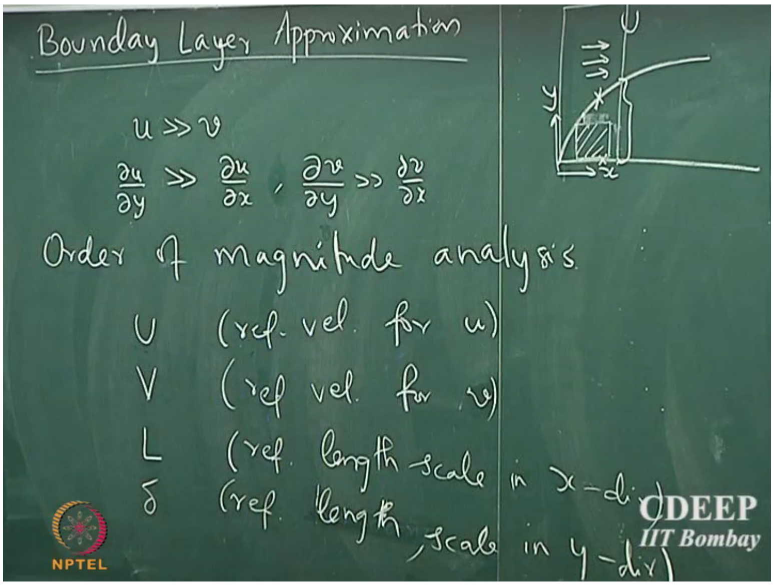
Heat Transfer

Prof. Ganesh Viswanathan
Department of Chemical Engineering
Indian Institute of Technology, Bombay

Lecture - 21
Order of magnitude analysis

(Refer Slide Time: 00:15)



So, we stopped in the last lecture by looking at the gradients in the boundary layer. So, we introduce the boundary layer approximation, where we said that x component velocity is much larger than the y component velocity and we said that d u by d y is actually larger than d u by d x and d v by d y is larger than d v by d x. Note that I made a typo this should be x and not this should be y and not x.

So, anyway. So, the way to introduce the boundary layer approximation, I suppose that it is being taught in your fluid mechanics course. So, you have to what is called the order of magnitude analysis.

Student: (Refer Time: 01:23), but they are present in y direction (Refer Time: 01:25).

Yeah. So, let us assume for now you will actually see it, because if the, if the force convection, if the flow rate is significantly larger than body force is do not play much role here. So, we will actually see that. So, a particularly body forces play an important role when you have natural convection not in these kinds of force systems alright. So, supposing if you say that the free steam velocity is u. So, that is the velocity of the fluid outside the boundary layer. So, that is the velocity of the fluid outside the boundary layer and suppose, we say that the V is some representative velocity v y component direction.

So, that is the mu, is the reference velocity free steam reference velocity in the for the x component velocity. So, this is the reference velocity for u that is the velocity in the x direction and if this is the reference velocity for v, this you do not know what this capital V is. We know what U is; U is the free steam velocity and L let say it is the length of the plate. So, that is the reference length scale n in x direction.

So, I define coordinate as x and y. So, remember, we talked about length scale in conduction. Similarly, L is the length scale here, for this problem and delta which is the boundary layer thickness, because we said that the velocity of the y component velocity is 0 at y equal to 0 and it is also 0 in the free steam location right. So, which means that the fluid is moving in the y direction and it is bounded between y equal to 0 and y equal to delta right. So, the fluid stream outside the boundary layer is primarily going to move in the x direction.

Student: (Refer Time: 03:57).

Outside the boundary layer no.

Student: Can I (Refer Time: 04:05).

No not really. So, if you look at the, if you look at far away in the free stream, it will be primarily in the.

Student: (Refer Time: 04:17).

Outside the boundary layer.

Student: Outside the boundary layer

Right, but that is, because.

Student: (Refer Time: 04:27).

No, but that is, because your boundary layer defined as 99 percent of the energy. So, supposing if you take 100 percent.

Student: Yeah (Refer Time: 04:32)

Then you will have zero y component velocity, because it has to be free stream, it has to be.

Student: (Refer Time: 04:40).

Because whatever fluid that is actually moving in the y direction here, that is actually compensated by the retardation of the x component velocity, that is what actually brings the continuity. So, if you actually do the continuity balance you will see that the y component velocity has to be 0 in the free stream.

