid, then the density does not change, so I can pull the density out of the integral. So, u m which is the cup mixing velocity that will be 2 by r naught square integral 0 to r naught u rdr that is the definition of the mixing cup or the cross sectional average velocity.

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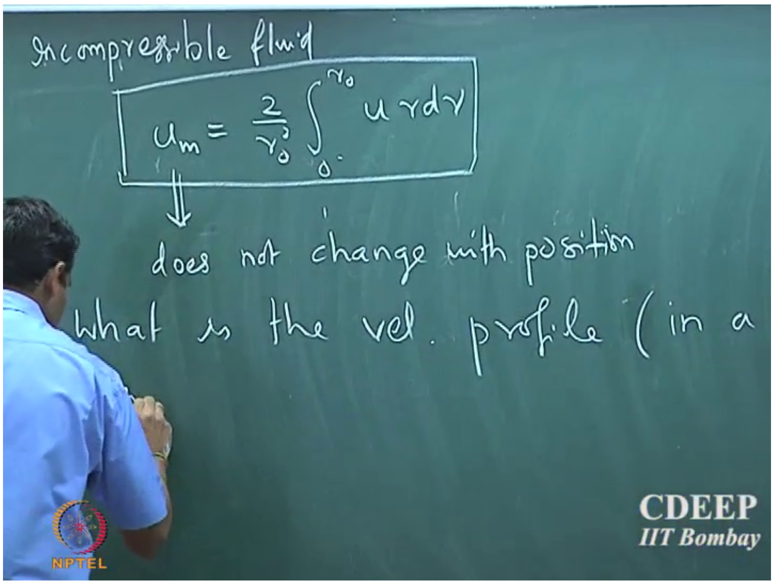

Now, this is valid at every location if it is in a fully developed regime then u is not a function of x position after the fully developed regime, but before the fully developed regime u is the function of both x and r and therefore, you have to consider the whole dependence of radius at all locations starting from 0 to if l is the length of the tube.

So, u m is a quantity which actually does not change with position after in the fully developed regime. So, the average it is not change with x position and. So, it is a good reference velocity, but if you need to find u m you need to know what u is right. So, if I do not know what the local velocity is at least in fully developed regime then I will not be able to find out what is the average velocity. So, the next exercise is to find out what is the velocity profile at least in the fully developed regime, what is the velocity profile in a fully developed regime, at least in a fully developed regime.

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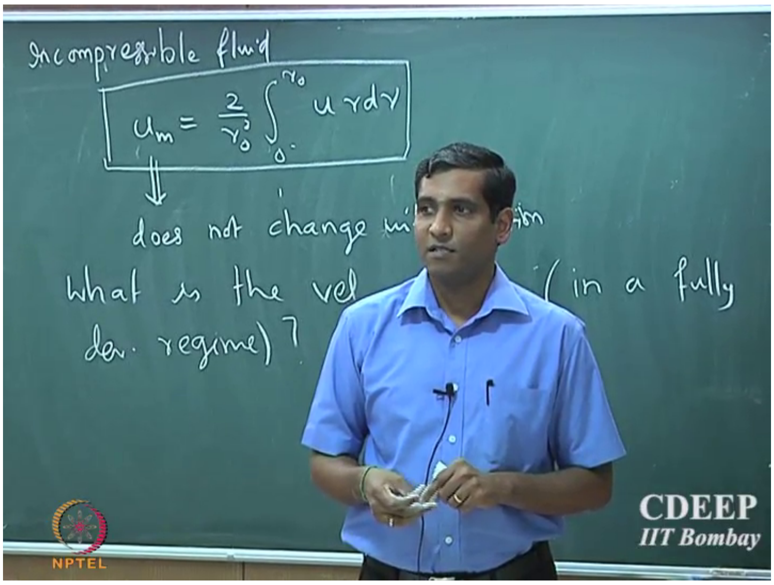

How we find this?

Student: (Refer Time: 19:18).

Yeah.

Student: (Refer Time: 19:22).

Navier stroke equation, well that is too compact that is true navier stroke equation is valid everywhere in the tube which is solve them you should be able to get some simple ways to do that. What are the forces that are acting in a fully developed regime? Let us draw, let us draw pipe.

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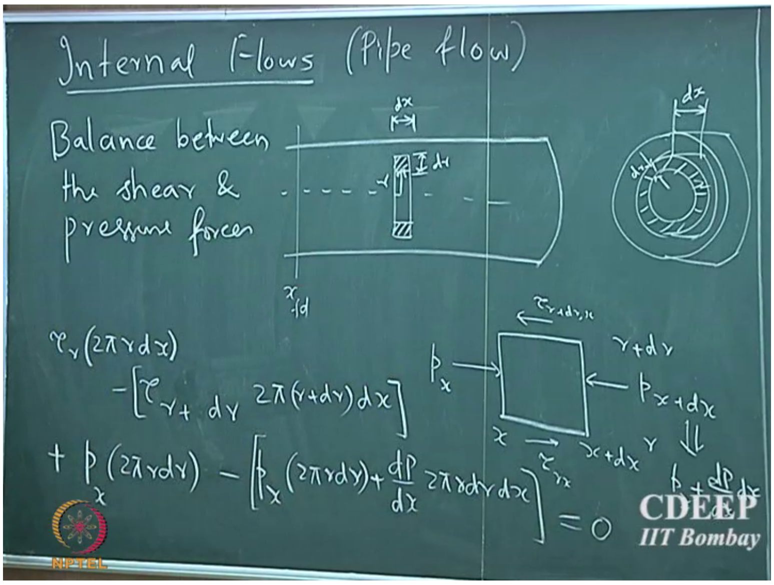

So, suppose this is my fully developed regime what are the forces that are acting on the fluid particles in the fully developed regime - pressure and viscous force right. So, if I can make a balance between the tube, I am done. So, what we going to do is we going to make a balance between the shear and pressure force. If I can make a balance am done right.

So, now, if I take a shell angular I will draw the cross section. So, if I have an angular cross section and let us say that the, so this is dx and this is d r, equivalently here this is dx and this is dr and this is located at some r position. So, if the small shell is located at some r location and I can make a force balance. So, supposing if I take this box here, if supposing p x is the pressure that is acting on that element in that location and if the pressure that is acting here is p of x plus dx so this I can write as t x plus d p by dx into dx using Claurin series expansion and if the Shear force acting is tau r tau r x and this will be tau r plus dr x. So, I can make a force balance now. So, what will be the balance? The tau r is the acting on what area, what is the area? 2.

Student: (Refer Time: 22:49).

Yeah.

Student: (Refer Time: 22:50).

Be careful it is an (Refer Time: 22:52). So, it is a circular angular right. So, the inside the area is 2 pi r into dx right. So, the area in which tau r is acting is 2 pi r into dx minus tau r plus d r, I can write it as tau r so that will be 2 pi r plus dr sorry 2 pi rdr into dx. So, that is the shear at the outer end of the shell plus I can write the pressure forces. What is the area at which is acting? 2 pi rdr. So, note that it is a angular right. So, 2 pi rdr is the area of this angular shell. So, that will be 2 pi rdr minus p x into 2 pi rdr plus d p by dx into 2 pi rdr into dx. So, that is the, and that should be equal to 0 under steady (Refer Time: 24:11).