Heat Transfer

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Lecture – 23
Relation between momentum, thermal & concentration boundary layer

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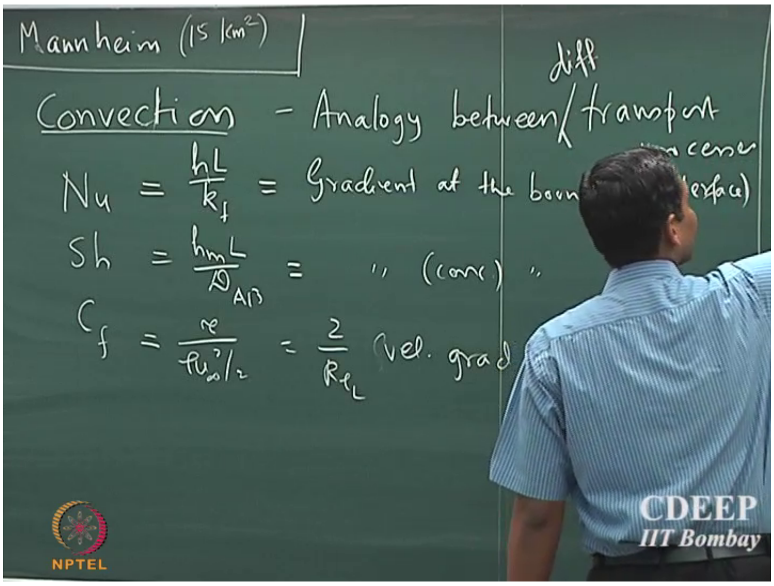

And it is actually located in very, very densely populated area it is a city called Mannheim. Mannheim is a place where it is located, is a very, very large city and it is so amazing I mean, how they are able to maintain it is it is more than 100 years old, and they have an interesting history that that is the only plant in the world there is not been a single explosion. That is amazing I mean they are able to we have made sure that their processes are so very well understood and their processes are so very well designed, whatever transport processes we have been learning in this course, they have understood it so very well for their process they are able to control it so very well, that is quite amazing I mean it is very unusual to hear that there is not even a single small accident in the industry since it is inception more than 100 years ago. It is a 15 square kilometer plant. Anyone who happens to be anywhere around that place.

So, apparently, they have a tour for open for public one time in a week; I think Sundays I think Sunday mornings they have a an open tour for all the people from outside. So, anyone who is actually going around that side even if your; Frankfurt Stuttgart any of these places it is just half an hour train ride. So, you should go and visit this plant and it is really a very pleasant experience alright.

So, we have been discussing convection. And so, let us little bit of let us little bit catch up a little bit, and then proceed further. So, we start by observing some of these dimensionless quantities, we have been discussing the analogy between different transport processes. So, we said that we define Nusselt number have anyone remember?

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Yeah.

Student: (Refer Time: 02:56)

HL by k what is that? Yeah.

Student: it is the mixture of (Refer Time: 03:08)

Minus. No, it is not it is the positive quantity.

Student: (Refer Time: 03:15)

What is it? It is a gradient at the boundary. I should rather say interface. It reflects the gradient at the interface. And similarly, we could write and that is the gradient concentration gradient at the interface. What about C f? Yeah, tau by. It is tau by rho u square by 2 and there will be.

Student: (Refer Time: 04:13).

2 by Reynolds number multiplied by.

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Velocity gradient.

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at the interface.

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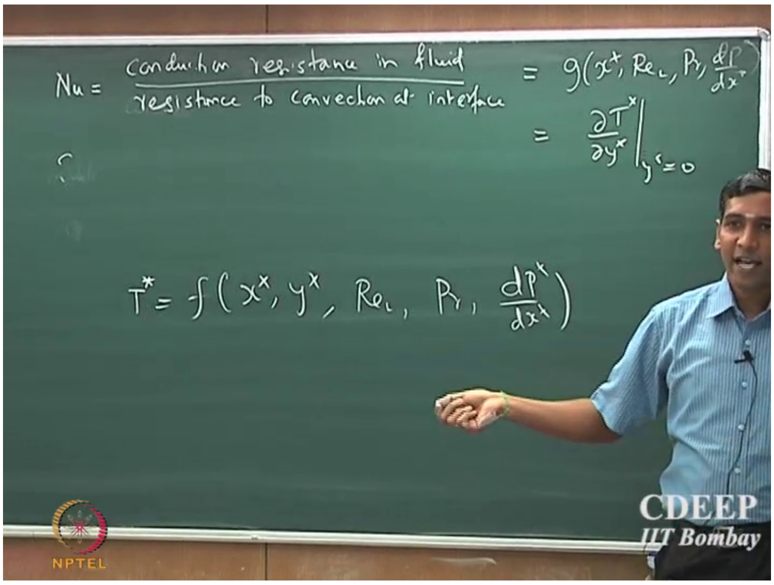

When we also define this is the ratio of Nusselt number is a ratio of?

