Heat Transfer
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Lecture – 24
Reynolds and Chilton-Colburn analogies

So, for this particular case we could assume that C f into Re L by 2 equal to equal to g 1 x star comma Re L, now note that this is only when Prandtl number and Schmidt number almost equal to 1.

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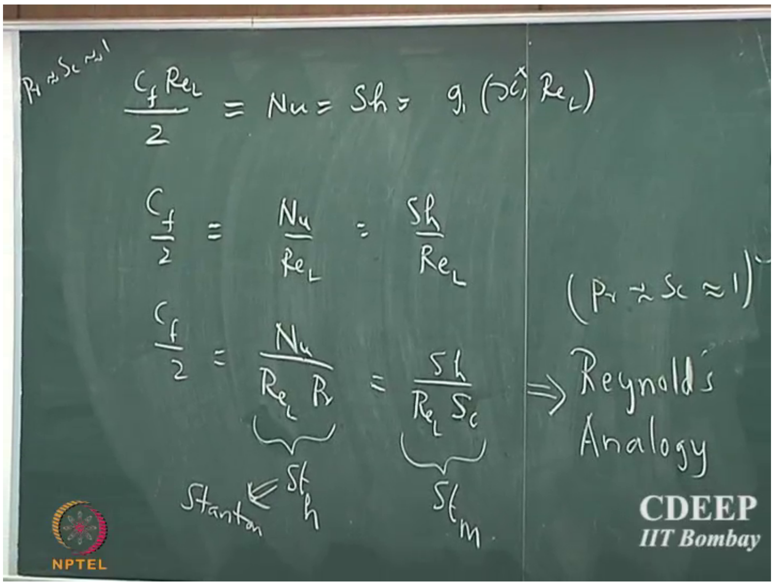

So, we can rewrite this as C f by 2 equal to Nu by Re L equal to Sherwood number by Re L and because we assume this could be 1, we could also rewrite this as Nu by Schmidt number. Simply because we assume Prandtl and Schmidt is almost equal to 1. So, this specific case of Prandtl and Schmidt equal to 1, this provide an idea as to what should be the relationship between the different characterizing numbers in these three boundary layer. So, this number once again this these 3 put together is called as the Stanton number for heat transport and these 3 put together is what is called Stanton number for mass transport, this is Stanton s t a n.

So, you will start seeing all kinds of dimension list numbers that you will start propping up here after you are not expected to remember all the expressions for these dimension list numbers, but what would be useful is if you attempt to understand what does these of each of these numbers signifies. For example, Nusselt number is the resistance to conduction the fluid divided by convection resistance to convection across the interface. Similarly Prandtl number is the ratio of momentum diffusivity to thermal diffusivity So, if you know what, if you understand what these different numbers characterized that is good enough we should actually be able to systematically find out what is the expression for each of these dimension list quantity.

So, this relationship that C f by 2 friction coefficient equal to the heat transport Stanton number equal to the mass transport Stanton number is what is called as the Reynolds analogy, called the Reynolds boundary layer analogy or simply Reynolds analogy. So, this is very very commonly used in different types of heat mass and momentum transport calculations because it is very handy. If you know you are able to measure the momentum boundary layer properties and are able to find the friction coefficient you are done.

You know the Stanton number, you know the mass transport Stanton number and all you need to know is Reynolds and Prandtl you are don. Your heat transport coefficient and mass transport coefficient also free because Reynolds number depends upon the properties of the fluid and the properties or the length of the plate that you looking at and Prandtl number and Schmidt number essentially properties of the fluid. If you know the properties and if are able to measure either of these 3, anyone of these 3 the other 2 comes for free.

So, this provides a powerful method for calculating the heat transport coefficient and mass transport coefficient. Moment you know what is Nusselt number and what is Sherwood number. So, remember that these two are characterizing numbers for heat transport coefficient and mass transport coefficient. So, if you know Nusselt number if you know Sherwood number you done. You found the heat transport coefficient you found the mass transport coefficient.

Remember we have not solved the equations yet, we have not even solved the equation. Without solving all of these three equations if you know one of them if you are able to even experimentally measure them, let us say we do not even solve the equations able to experimentally measure either of these three quantities you are done the others come for free. So, that is a very very powerful method. In fact, that is the power of these analogy.

