Heat Transfer

Prof. Ganesh Viswanathan
Department of Chemical Engineering
Indian Institute of Technology, Bombay

Lecture – 22
Transport coefficients

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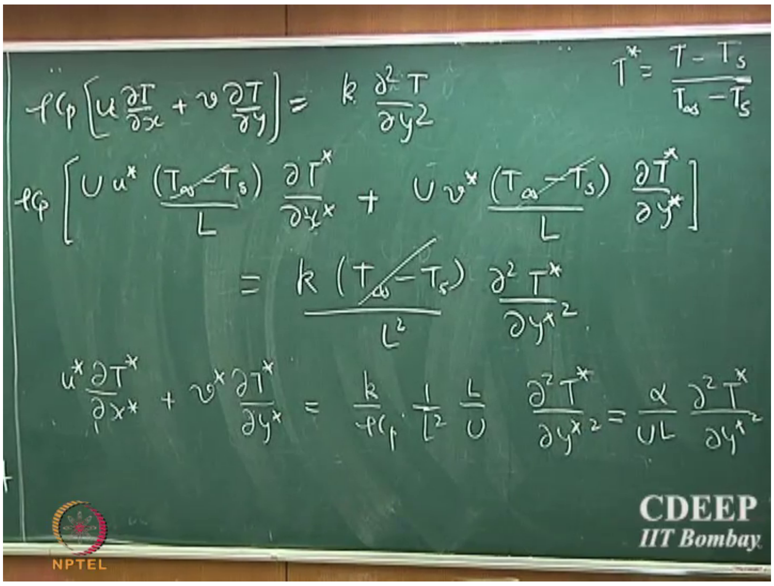

So, there will be u d T by d x plus v by d y that is equal to k, there should be a rho C p here, k into d square T by d y square. I missed out a rho C p in my earlier model that I wrote, there should be a rho C p in the front. So, that will be so the scaling will be U into u star, so now, I said that we scale it with T infinity, but remember when we actually wrote the heat transport coefficient right, when we define heat transport coefficient we said that its always a function of the temperature of the surface. So, now, it is better to incorporate the temperature of the surface also in the scaling.

So, the better way to define a scaling for the temperature is T s T minus T surface divided by T infinity minus T surface. So, that is a better scaling to introduce and so if we introduce that scaling it will be T infinity minus T s rho C p into d T star divided by L y star plus U into v star T infinity minus T s divided by L into it should be x T star by d y star, then that should be equal to k into T infinity minus T s divided by L square into d square T star by d y star square.

Now, if I do the algebra so that will become u star into d T star by d x star plus v star d T star by d y star that should be equal to so the T infinity minus T s will cancel out that will cancel out and so you will have k by rho C p into1 by L square and so we get L by v from the other side, so it will be L by u into d square T star by d y star square what is k by rho C p.

Student: (Refer Time: 03:05).

It is1 by.

Student: 1 by (Refer Time: 03:09).

K by rho C p is thermal diffusivity right.

Student: (Refer Time: 03:13).

Rho c p by k is thermal diffusivity no; it is k by rho C p is thermal diffusivity. So, this will be alpha divided by u into L d square T star by d y star square. Is it dimensionally correct? No why? What is alpha? What are the units of alpha? Meter square per second u into L is meter square per second. So, in non dimensional all the terms should have no dimension right, good sign.

What is alpha by u L? What is alpha by u l?

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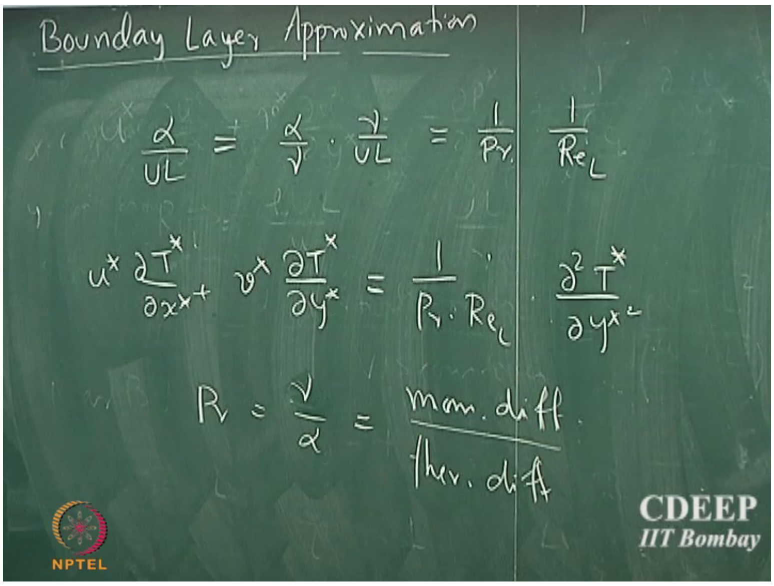

