Heat Transfer
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Lecture – 25
Forced Convection: Introduction

So, we are going to move into the next topic of forced convection. So, when I when we discussed about convection, we said there are 2 types of convection. One is the forced convection where you have a definite pressure gradient which is fluid is flowing fast in certain object.

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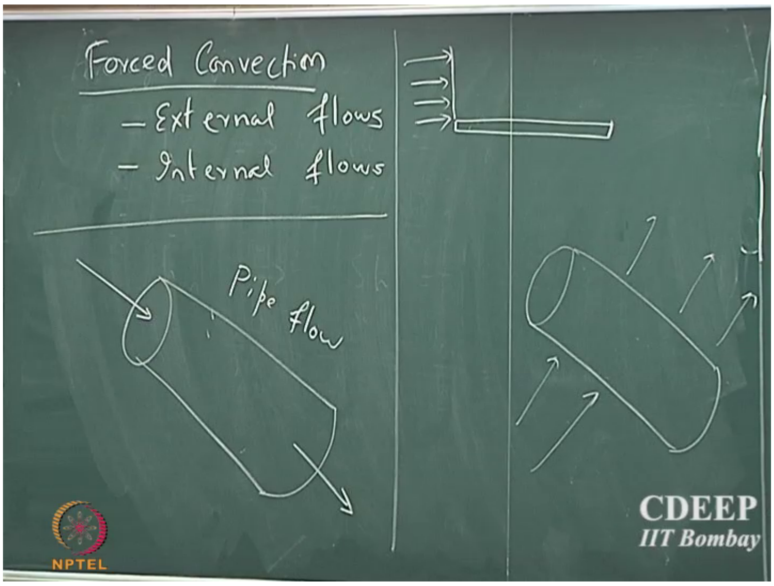

And that could be different geometric, it could be a flat plate it could be a cylinder. It could be inside the cylinder etcetera. And the other one parallel is the natural convection, which we will see after completing the forced convection topic. So, here you can have 2 types. One is the external flows, and internal flows. You have 2 types. So, external flow very simple example of that is you have a flat plate, in you have fluid which is flowing faster this flat plate, that is an external flow.

It will be very simple idea. So, if you look at; let say fluid which is flowing fasts stable. That is like flowing faster flat plate. You could also have a cylinder, you also have a cylinder, where the fluid is actually flowing perpendicular to this cylinder; that is one possibility. Where the fluid is now flowing at the along the curved surface. Along the curved surface of this cylinder. You could also have fluid which is actually flowing in the axial direction, parallel to the curves surface of the cylinder. In fact, that is what you have in your laminar flow experiments. And turbulent flow experiments, where the fluid is actually flowing fast the external curved surface of the in interior cylinder.

Now, you could have internal flows, where you could have a cylinder, where you have fluid which is flowing through a tube, call the pipe flow. You can have a pipe flow, where the fluid is flowing through the inside the tube. And you can have various condition. You can have laminar condition, you can have turbulent conditions etcetera. So, we going to see these 3 geometry, and few other simple case which we going to see in these external and the internal flows.

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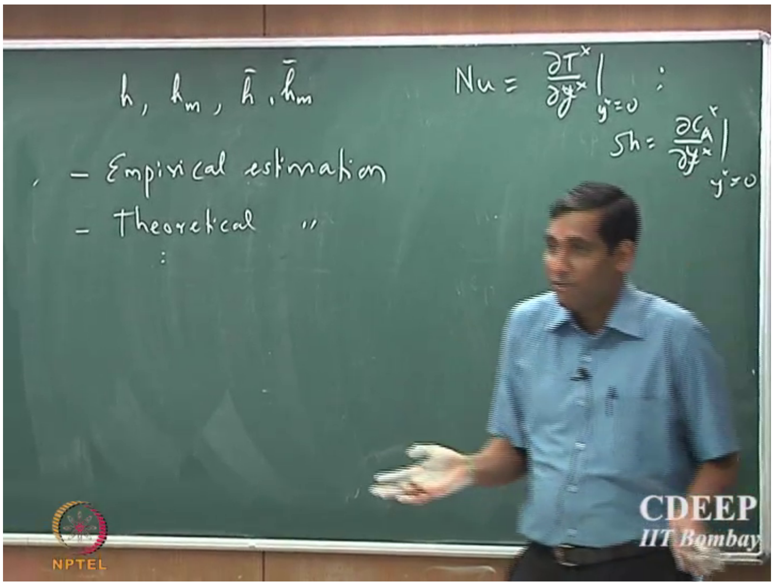

