Heat Transfer

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Lecture – 28
Flow past cylinders

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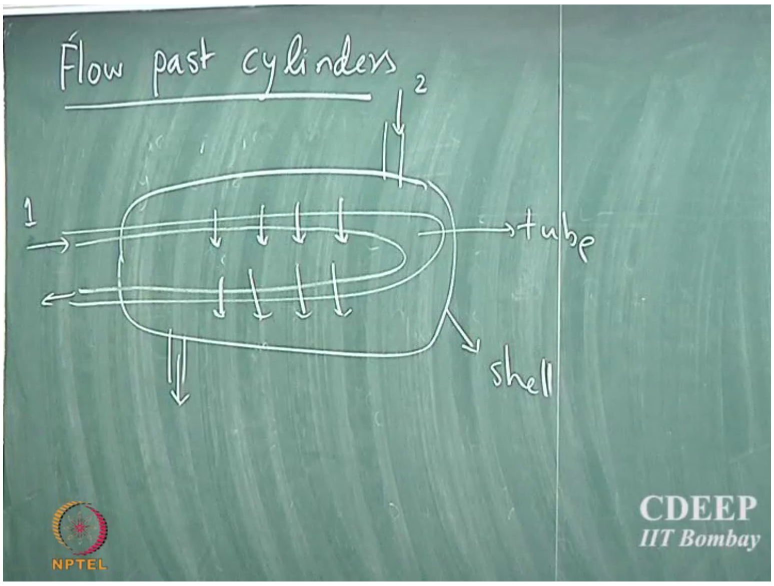

So, we going to move to the next topic flow past cylinder. Now, flow past cylinder is actually much more common than flow past of flat plate. So, for example, if you look at the humidifying towers there is a humidifying tower behind the library if you look at the humidifying tower there are tubes which are going inside and. So, there will be fluid which is flowing passed these tube if you look at heat exchangers most of you have visited these r c f right. So, you must have seen these heat exchanger. There are these shell and tube heat exchanger. So, where you have in shell and tube heat exchanger the typical sketch of that is we have an index stream.

So, we have 2 fluids fluid one is actually going through this tube. So, that is the tube. So, this is called the tube and this is called the shell that is why it called the shell and tube exchanger. So, you see a tube one of the fluid is flowing through the tube and this is another fluid which is flowing through the shell side now it could be that fluid one is a hot fluid and fluid 2 is a cold fluid. So, there is heat exchange between fluid one and fluid 2 as it goes through this shell and tube heat exchanger. So, therefore, you can see the fluid tube is actually flowing past these circular geometry and. So, it is important to understand how to find heat transport coefficient when a fluid is actually flowing past a circular object or a cylinder.

So, that is what we are going to see in to today in rest of todays and the next lecture.

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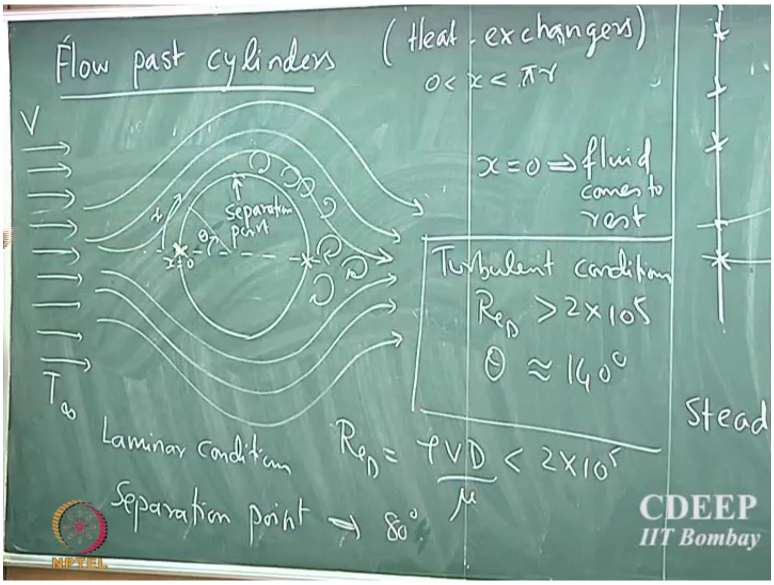

