Heat Transfer
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Lecture – 20
Energy and Mass Balance; Boundary layer approximations

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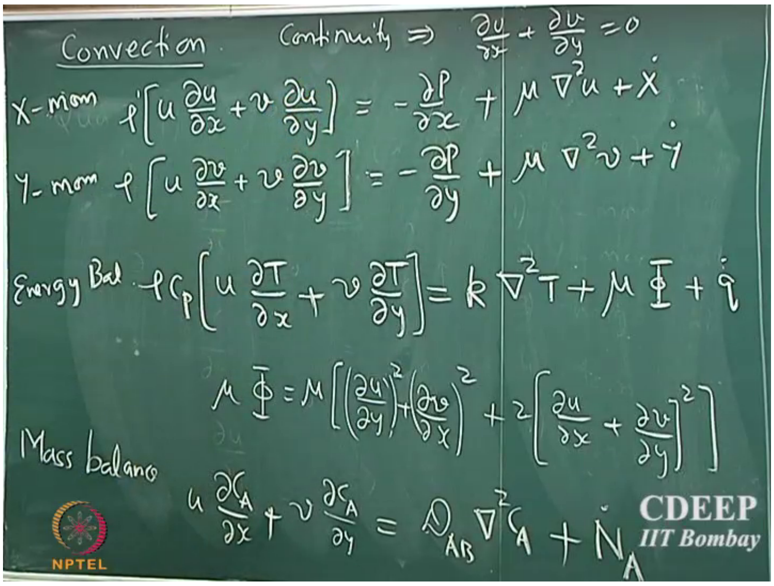

Next we are going to write energy balance. What are all the different terms involved in energy balance different processes conduction.

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Instead of shear stress it is conduction, what else will there be a pressure gradient no of course, not what else viscous dissipation.

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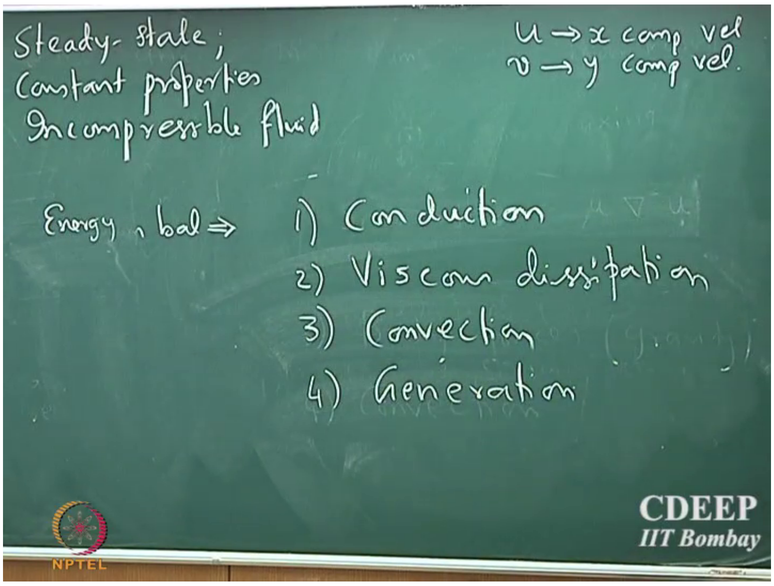

You will have viscous dissipation what else do we have convection yes or no we have convective mode of heat transport of course, yes of course, yes what else.

Student: (Refer Time: 01:35).

Excuse me work done by ok.

We can call it generation term. If it is a positive sign if it is generation it is a negative sign if it is a synch term. These are the 4 processes. So, We can easily write down these balances rho cp u dou t by dou x.

Student: (Refer Time: 02:03).

Viscous dissipation yeah.

Student: (Refer Time: 02:07).

What shaft.

Student: (Refer Time: 02:10).

Yeah.

Student: (Refer Time: 02:15).

Thats what convection accounts for that is what convection is already taken into account right dou T by dou y. That is your convection term and that should be equal to the conduction del square T laplacian is the mathematical representation for conduction plus the viscous dissipation I put capital phi as the viscous dissipation term plus the heat generation per unit volume viscous dissipation is given by mu dou u by dou y dou v by dou x square of that plus 2 times oops dou u by dou x plus dou v by dou y the whole square, that is viscous dissipation.

This sort of completes the energy balance again you can get the same thing from navier stoke. All that we have done you can use shell balances and reach the same expression you can write the navier stroke and approximate I mean reduce the navier stokes you will get the same expression and you can also start by describing different processes. It is all equivalent if you do it properly that is very important if you do it properly. Similarly we can write mass balance N what is the equivalent convective term for mass balance u into dc a by dx plus v into dc a by d y.

