Heat Transfer

Prof. Ganesh Vishwanathan
Department of Chemical Engineering
Indian Institute of Technology, Bombay

Lecture - 27
Flow past Flat plate II: Correlations for heat & mass transport

So, similarly one can do for the energy balance and the mass balance. So, what you will you see for energy balance? You can now write energy balance in terms of the modified or new variable.

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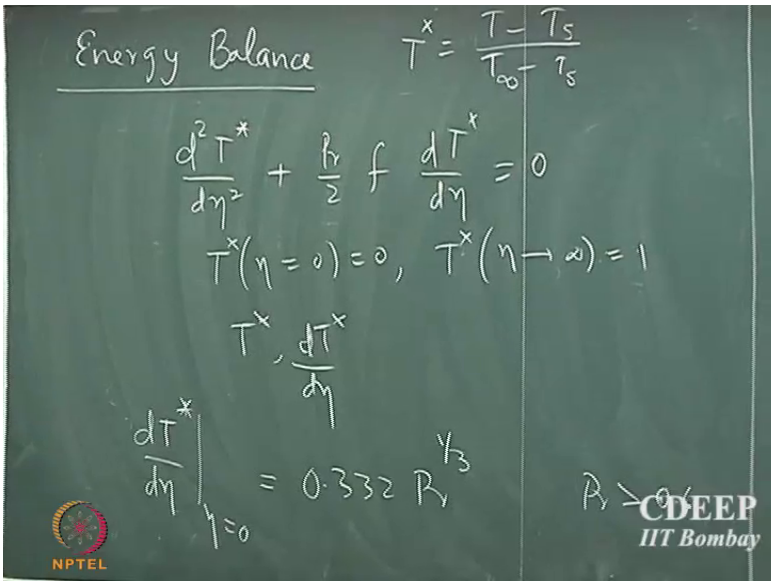

While defining T star as usual as T minus T s by T infinity minus T s i. So, defined this dimension less temperature, and that I can find that the momentum balance will reduce to. Sorry the energy balance will reduce to d square T star by d eta square plus Prandtl number by2 into f into d T star by d eta is equal to 0, and T star at eta is equal to 0 is 0 and T star as eta going to infinity that is equal to 1. So, that is the functional form. And at once again you can find T star d T star by d eta.

So, if you know these two quantities then we should able to find the heat transport coefficient. So, what is important here, once again is to find the heat transport coefficient. So, what was found is that d T star by d eta at eta equal to 0. So, this scales has 0.332 into Prandtl number to the power of 1 by 3. So, note that we never knew what is exponent N is suppose to be. So, by solving the actual equations what has been found, is that the gradient at the boundary scales as Prandtl number to the power of 1 by 3.

To the scaling for Reynolds number comes from eta, and the scaling for Prandtl number actually comes from the gradients. So, far I asked to assume what is the functional form, but actually the correct proof for the functional form is that the gradient scales as Prandtl number to the power of whatever exponent, and that exponent tends out b 1 by 3. By the way this is only when Prandtl number is greater than 0.6. So, less than 0.6 you have a different functional difference. So, this N is going to be different when Prandtl number is less than 0.6. What is it mean and yeah.

Student: (Refer Time: 02:52).

That is the property, because not that this is the solution of non-linear equation. You cannot say that you will have a linear dependence on Prandtl number. so; obviously, its the its the function of.

Student: (Refer Time: 03:03).

That is what I am saying. If it is linear dependence then you could intuitively guess that it, it is going to have similar functional form with respect to Prandtl number, but then this exponent 1 by 3, exponent 1 by 3 has been observe only when Prandtl number is greater than 0.6, and that is not difficult to understand. What is Prandtl number?

Student: (Refer Time: 03:26).

Whats Prandtl number. Yes Prandtl number is mu over alpha, it is actually not very difficult to understand why that’s, because Prandtrl number is mu over alpha.