Student: Prandtl number.

Prandtl number is it.

Student: (Refer Time: 04:16).

A speckle number. So, what you can do is you divide this by nu. What is nu? Nu is the kinematic viscosity, multiplied by nu by u L. What is alpha by nu? So, this is 1 by prandtl number what is nu by u L.

Student: (Refer Time: 04:50).

One over Reynolds number right, so this is 1 by prandtl number into 1 by Reynolds number this is 1 by prandtl number, what prandtl?yeah.

Student: (Refer Time: 05:06).

1 over speckle number prandtl and reynolds speckle number, anyway speckle number is not important now, we will actually look at it when we talk about mass transport, we will actually see some importance of speckle number there. So, this is 1 over prandtl into1 over Reynolds number.

So, the model equation be u star d T star by d x star plus v star d T star d y star that is equal to1 by prandtl number into Reynolds number d square T star by d y star square. So, prandtl number is alpha by nu by alpha, nu is momentum diffusivity alpha is thermal diffusivity right, so this is momentum diffusivity divided by thermal diffusivity. So, that is what is prandtl number?

So, it tells you supposing if there is simultaneous momentum and heat transfer, it tells you, what is the extent of momentum diffusion versus the extent of thermal diffusion and both of these are happening simultaneously. What will be the mass balance is non dimensional form, can we guess I want to find I want to do the same non dimensionalization for mass balance also, right. You done for x and the y component momentum balance, done for the energy balance I wanted to do it for mass balance also. Can we guess from here yes or no?

Student: (Refer Time: 07:03).

Should be, because the energy balance and mass they have similar structure. Remember in last lecture I told you that these 3 balances they actually have very similar structure, so if you know one you should actually be able to guess what the other one is.

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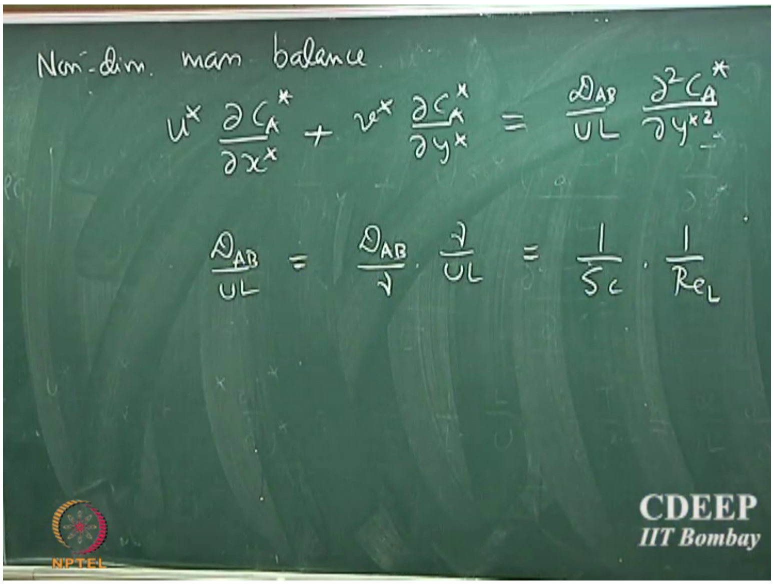

So, their mass balance non dimensional mass balance would be u star d C A star by d x star plus v star d C A star by d y star that should be equal to should be equal to, so let us start from the 1st one, let us not worry about prandtl right now. So, instead of alpha it is, it is mass diffusivity so that will be D A B divided by U L into d square C A star divided by d y star square. So, what is D A B by U L, so very similar to what we did here, we multiply and divide by the kinematic viscosity. So, there will be D A B divided by nu into nu by U L, so there will be 1 by Schmidt number into1 by Reynolds number. So, this term here is 1 by Schmidt number into1 by Reynolds number.