So, here a direct application is heat exchanger that is the direct application of flow past cylinders. So, suppose I take a cross section I take a cross section of this cylinder any cross section and let us say that there is fluid which is flowing past this cylinder let us say v is the upstream velocity which is the velocity very far away from the cylinder. So, here is a cylinder which is actually going inside to outside of the board and this is the cross section and you have a fluid which is flowing at a certain upstream velocity where v is the velocity which is very far away from the cylinder let us assume that the temperature of the fluid is t infinity.

So, let us say that this is the; it say I call this as x equal to 0 allocation x equal to 0 and I define my coordinates x as the curved surface location I can always give that. So, now, x really goes from 0 to pi, pi r 2 pi r is the 2 pi r is the circumference also pi r is the distance going from this end to this end and it is symmetric. So, if I know what happening on the other side, I can a easily find out what is happening on the lower bottom of the circular. So, it is a symmetric system x symmetric system and now what will be the velocity of the fluid at x equal to 0. So, the fluid actually comes to rest ok

In fact, if you look at the profiles you will see that the fluid actually never reaches this position x equal to 0 because they are actually going past this cylinder. So, this is the fluid is actually taking this path right. So, really the fluid is never going to encounter this location x equal to 0, it is never going to experience that location or rather the cylinder is never going to experience the fluid at that location x equal to 0. Now if you look at the fluid which is flowing well past this cylinder they are not in direct contact with the cylinder at all the only way it feels the presence of the cylinder is because of the deceleration that the fluid experience because of the presence of the cylinder far away ok.

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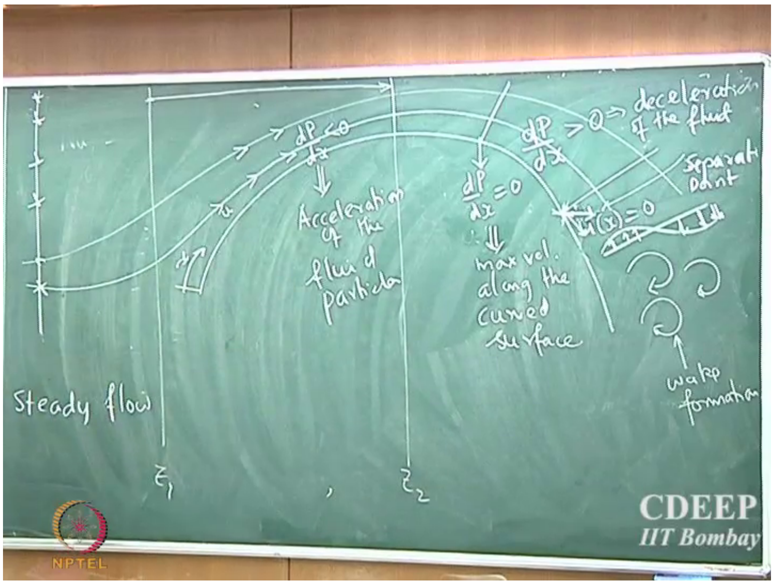

So, now; so, suppose I zoom up the upper half of the cylinder and we have a fluid which is flowing past this upper half. Now what happens to the velocity if I want to know what is the velocity of the fluid at that location compare to the velocity of the fluid which is flowing well above this cylinder which one will be higher well above or well below. So, remember supposing I take a fluid stream which is flowing in this location now the distance that it has to travel supposing I mark this as some location z 1 and I mark this as some other location z 2.

So, the distance that the fluid travels the fluids stream which is present well above the cylinder the distance is travels to reach z 2 is actually smaller than the distance that the fluid particle which is presented this location although they would have started at the same plane right. So, if you look very far away upstream, all of these particles would have actually started at the same time.