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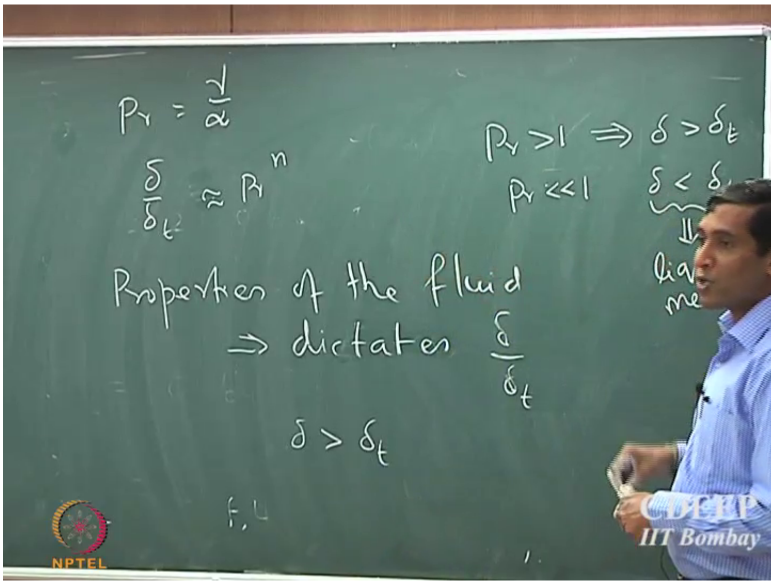

So, note that we also said that delta by delta t which is the ratio of the boundary lay, momentum boundary layer thickness to be thermal boundary layer thickness. So, that is scales as Prandtl number to the power of n. So, the boundary layer thickness that you are going to get the momentum and thermal bound rate it this, is now going to be dependent on the properties of the fluid. So, if the properties of the fluid. So, properties of the fluid is now going to dictates the delta by delta T fine. So, suppose if delta is greater than delta t. suppose if delta is greater than delta t, what will be the nature of the Prandtl number.

Student: (Refer Time: 04:38).

Yeah, it will be greater than 1 or less than 1.

Student: Greater than 1.

Greater than 1, but remember there is an exponent 1 fine. So, so if Prandtl number is greater than 1. If Prandtl number is greater than 1, you will always see that delta is greater than delta c. What does it mean? To be set that the momentum diffusivity in the y direction that is.

Student: (Refer Time: 05:05).

Oops sorry t that should be thermal boundary. So, the [ma/momentum] momentum transport depends upon the boundary layer thickness, and the heat transport depends upon the thermal boundary layer thickness. So, the length scale of diffusivity of momentum and the thermal boundary layer. I mean thermal diffusivity. These two are going to strongly depend upon the boundary layer thickness.

Now, suppose if delta is less than delta t. In fact, there are situations where this is the case. This is true for liquid metals, the place where it is used is, the way aluminum is extracted as you have a very high temperature process, where the aluminum is actually manufactured at a molten stage. So, you have a liquid metal which is presents. So, in those cases, you will see that the Prandtl number is actually much smaller than 1.

So, there the boundary, the heat transport process is completely different, because the boundary layer thickness of the momentum boundary layer is going to be, it is going to be smaller than the thermal boundary layer. Therefore, the heat transport properties are completely different. In fact, that is why the exponent is different. To this 0.6 is the special case where, if you have a liquid metals where you have a very high temperature process, then the heat transport mechanism is slightly different. And therefore, you will see there the solution falls out to be different exponent, and we will see that in a short while, may be not todays lecture. I will give you; what is the expression for the exponent N when Prandtl number is actually less than 0.6 ok.

So, with this actually we can go and find out what is the heat transport coefficient. So, the heat transport coefficient is minus k d dou T by dou y at y equal to 0 divided by T s minus T infinity.