If a is the species and we are monitoring the concentration of the species. That is the convective term and that should be equal to the diffusivity multiplied by del square Ca. That is the mass diffusion there is no viscous dissipation. All that we can have is a generation term if I put n dot as the mass generation term it could be by chemical reaction etcetera it could be many different ways, but let us say Na dot is the volumetric mass generation term.

These are the 4 balances and of course, you can write continuity equation that is dou u by dou x plus dou v by dou y that is equal to 0. The continuity moment you say it is an incompressible fluid the continuity equation is naturally satisfied right. We really do not have to solve for continuity equation because the fluid is incompressible and it is a steady state system we are looking at steady state incompressible fluid. The continuity equation will always be satisfied. You do not have to worry about solving the continuity equation. So all we will have to worry about is solving these 4 equations is this linear or the set of equations is it linear why is it non-linear.

Student: (Refer Time: 05:59).

Yeah.

Student (Refer Time: 06:03).

Convective term and viscous dissipation in all the equations are only some of them some of them. So, which one is linear which one is non-linear.

Student: (Refer time: 06:15).

Yeah all are all are non-linear mass balance is linear what makes you think that it is linear.

Student: (Refer Time: 06:29).

What is the definition of non-linearity when do you call an equation linear.

Student: (Refer Time: 06:44).

That is 1.

Student: (Refer Time: 06:47).

Fine and then what else what else, note that u and v, all these equations have to be solved simultaneously right the only difference is that in principle.

Student: (Refer Time: 07:13).

Yes it does except that we have not written the constitutive equation. Supposing, it depends upon the concentration level for the species that you have supposing I have a species whose concentration is let us say CA 1.

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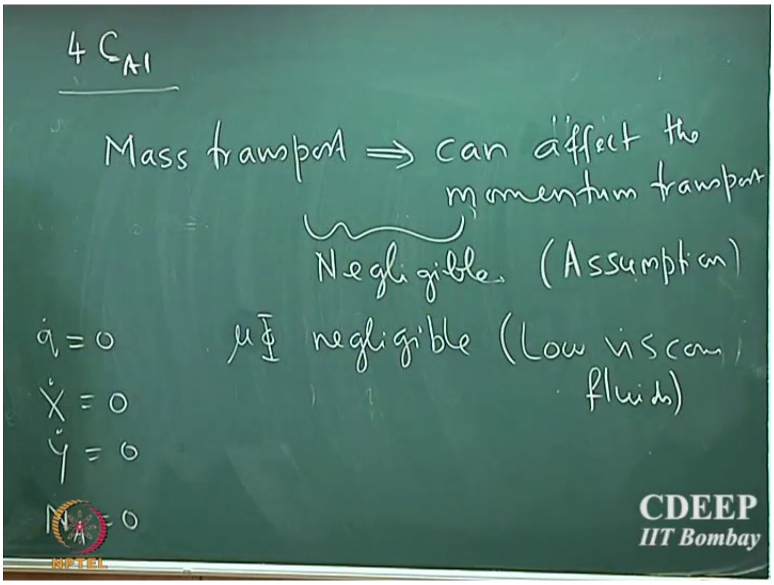

Now, I suddenly have from the solid or from the interface between the solid and fluid let us say that the species now has transported and it has become 4 times CA 1. Now, the density has changed right density of the fluid has changed.

Student: (Refer Time: 07:57).

Correct, so moment you, the first thing is that if you assume that the transport the velocity affects because of the concentration change is negligible right you can still have incompressible if you have concentration if you have 4 times the species that is added you can still have by actually tweaking with the velocity you can have a higher velocity flow you can have gradients you can play with the gradients d u by dx and d u by dy and still maintain incompressible fluid. Incompressible does not mean necessarily that the concentration does not affect the velocity.

In principle the mass transport it can alter or can affect the momentum transport. In fact, you may recall and your transport course the flux of mass transport was actually called as j it is defined as the diffusive flux plastic contribution from the conductive loss right. Anytime you have mass transport in principle it can actually affect the convection of that that species in that direction.

Student: (Refer Time: 09:33).

Correct right, but this 1 tells you that momentum transport affects the mass balance mass transport here. They are of course, couple. For the course purposes we are going to assume that this coupling is negligible we are going to assume that the mass transport does not affect the momentum transport this is an assumption that we are going to make and. In fact, some of these assumptions would be relaxed in the advanced course we will not do that here.

Therefore, this equation if NA dot is a linear term if it is linear then the mass transport equation will become lean all right. For to sim, further simplify we are going to assume that u dot is 0 we are going to assume that there is no heat generation term and then we are going to assume that the body forces are 0 to start with we will actually include body forces when we talk about natural convection, but let us assume for now that it is 0 in order to understand the simultaneous heat mass and momentum transport we will make these approximations and then we will assume that NA dot is also 0.