Now, before we going to the details of this. There are 2 method. So, note the objective is to find the heat transport coefficient, and mass transport coefficient local and the average. So, that is what we want to find. That is the objective for all of these problems. So, before we going to that, we going to look at 2 possibility. One is the empirical estimation. Going to look at the possibility of empirical estimation. And you going to look at the theoretical estimation. You going to look at both these possibilities. In the empirical estimation, we going to look at different kinds of experiments there is been perform. And based on these experiments, how to extract the heat transport and mass transport coefficient, and there will be a small topic which you will seen in the next lecture.

And after that we will go into the theoretical estimation, note that when I say theoretical estimation. We are not always trying to find complete analytical solution. All we want to is we want to find the Nusselt number and Sherwood number. You remember that Nusselt number and Sherwood number simply relates the gradient at the boundary. So, if we know the gradient we are done. You remember that Nusselt number is nothing but d T star by d x star at y star equal to 0. And similarly, Sherwood number is d C A star by d x star at y star equal to 0 dy star sorry that should be y. So, all we need is you only need the gradient at the boundary. So, we do not need to solve everything and find the complete temperature profile and concentration profile. If we are able to find the gradient we are good. We are able to achieve the objective of what we wanted to get.

So, what you going to see is instead of solving everything, you going to see how to find these gradients for various kinds of problems. It is possible that in some cases you will have to solve the full equations. And you will have to generate these gradients at various properties. And some cases you may be able to find the gradient directly.

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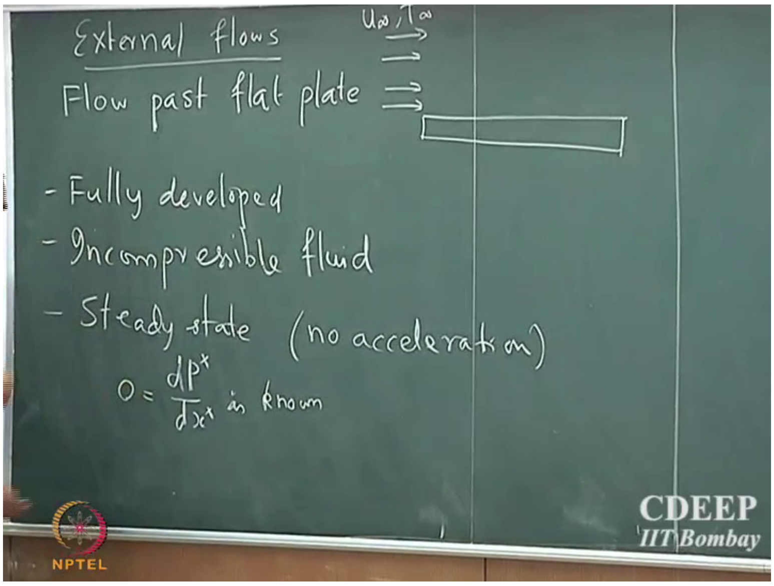

So, let us start with a case of a flow past flat plate. Flow past flat plate. Supposing there is a fluid which is flowing at u infinity velocity. And let us say the temperature is T equal to let say. And it going to be a boundary layer. So, if we assume that the flow is fully developed. Fully developed, and it is an incompressible fluid. Assume that it is an incompressible fluid. And if we also assume that it is at a steady state, not just that if we assume that there is no acceleration. In fact, if you want to maintain a steady flow, right. You want to maintain a steady flow, then you want to maintain a constant pressure gradient. Where you do not maintain a there is no acceleration.

So, therefore, we know what a pressure gradient is. We assume that we know d P star by d x star is known. In fact, if we scale it out properly, you can actually maintain that certain 0 value d P star by d x star. Note that it is a scaled pressure. Now this is if you want to maintain 0 acceleration. So, if you scale your pressure, we will come to that in in a short while. So, let us assume that it is known at it is constant. And you have a fluid which is flowing at a constant velocity. It is flowing at a constant velocity. And so, the net acceleration and therefore, the net pressure gradient if you scale them properly 0.