Student: (Refer Time: 05:02) actually.

Otherwise the fluid is going not to move the primary direction of the fluid is not going to be in the x direction.

Student: (Refer Time: 05:07) large (Refer Time: 05:08). So, from where having data (Refer Time: 05:13) the upper surface and large.

When you say large what you mean large?

Student: (Refer Time: 05:18) over the (Refer Time: 05:20) x equal to 0 and (Refer Time: 05:26) from x it is not balanced.

No what is in y direction.

Student: (Refer Time: 05:32).

Yeah is it going to be within boundary layer or outside boundary layer it is, if it is outside boundary layer.

Student: I did not have marked the (Refer Time: 05:41).

No you will not.

Student: Yes.

Why you will not; because if you actually write a continuity equation supposing, you say write continuity equation.

Student: (Refer Time: 05:52)

Just listen just wait. So, if you write continuity, if you say this is your control volume you write your continuity equation here. Now, here you definitely going to see fluid which is actually moving out of the control volume right, because now, this is inside the boundary layer and a fluid stream is still experiencing the friction which is offered by the interface.

But supposing, if you write a control volume here.

Student: Sir, I do not have mass flux (Refer Time: 06:18)

You will have a mass flux not here; you will have a mass flux here.

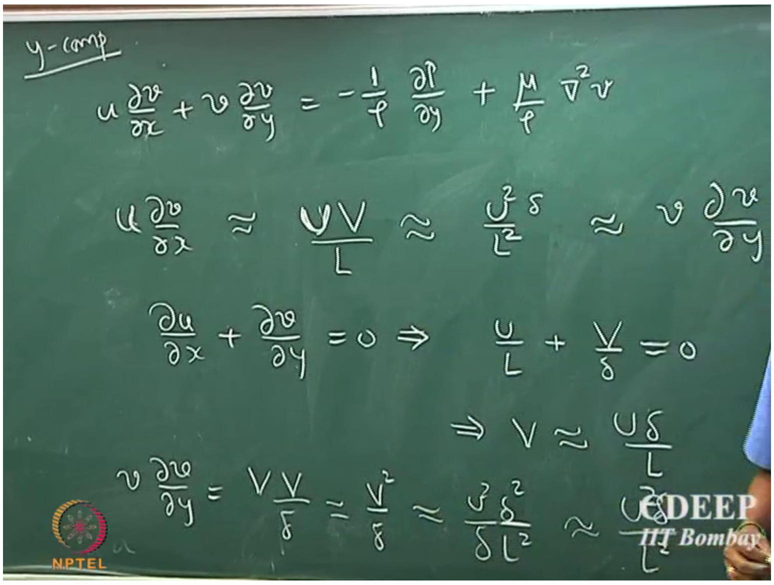
Student: Sir I did not have also there the mass flux.

Now, you will not. So, you write it carefully, you will not, you will not have, you will not have a mass flux.

Student: (Refer Time: 06:31) it was then few lectures back (Refer Time: 06:34).

Well outside the boundary layer, no will not be, you will not have. Well outside the boundary layer, you will not have mass flux then, which means that the fluid is not, is no more moving in the x direction, you going to have a next pressure gradients in the y direction. In fact, you going to see for a order of magnitude analysis that the net pressure gradient in the y direction has to be 0, we will see that in a short while. So, anyway, if delta is the reference that is the reference length scale in y direction. So, now, if you look at the y component momentum balance, we will come to the x component in a short while.

(Refer Slide Time: 07:21)



