**Heat Transfer**
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**Lecture – 19**
**Laminar & Turbulent flows; Momentum balance**

So, we defined the heat and mass transport coefficient yesterday. So, supposing if it is a flat plate.

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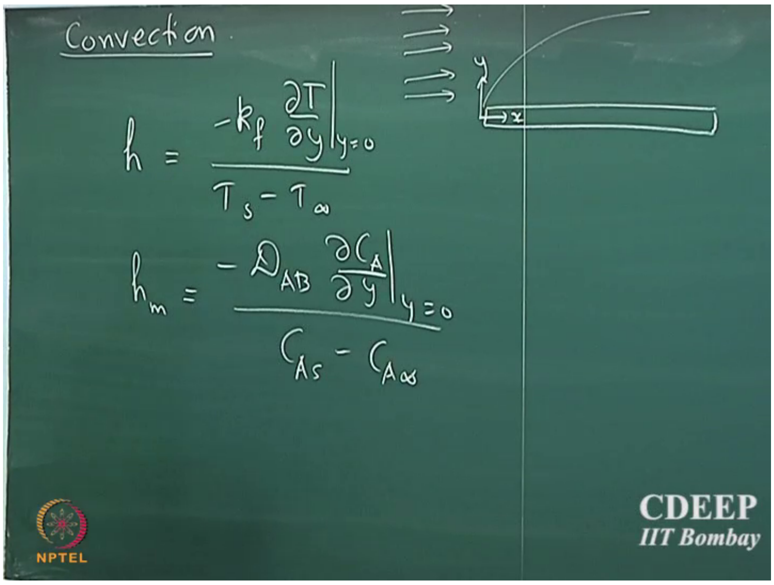


I have a boundary layer and you have fluid which is flowing past the object. What is h? what is h? Yeah.

Student: (Refer Time: 00:41).

Yeah heat transport.

Student: (Refer Time: 00:47).

So it is flux which is minus k f dT by dy. So, if this is y direction, evaluated at y equal to 0. So, that is the flux of heat that is transported at the boundary at the interface divided by the temperature difference, bulk temperature difference. And similarly one could define mass transport coefficient as diffusivity multiplied by, divided by C A, ok.

So now, let us go a little bit more and into the details of what is boundary layer and describe some of these processes and attempt to quantify them. So, before we do that, we need to understand some of the flow properties right. So, remember that there is fluid which is flowing past this object. So, therefore, any transport process inside the boundary layer is going to be a strong function of the nature of the flow itself ok.

Student: (Refer Time: 02:02).

T depended on A, of course yes Of course.

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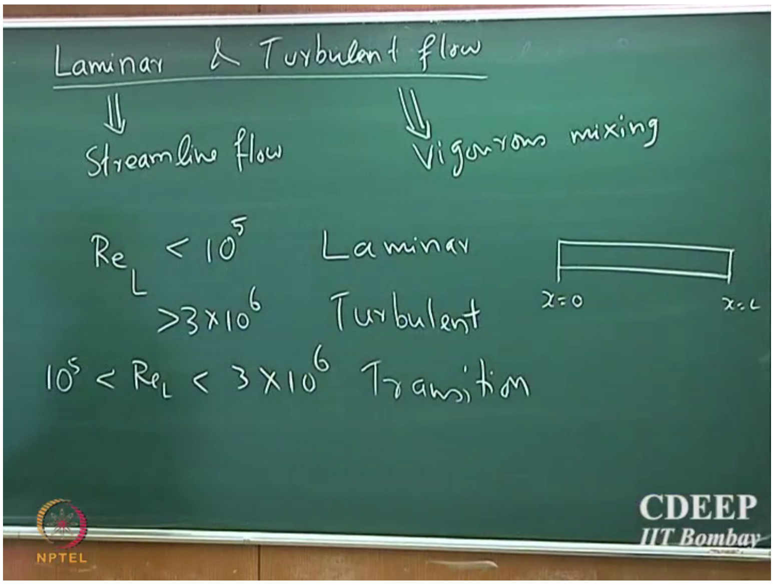


So, there are 2 classes of flow types called a Laminar and Turbulent flow. So, there are 2 classes, these are the 2 different nature of flow of a fluid. The Laminar flow is has an ordered flow where things flow in streamlines. And Turbulent flow is where there are fluctuations and there is cross mixing of fluid particles. And so there is no stream line flow. So, laminar flow is essentially a Stream line flow while there is vigorous mixing, vigorous mixing of fluid particles in the turbulent regime.