So, if I write down all the balances, so you can see that there is a strong similarity between the heat transport, mass transport and the momentum transport. So, there is 1 caviar to it, which will see in a short while.

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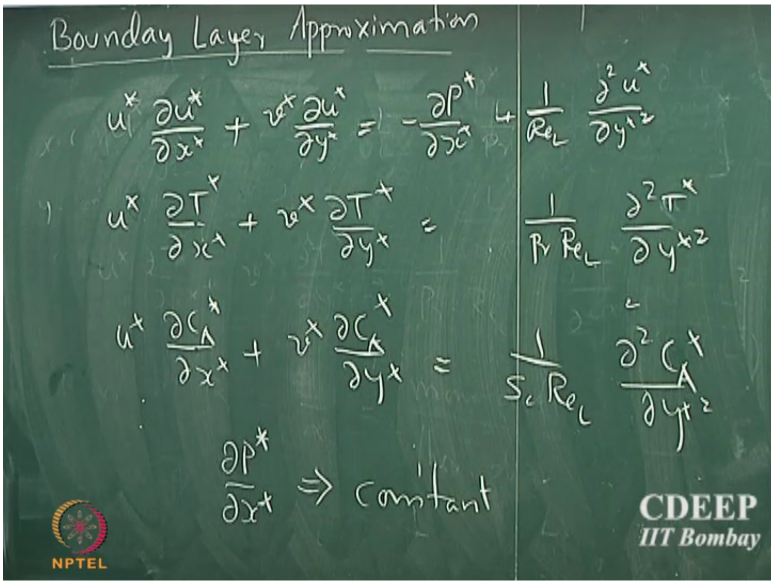

So, if we see the momentum balance u star d u star by d x star plus v star d u star by d y star that is equal to minus d P star by d x star plus 1 by Reynolds number into d square u star by d y star square and the energy balance is d star u star d T star by d x star plus v star d T star by d y star that is equal to 1 by prandtl, Reynolds number d square u star T star by d y star square and you have u star d C A star by d x star plus v star by d y star square.

So, if we look at the 1st 2 terms in all 3 equations they are very similar right, except that here u is replaced by T and here it is replaced by C A. Similarly the 2nd order derivative term also, you can see that they are similar. What about d P by d x. So, if I have to say that these 3 equations are similar then I need to know something about this right. If that is a function of position, if it is a function of position, then I cannot say that these 3 are similar.

So, only if I know what is the, what is d P star by d x star, then I should be able to say that these 3 are similar equations. So, if I know what d P star by d x star which is actually not a, so note that I am pumping the fluid, right. So, it is a forced convection where I am pumping the fluid. So, if I assume that it is a steady state, where the fluid is actually flowing at a constant pressure drop, then it is prior to assume that the pressure gradient is constant. Then these 3 equations are similar modulo, a constant which is d P by d x.

So, it is fair to assume in reality that the pressure gradient is known, because I have pump which is actually flowing the fluid. So, it is a measurable property, so if I know what the pressure gradient is I am done. So, these 3 equations are very similar, which means that if I know the solution of one of them I should be able to guess what the solution of the other 2 equations are, plus they are structurally similar.

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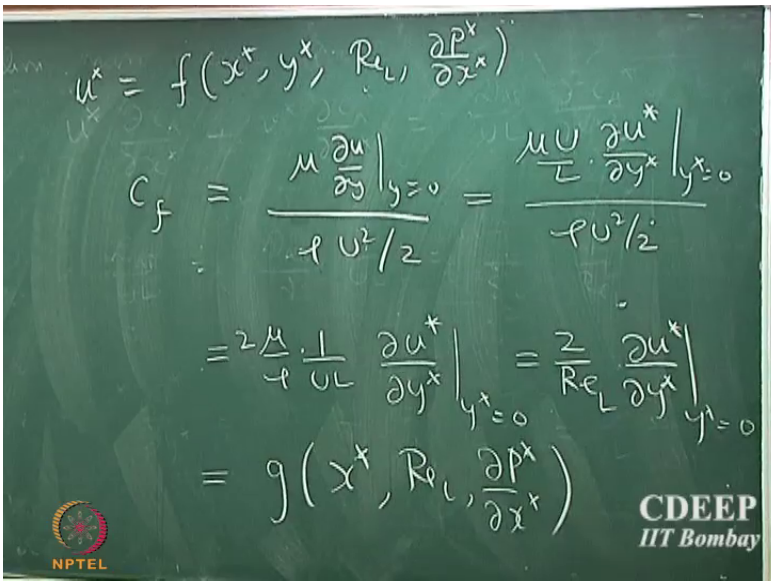

