Heat Transfer
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Lecture - 26
Flow past Flat plate I: Method of Blasius

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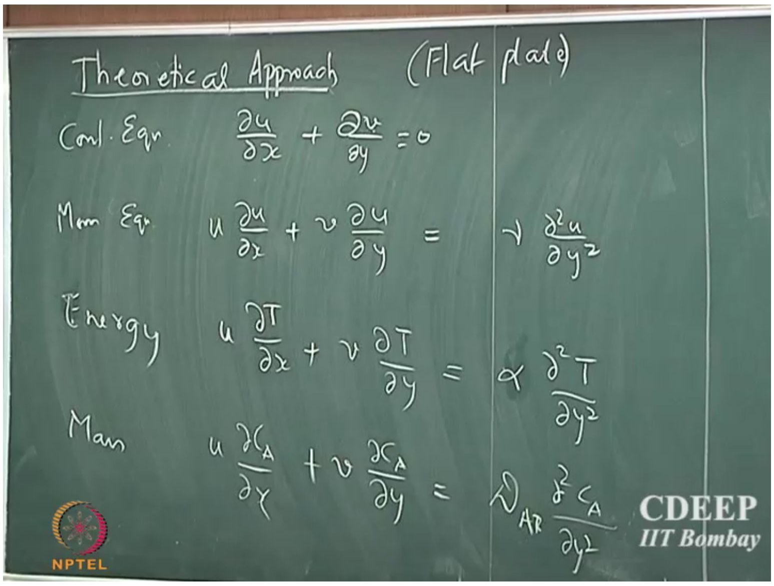

So, let us move to theoretical approach form once again for a flat plate. So, the governing equations are I have continuity equation by d x, d v by d y that is equal to 0, u and v are the x component of the y component velocities when you have momentum equation that will be u d u by d x plus v d u by d y that is equal to we said that d p by d o can be approximated to 0, will have nu into d square u by d y square ok.

And then you have energy balance that will be u d T by d x plus v d T by d y, that is equal to alpha into d square T by d y square, and the mass balance would be u d CA by d x plus v d CA by d y that will be d AB d square CA by d y square. So, those are the governing equations what are the boundary condition? u is 0 at y equal to 0 then. So, boundary conditions are u at y equal to 0 is 0 u at y equal to infinity is u infinity that is the free stream velocity.

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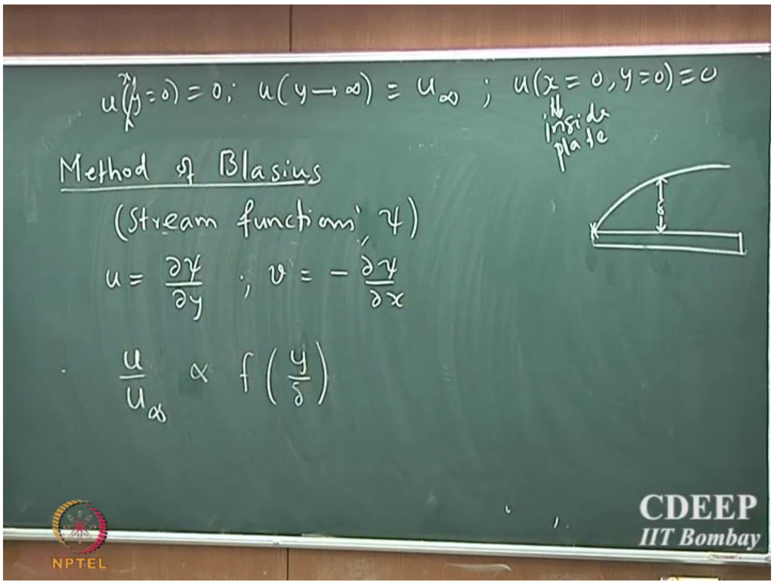

Student: (Refer Time: 02:30).

D u by d y we do not need that we are happy with two boundary condition 1 y what about x?

Student: (Refer Time: 02:42).

Which one?

Student: (Refer Time: 02:43).

Yeah.

Student: (Refer Time: 02:52).