So, any fluid flow you can have either Laminar or Turbulent and somewhere in between is the transition region. So now, what is the characterizing number which tells you whether it is Laminar or Turbulent? It is called the Reynolds number. Now what is the Reynolds number? For what is the Reynolds number range for a flat plate where you will have laminar flow? Yeah up to what Reynolds number is the flow laminar?

Student: (Refer Time: 03:48).

2100 that is for which system?

Student: (Refer Time: 03:53).

That’s for pipe flow. This is not pipe. That is for flow through a tube right That is for pipe flow. What is it for flat plate? So it is 10 power 5. So, the length at which the Reynolds number reaches 10 power 5 is actually the limit for Laminar flow regime. And for Turbulent flow, anything less than 10 power 5, it is Laminar. So, note that in pipe flow which you have already studied the characteristic length scale is the diameter of the pipe. Here the characteristic length scale is the length up to which from the leading edge of the plate. So, if you have, if you have a plate here; this is x equal to 0 and this is x equal to some l.

So, the length of the location from the starting point, that is x equal to 0 of the plate. So, that is the length scale that you should use for defining this Reynolds number. So, that tells you which location along this plate, the flow is Laminar and which location it is Turbulent. So, typically 3 into 10 power 6, that is Turbulent regime and anywhere in between is called intermediate regime. So, it is neither Laminar not Turbulent. So, anywhere in between, it is called Transition regime.

So, you can see features of both Laminar and Turbulent flow in this region here. So, now, supposing I look at, I stare at the heat transport coefficient expression, now what is the, what can we say about the gradient as a function of.

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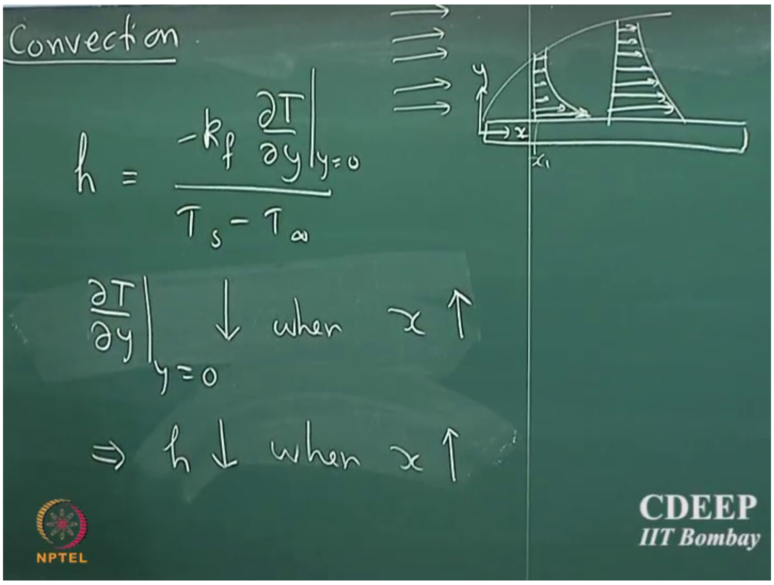


So when you go into the flat plate, you expect the gradient to decrease or increase? What do you expect?

Student: (Refer Time: 06:27).

Yeah, decrease or increase?

Student: (Refer Time: 06:33).

Why?

Student: (Refer Time: 06:34).

So the residence of or the contact time of the fluid with the flat plate increases as the fluid particles are actually traversing through these flat plate right, flowing faster. Therefore you would expect that the fluid would have gained more heat from the soil, from the flat plate; and therefore you would expect that the gradient will actually decrease when you increase the, when you go further into the flat plate. So, this is the gradient at x equal to, but y equal to 0.