So, right way to put that is suppose I say that the velocity u star is a function of x star actually it has to be a function of x star and y star, because you have x component velocity which is varying in both x and y and it has to be a function of the Reynolds number, because that is the property of the fluid which is appearing in the equation and it has to be a function of d P star by d x star, have not solved the equation, note that I have not solved the equation.

Student: (Refer Time: 12:47).

Which one, yeah it is a constant, but still it should be a function, right. If the constant can be different number depending upon your system, but it has to be a function of that constant, yes it is a constant it does not change.

So, without solving I will simply assume that it is a function of these 4 quantities. Now what do we want to find, the purpose of writing these balances is to quantify these processes, right. So, if I want to quantify the process the momentum diffusivity is quantified by the friction coefficient C f. So, remember that we define 3 quantities, one is the friction coefficient, the other one is the heat transport coefficient and the third one is the mass transport coefficient, because the objective or the exercise is to find these 3 coefficients. So, if you want to find C f is defined as what is the definition of c f.

Student: (Refer Time: 13:46).

It is nu d u by d y at y equal to 0 divided by rho U square by 2, right. So, that is the definition of friction coefficient. So, now, if I convert this into dimensionless quantities that is mu, mu the scaling for U is capital U divided by L into dou u star by dou y star evaluated at y star equal to 0 divided by rho U square by 2, right. So, now can rewrite this mu by rho into1 by u L into d u star by d y star at y star equal to 0, what is mu by rho and1 by u L?

Student: (Refer Time: 14:41).

There is 1 by Reynolds number, right. So, this is nothing but I forgot a 2. So, there will be 2 by Reynolds number into dou u square by dou y square y star, y star equal to 0. So, if u star is a function of x star y star Reynolds number and d P star by d x star, what about C f?

Student: (Refer Time: 15:11).

C f will be, same thing evaluated at y star is equal to 0. So, remember that you take the derivative of this and evaluate at y star equal to 0. So, the dependence on y star is gone, because you are actually evaluating at y star equal to 0. So, which means that this should only be a function of x star Reynolds number and d P star by d x star, why, because you are evaluating this function at y star equal to 0, here removing the dependence on y star, because you are interested in the friction coefficient at the interface, that is what we want to know. So, the friction coefficient is now function of only x star Reynolds number and d P star by d x star.

So, suppose we perform the same exercise for the heat transport coefficient, to note that again, because these 3 equations are similar. So, T star will also have the same functional form, except that it will now be a function of prandtl number and d P star by d x star. Why is it function of d P star by d x star?

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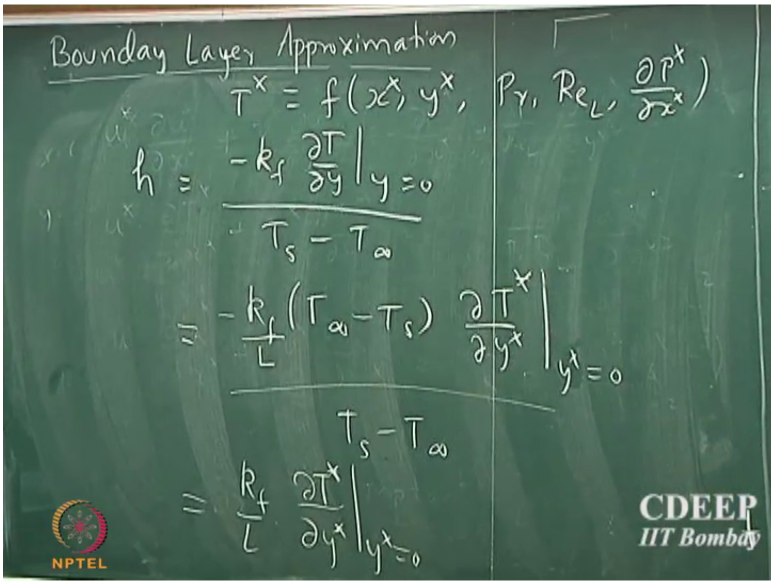

Student: Because (Refer Time: 16:32).