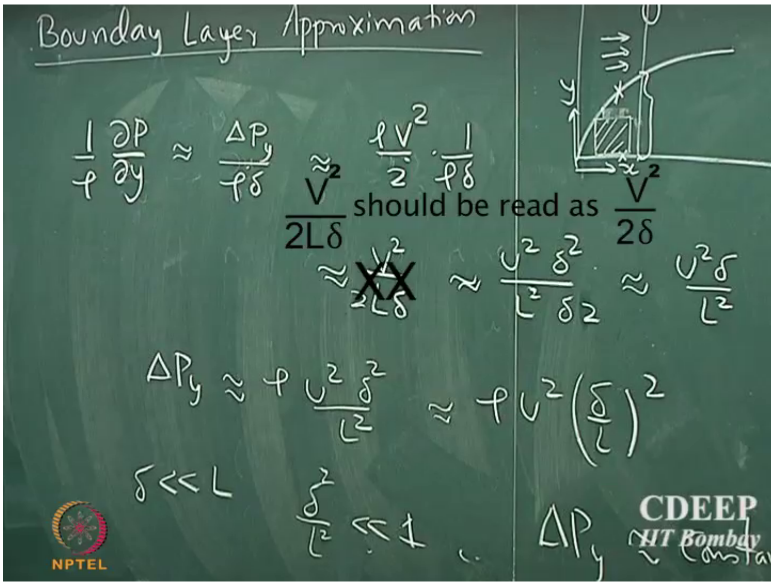
If you look at the y component momentum balance, you will have u d v by d x plus v d v by d y that should be equal to minus 1 by rho d P by d y plus mu by rho into del square v. So, that is the y component momentum balance. Now, the order of magnitude of u is the free stream reference velocity. So, u d v by d x would actually scale as u that is the order of magnitude for the x component velocity multiplied by v, which is the order of magnitude for the y component velocity divided by l.

So, note that we still do not know what v is. So, how do we find v use continuity equation? So, from continuity equation you will have d u by d x plus d v by d y that is equal to 0. So, from here you will see that u by l, which is the order of magnitude for u and L is the order of magnitude for x plus v by delta that should be equal to 0 fine. So, from here, we find that v should actually scale as u into delta by l. So, the order of magnitude of the y component velocity is scaled as the free stream velocity multiplied by the boundary layer thickness alright.

So, now with this, which means u into d v by d x would actually scale as u square delta by L square. Now, if you look at v d v by d y v scales as capital v and d v by d y scales as v by delta. So, that will be v square by delta and that will be u square delta square delta square divided by delta into L square. So, that will be u delta by oh I missed an L u delta by L square, which means that oops u square delta by L square ok.

So, which means that u d v by d x and v d v by d y are actually of same order of magnitude. You can clearly see from here, this is u square delta by L square exactly that is the same scaling. You get for v into d v by d x. So, that is an important observation that is the v into d v by d y. So, the first two terms in the y component momentum balance. They actually scale to the same order of magnitude. So, let us look at the pressure term now.

(Refer Slide Time: 10:42)



So, d P by d y. So, suppose, the suppose if delta P is the pressure gradient in the y direction. So, delta P y, this will scale as RHO into delta. So, y direction scales as the boundary layer thickness now delta P y. So, that is the. So, delta P y is given by the inertial. What are the pressure terms in terms of velocity? How do we write pressure delta P in terms of velocity?

Student: Knowledge equation.

Knowledge equation, we get. So, that scales as RHO v square by L right into 1 by RHO into delta.

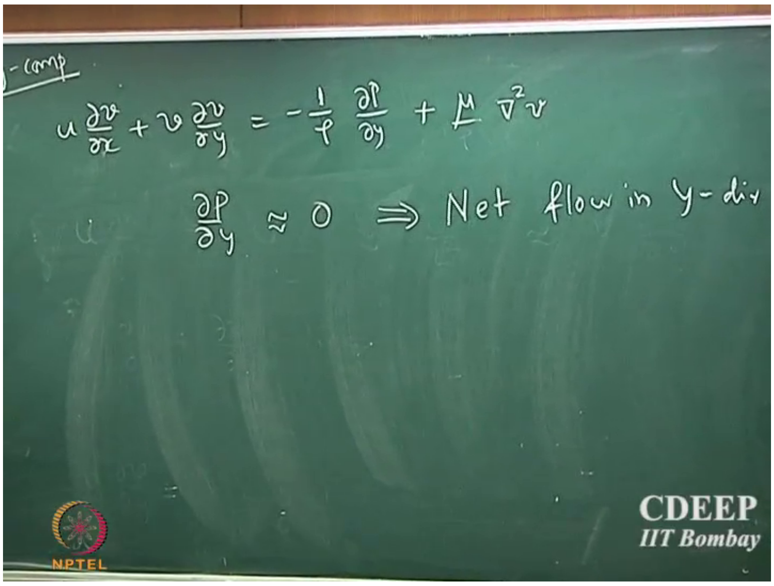
So, that will be v square by L into delta, but we know that v scales as u into delta by L right. So, that will be u square delta square by L square into delta into L oops, I put an extra, L bit term sorry wrong. So, this should be RHO v square by delta P scales as RHO v square by.