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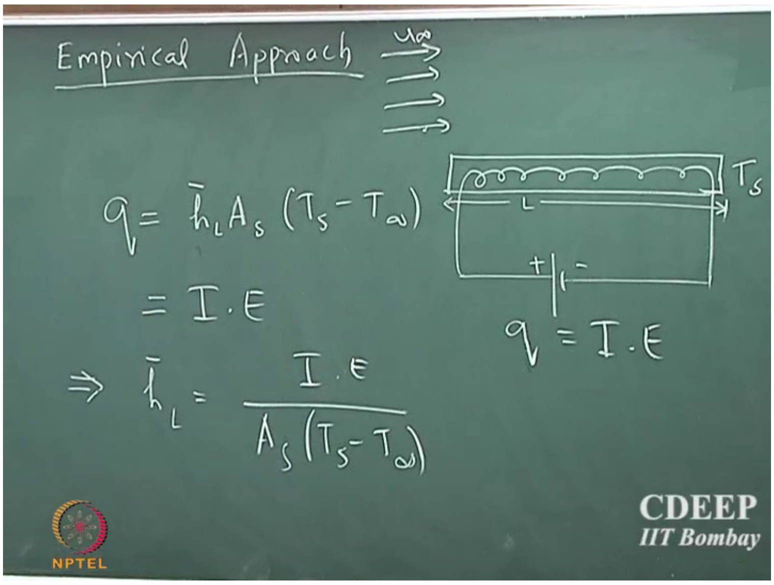

So now we said there are 2 approaches. One is the empirical approach. So, we will start with empirical approach first. And then we will move to the theoretical approach later.

So, suppose to the construction for empirical approach is; suppose I have a heating element. So, let us say I know how to heat the plate. This is not a bad thing do it is very easy to implement these experiment. And it is connected to a power source. So, I am going to heat the plate at a certain rate. So, the rate at which is heated is given by I time E where I is the current, and E is the potential difference, but supposing if there is a fluid which is flowing part this flat plate at a certain velocity. Now at steady state, you expect that we should maintain the temperature of the flat plate. So, let us say that the temperatures of flat plate is maintain that T s. So, the amount of heat that is supply to the flat plate, it is controlled in such way that the temperature of flat plate is maintain that temperature T s.

So, therefore, the amount of heat that is transferred from the plate to the fluid at steady state is given by h into whatever is the surface area of heat transport multiplied by the temperature gradient. This h should be average or local? Yeah.

Student: (Refer Time: 09:06).

It should be average. Because what is the amount of heat that you are supplying here is the total amount of heat that is supplied to the plate. So, if you want to look at the total amount of heat that is transferred from the plate to the fluid. We should actually use the average heat transport coefficient. Then that should be equal to I times E, that is not difficult to see that. So, from here experimentally one could estimate h bar. So, if I put a subscript L if L is the length of the plate. So, if I say this is average heat transport coefficient based on the length of the plate. So, h L would be I times E divided by T s minus T infinity. Well enough so, we know how to calculate the heat transport coefficient experiment.

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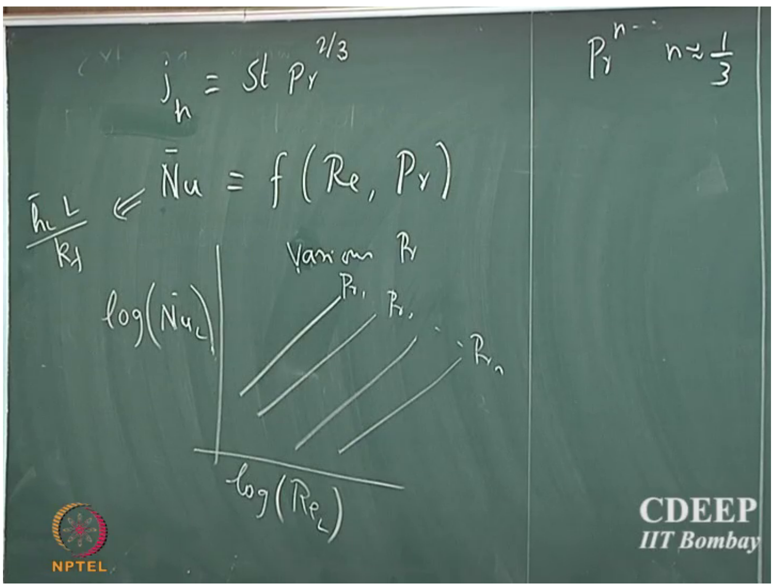