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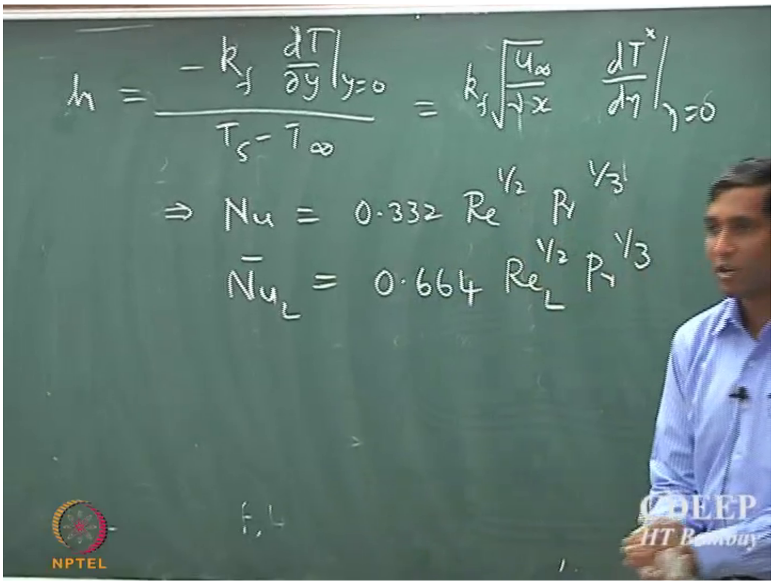

So, we introduce all the scaling, and that will trun out to be k f into square root of u infinity by mu into x into d T star by d eta at eta equal to 0 and. So, from here you find that the Nusselt number is equal to 0.332, Reynolds number to the power of half into Prandtl number to the power of 1 by 3 ok.

So, this half comes from the scaling that you used for eta, and 1 by 3 comes from the gradient of temperature at the boundary. So, the first time you are seeing a complete functional form for Nusselt number. And similarly you could find out what is the average Nusselt number. So, that will once again turn out to be 0.664 into R e to the power of half. This is based on the length of the plate multiplied by Prandtl number to the power of 1 by 3 ok.

So, he showed that the friction coefficient is given by 0.664 divided by the square root of Reynolds number, local Reynolds number.

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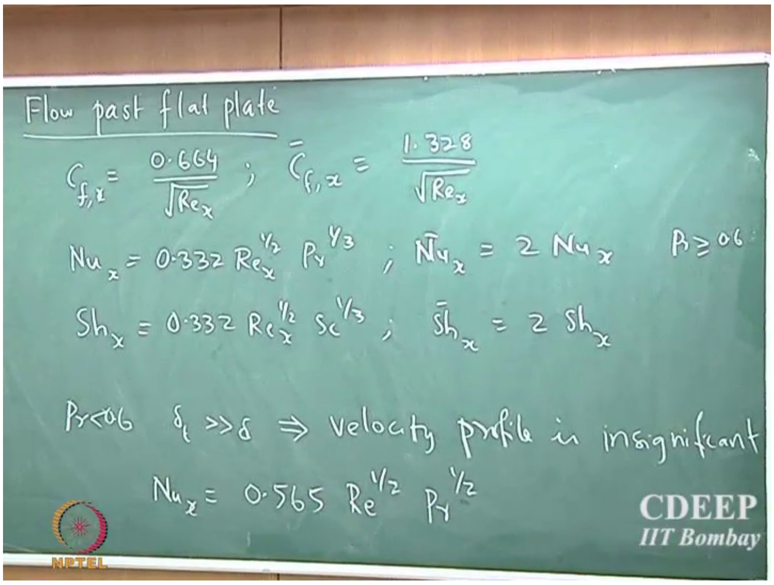

And we have also said that the average friction coefficient is given by one average friction coefficient in that location. Then similarly we said that Nusselt number, the local Nusselt number is given by 0.332 Reynolds number to the power 1 by2 and Prandtl to the power of 1 by 3. And these scale with power of Reynolds number comes from the scaling arguments, and the Prandtl number to the power of 1 by 3 comes from the gradient at the boundary and. So, from this you can see that the average Nusselt number, till any location is given by twice that of the local Nusselt number.

Can similarly we could write for the flaws transport problems, simply by using the boundary layer analysis. So, we said that this is valid only when Prandtl number is greater than 0.6. So, when it is less than 0.6. So, what happens is that the, with thermal boundary layer thickness, is usually larger than. So, very small Prandtl number. The thermal boundary layer thickness is much larger than that of the momentum boundary layer thickness, which also implies that the velocity profile is insignificant. So, it is almost like a flat velocity profile.