So, the distance that the fluids stream well above covers in order to reach the same plane is much smaller than this fluid particle that has to reach this location. So, therefore, the fluid particles which is present close to the cylinder or which is experiencing the presence of the cylinder; they have to accelerate in order to catch up.

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Supposing if it is steady state.

Suppose you have a steady flow supposing you have a steady flow where the volume of the fluid that you are actually pumping past the cylinder is constant. So, whatever fluid particle that is started at this location, they have to catch up, before they leave the cylinder. So, in order for it to catch up, these fluid particles, they have to accelerate and they have to catch up with this fluid particle which is flowing well above the cylinder.

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Same thing can be said for.

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Yeah.

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But the distance travelled is the same.

So, remember that the fluid particles in a flat plate the distance travelled is the same there the velocity no, no, no, there the velocity difference that you seen in your flat plate is because of the friction that is experience by the flat plate friction that the fluid experience because of the presence of the flat plate and you will see the same thing here in a short while, but supposing even if you have a even.

If you have a friction even if you have this cylinder in order for the fluid to leave at the same time they have to catch up that is not true in a flat plate in flat plate; what happens is that the momentum which is present before the experience of the flat plate is distributed because the fluid particles are now moving in both x and y direction if you have fluid which is moving only in the x direction before the plate is experience, but in the boundary layer because of the presence of the flat plate its moving in both direction, but the effective mass flow is the same.

Unlike in cylinder what happens is that if you want to have same mass fluid or the same fluid particle which started at some location and they have to cross the location in the same point then these fluid particle which is travelling at a larger distance they have to accelerate in order to catch up. So, therefore, if you look at the pressure gradient to note that here x coordinates the curve linear coordinate not the linear coordinate. So, x direction is the; it is the location on their along the curved surface and. So, d p by d x along the curved surface that has to be less than 0 in order for the fluid to accelerate.

Now, that is not true everywhere this happens only at certain location. So, what happens is that the fluid particle will accelerate and the momentum that it carries will actually accelerate up to a certain location and at some point it is not able to manage depending upon the upstream velocity whatever is the magnitude of the upstream velocity, it is not able to catch up with this speed that it has to and. So, what happens is that the pressure gradient will go to 0 at some location. So, let us say at this location is not always at this location. So, let us say at some location the pressure gradient will go to 0.

So, that is results in the. So, that is results in the maximum velocity. So, there will be a; this corresponds to the maximum velocity along the curved surface. So, remember that the distance that it travels along this location now because the distance is much larger the fluid particle has to accelerate, but then when it leaves the cylinder when the fluid particle leaves the cylinder it has the cannot have any acceleration because it is now going to it has its travelling the same distance as that of the other fluid stream after the fluid have left the cylinder. So, therefore, there has to be location where the fluid particles are decelerated so that the increase in the velocity in order to catch up must be decelerated.

So, that it reaches to same velocity as that of the upstream as that of the free stream fluid otherwise there is going to be a relative motion between the fluid and you cannot have a steady flow. So, therefore, after the location where the maximum velocity e occurs there will be a deceleration of the fluid and so, you experiences the situation where there is a an adverse pressure gradient this results in the deceleration of the fluid this results in the deceleration of the fluid particles and this results in the acceleration of the fluid particle.

So, remember that the location where you get a maximum velocity; where the pressure gradient goes to 0, but depends upon the upstream velocity; it does not; it is not true that the location where you have d p by d x is 0 or a maximum velocity it occurs only at one location. So, it completely depends upon the upstream velocity and how much it is able to bare the acceleration of the fluid.

So, now at some location at some location what happens is that the fluid is now decelerated, but then it is decelerated to a certain extent that the velocity would come to 0 to the deceleration. So, the deceleration is. So, much that at a certain location what happens is that the fluid particles will is no more able to sustain the adverse pressure gradient and therefore, the fluid particles will come to rest now what happens when it comes to rest it is not able to move further when the fluid particle comes to rest when the velocity is 0 is not able to move further anymore and so, a separate sound from the cylinder. So, this point is called this separation point.