So, we can sketch it. So, if that is the profile at let us say this location x 1, some other location. Because the fluid is now exposed to the plate for a longer period of time, more heat would have got transported. So, you would expect that the profile would slope look something like this, where the gradient has now decreased. Because the fluid particles which are actually, let us say well above y equal to 0 would also reach temperatures which is close to the plate temperature.

So therefore, you expect that the gradient would decrease as you go into the plate. And that simply means that, the h, the heat transport coefficient would actually increase as you, this directly translates into the reduction in the heat transport coefficient. Note that this is local heat transport coefficient. There is a reduction in the local heat transport coefficient as you actually go into the plate. So, T s and T infinity is our measurable quantity and we can assume they are constants and k f is an intrinsic conductivity of the fluid. It is an intrinsic property and we can assume that also as a constant.

So, clearly the heat transport coefficient, the nature of local heat transport coefficient depends primarily on the temperature gradient at y equal to 0. So, the whole of this exercise that we are going to do in convection topic is essentially to find out for various kinds of system; what is this temperature gradient. So, all the quantification process that we are going to actually discuss in the next several lectures for various kinds of geometries, various kinds of flow properties is essentially to find what is this gradient. If we know this gradient, we are done. Because this is a definition of heat transport coefficient and a similar way for mass transport, if we know the concentration gradient at y equal to 0, we are done.

So in principle, the momentum heat and mass transport, they can in principle occur simultaneously. So, there could be a system where you have a boundary which is maintained at a higher temperature than the fluid which is flowing past it. And the there could be some species which is simultaneously moving from the solid phase to the fluid phase. It could be like a membrane for example. An excellent example of that is the cholesterol which actually deposits into the arteries of the of the blood system right, of the blood circulation system.

So, the blood actually carries cholesterol. So, whatever food that we eat, it is processed to me, it is the nutrients are extracted in the intestine, the small and the large intestine. And so the way the nutrients are carried is the bloodstream, they actually go over there and then they carry the nutrients and these nutrients are now circulated in the bloodstream, whatever cholesterol particles or fat material that you eat, whichever is absorbed by the intestine, it is actually circulated and into the bloodstream and then these they go on deposit in the arteries. Now you could, it is a reverse process of what we saw here where the concentration in the solid phase is higher and the liquid phase is lower in the in the example that we are considered. But in the deposition of cholesterol, the cholesterol concentration will be higher in the fluid stream and you have deposition of cholesterol on the surface ok.

So, it is a very strong application of some of the things that we are going to see. And in fact, people have actually found out what is the mass transport coefficient for cholesterol deposition. And that has a strong implication on a perfect diagnosis based on the concentration of cholesterol in the blood. See it is a very difficult thing to see. One cannot see how much cholesterol is being deposited in the arteries; there is no in situ way of doing that, but there are some ways now, but there is no clean way to do it. So, if we can find out what is the approximate mass transport coefficient and based on the concentrations we should be able to estimate the rate at which the cholesterol is being deposited in the arteries.

So, this has strong implications in deciding the, you know, the medical strategies or what kind of invasive treatment that one has to have, if there is a strong cholesterol deposition. So, there is a lot of applications for some of the things that we have seen. And in fact, it depends on temperature. You put your hand in front of the infrared camera; you will see that the temperature or different location is different. And that is true for the blood stream also. So, the local temperature of different location of the body will be different. So, it will be like a simultaneous heat, mass and momentum transport. All 3 are occurring simultaneously.

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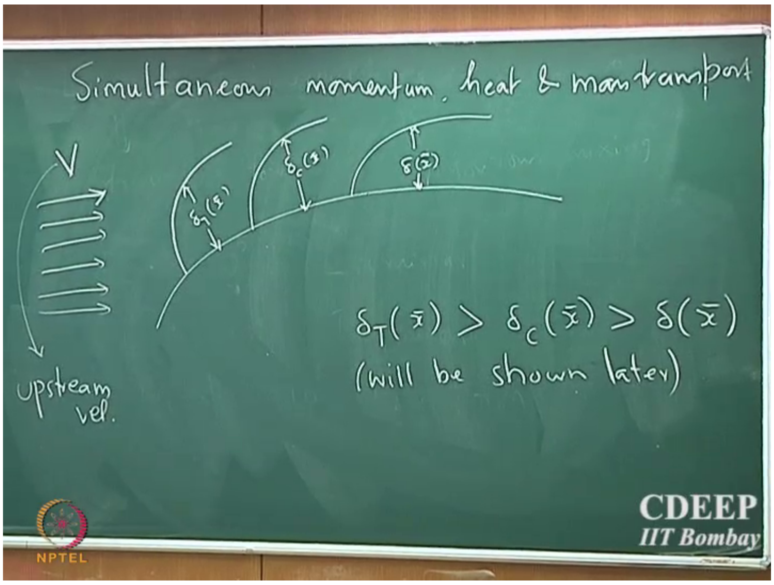