Now, remember that this Reynolds analogy is valid only when Prandtl and Schmidt are equal to approximately equal to 1. So, the question is real system for; obviously, not the case Prandtl and Schmidt are; obviously, not 1 because we are not always looking at dilute gases we are looking at other system too.

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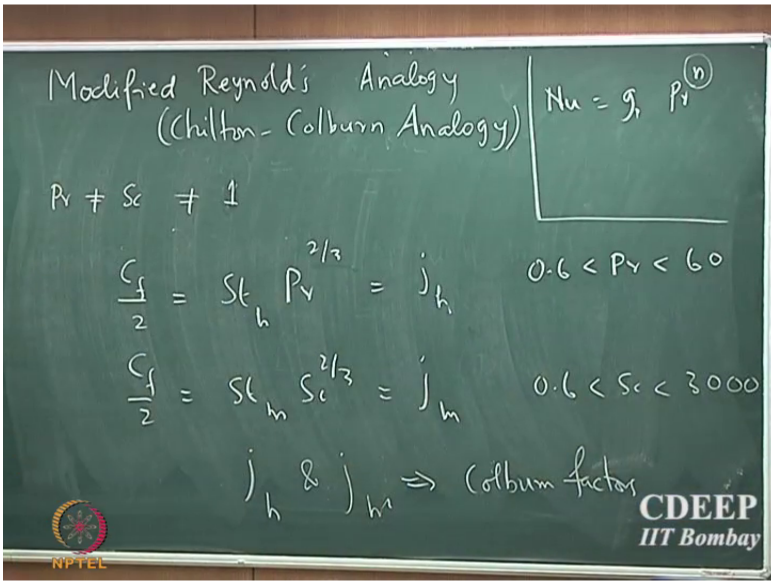

So, that required next that takes us directly to the next topic which is the modified Reynolds analogy or it is also called as the Chilton-Colburn analogy, also called as the Chilton-Colburn analogy where the analogy assumes that Prandtl number is not equal to Schmidt number and obviously, it not equal to 1. So, there is no analytical way of finding out what is the equivalence between the 3 transport mechanisms in the boundary layer, if this is the case where the Prandtl number is not equal to Schmidt number. But then observing the functional form you look at the functional form and then one can define what is called the Colburn factors. So, the functional form helps in deciding and this is valid for. So, this is what is called the Chilton-Colburn analogy or modified Reynolds analogy.

So, what Chilton and Colburn independently did is they took the Reynolds analogy, the functional dependence of this friction coefficient and these Stanton numbers and observing that the, observing that Nusselt number goes as g 1 something into Prandtl to the power of n. So, the now the question is what does n gives. So, they found and these were done by all kinds of experimental studies and correlations where they look found out that is n actually goes as 2 by 3.

Now not just that we are also going to find, this is the analogy Prandtl number to the power of 2 by 3 multiplied by Stanton number gives you what is called the Colburn factor. So, this j is called the, these are called the Colburn factor, these two are called the Colburn factors.

So, if you know the friction coefficient then you should be able to estimate what the Colburn factors are and once again if you know the Colburn factors you done. So, this is for a real system with certain range of validity for Prandtl and Schmidt number which sort of encompasses most of the systems that you would probably experience on the real system ok. One second, yeah what is your question?

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Right, right.

Student: (Refer Time: 08:38).

We going to see that, we going to see what is the functional form of Nusselt number very soon, there you will see that it actually scale as 1 by 2 and sometime its actually scales a 0.4, 0.33 and 0.4 we going to see that in a short while. Mostly in the next lecture, alright.