Student: (Refer Time: 04:44).

Right. So, it is the conduction resistance in fluid divided by resistance to convection at rather across the interface, cause you are really looking at transport across the interface. And similarly, one could have a definition for Sherwood number we are not get into that it is anyway similar.

Now, we also said what is the functional form of Nusselt number? If you remember we said that the temperature T star the dimensional temperature is a function of x star which is the precision inside the boundary layer right. So, this is Reynolds number, Prandtl number and if you know the pressure gradient is typically a constant. So, Nusselt number will be a function of

Student: (Refer Time: 06:15)

X star, Reynolds number, Prandtl number and dP star by dx star, because it is evaluated at the boundary you take the first derivative. So, this is nothing but dT star by d y star at y star equal to 0. So, the Nusselt number is not a function y position anymore it is only a position of x position plus you are interested in the in the gradient at the interface which is y star equal to 0.

So, what we are going to see today is. So, so far you have not solved any equation, the question is can we get any further insights without solving the equation? We have to solve them eventually, but we going to first let us try to extract as much information as possible before solving the governing equations.

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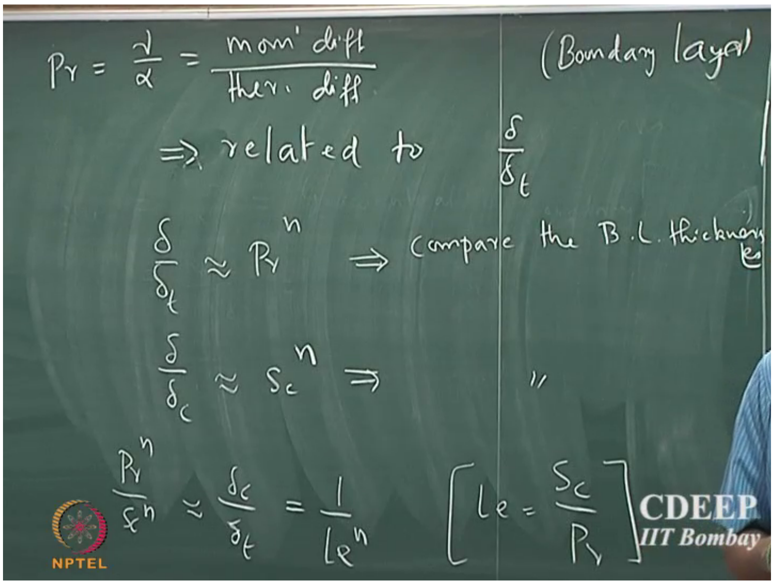

So, the first observation is that the Prandtl number. Prandtl number is the ratio of nu over alpha which is the momentum diffusivity divided by thermal diffusivity. Now note that all. So, these are happening in the boundary layer right. So, you are looking at characterizing the processes of the boundary layer. So, the momentum diffusion and the thermal diffusion they have to somehow be related to; they have to be related to the delta time, what is delta? Delta is the boundary layer thickness. So, all the process is the momentum diffusion is occurring in the boundary layer and it has to be a function of the boundary layer thickness and the thermal diffusivity has to be a function of the thermal boundary layer thickness, because that is the domain in which these 2 processes are occurring.

So therefore, so this is the general functional form we do not know what that n is still now we are going to figure that out why we discuss different aspects of conjunction. So, it is safe to look assume that the ratio of the momentum boundary layer to the thermal boundary layer thickness scales as Prandtl number to the power of n, we still do not know what are that n is in that some of you have already done this laminar and turbulent fluid experiment in the lab you must have seen some something some experiment like this. And so, we are going to see what are exponent is.

Similarly, one could express delta c as Schmidt number to the power of n. So, this also gives you provide the mechanism to compare the boundary layer thickness, provides the provide the mechanism or provides the method to compare the boundary layer thicknesses, and same here right. So, immediately we could define to be I could take a ratio of these 2, I can say Prandtl number to the power of n Schmidt number to the power of n. So, that goes as delta c by delta t we make sure that you get the numbers right. So, that is called Lewis number. So, so this is basically 1 by so, defined as Lewis number is defined as Schmidt divided by Prandtl number. So, if we define a number called Lewis number what would be the definition for Lewis number? You remember Prandtl is momentum by thermal diffusivity Schmidt number is momentum by mass diffusivity. So, Lewis number will be thermal by mass diffusivity that is easy to see that.