Because velocity appears in the convectional term, so therefore, T star is now a function of all these 5 quantities x star, y star which is the dimensions, prandtl and Reynolds number which is appearing as a scaling in the diffusion term and d P star by d x star are the is the pressure gradient. So, heat transfer coefficient is defined as minus k f, d T by d y divided by T s minus T infinity.

So, now if I introduce the non dimensional quantities, so that will become minus k f, T infinity minus T s divided by L, T star by d y star, at y star equal to 0, divided by T s minus T infinity. What is that, so that is, this cancels out with a minus sign and so that will be k f by L into d T star by d y star, y star equal to 0. All I have done is, I have just cancelled out these 2 terms and pull the minus sign inside and so there will be k f by L into d T star by d y star.

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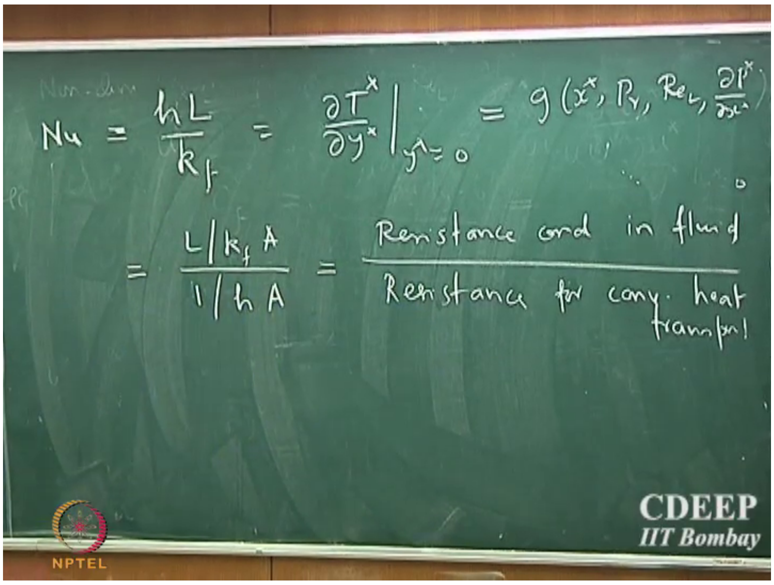

So, this equation we can now rewrite as h into L by k f that is equal to d T star by d y star, y star equal to 0. Now this is only a function of, because you are evaluating at y star equal to 0, so, this is only a function of x star prandtl number, Reynolds number and d P star by d x star. What is this h l by k f? nusselt number. So, this is what is called nusselt number, I am sure you must have seen this in your transport course in last semester. What has it signify?

Student: (Refer Time: 18:50).

So, this is L by k f into A, which is the area of heat transport; local area of heat transport divided by1 by h into A, right. So, you can rewrite it as L by k f into A divided by1 by h into A. What is L by k f a?

Student: (Refer Time: 19:19).

That is the resistance for conduction in the fluid. So, that is the resistance for conduction in fluid divided by the resistance for, that is1 by h a?

Student: (Refer Time: 19:41).

So, it is the resistance for conductive heat transport. So, note that here, conductive heat transport means, what is the extent of transport of heat, because of the presence of convection from the solid to the fluid phase. So, here we are not referring to convection because of the flow of the fluid, here we are referring to what is the extent of heat that is transported from the solid to the fluid phase in the presence of convection, so that is what it signifies. Now note that, it appears very similarl, the functional form or the quantities look very similar to what you saw for the Biot number in conduction. What is the difference?

Student: (Refer Time: 20:33).

So, in Biot number.

Student: (Refer Time: 20:36).