But when you say d u by d y at y equal to infinity is 0, what is it mean? It means that the velocity there has to be constant and that has to be equal to u infinity, otherwise we do not have a; we do not have no acceleration fluid.

Student: (Refer Time: 03:10).

No, but see boundary conditions are not based on the solution that we would get, we have to use the correct boundary condition based on what is this physical problem the solution comes later. We never write a boundary condition for finding a solution, you first find out what is the correct boundary condition then you go and find the solution that is not a correct approach. And the other one is u at x equal to 0 what is it? U infinity or 0.

So, u at x equal to 0 inside the plate. So, this is very important outside the plate it is; obviously, u infinity, but inside the plate what is it?

Student: (Refer Time: 04:06) the continuity.

By continuity, but that is not continuous at the edge of the plate. So, remember what happens at the edge of the plate the fluid suddenly comes to rest, both the x component and the y component velocity is 0 in all directions. Because the fluid is suddenly experiencing the flat plate and. So, just inside the flat plate the fluid is come to complete rest at the boundary. So, therefore, u at x equal to 0 inside the plate has to be 0.

You cannot be cannot be moving, if it is moving then there will no meaning to have a plate here where which means it is a friction less plate.

Student: It is not be.

Why.

Student: (Refer Time: 04:50).

Right, but that is x equal to 0 at sorry that should be y equal to 0, I was right that should be y equal to 0 that is the boundry condition. So, now, obviously, this is non-linear look at the velocity equation, it is a non-linear equation and you will not be able to solve them solve them analytical, but then there are some (Refer Time: 05:16) ways of reducing the equations to a form which is much more amenable to these standard analytical techniques, and that is given by what is called the method of Blasius.

So, Blasius is actually one of the very very well known scientist who looked at these kinds of problems many many years ago and so, he came up with this method to solve the not actually solving and finding exact analytical solution, but at least there is a method to find the solution for the velocity profile and the and particularly to find the fiction coefficient and the heat transfer coefficient so.

Student: (Refer Time: 06:02).

First one does not include.

Student: (Refer Time: 06:08).

This is 0 at any x.

Student: (Refer Time: 06:12).

Right, but it is important to provide this boundary condition you will see because probably transformation you make in this Blasius solution, you will actually see that it will account for both simultaneously. It is important to observe that this particular condition exists you know that is all. Because when we started looking at flow past a plate we said that the fluid suddenly comes to rest at that particular location. So, if I say that your indeed right and saying that if I simply say at all x this condition will automatically satisfy, which you will see has been actually used by Blasius in one of the approximations.

So, the way to use is call this stream function, here some of you must have heard I think most of you must have heard this word in a fluid mechanics class. So, you say u is d psi by d y. So, if psi is the stream function, which describes these stream lines with which the fluids are fluid is moving and v is minus d psi by d x. So, moment you assume this the continuity equation is automatically satisfied. So, we do not have to worry about the continuity equation. Then the most important observation that was made by Blasius is that the ratio of u by u infinity in the boundary layer ok.

So, ratio of u by u infinity in the boundary layer you somehow a function of the ratio of the y by delta, where y is any y coordinate the location in the y direction and delta is the corresponding boundary layer thickness at that location. So, if delta is the boundary layer thickness here. So, what was observed was that the ratio of this velocity somehow has to be proportionate with the location of the boundary layer, that is not hard to see now, but it is very hard to see that time ok.

Well he said suppose we assume that we define a new function called eta which essentially sort of characterizes the captures the effect of y by delta.

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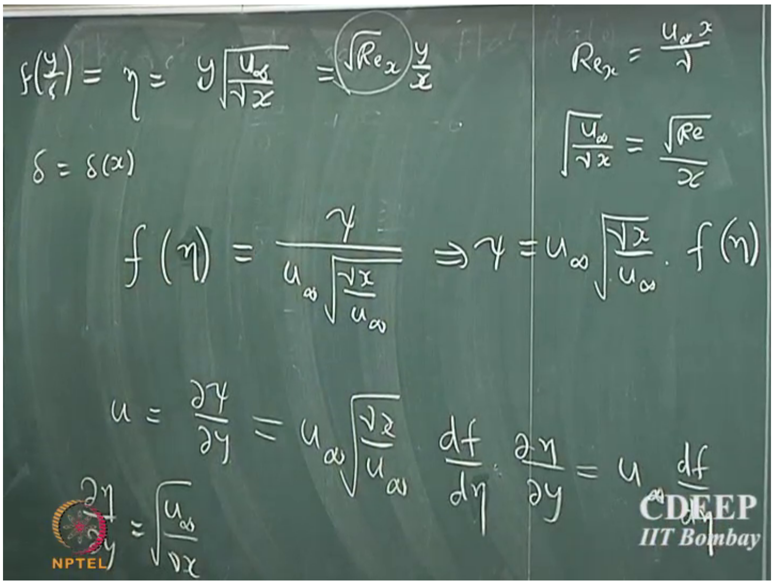