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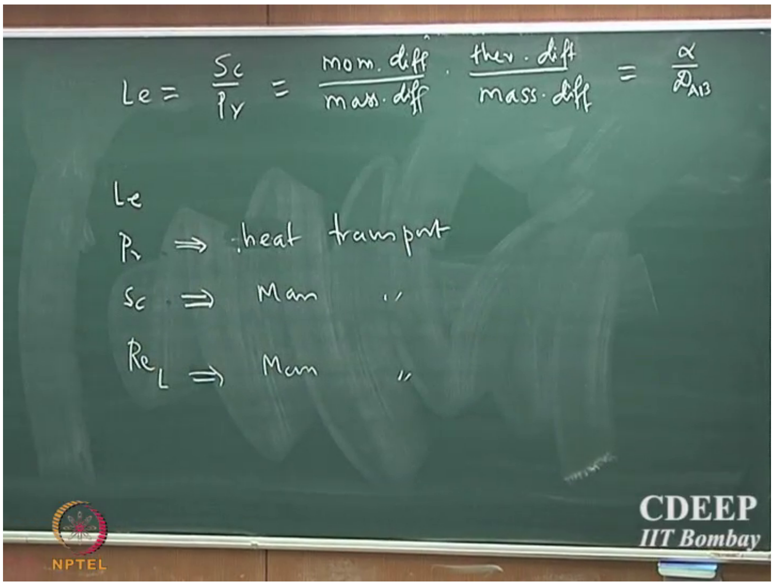

So, Lewis number is given by that is momentum diffusivity divided by mass diffusivity, multiplied by thermal diffusivity divided by mass diffusivity. So, that will be alpha by DAB, where alpha is the thermal diffusivity of the fluid that you are look that you are considering and DAB is the equimolar mass diffusivity of the species that are diffusing that is that particular stream. So, really what we have found is that Lewis number, Prandtl number, Schmidt number and Reynolds number it is based on the length of the plate because you look at flat plate you could always consider other geometries.

So, these 4 numbers essentially characterize the diffusive properties of all 3 different transport mechanisms. You get a way by which you can compare all these which means that if you know one of them, if you know Reynolds number there has to be some relationship between the heat transport and the mass transport processes, each of these numbers Prandtl Schmidt and Reynolds number they independently characterize each of the transport processes. So, this characterizes the thermal transport of the heat transport processes, this characterizes the mass transport process, and this characterizes the momentum transport process.

Now, we may also recall that the governing equations that we wrote for these 3 transport processes if we introduce the boundary layer approximation they all looks similar right? They all have same terms they have a conjunction term they have a diffusion term plus the momentum boundary layer equation has a constant which is the pressure gradient. So, the functional form.

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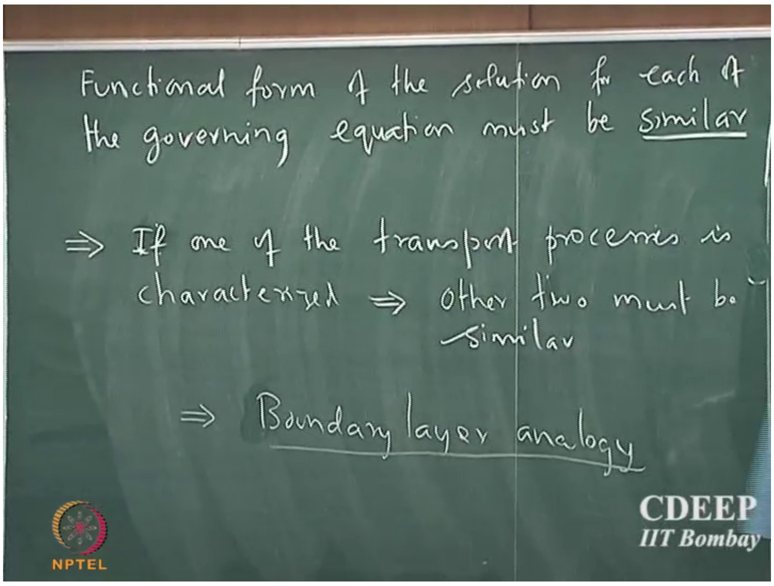

So, the functional form; of the solution for; must be similar. So, it is very important to understand this it is not same it is similar, because the momentum boundary layer equation does not depend upon the concentration and temperature, while the concentration and thermal boundary layer equation they depend upon the velocity although their functional form is same. So, therefore, the solution that you get the functional form of the solution that you get they have to be similar to each other which also means that; this also implies that if one of the processes is characterized, then other 2 must be similar.