So, it is a very important aspect. So, this is Simultaneous momentum heat and mass transport. In most real system, you will see that it occurs simultaneously. So, supposing if we have an arbitrary object, where there is a fluid which is flowing past this object, now if I define V as the upstream velocity. So note that I am going to introduce some key definitions. So, V is the upstream depth velocity. So, this is the velocity of the fluid before it sees the object. Then when it sees the object, obviously you are going to have a boundary layer, a velocity boundary layer or a momentum boundary layer.

So, let us say that this is the velocity boundary layer. So, it is a, it could be in all 3 dimensions. So, I put an x bar for vectorial representation. And, one could have a concentration boundary layer and one could have a thermal boundary layer. So, all 3 will exist simultaneously. And there is a fluid which is flowing past an object and there is heat and mass transport which is occurring simultaneously. So, the thermal boundary layer thickness is actually larger than the concentration boundary layer thickness and that is larger than the momentum boundary layer thickness. In fact, mostly not todays lecture, maybe in the next lecture or the 1 after that you will actually show rigorously that this is true.

So, right now you assume that the thermal boundary layer thickness is larger than the concentration boundary layer and that is larger than the momentum boundary layer. We will actually show this later. We will be shown later as to why that is the case. In fact, the governing equations will, it will fallout from the governing equations that this is really the case; all right. So now, the question is how do we quantify this process. How to quantify this process?

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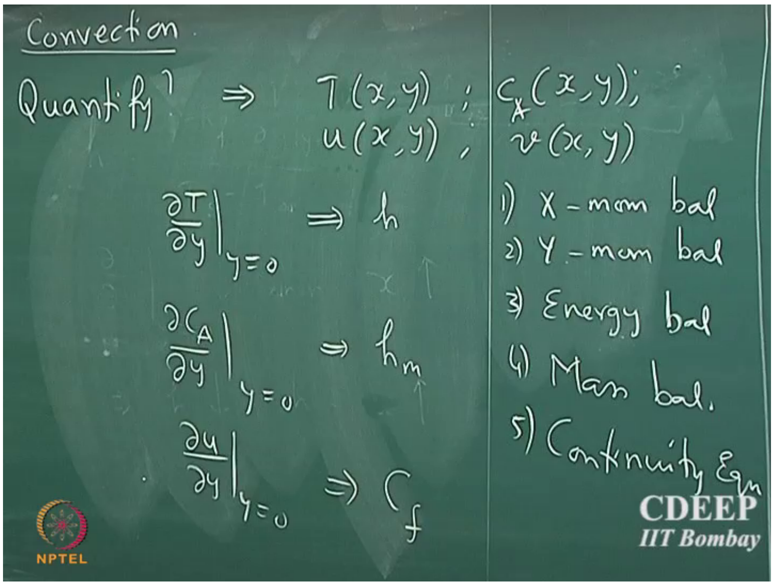


So, note that the objective of the course is to quantify these different heat and mass transport processes.

So, if they are occurring simultaneously, how to quantify them? How do we find, when I say quantify what I mean is, what do I mean? What did we mean by saying quantification when it came to conduction.

Student: (Refer Time: 15:39).

Yeah, you wrote the governing equation.

Student: Get the.

Get the temperature profile. So, when you say quantify, you need the temperature profile as a function of x y. And you need the concentration profile as a function of x and y. And you need the velocity profile as a function of x and y. And you have 2 components if it is a 2D system you have, 2 velocity components u and v right. It is the x component velocity and v is the y component velocity. So, if we know all of these, then you are done. Because the objective is to find dT by dy. That is what we need to find. We need to find dT by dy. This will give you the heat transport coefficient dC A by dy, at y equal to 0. That will give you the mass transport coefficient. And du by dy, what does this give? du by dy at y equal to 0. Yeah what does it give?