We will also assume that the fluid that we are actually dealing with is a low viscous fluid and therefore, the viscous dissipation is also not significant. With that assumption these 2 guys will go away these 2 guys will go away and this guy will go. Now, which are linear here which equations are linear mass balance is linear what about energy balance that is also linear what about U. If we know u and v then we should be able to find T and CA in principle it is not linear. You and b if we assume that the viscous dissipation is 0 then you and we are completely independent of the temperature right if we assume viscous dissipation is negligible then u and v are independent of temperature and therefore, these 2 equations are linear with respect to temperature and concentration.

What real task is to find what is u and what is v if we know that we know exact solutions of this.

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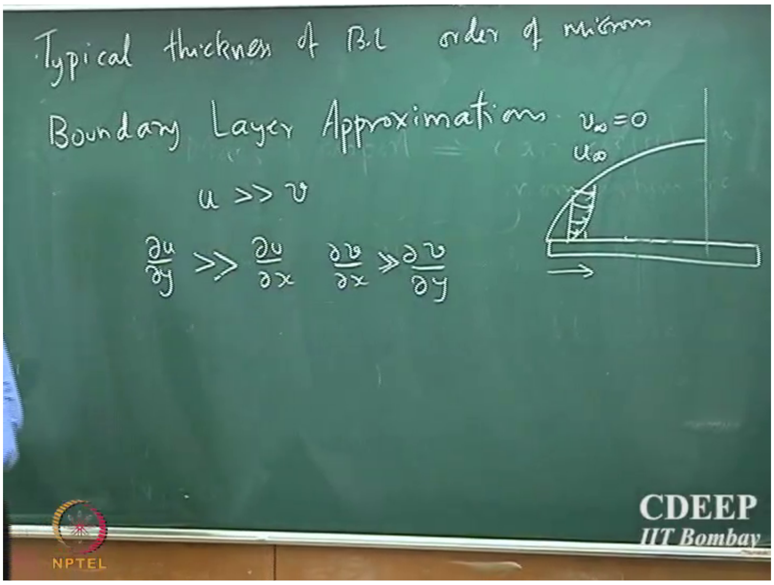

The next part we are going to do is, the typical thickness of boundary layer you about order of micron. That is the typical thickness of the boundary layer momentum heat and mass transport boundary layer. Because the thickness of the boundary layer is much smaller than the length of the plate or the system that you are looking at, we can introduce something called boundary layer approximation.

Student: (Refer Time: 13:39).

Yeah that is the typical order of magnitude right it is very small.

Student: (Refer Time: 13:44).

Hmm.

Student: (Refer Time: 13:46).

But still the order of magnitude is very small right. The boundary layer approximation, supposing if we know that there is. So, much disparity in the length scale of the boundary layer compared to the actual length of the system that you are looking at one could make certain approximations. Note that these are approximations and we make this. That we can actually get some insight as to what happening in this problem 1 could actually go and do is for all this numerically no 1 stops anyone from doing that 1 could actually solve them all numerically, but you do not get any major insight by simply going and brute force solving them.

We can introduce boundary layer approximations and the first approximation is that the x component velocity is much larger than the y component velocity the y is this justified why this reasonable approximation is. Let us draw the boundary layer supposing we take a flat plane what is the x component velocity above the boundary layer.

Student: (Refer Time: 15:03).

It will be the free stream velocity. Supposing I call this u infinity what about y component velocity y component velocity it will be may be 0 because most of the fluid is actually moving in the x direction.

The v infinity is actually 0, the y component velocity is really present only inside the boundary layer. The thickness is very small while the x component velocity is present in x direction and this length scale is much larger and also it is the flow is because of the pressure gradient right because there is an external pump which is actually forcing the fluid to flow past the object. Therefore, the ux component velocity is much larger than the y component velocity. Note that there is no pumping the y directions only the x direction flow what about d u by dy what can we say about the gradients.

Student: (Refer Time: 16:22).

What can we say about the gradients in boundary layer let us say d u by dy du by dx dv by dy dx and dv by dy what about order of magnitude of this 4 v it iself is very small right. These 2 gradients are expected to be very small what about d u by dx compared to d u by dy which 1 is very larger.

Student: (Refer Time: 17:06).

D u by dy is larger why is it larger why is it larger yeah.

Student: (Refer Time: 17:18).