So, that also means that because the function form is same. So, if one of the transport processes is characterized if I know how to solve one of them and I found the functional form and I found this dimensional numbers then I am done I should be able to find the solutions for the other 2. So, it is this property which leads to the concept of the boundary layer analogy. So, we are going to capitalize of this observation that the functional form is similar and we are going to identify how to relate these 3 transport processes and therefore, by characterizing one of them we should be able to characterize the remaining.

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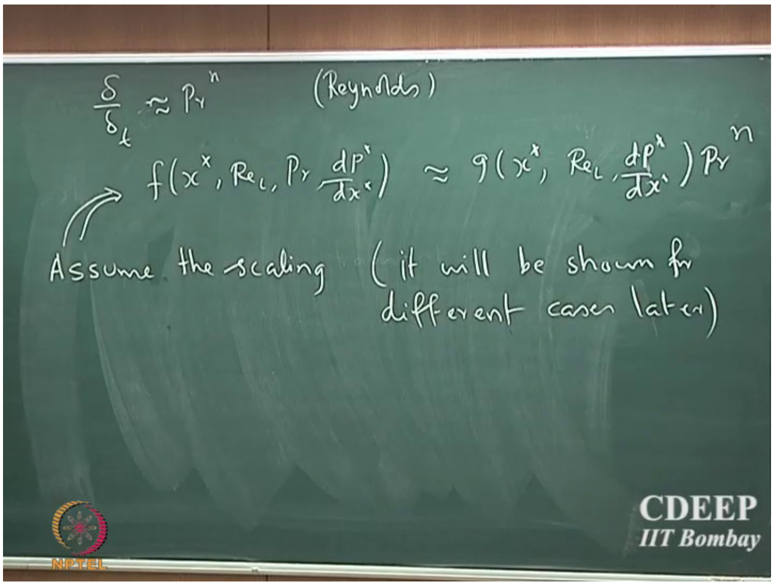

So, because we said that the delta by delta t which is the ratio of boundary layer thickness that is scales as Prandtl number to the power of n. So, there been lots of experiments particularly primarily due to work done by Reynold; where is shown that the functional form dx star somehow it is found to be in fact, we will see in the future lectures when we take specific cases we will see the kind of scaling that you will get. Prandtl numbers to the power of n. So, it is scales as some function multiplied by the Prandtl number to the power of n, and in fact, that observation is because of the positing that the ratio of boundary layer thickness is scales as the Prandtl number to the power of n.

Student: sir.

And in this ap may appear very heuristic right now, but you will see that in the subsequent lectures that almost all the geometries and all the problem you consider this is the kind of scaling that you would usually get.

So, for now we will assume that this is the scaling that we get, assume the scaling. In fact, for different cases later by the way this is been experimentally shown very well. So, there are some nice cute experiments it is being performed where you can see this kind of scaling has been observed experimentally for different kinds of problems.

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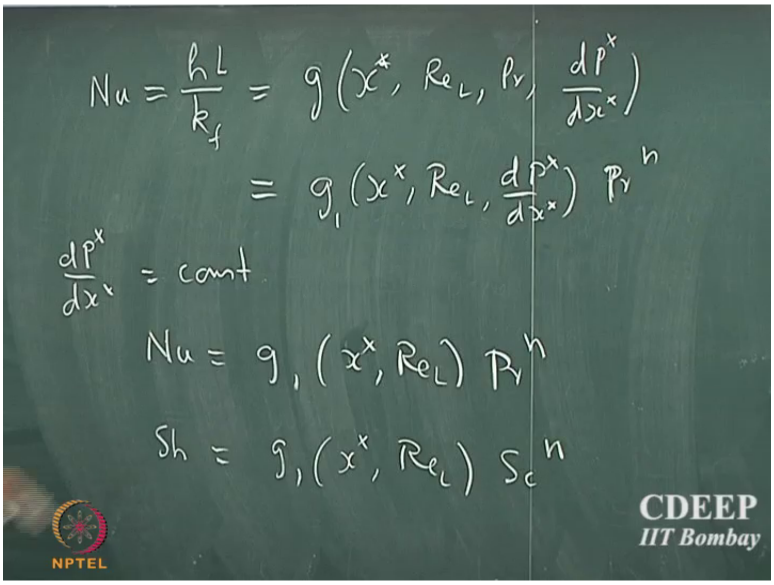