So, the boundary level velocity profile is completely rid off, not very significant and therefore, it is not going to affect the heat and mass transport process and. So, the correlation corresponding to that would be. I will copy from it, while d 0.565 and Reynolds number to the power of 1 by 2 and Prandtl to the power of 1 by 2. So, you get a different scaling for Prandtl number, because the velocity is now not playing a significant role in the heat transport process.

Student: (Refer Time: 11:10).

By solving the equations. You have to make an approximation. Do not by you solve the momentum equations. Now you do not have to solve the momentum equation assuming that the velocity is constant right, you say it is a flat velocity profile. So, now, for all values of. So, for any values of P R Prandtl number, a general correlation would be all values of Prandtl number, and that would be every 0.87 1 by 2 Prandtl 2 the power of 1 by 3.

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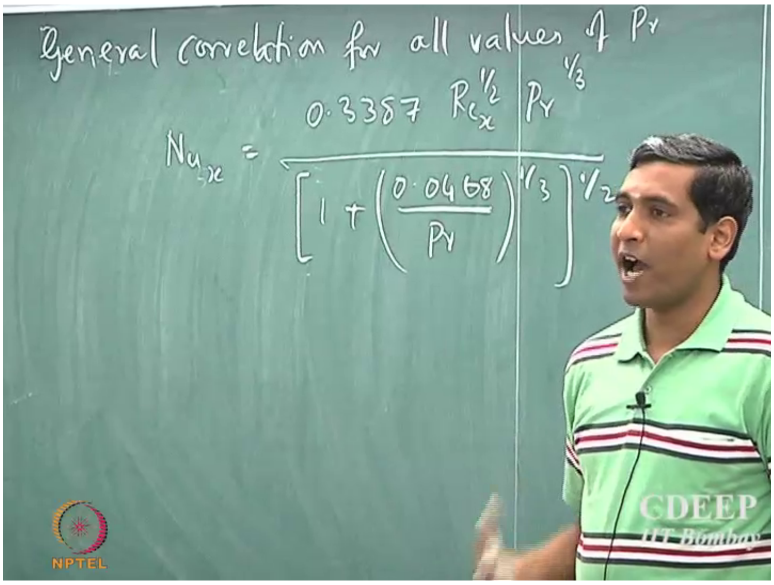

And 1 plus. So, remember that here we said that we. So, if you want to use correlation for any range of Prandtl number.

Now, this becomes important, particularly when you have fluid which is moving of when you have multiphase systems where different fluids are moving, then you have a different kind of problem and. So, you need to have affective Prandtl number, which is based on the affective properties of these two fluids and. So, in those cases you will have different ranges of Prandtl number and. So, it is important to have a general correlation which works. Now this is primarily for the purposes of, somebody who is working in an industry. He doesnt know what we cannot differentiate between Prandtl number of 0.6

less or 0.6 greater than 0.6. So, it is just for a general correlation, which is useful to you, useful in certain calculation purposes.

But for the course purposes, what is really important is these two individual correlations which is valid for different ranges of Prandtl number all right. So, one could actually do a similar exercise for turbulent conditions.

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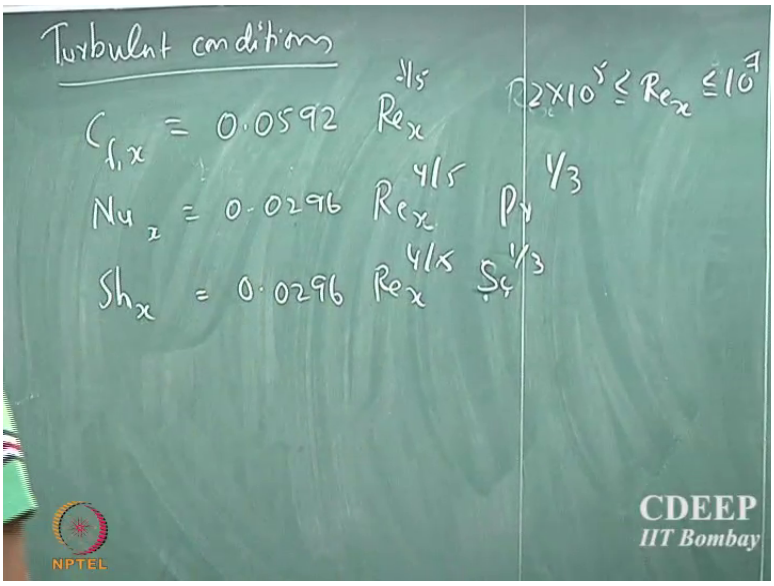