Student: 2.

2 correct right. So, that scales as RHO v square by 2. So, if I take modulo constant. So, this really scales as u square delta by L square. So, that is the scaling for delta P by RHO into delta.

So, therefore, delta P y that is scales as RHO u square delta square by L square. So, now so, that is RHO u square into delta by L the whole square. So, in boundary layer approximation, we said that delta is much smaller than L right. So, therefore, because delta is much smaller than l, delta square by L square will be significantly smaller than 1. So, therefore, you approximate therefore, you expect that the delta P y the pressure actually, pressure change in the y direction, should actually remain approximately constant.

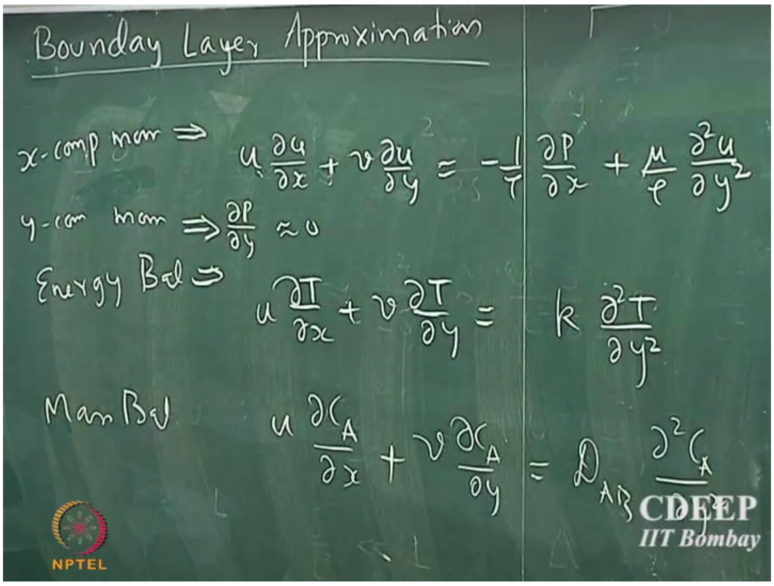
(Refer Slide Time: 13:40)



So, which means that d P by d y should be approximately 0. So, we expect that the pressure gradient in the y direction approximately remains constant, the pressure change remains constant then you expect that the pressure gradient is 0, which means that there is no net flow remembers that it is only a net flow. This does not mean that the y component velocity is 0. It already means that there is no net flow of the fluid in the y direction, all the primarily direction of flow is in the x direction alright.

So, similarly we can do an order of magnitude analysis for the x component velocity momentum balance is very similar. So, you do exactly the same analysis.

(Refer Slide Time: 14:34)



So, you will find that the x component momentum balance will essentially reduce to u d u by d x plus v d u by d y that is equal to minus 1 by rho d P by d x plus mu into mu by rho d square u by d y square. So, we have to do exactly the same, the tough part is the y momentum balance, x momentum balance is very easy to see, why the second gradient is should not be present here, you simply introduce the scaling variables. You will see that order of magnitude is much smaller than all the other terms. So, you ignore it.

And similarly, we can do for energy balance again, the idea is same d t by d x equal to k into d square t by d y square and mass balance at the u d C a by d x plus v d C a by d y that is equal to d a b into d square C a by d y square. So, that is the mass balance. So, this is after we introduce the boundary layer approximation, then the gradients in the y direction are significantly larger than the gradients in the x direction.

So, once you introduce those approximation, this is the reduced equation that you will get for the dynamics of the velocity temperature and the concentration of the boundary layer of course, d P by d y is 0. So, that is your y component momentum balance.

Student: (Refer Time: 16:44).

Rest of the term there not. So, you see there is an extra delta.

Student: (Refer Time: 16:48).

There are not same order.

Student: (Refer Time: 16:50).

Right. So, delta P by RHO right. So, you have to divide by density.

Student: Sir I also have to divide that delta.

Right.