So, note that delta is the function of x position; delta is now function of the location depending upon where you are located the height of the boundary layer is different. So, you define a new quantity which is mu into x. So, that is the new quantity, it is not by magic that he came up with this approximation it is after a several trail and errors he found that this is the correct scaling that you have to use. In fact, you will see in a short while why that is the correct scaling, when we actually find out what is the boundary layer thickness at different locations ok.

So, you can actually see that there is; what is u infinity by nu into x. Any guess what is u infinity by nu into x? So, you can write this as. So, u infinity by nu into x square root can be written as square root of Reynolds number right you can write it as square root of Reynolds number divided by x. So, it has some functional form of Reynolds number. So, that is nothing, but square root of Reynolds number into y by x.

So, it is not a magic that he found this form it is actually related to the Reynolds number, it is related to the properties of the fluid flow now. In fact, you may recall that we said that (Refer Time: 10:26) number has a certain functional form which is related to Reynolds number and Plantil number in fact, that functional form of Reynolds number comes from here. So, you may recall that some of the expression you may have seen in your; those who have done the experiments.

You will see Reynolds number to the power of 1 by 2 that actually comes from here you will actually see rigorously you will derive and we will show that the functional form is Reynolds number to the power of 1 by 2, but I just wanted to mention that it comes from this approximation or this transformation that was started by Blasius all right.

So, from here you define another function called f, which is the function of eta, and that is defined as psi u infinity into square root of. So, that is the functional form for stream function, you scale the stream function also with some form of Reynolds number because Reynolds number captures the hydrodynamic conditions of the fluid flow. So, from here we can find out what is u. So, u is d psi by d y. So, from here psi is u infinity, into square root of nu x by u infinity into f. So, f is now a function of eta and. So, d psi by d y is u infinity into square root of nu x by u infinity, and I use chain rule d f by d y d f by d eta into dou eta by dou y ok.

So, I will have done is I am simply using the chain rule, and d eta by d y you can get from here that is nothing, but d eta by d y that is square root of u infinity by nu into x. So, I can substitute that here. So, that will be u infinity into d f by d eta. So, that provides an interesting functional form for the ratio.

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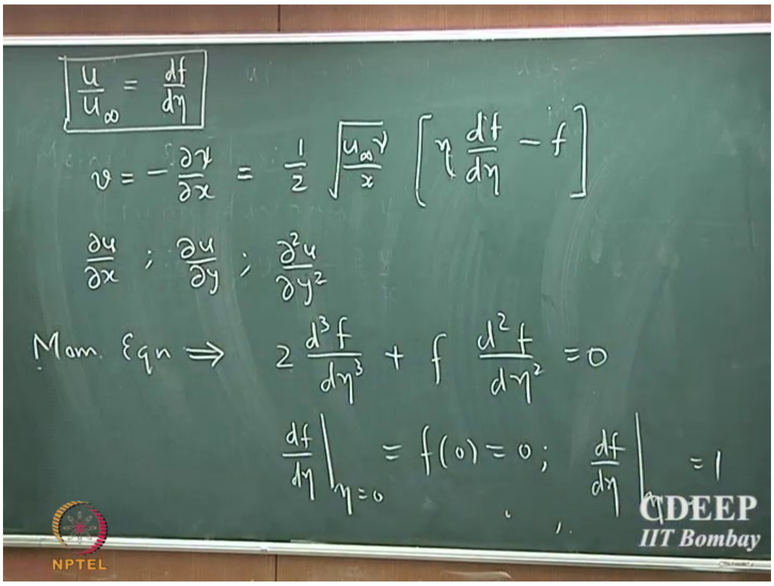

So, you can see u by u infinity is nothing, but d f by d eta. So, it is not that this functional form actually he was got by magic, because basically if you come up with the very nice way of looking at the velocity profile and looking at temperature profile which we will see shortly.