Now so, we said that the functional form in the Clinton coal burn analogy, that is j h factor is Stanton number multiplied by Prandtl to the power of 2 by 3.

So, from here one could guess that the Nusselt number should be a function of Reynolds number and Prandtl number. So, of course, one could guess that the exponent that you will get P r to the power of n y. So, that exponent should actively be 1 by 3, we could guess from this Clinton coal burn analogy. But then let us assume that there is a general function form. I am doing an experiment, there is no reason why I should assume that there is a certain functional form. If there is a certain functional form, it should actually come out of the experiment. Remember that any time you write a model or you write a you conduct an experiment it is always better to start with assuming nothing.

Student: (Refer Time: 11:12).

Excuse me? Yeah, this should be the average, right.

So, because that is what we are going to measure. So, it is better to start with assuming nothing. So, let us say we do not know what this functional form is, and can we detected from the experiments. Any answer is yes you can. So, although it is not a very correct thing to say that the person who did the experiments and found the functional form, did it without any basis, that is not correct actually. So, he has actually done the analytical theoretical approach and found out what should be correct way to plot these data. So, do not worry you will not be able to get the functional form. Until you know that Nusselt number and Prandtl number are important right. So, what they would have done is I would have first made a plot of Nusselt number versus Reynolds number. At various Prandtl numbers, various Prandtl number values

So, note that Prandtl number is the property of the fluid. So, if you can change the fluid, you conduct the experiments with different fluids or different temperatures, where you are now able to control the thermal diffusivity and the momentum diffusivity of the fluid. So, you can do repeat these experiment n number of times. And so, what they did was they made a plot of log of this average Nusselt number.

So, note that average Nusselt number is h L bar into the length of the plate divided by k f the conductivity of the fluid. So, you know k f you know the length, and you know the average heat transport coefficient from your experiments. And you know the Reynolds number of the flow. So, it is not by chance that they found you have to plot log. It is actually some of these theoretical coal burn analogy was use to understand what should be the functional form that you have to use to extract information from these data.

So, they made a plot for different values of Prandtl number. P r 1 P r 2 etcetera. Different values of Prandtl number. And then, what was done was they found at there is some correlation between Reynolds number and Prandtl number.

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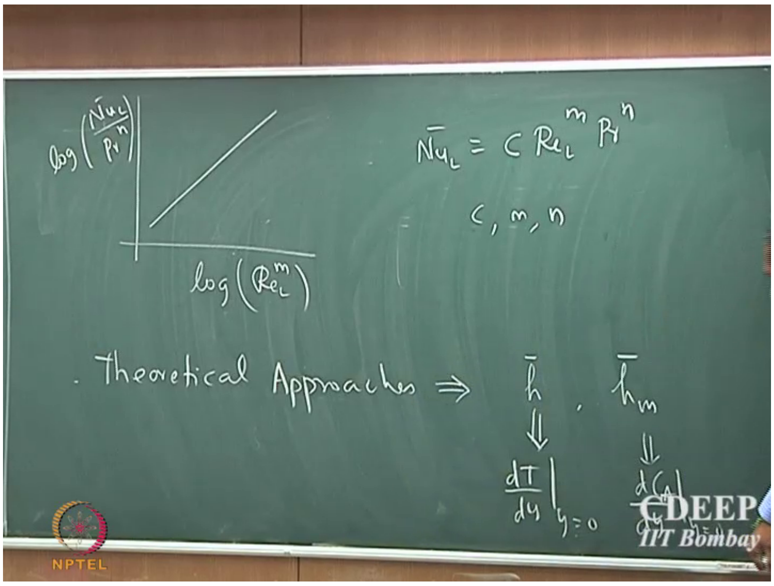

When the next stage was they set, let us try to look at log of Nusselt number by P r to the power of n, we still do not know what the exponent is. And log of Reynolds number Reynolds number to the power of n.