One could take similar expressions for turbulent conditions. So, the friction coefficient will be 0.0592 into Reynolds number to the power of minus 1 by 5. So, we assume that the Reynolds number is between 2 into 10 power 5, and the Nusselts number is given by 0.0296 into R e x 4 by 5 1 by 3.

And similarly. So, that is for the turbulent conditions. So, note that there is no ways to solve even to get insights into the nature of the problem. So, turbulent conditions, its impossible to solve them. So, problem even for the asymptotic analytical conditions. So, to really it just all of these have been a pain by solving the equations, either using series solutions or numerically in order to obtain these correlations.

Student: (Refer Time: 14:59).

No.

Student: (Refer Time: 15:02) how do you get out the (Refer Time: 15:04).

Some of the approximations do change. So, some of the approximation you make in laminar flow conditions it would be different in turbulent flow. Suppose I have a flat plate where does the fluid which is flowing after this flat plate, and you have some region which is laminar.

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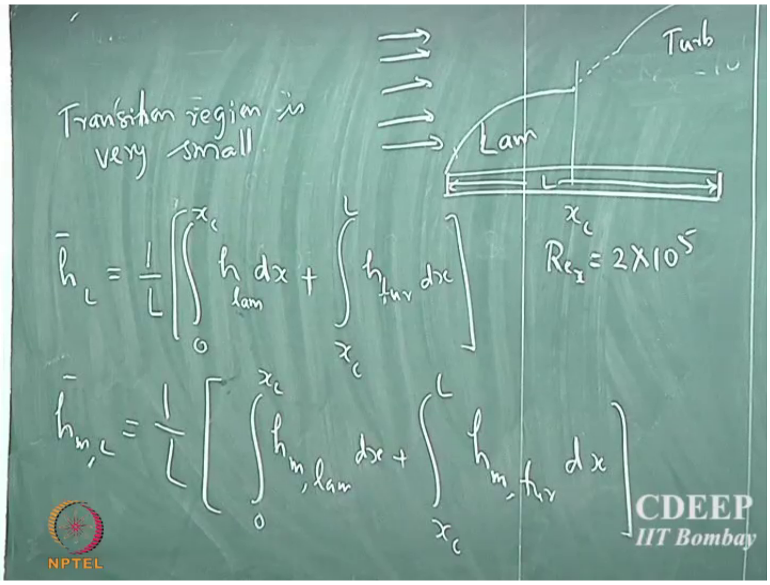

Yes we have laminar conditions here, and then of course, have a transition, and then you can have a turbulent regime. So, this is the critical western at which the Reynolds number is, what is the Reynolds number

Student: (Refer Time: 15:46) .

Critical Reynolds number

Student: (Refer Time: 15:49).

Yeah.

Student: (Refer Time: 15:52).

2 into 10 power 5. So, that is the critical Reynolds number. So, one could find. So, remember that, what we are interest is only the average heat transport and average past transport coefficients. So, you would really required to calculate the average heat transport coefficient for the full plate. So, suppose if we assume that the transition region is very small. Remember that transition region is a flow condition which has not been

characterized, and is not been fully understood as to what is the hydrodynamic conditions in the transition region. So, if we assume that it is very small. Then want to define average heat transport coefficient for mixed hydrodynamic condition, that is we have laminar and turbulent, simply as 0 to x e the x plus x c to l. So, this is laminar heat transport coefficient, and turbulent heat transport coefficient ok.