It is the conduction of the solid, so here, there you are comparing the resistance to conduction in the solid versus the resistance to heat transport because of convection from the solid to the fluid. Here you are comparing the resistance of conduction in the fluid phase versus the resistance for transport from the solid to the fluid phase. So, there is a fundamental difference in the quantities and that is why they carry different names.

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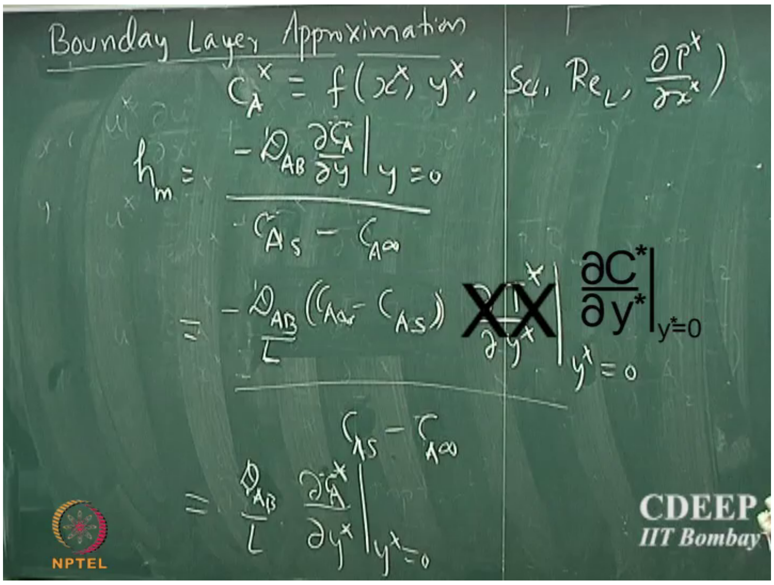

So, now, one can actually do the same exercise for mass transport, so if I call this C A, instead of prandtl I replace it by Schmidt number, there will be h m, D A B, there will be C A and C A s, C A infinity. So, I can do the same exercise, so this will be D A B and c d C A star by d y star, evaluated y star equal to 0 and this will become what is that number, h m, D A B, d C A star by d y star and that is a function of Schmidt number. What is this number called as? No.

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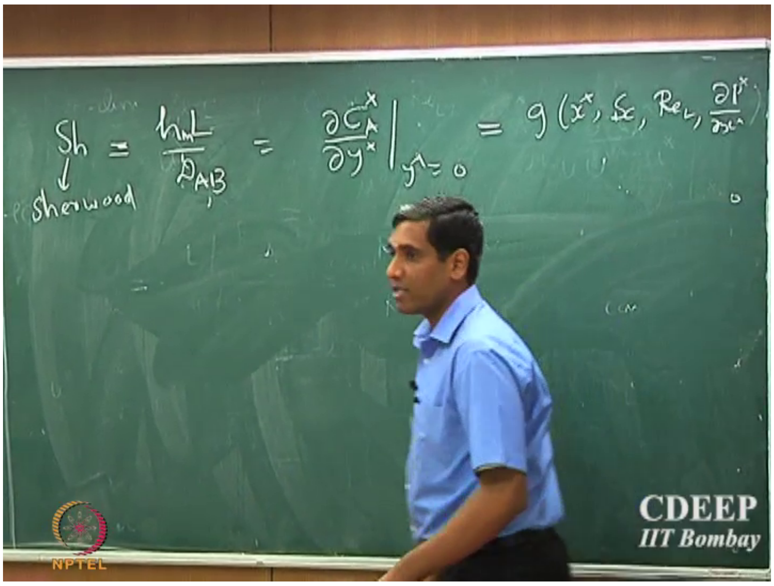

So, this is called the Sherwood number, if you are not heard of this. So, it is the Sherwood number which is h m into L by D A B, once again it defines the ratio of the resistance to diffusion, mass diffusion in the fluid phase divided by the resistance for mass transport due to the presence of convection from the solid phase to the fluid phase.

So, you can see that, without solving the equations note that we have not solved anything, all we have done is we have introduced some approximations simply based on intuition as to how the fluid flow is behaving in the boundary layer, how mass transport and heat transport would occur and we are able to come up with some really comprehensive numbers which sort of characterizes the heat mass transport and momentum transport in the boundary layer, so will stop at this point.