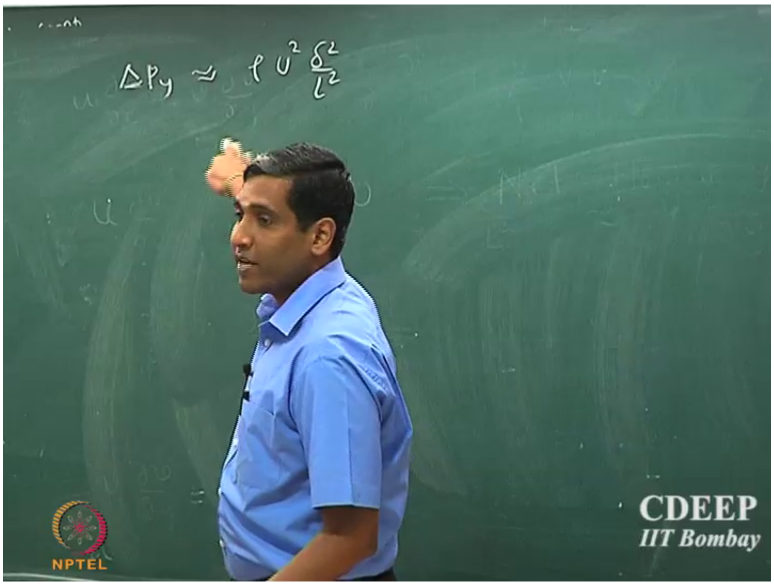
Student: (Refer Time: 16:59).

So.

Student: ((Refer Time: 17:01).

Correct correct. So, if you actually note that the. So, you have to factor of the delta into account, if all of them are equal order of magnitude and, because delta P.

(Refer Slide Time: 17:13)



So, because delta P y scales as RHO into u square delta square by L square, you expect that should be constant and that is, because the scaling as is d P by d y. So, we have a 1 by delta right. So, that is why. So, all of them are of same order of magnitude. Now, you can not cancel out any terms. So, all you can say that is only that the pressure is approximately constant that is all you can say.

Student: (Refer Time: 17:44).

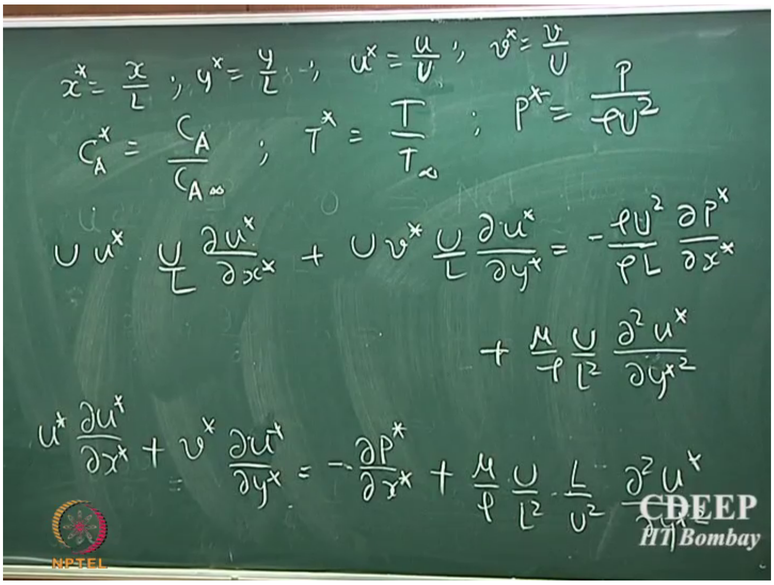
No, if you assume that the delta P y, which comes out from the scaling that it is approximately constant, then you put reduce into the factor of d P by d y is 0 right, because all the other terms although they are comparable, but they are very small and therefore, d P by d y has to be 0. So, if you, if the order of magnitude analysis tells you that the pressure change is constant in y direction, then you would naturally expect that d P by d y is approximately 0, again it is approximate.

Student: (Refer Time: 18:20).

But y it is y scaling is very small right. So, if the pressure changes very small in that direction and the net pressure gradient has to be approximately 0, otherwise you going to have a net flow in y direction. So, there is no net flow in y direction, the net flow is only in the x direction, there is a y component velocity in the boundary layer, because the fluid sees it. It experiences a friction with the wall, it experiences a friction with the interface, but the net pressure gradient in y direction should be 0. I mean approximate very important only approximately 0 alright.

So, Now I am going to introduce some scaling. So, suppose I say that x star is my new variable and I scale it with the length of the plate.