Then we need to find v, v is minus d psi by d x and you can do all the chain rule business. So, I am not going to do the chain rule here. So, there will be half square root of u infinity nu by x, multiplied by eta d, d f by d eta minus f. So, I would like to all of you to do the chain rule and convince yourself that this was the correct expression for v in terms of the stream function. So, it is not very hard, it just a couple of minute exercise. So, now, I can find out the u by d x, I can find out d u by d y, you can find out d u by d square u by d y square. If I know these three derivatives I am now transformed all the all the all the components of the momentum balance ok.

So, the momentum balance will now if you turn out to be 2 d cube f by d eta cube, plus f into d square f by d eta square that is equal to 0, and d f by d eta, eta equal to 0 x equal to f of 0 and d f by d eta equal to 1. So, that is the boundary condition. So, momentum balance essentially reduces to this ordinary differential equation. What you had earlier was the a p d e which was the function of x and y, and now you crashed into a an o d e and note that you also had the x component and the y component velocity in the previous equation.

That because you use the stream function formulation. So, all that has been crashed into a simple o d e third order o d e. I first just write all the boundary condition you will see that.

Student: (Refer Time: 15:38).

You can also bring actually another boundary condition for v; you see that you will actually get it. So, of course, it is non-linear that is not the linear equation and it is not the surprise and we are not going to get the linear equation for non-linear equation anyway.

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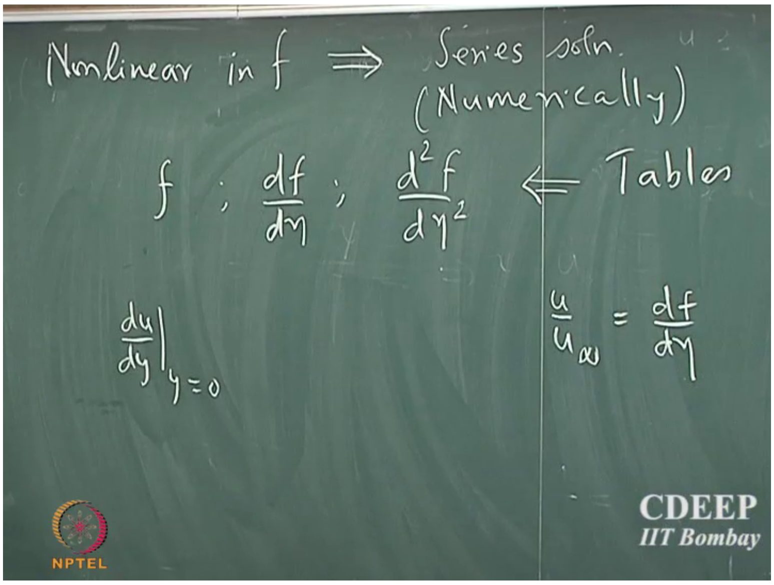

So, it is a non-linear equation, it is non-linear in f of course, these days you would be tempt to solve it numerically, but what (Refer Time: 16:10) that was he use the series solution one could of course, solve it numerically and so, it has been. So, he is worked out the is now we have table which gives you f d f by d eta and d square f by d eta square.

So, now their standard tables available for. Again these tables did not come by magic somebody has actually painstakingly worked out all these numbers for this differentially equations and. So, we have; we now have tables which gives you these numbers, it is there in your text and it is also there in other references. So, if you know this gradients you are done. So, remember that what we want really it is to find d u by d y at y equal to 0 that is what we want to find. And that is what we want if you want to find the friction coefficient because that is what we want to find we are not interested in the whole profile, it is very boring to see the whole profile. So, what is really of interest is to find the friction coefficient, and the heat transport coefficient and mass transport coefficient.