To the fluid is no more able to move forward because the adverse pressure gradient to. So, much that the velocity comes to 0 the moment it comes to 0 it separates out from the cylinder it is not able to move past the cylinder anymore because the adverse pressure gradient into so much that it is not able to manage to move past the cylinder. Now once in separate sound once it reaches the separation point. So, this is the separation point where the velocity of the fluid stream at the interface is 0. Now once is separate sound the pressure gradient has not changed the pressure gradient is still adverse pressure gradient while the fluid particles above or moving forward. So, in order to keep up with the pressure adverse pressure gradient what the particles will do is they will start moving in the backward direction.

So, you will start having a profile a velocity profile where the fluid which is close to the cylinder. So, note that this is at the leading edge of the cylinder is not at the lagging edges is the leading edge where the fluid particles now very close to the surface will start moving in the reverse direction because it is not able to sustain the adverse pressure gradient and some of these particle will start moving which is far away from the surface will continue to move in the forward direction. So, you will have some particles which is moving in the reverse direction of the backward direction and some fluid particles which is move in the forward direction. So, if this continues then what happens is the circulation of this fluid particle. So, this leads to what is call the wake formation or internal circulation of fluid particles in the local gradient. So, this is called the wake formation ok.

So, there is a series of interesting events that actually occur to these fluid particle which is flowing past the cylinder. So, it first experience to an acceleration and then it is not able to sustain the acceleration. So, the velocity goes to a maximum and stops falling starts falling down and there is an adverse pressure gradient in order to catch up with the rest of the fluid stream and it is not able to manage the adverse pressure gradient and therefore, the velocity comes to 0 and so, it leaves the surface and then its start circulating the cylinder.

So, that is exactly what happens in this location here. So, what you will see in this location is that the fluid particle will start rotating and these will be wake formation similar to what you would see in a turbulent flow where you have constant mixing and circulation of fluid you will see that in all the hatrodynamic conditions both laminar and turbulent conditions then there is flow past cylinder.

Now, the only different between laminar flow and turbulent flow here is that the location where the separation other. So, this separation point for laminar flow is less than ninety degree. So, if I call this as phi theta for laminar condition once again it defined by the Reynolds number. So, here the Reynolds number is defined based on the diameter of the cylinder d is the diameter of the cylinder. So, Reynolds number is defined based on the diameter of the cylinder. So, if this is less than 2 into tenth of five this is less than 2 into tenth of five considered as laminar conditions an under laminar conditions the separation point is approximately at eighty degree angle.

So, its somewhere in the 80 degree angle in the laminar conditions while under turbulent conditions I can write here under turbulent conditions the angle of separation is about 140 degree oops that is not a c not temperature sorry. So, the angle of separation the separation point occurs at 80 degrees; 80 degree angle when it is laminar condition while it is turbulent condition it occurs at 140 degree.

So, this is interesting because the wake formation that you would observe in laminar condition, you will start seeing wake formations the right next to the cylinder at for a for a pretty good locate pretty good distance when the fluid is flowing past the cylinder while under the turbulent condition you will see that the wake formation is only toward the leading edge of the cylinder and not very close to top surface.

So, this is a very very different phenomena compare to what you see in a flat plate case where you will never see a wake formation in the laminar conditions while you will see recirculation of the fluid even under laminar conditions when the fluid is flowing past the cylinder and that is because of the geometry.

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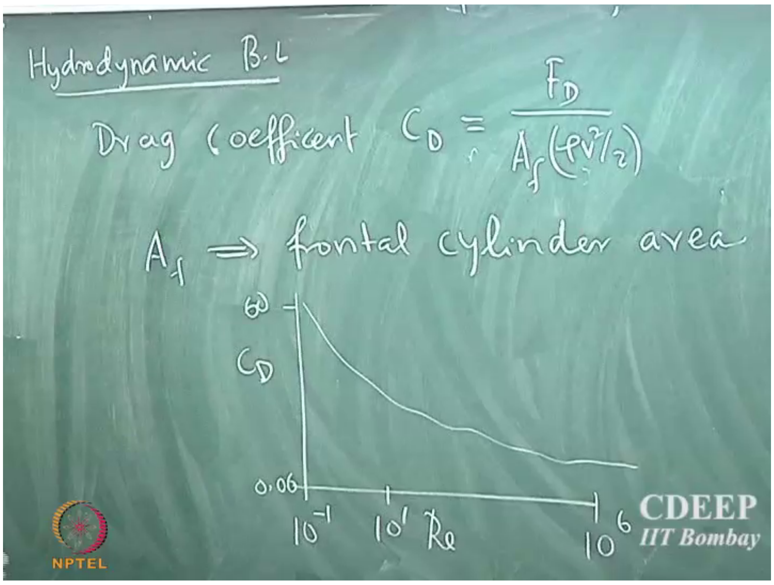