So, where the length of the plate is l; that is the length of the plate. So, one could simply integrate the local heat transport coefficient in laminar conditions up to the critical point, and integrate the same in the turbulent condition up to the full length of the flat. That will give you what is the average heat transport coefficient when you have mixed conditions of both laminar and turbulent conditions. And one could the same exercise for mass transport coefficient as well. So, if we know Nusselts number we should be able to calculate the heat transport coefficient under laminar conditions. Similarly heat transport condition under turbulent condition; so plug that in and you can integrate.

So, not going to do the integration here, in spite easy to do this integration, it is not very hard. It takes about 5 minutes to do the integration. So, you should all do this. So, so the average Nusselts number, which is defined as h into l by k f that is given by 0.664 into Reynolds number with critical Reynolds number to the power of 1 by 2 plus 0.037 into Reynolds number to the power of 4 by 5 minus to the power of 4 by 5 multiplied by Prandtl number to the power of 1 by 3.

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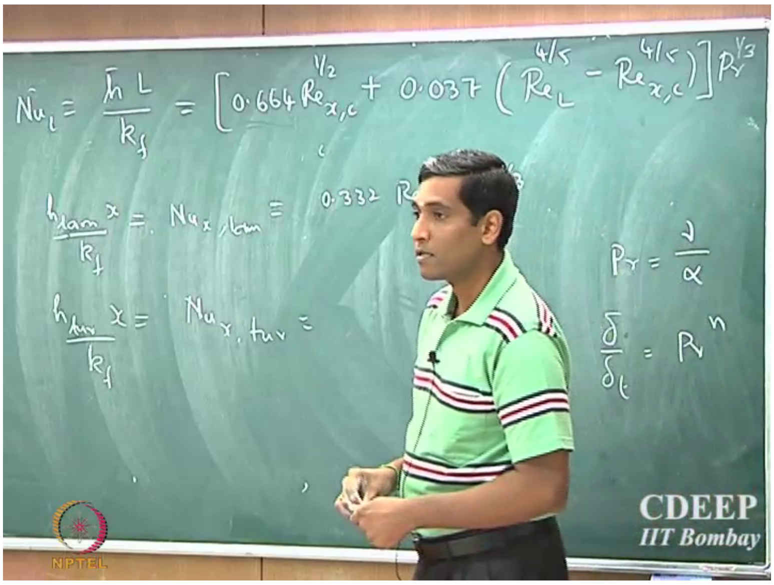

So, all we have done this, I have taken the expression for Nusselts number, local Nusselts number that is N u laminar that is 0.332 R e x Prandtl to the power of 1 by 3. So, this is nothing, but h laminar into x by k f. So, I have found out what is h laminar from this expression, and then I integrate with respect to the x coordinates, up to the critical location. And then similarly I take the expression of Nusselts number for turbulent flow that will be, by k f and then you integrate that with respect to x coordinate going from the critical location to the length of the plate.

And. So, clearly you can see that the second term here, corresponds to the integration in the turbulent regime, and the first term corresponds to the integration in the laminar regime. You clearly see that there is the 0.664 that is comes here, that is nothing, but the average Nusselt number in the laminar condition, and this is the average Nusselt number with the turbulent condition.

Student: (Refer Time: 20:34).

If you assume that the properties are constant they doesnt, but supposing if you say that the properties change.

Student: (Refer Time: 20:45).

With the hydrodynamic condition.

Student: (Refer Time: 20:46).

No no, but Prandtl number is simply defined as mu or alpha fine. So, now, if he said that delta by delta T is Prandtl number to the power of n. This N is different for laminar and turbulent. So, that N takes care of the difference in the height.

Student: (Refer Time: 21:10).

Excuse me.

Student: (Refer Time: 21:11).

Yeah that is true. So, it turns out for flat plate case, it is a same. So, note that it is not just the hydrodynamic boundary layer which changes, this is the ratio be careful it is a ratio, it is both the hydrodynamic and the thermal boundary layer actually is changing. Now when it comes to Prandtl number less than 0.6, the variations that you get in turbulent flow is very different. So, you will be careful how it is defined. This problem comes only when the Prandtl number is less than 0.6.