So, if we know d u by d y we are done. So, do not know the expression for u, u by u infinity is d f by d eta. So, if you want d u by d y you each know; what is d square f by d eta square. So, it is for that purpose the tables have been drawn in these three categories.

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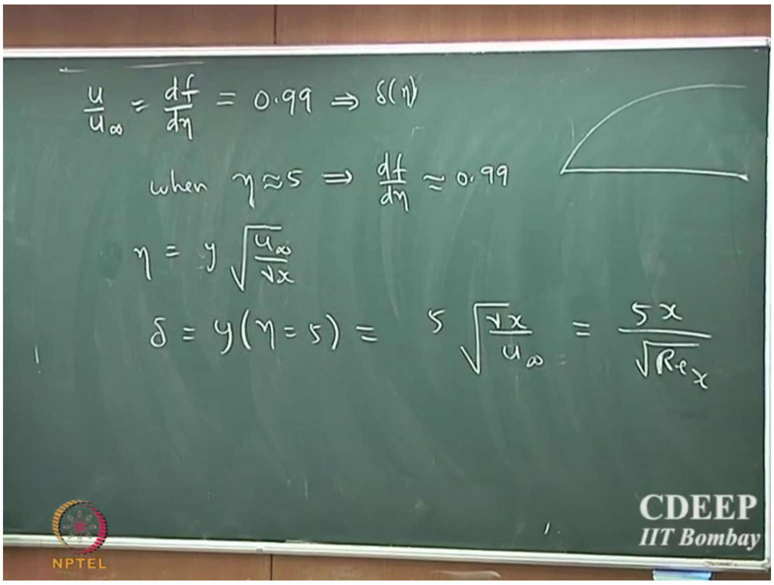

So, before we go into finding a friction coefficient. So, we said u by u infinity is d f by d eta. So, what is the definition of boundary layer thickness?

Student: U tends to 0.99.

U tends to 0.99 so, boundary layer thickness that corresponds to the corresponding boundary layer thickness right and here it will be a function of eta fine. So, whatever is the value by d f by d eta at which it is equal to 0.99 tells you what is the corresponding boundary layer thickness fine. So, it was found that when eta equal to 5, approximately 5 d f by d eta where approximately 0.99. So, that eta equal to 5 actually defines the boundary layer thickness.

So, it is the. So, eta is given by y into square root of u infinity by mu into x fine. So, boundary layer thickness is that y value for which eta is equal to 5. So, therefore, delta equal to y at eta equal to 5, that is given by 5 into square root of nu x by u infinity. What is square root of nu x by u infinity it is x by square root of Reynolds number right. So, this is 5 into x by square root of the local Reynolds number define based on the corresponding location. So, the boundary layer thickness, now if we got an expression for boundary layer thickness as a function of the x coordinate. So, that is very nice. So, now, you know to supposing I want to find out what is the boundary layer thickness.

I have got an expression to do that. As long as I know what the position is I should be able to calculate; what is the corresponding boundary layer thickness at that location. So, that is an important piece of information. So, remember it is its very difficult to measure experimentally what the boundary layer thickness is, but we got an estimate now, we got the theoretical estimate of what the boundary layer thickness has to be as a function of position ok.

Keep it that now we can go and find out what is the friction coefficient. So, friction coefficient is defined as tau at y equal to 0 divided by rho u infinity square by 2.

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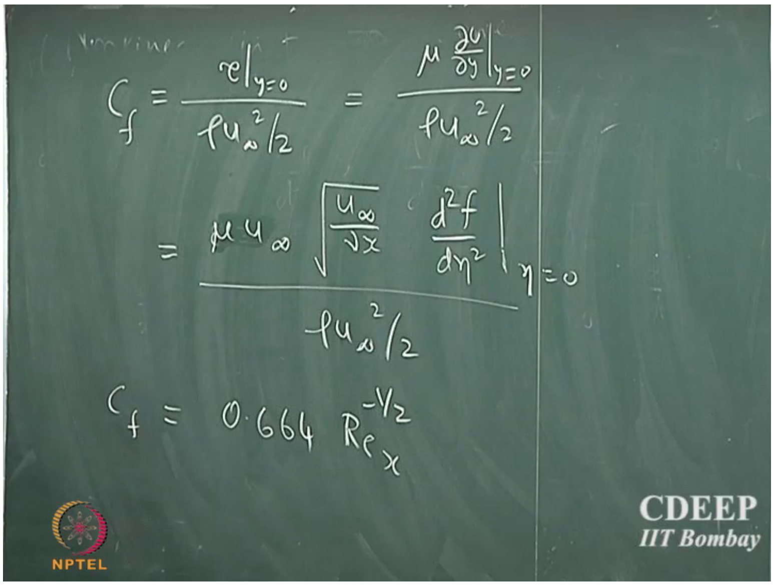