(Refer Slide Time: 19:18)

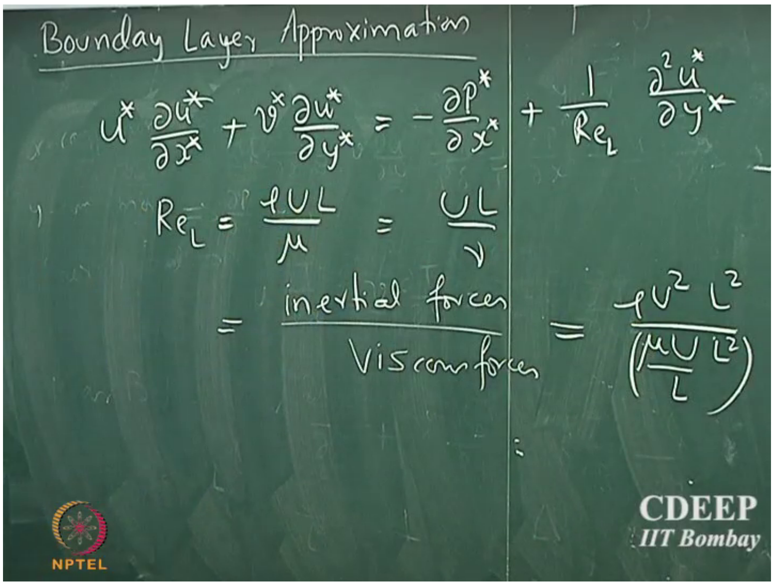


And y star also, I scale it with the length of the plate, because that is the measurable property. So, the boundary layer thickness is not a measurable property. It is not something that you can measure and then I scale U star U as the x component velocity divided by the free stream velocity, it is capital U. So, if capital U is the free stream velocity and if I say v star is v by capital V capital U. So, I use the same velocity free stream velocity in order to scale the x and the y component, because that is the only measurable quantity that I have.

So, similarly I could scale C a star the concentration of this species with the free stream concentration and the temperature with the corresponding free stream fluid temperature. So, if I introduce these scaling. So, I am going to non dimensionalize these equations. So, that will be u into u star u into b u star by L into d x star plus u into v star u into v u star by v y star t hats equal to minus 1 by RHO. Now, pressure you can actually scale with RHO v square. So, if I introduce P star as P by RHO v square. So, if that is the scaling that I introduce for pressure. So, that will be RHO v square into d P star RHO u square thank you divided by L into d x square.

So, that is the scaling for pressure plus mu by RHO into mu into d square u star divided by L square into d y star square. So, now, if I cancel out all the like terms and. So, this will become u star into d u star by d x star plus v star into d u star by d y star that is equal to. So, now, I bring L by u square. So, L by u square goes out RHO will cancel out. So, it will be minus d P star by d x star plus Now, I have L by u square. So, that will mu by RHO into mu by L square into L by u square into d square u star by d y star square what is this term this is one number, Reynolds number .

(Refer Slide Time: 23:05)



So, that will be u star d u star by d x star plus these star d u star by d y star that s equal to minus vb P star by b x star plus one by Reynolds number defined based on the length of the plate into d square by u star by d y star.

So, Reynolds number is nothing, but RHO into U into L by mu. So, if I know the kinematic viscosity, which is mu by Rho. So, Reynolds number is U into L by mu. So, this is the Reynolds number for the flat plate, this is defined based on the length of the flat plate. So, this Reynolds number will tell you what is the translation from laminar to turbulent flow. So, this is the ratio of Reynolds number is the ratio of

Student: (Refer Time: 24:14).

Inertial over viscous forces. This is the ratio of inertial over the viscous forces. What is the inertial force?

Student: Half RHO U square.

Half RHO U square that is it, that is pressure divided by L.

Student: (Refer Time: 24:52).

Hm

Student: Multiplied by L square.

Student: Yeah (Refer Time: 24:58).

Yes this is stress ok.

Student: Multiplied by.

And what about the viscous forces.

Student: (Refer Time: 25:08).

Mu mu.

Student: (Refer Time: 25:13).

U ok.

Student: U over L.

U over L multiplied by L square so that, what gives you the ratio of inertial to the viscous forces. So, once you have this. So, it tells you what is the effect of inertial and viscous forces on the fluid flow that is what Reynolds number characterizes. So, if you know Reynolds number then the properties of the fluid will tell you what is the nature of the flow. So, it captures it, incorporates all the properties of the fluid like density and viscosity, which effects the fluid flow and it characterizes as to what should be the nature of the flow.