So, let us try to make a plot, because they got some insight from Clinton coal burn analogy. And they made a plot of this curve, and then you get some sort of a straight-line relationship. So, note that this was 100 years ago, more than 100 years ago. And so, in those days you know getting a straight-line plot is like a bingo. We always want to see what can we do so that you can get things towards the straight line. They never had excel. Excel was the concept of excel itself did not exists in those days

So, an concept of computer itself did not exist. So, they whatever solutions they made, whatever problems they looked at, they want to try to bring it to a form which can be fit manually by using your hand calculations. So, they try to bring it to the straight-line form not by assuming anything, but by looking at the functional form that one would get using the Clinton coal burn analogy. So, then therefore, they found that should be some constancy multiplied by power n and Prandtl to the power of n. So now, the exercise will just to find out what these constant c m and n r. So, from the experimental data if you have statistically enough number of data. So, one could always question what is mean by statistically enough, but let us assume that what we know what is statistically enough number.

So, we have enough amount of data you should be able to calculate the m and n. It is a very boring task, but somebody has done this boring task. It is like you know keep a plate to put a fluid run at measure this put a put a another fluid change the temperature measure it, the very boring things to do. But somebody has done it for us, may not going to do this. And I do not think such kind of an experiment any of you will be doing ever. So, people have done all these thing. In fact, people have done this for different kinds of geometries.

But what is more interesting is to look at what are the different theoretical approaches.

Student: (Refer Time: 16:16)

Yes, it does.

Student: (Refer Time: 16:25).

Yes, it does. Except that these exponents are different.

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Yes, it does. And in fact, you will see that for different kinds of geometries is a very, very similar functional form that you will see, whether it is external or internal flow. An excellent example of that is those who have done laminar turbulent flow experiments, you will see this functional form that is given to you for both external flows and internal flows. Whether it is flow through a tube, whether it is flow fast a tube, flow fast a flat plate whatever kind of. So, it is a pretty ubiquitous functional form that you will see in all kinds of geometrics. And the reason why it is ubiquitous is because of Clinton coal burn analogy. We didn’t assume we only said that it is a fa flat plate, but if let us say if it is cylinder instead of having a Laplacian and cartesian coordinates, you are going to have a Laplacian in cylindrical coordinates.

So, what? The functional form still reminds to same. In fact, when we make this analogy. It is a very general analogy which is valid for pretty much all the geometry is that we did not think off. So, what is more interesting for the course purpose. So, it is theoretical approach. Although you must know what the empirical approach that was used in order to verify some of the theoretical predictions. So, hereafter what you will see by enlarge is going to be the theoretical approach that has been used. And what is important to also note is that the equations that you got the governing equations that you got earlier they were all non-linear. So, note that the temperature in the concentration equation, they were non-linear because it has both temperature and also the velocity function. So, it is not a linear equation, and not always you will be able to find analytical solution. Unlike what we have see in most of the course, you will not be able to find analytical solutions for all the model equation that you would be write in here after.

That does not mean that you cannot get insights into what is happening. You will still be able to make some approximate solutions. So, note that the these things were done several decades ago when computers did not exists. So, they have some innovating ways of finding solutions. For example, series polynomial solution is one. So, people had looked at different methods, and solve these non-linear problem using different mathematical techniques. And of course, now you could always go and brute force solve it in a in math lab or any other computer software. And you should be a able to find similar solution.

Now, what is also important to note is that the objective? So, remember that the question that we are trying to ask in all of these problemses what is the average heat transport coefficient, what is the average mass transfer coefficient. So, we really do not need the full temperature profile and full concentration profile. We have it good well and good, but that is not really important. What is important is only the average. And this depends only on the temperature gradient at the boundary, and the concentration gradient at the boundary. So, if we are able to find these 2 quantity for a given problem we are done. So, the methods that people have developed series solutions etcetera they concentrated on finding these 2 quantities for any kind of geometry that one was interested in.