Fine that is the free stream a u infinity is the free stream velocity and so, that is mu into d u by d y at y equal to 0 divided by rho u infinity square by 2. So, now, I can calculated in terms of my modified variables and. So, that it will turn out to be. So, that will be about 0.33. So, it will be mu into u infinity, square root of u infinity by nu x into d square f by d eta square at eta equal to 0 divided by rho u infinity square by 2 ok.

So, if I know the second derivative of my function f with respect to eta I am done. So, that will be 0.664 into Reynolds number to the power of minus half. So, that is the functional dependence of the friction coefficient on the Reynolds number. So, that is the local friction coefficient, but what I am more interested is the average friction coefficient. Because from experimental point of view what I really require is the average quantity and the local quantity is not of much use ok.

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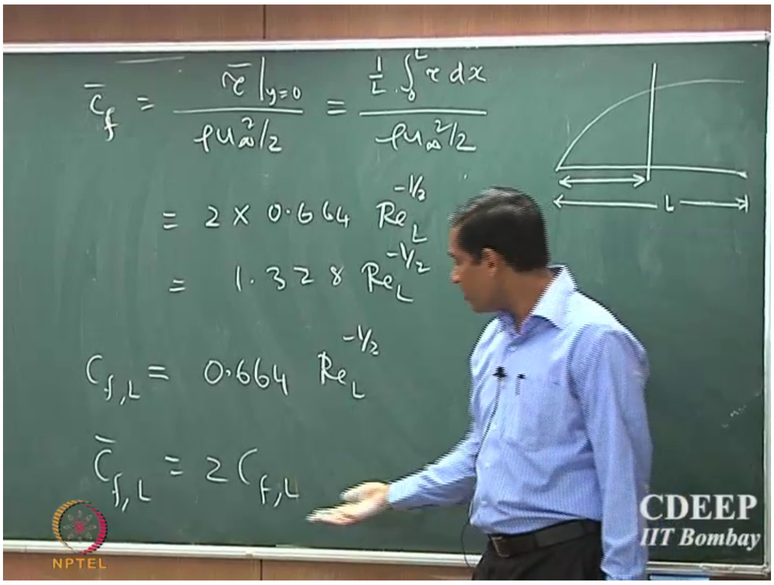

So, I can find out the average friction coefficient which is given by the average shear stress at the boundary, divided by rho u infinity square by 2 and. So, if you work out the integral it is not very hard to do that. So, that will be integral 0 to L 1 by L tau into d x divided by rho u infinity square by 2. So, you can work out the integral. So, that it will be 2 times 0.664 into Reynolds number based on the length divided by 1 to the power of minus 1 by 2 and. So, that will be one point how much is it is 1.328 minus 1 by 2 ok.

So, that is interesting. So, if I find the friction coefficient at length. So, that is the local friction coefficient at the end of the plate its. So, that will be 0.664 into Reynolds number based on the length of the plate to the power of minus 1by 2. So, that is interesting. So, the average friction coefficient for the whole flat plate of length L, it is just the twice of the local friction coefficient at that location. So, that is an important observation will be twice of the friction coefficient based on that length.

Now, the some; the interesting thing you can actually extract from here, suppose I want to find out let us say I have another plate supposing I have a plate which is I have done all my calculations on experiments based on the plate whose length is L. Now if I want to repeat the same experiment let us say half of the size. I do not need to redo the whole calculations I can simply use this property in order to extract all the average quantity is it I wanted to get. So, if I know the local friction coefficient, then I should be able to estimate what is the average friction coefficient up to that location. So, that is an important piece of observation.