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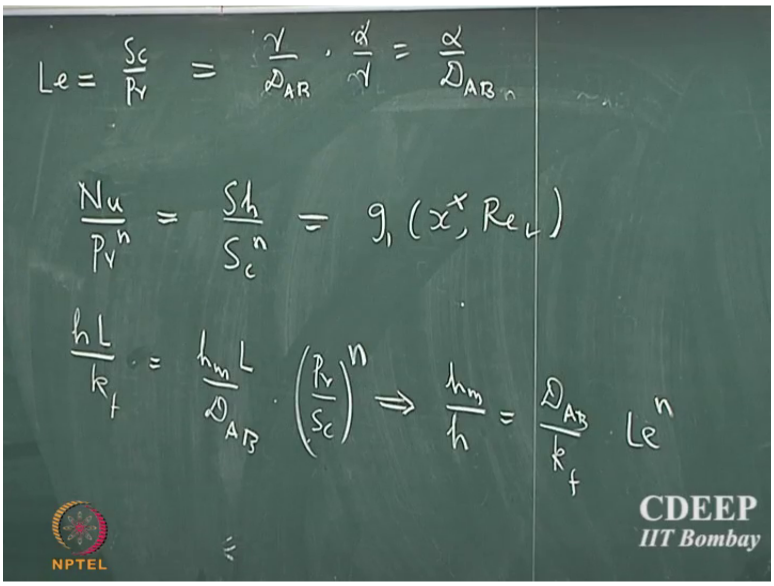

We said that Lewis number, the Lewis number is the ratio of Schmidt number to Prandtl number Nu by D AB thank you, into Nu by alpha oops alpha by nu, so that alpha by D AB and ok

So, now we said that Nusselt number divided by Prandtl to the power of n that should be equal to Sherwood by Schmidt number to the power of n and that has the same functional form that has the same functional form. So, if I open up the expressions here Nusselt number is h into L by k f that is the heat transport coefficient multiplied by length of the plate divided by the corresponding conductivity that is equal to h m into L by D AB into Pr by Sc to the power of n right.

Now, from here h m which is the mass transport coefficient divided by heat transport coefficient that ratio is now given by D AB by k f into Lewis number to the power of n. We know that Lewis number estimate by Prandtl and so that simply gives you that D AB by k f into Lewis number to the power of n.

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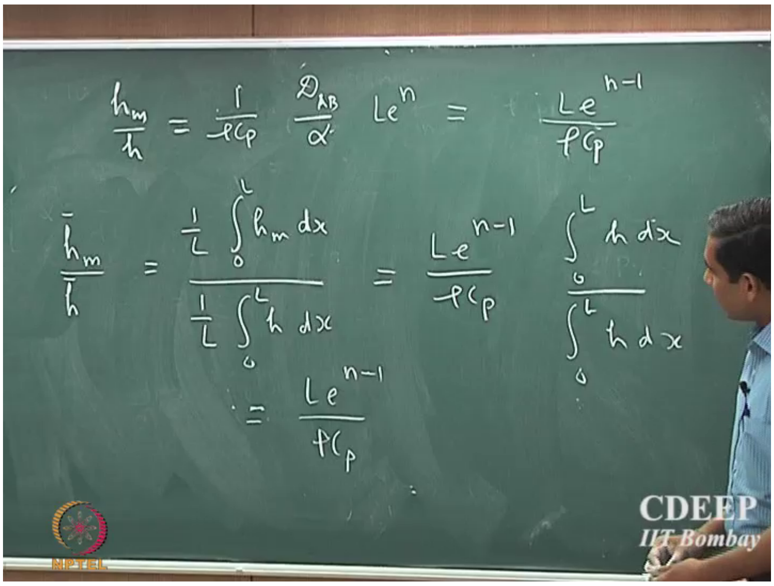

So, h m by h now Lewis number is defined as alpha by D AB right, so now we can multiply and divide by rho C p D AB by alpha right all I have done is I found k f by rho C p. So, alpha is conductivity divided by rho C p that gives you thermal diffusivity. So, I have introduced a definition into Lewis number to the power of n. But we also know that Lewis number is alpha by D AB right, from the expression that we have written there.

So, that will be rho C p Lewis number to the power of n minus 1, 1 minus n, n minus 1. So, again right nothing is wrong should be 1 by rho C p oh sorry.

So, ratio of heat transport coefficient and mass transport coefficient is Lewis number to the power of n minus 1 by rho C p that is an important observation. So, Lewis number is alpha by D AB and rho C p these are the properties of the fluid because of the analogy you see that the ratio is now going to remain constant. It is going to be only a function of the properties of the fluid not just that supposing if I look at the average mass transport coefficient and average heat transport coefficient, what is a definition of average mass transport coefficient for a flat plate? 1 by L, integral 0 to L h m d x divided by 1 by L h into dx.