So, similarly you could assume. So, therefore, we said that the Nusselt number which is h into L by k f. So, that is given by some functional form if I call this as maybe I should use a different nomenclature, I call it g1. So, x star ReL and Prandtl number. So, that turns out that it is scales as dP star by dx star into Prandtl number to the power of n. So, that is a because of the observation that the functional form the Prandtl number to the power of n scales out as a separate entity in the functional form.

You should be able to write Nusselt number in simply as some function g1 which is going to be function of x star which is the x th position and the Reynolds number and some pressure gradient multiplied by Prandtl number to the power of n. So, supposing if I assume that dP star by dx star is constant it is a fair thing to say. So, Nusselt number can simply be written as x star ReL into Prandtl to the power of n. If similarly, we could say that Sherwood number will be same functional form x star multiplied by Schmidt number to the power of n any question? Because the heat transport and mass transport of the boundary layer are related to each other.

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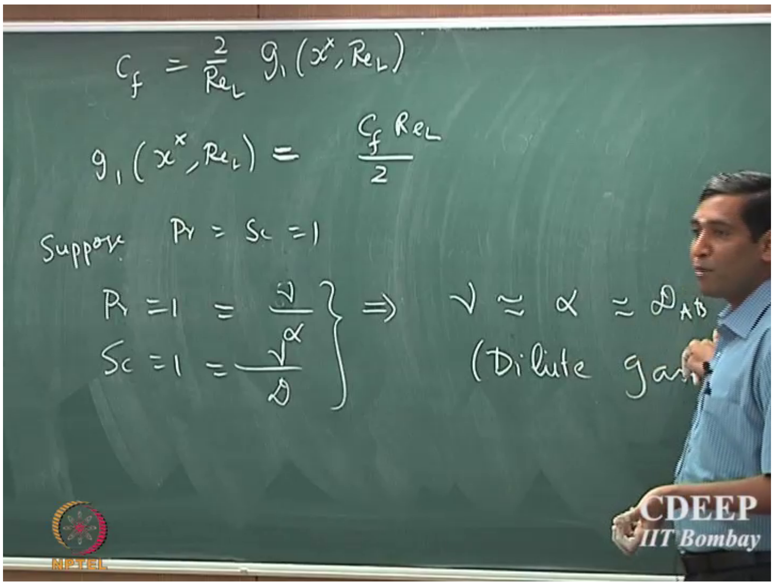

For the momentum boundary layer, we said that the friction coefficient. So, that is given by 2 by Reynolds number into the same functional form into g1 into x star ReL now if I am going to relate these 3.

Student: sir how (Refer Time: 21:11)

How do I know what?

Student: (Refer Time: 21:12)

This functional form would be just g1. Now the reason is that if you look at if you remember the model equations, you will see that the scaling that you get in front of the diffusion term the momentum equation is 1 by ReL in heat transport equation is 1 by ReL into 1 by Prandtl number that is why, and in mass transport equation is 1 by ReL into 1 by Schmidt number that is why the functional form will be same.

So now we can relate these functional forms. So, we said g1 x star ReL. So, that should be equal to there will be C f ReL by 2. And suppose the Prandtl number and Schmidt number are one when can Prandtl number and Schmidt number be one.

Student: when delta (Refer Time: 22:15)

When can that be when can you have delta t and delta same? So, when Prandtl number is equal to 1, you remember that it is nu by alpha right? So, momentum diffusivity by thermal diffusivity. So, this is one which means that the momentum diffusion it is diffusivity is almost equal to thermal diffusivity, when is that the case what kind of systems will have momentum diffusivity and thermal diffusivity equal?

Now let us say momentum diffusivity thermal diffusivity and mass diffusivity I said Schmidt number also is equal to 1, what kind of systems will have; I am saying that Schmidt number equal to 1 which is nu by d has anyone know? Any example when can it be the case that the thermal diffusivity and mass diffusivity are almost equal. It is dilute gasses when you have very, very thin gas stream. So, if you measure the properties of this thin gas stream it turns out that these 3-momentum diffusivity is a almost equal they are not exactly the same, but they are almost equal. So, in those cases the Schmidt number and Prandtl number are almost equal to 1.