So, the hydrodynamic condition boundary layer similar to friction coefficient, if here what you have is called the drag coefficient. So, very similar to the friction coefficient; that was defined earlier, it is called CD and that is given by the drag force divided by A f rho v square by 2. So, A f is the surface area of the front side of the cylinder. So, this is the;

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That is the frontal area that is the front curved surface of the cylinder. So, the based on that the drag coefficient is defined and if I look at the drag coefficient as a function of Reynolds number as a function of Reynolds number. So, you will have a slightly decreasing, it is a decreasing function and the reason is very simple. So, remember that when you have laminar conditions the separation occurs at a much smaller angle and that because the drag is much higher and a turbulent conditions because the velocity is much higher the drag is not that high and therefore, the drag coefficient is much smaller also I can put some numbers here.

So, this is tenth on minus 1, it is a low plot tenth on one about tenth on 6 and this goes from 0.06 to 60. So, that is the profile for drag coefficient. So, when we do the when the boundary layer analysis which will not do in this class.

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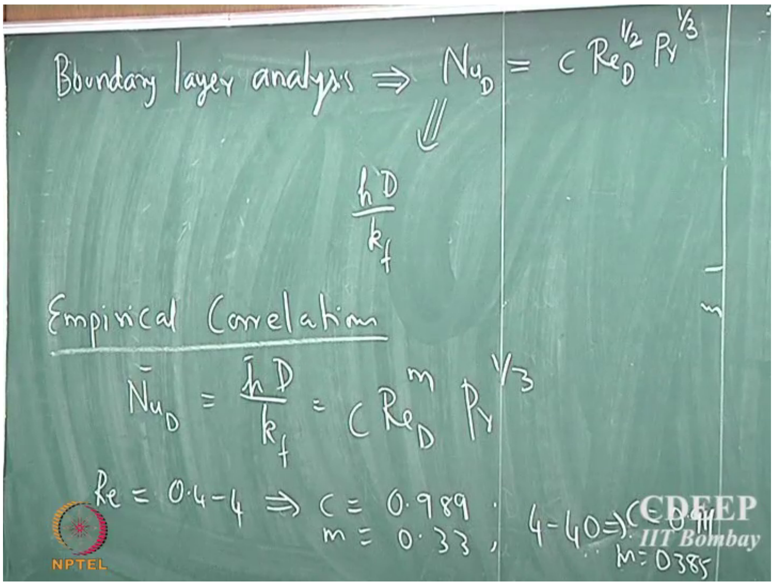

So, the boundary layer analysis suggested the boundary layer analysis suggested that the Nusselt number that is defines now based on the diameter of the cylinder. So, this is given by h into d by k f defined based on the diameter of the cylinder. So, that is now some function of the power of half and Prandtl number to the power of 1 by 3.

So, that is again based on the boundary layer analysis and. So, there has been empirical correlation which is available for this problem. So, there are no clean theoretical predictions which is available. So, one has to rely on the empirical correlation which has been observed by performing repeatedly several experiments. So, the empirical correlation is one by three and. So, if C is C Reynolds number 9 m is 0.33 and for between 4 and 40 C is 0.911 and m is 0.385. So, that is the correlation that is available for Nusselt number and so, a general correlation is call the Churchill Bernstein.