Now we can use this expression for local heat and mass transport coefficient and. So, we can rewrite this as Lewis number to the power of n minus 1 by rho C p into integral 0 to L h dx, h dx also that will be by rho C p right So, that is an important observation.

Not just that the loc ratio of local mass and heat transfer coefficient depends only on the properties and constant and that is also equal to the ratio of average heat transport coefficient and mass transport coefficient. So, that is an important observation. So, if you know the local quantities then you should be able to relate the ratios of the average quantities now this we have shown for a flat plate case you could actually shows similar expressions for other geometrics too. So, if you know the local ratio of the local heat mass and heat transport coefficient you should be able to find out what is the ratio of average heat transport coefficient and average mass transport coefficient.

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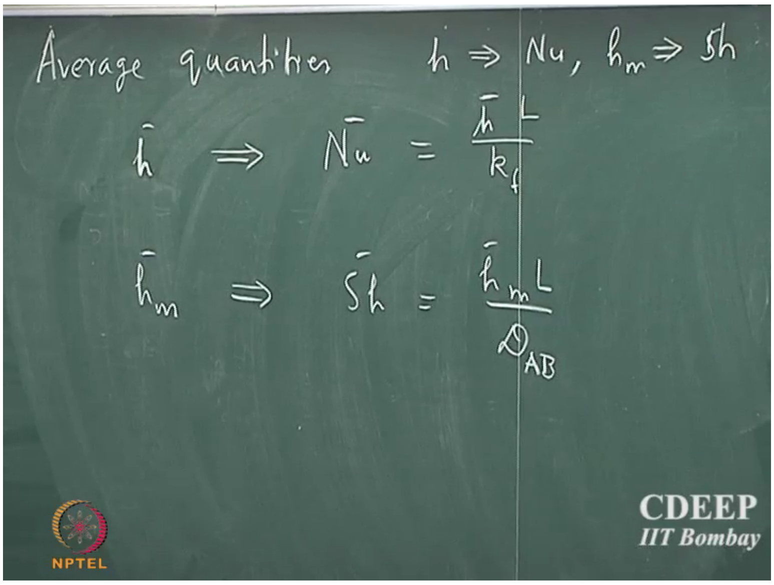

So, when we define these average mass transport and heat transport coefficient I briefly alluded to the fact that what is really important from practical point of view is these average quantities. Although we are trying to estimate all the local heat transport and mass transport coefficient, but ultimately what you would be using in your experiments which is some of you have already done in your laminar flow and turbulent flow experiments and others who do that through the rest of the semester in your lab course is that what you would actually be using is actually the average heat transport and mass transport coefficient. The reason why you would use the average is what you can measure supposing, if you have a double pipe heat exchanger they have concentric pipes there is fluid flowing through the inside pipe and there is fluid flowing between the two pipes and there is heat exchange between them.

So, what you would actually measure the temperature of the fluid that is inlet to both these streams and outlet to both these streams. You really cannot measure the temperature at every local point inside. So, what is of real practical importance are these average quantities. So, ultimately what we will see over and over again many number of times is how to find average mass transport coefficient and average heat transport coefficient. But we said that the local heat transport coefficient we need to find Nusselt number and if you want to find the local mass transport coefficient you will have to find the Sherwood number.

Therefore, we will have to define Nusselt number and Sherwood number based on these average quantities. So, if you want to find average heat transport coefficient then you need to know what is the average Nusselt number and that is given by h bar L by k f and similarly you will require the average Sherwood number which is average mass transport coefficient divided by multiplied by L divided by the corresponding diffusivity. So, that is what you will have to find. So, for all the different geometry is that we will discuss in the next several lectures the ultimate goal is to find out this average Nusselt number and average Sherwood number.

So, with this we sort of finish the basic boundary layer approximation and analogies and we going to move into the specific different geometries and this is where we are going to attempt to solve some of these model equations. Remember that so far we have not solved any of them, we have only look at the functional form structure of the equations and some intuition that we use for boundary layer of approximation, based on that we are able to get all kinds of insights and information about the processes that recurring in boundary layer.