Student: (Refer Time: 16:54).

Coefficient of viscosity No. It is not coefficient of viscosity. Shear?

Student: Shear stress.

Shear stress, it gives you the friction coefficient.

We defined this in yesterday's lecture. So, if you know these 3 gradients, you actually quantified or you have identified the characteristic properties of the 3 boundary layers; which means that we have identified the characteristic properties of the 3 transport processes that are occurring simultaneously. So, in order to find these, we need to write for u and v, we need to write the X momentum balance, we need to write the Y momentum balance, we need to write the Energy balance and we need to write the Mass balance. So, these are the 4 balances that we have to write. And of course, Continuity equation has to be satisfied.

So, these are the 5 different pieces that we have to write. So, that will essentially, these are the equations which will capture the profiles of temperature, concentration and velocity ok.

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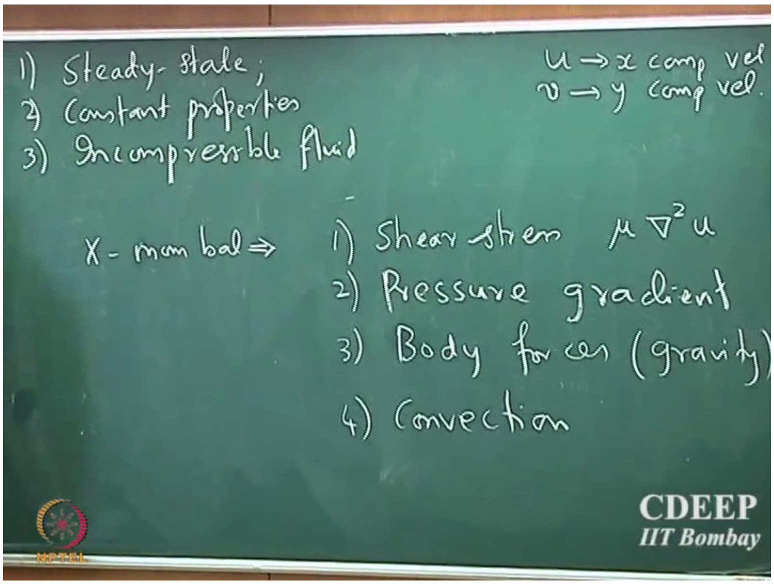


So, suppose I assume that it is a steady state process, I assume that the Steady-state process. And if I assume that the properties are Constant, this is not a bad assumption to make; if I assume that the properties are Constant and if I assume it to be an Incompressible fluid. So, we will not deal with compressible fluids in this course. There is an advanced transport course. So, some issues related to compressible fluids and quantification of compressible fluid systems will be dealt with in that course.

So, this course will assume that all the fluid is incompressible all right. So, so we need to write the X momentum balance. So, what are all the processes which are likely to occur? So, remember that whenever you write balances, you must identify the individual processes which are occurring in that problem. And then you find out what is the mathematical representation and then put the correct signs to it, you are done. Of course, you can do it by writing shell balances, that is the rigorous way of doing it, but you must also know what is the non rigorous way, otherwise you do not know if we made a mistake in your shell balance. You must also know what is the non rigorous way of doing it ok.

So, what are the different processes which are occurring here, let us say in momentum balance?

Student: Shear stress.

Shear stress. So, that is the; why only wall? It is only at the wall? There can be shear stress between different fluid elements right. So, in general Shear stress. What is the mathematical representation of shear stress, mu into del square.

Student: (Refer Time: 20:50).

So will be, del square u if it is an x component momentum balance. So, note that u is the x component velocity. u is the x component velocity and v is the y component velocity. What else?

Student: Pressure gradient.

Pressure gradient, because it is the forced system, you are pumping fluid. They are actually pumping the fluid to flow past the object. So there has to be a Pressure gradient otherwise there is no flow right. The Pressure gradient is 0, there is no flow. What else? Yes.

Student: Gravity.