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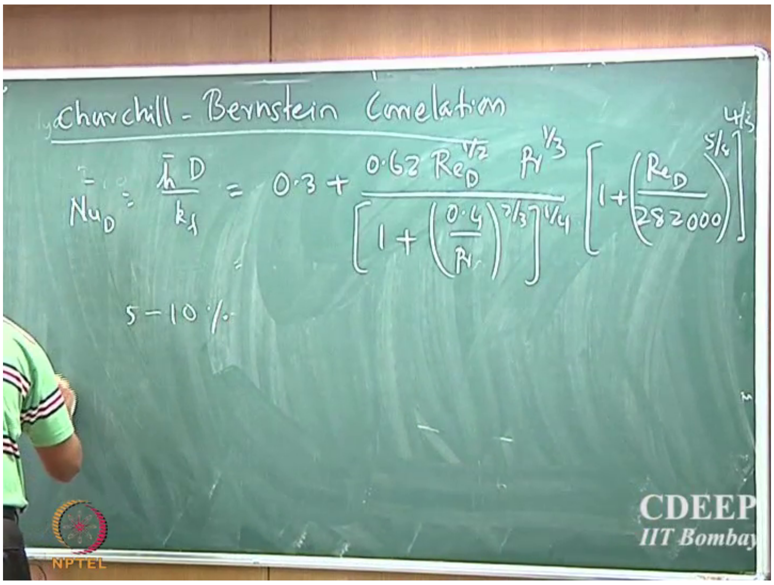

So, Churchill Bernstein correlation which are; which is useful for estimating the heat and mass transport coefficient for flow past cylinders. So, that is given by. So, note that it is a correlation based on experiments to the best you can get is the average Nusselt number you will not able to find the local quantities anymore plus 0.62 R e 3 1 plus 0.4 and by 4820. So, you will start seeing all kinds of correlations because one cannot get analytical solutions for some of these problem and one cannot even get approximate analytical solution. So, even numerical solution become a very very non formidable also one has to result to these kinds of correlation and note that these kinds of correlation do not always give a very good prediction.

I would say that the error is somewhere between five and ten percent again this is been verified for some experimental conditions. So, there are always some errors in the prediction of the heat and mass transport coefficient of course, from because we know that the boundary layers are similar one could always replace.

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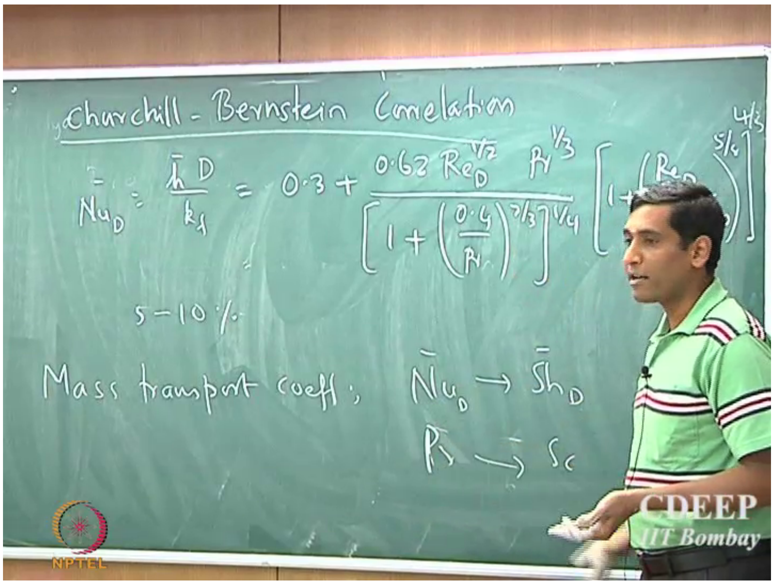

If you want to get mass transport coefficient; so, we simply replace the Nusselt number with the corresponding Sherwood number and Prandtl number with the corresponding Schmidt number you get the correlations for the mass transport problem. So, we there is no point in rewriting it is for mass transport except that you have to just replace Nusselt number with Sherwood and Prandtl number with Schmidt number.