The friction is actually present in the at the interface between the solid and the fluid right. Therefore, this friction is now going to introduce gradients strong gradients in the y direction right. The x component velocity the fluid particles which is actually moving in the x direction they are now retarded strongly because of the friction that is offered by this surface and therefore, you would expect that the there will be steep gradients therefore, because of that retardation the gradients in the y direction of the x component velocity is going to be much larger than the gradients that is offered in the y x direction it itself and the reason.

Why that is the case is that the retardation that might occur in the x direction of the x component velocity is only because of this shear of the fluid it iself it is not facing the friction of the wall which is much larger than the shear that is offered by the fluid through it is own particle and therefore, you would expect that the y comp y gradient of the x component velocity that is d u by dy is significantly larger than all the other gradients and. In fact, dv by dx actually is larger than dv by dy yes.

Student: (Refer Time: 18:53).

Yeah.

Student: (Refer Time: 18:56).

Correct then dvd u by dx is 0, still d u by d y is larger yeah, but that is only in the fully this is true even inside the boundary layer even before they fully developed regime. You are talking about what is the gradient after the fully developed regime where d u by dx is 0. That is very clear that d u by dy is going to be greater because the gradients in the y direction is not 0 we know that, but even inside the boundary layer d u by dy that is the y gradient of the x component velocity is always much larger than the x gradient of the x component velocity and all other gradients.

Student: (Refer Time: 19:35).

Because of the wall shear, that offers a significantly higher friction and than the shear that would be offered by the fluidity it is own particles and that is why this gradient is always larger.

Student: (Refer Time: 19:51).

Exactly see the fluid is moving here now because it is seen of all here the fluid a particles have come to rest at the interface. Now, you need to have a definite v dv by dy otherwise the continuity equation will not be satisfied. So, that forces in certain gradient in the x direction and that is going to be much smaller than the y component gradient I mean y gradient of the x component velocity any other question all right.

if we impose these boundary layer approximations. We can rewrite the model equation yes.

Student: (Refer Time: 20:33).

Yeah sure, supposing you take the wall is present here right, you have an interface now when the fluid is moving in the x direction. The pressure gradient is primarily in the x direction right because that is where your that is the direction in which the fluid is being pumped now. As to as soon as the fluid sees the flat plate it is exposed to the friction which is offered by this surface.

Therefore, it is going to retard the fluid particles above it in the y direction right, but then because some of the fluid particle velocity has now changed and some of them have actually come to 0 right the x component velocity has come to 0 the continuity equation will force that there has to be a dv by dy otherwise the continuity equation is not satisfied and because it is an incompressible fluid you must have well weak our y component velocity in order to satisfy the continuity equation.

Now, the slowing down or the gradients in the x direction that you will get is simply because of the shear of the fluid offered on itself. It does not feel the presence of the plate the gradient that you see in the x direction is not necessarily because of the friction that is offered by the plate, but the y gradient of the x component velocity is primarily because of the friction that is offered which is significantly higher and that is why this velocity gradient is larger than all of these and about these 2 is because the boundary layer thickness itself is very small. The y component velocity itself is going to be significantly smaller.

It has to go from some 0 at the boundary. Note that the v 0 here both x and y component velocity is 0 and if I actually go along this line.

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Let us say this is x one if I go along this line you see that the y component velocity at the edge of the boundary layer is also 0 because the it is primarily the free stream fluid which is flowing outside the boundary layer. The gradient is the velocity is 0 here and it is 0 here. Therefore, the gradients cannot be significantly larger the gradients are significantly larger then within small distance it cannot go to 0.

Therefore, the y component velocity is bounded with values 0 from the surface of the flat plate to the edge of the boundary layer that is because of yeah that is the mathematical way of looking at it. The gradients are not significantly higher that is why the these 3 gradients are actually much smaller than the y gradient of the x component velocity. we will include these boundary layer approximations with the model equation and we will start looking at some similarity. In fact, you can already see that these equations are similar if you know the pressure gradient. That is a measurable quantity right if I am pumping a fluid I should be able to measure the pressure gradient at which I am pumping the fluid.

If I know the pressure gradient that is a constant then if you stare at these 3 equations they are actually looking very similar. If this pressure gradient is a constant then all you have where is udu by dx plus v dv by d u by dy and that is very similar to the terms that you see here. It is convection equating to a diffusion term that is what you see in momentum transport that is what you see in my energy transport and that is what you see in mass transport also. You can already see that there is some similarity in the model equations that describes the velocity profile, temperature profile and concentration profile. They are going to capitalize on this similarity and we are going to show the next lecture as to how to actually calculate some of these important properties simply by capitalizing on the similarity of these 3 equations without solving them we are not going to solve these equations the way they look, but we are going to try to extract as much information and insight simply by looking at the structure of the equation.