Gravity forces. So, in classical literature it is called Body forces. So, all gravity etcetera will be coupled into body forces. What else? May not have heard this term, but it is what, that is what it is called as. It is called body forces. What else? What is missing in your, you have seen Navier's stokes equation right. Yeah.

Student: (Refer Time: 22:16).

Convective momentum. So, do you have Convection. Right So, it is essentially different terms what you would see in your name is Navier's stokes equation that you studied right. So, if we put them all together, what is the mathematical representation of momentum convective momentum term?

Student: Ovv.

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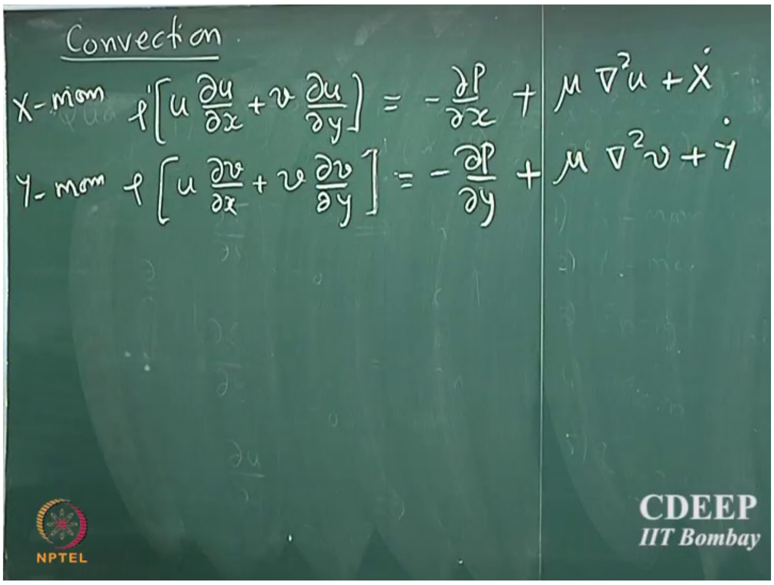


Ovv. So, there will be rho into, if I open it up, do u by do u x plus v into do u by do y. So, that is the momentum in the x direction. That is the momentum in the y direction. And that should be equal to the pressure forces will be minus dP by dx plus the shear term, it will be mu into del square u plus whatever body forcess. So, x dot is the body forces. One can actually get the same expression by starting from Navier stokes, you can actually start from shell balances I mean Navier stokes. In any way you can get from shell balances.

So, you can start from shell balances and take put the right approximations and you will see that you will get this. You can start from Navier stokes, you impose the approximations or assumptions, you should still get the same expression. And the third way is you simply identify different back processes which are involved, what is the mathematical expression corresponding to that and put the correct sign.

Student: (Refer Time: 23:59).

Because you know what is happening. So this is the, supposing you the fluid is moving in this direction so if I take this direction is positive, Convection is taking the fluid from x equal to 0 to x equal to l right. So, that is what let us say if this is my positive term here and dP by dx has to be.

Student: (Refer Time: 24:24).

Exactly. So, a moment you know what is your positive direction, everything else falls into place. What is the purpose of shear stress, it is going to retard the flow right. But there is going to be friction between fluid layers and that is why the shear stress comes here and body forces will come on the other. So, it is very easy put this.

So, a good exercise would be to go back to Navier stokes equation that was derived to you in transport course and stare at it and see what are the different terms and how does the sign actually forces the different terms to come in different locations in the equation. That will help you to identify what should be the sign that if you put for each of these terms. So, you must actually convince yourself that there are different signs that you were to put based on what are the different processes. In fact, if you look at the shell balances, the sign actually falls out naturally because you would have accounted the signs when you wrote the shell balance right. And you must convince yourself what these signs are. It is very important ok.

So, this is a quick way of writing the balance. Of course the laborious way is to write the shell balances and the intermediate way is to start from Navier stokes. All right. So, this is the X component momentum balance similarly we can write the Y component momentum balance, will be rho u dv by dx plus v dv by dy, that is equal to minus the Pressure gradient in y direction plus mu del square v plus y dot. So, if I assume y dot is the body force in y direction. So, that is the y component momentum balance.