Heat Transfer
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Lecture - 18
Boundary layer: Momentum, Thermal & Concentration

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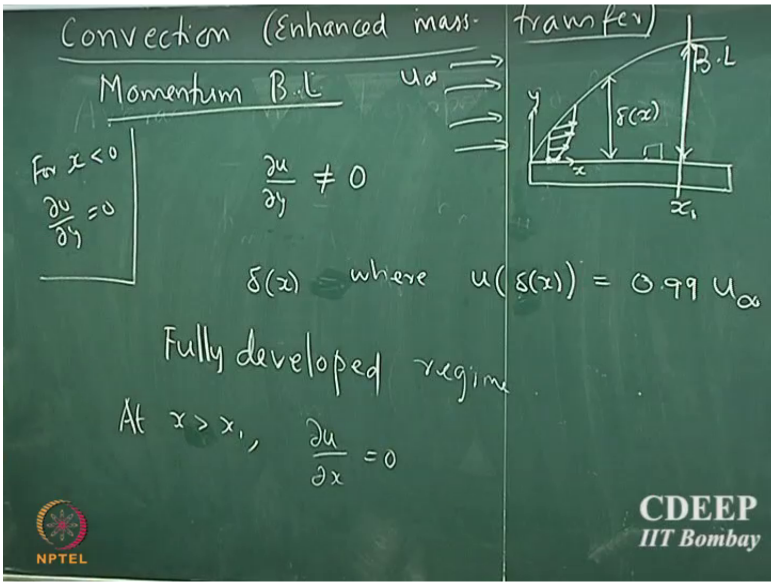

So, suppose I take a flat plate ok. So, I said that is going to be a boundary layer. So, let us look at the momentum boundary layer. So, this has been dealt with the to quite extensively in your fluid mechanic course. So, ill just briefly outline and will not go into the details of explaining all aspects of the momentum boundary layer. So, what happens is that the fluid which is coming pass this object. So, let us say the free stream velocity is u infinity. So, suddenly at y equal to 0 and x equal to 0 the fluid will suddenly come to rest at this location here so, but because of continuity. So, note that it is a continuous stream.

So, so the fluid which has come to rest here it is now going to retard the fluid particle which has actually go a flowing above it supposing we take a fluid particle here. So, that fluid particle before it it is felt by the flat plate it is moving at a velocity of u infinity, but then as soon as it sees the flat plate, the fluid particles below it has come to rest. So, now, because of continuity it is going to retard the fluid particle which has actually placed above it. So, therefore, you going to see here a temperature gradient and I am sorry velocity gradient and velocity profile in the boundary layer. So, this is the boundary layer.

So, therefore, d u if u is the x component velocity. So, d u by d y is not equal to 0. So, before the plate. So, for y less than 0 d u by d y is 0, cases very important because we assume that it is a flat velocity profile and. So, the velocity of every fluid particle is actually exactly the same and. So, there is no gradient in the y direction. But as soon as it sees the flat plate, now suddenly there is going to be a gradient in the y direction.

Student: Sir why does (Refer Time: 02:44).

Why does.

Student: (Refer Time: 02:46).

Oh I am sorry thanks you thank you very much x less than 0 that is right. So, for x less than 0 there is going to be a gradient of the x component velocity in the y direction 0 gradient, but when x is greater than 0, there is going to be a finite gradient and that is because the fluid particles which is at the surface, they have come to rest and because of continuity they retard the fluid particles above it ok.

Student: (Refer Time: 03:15).

Fluid is not.

Student: (Refer Time: 03:17).

That is true it has a viscous shear we going to talk about shear stress very soon, but note that it has viscous effects, now what happens is that because of viscous effect you will see that the gradient is going to come in both directions and. In fact, the retardation why does it retard because there is a definite viscosity and so, you when there is one fluid particle which is at rest, the other fluid particle is now trying to flow at a certain velocity whether it is going to offer a retardation because of the viscosity right so.

Student: (Refer Time: 03:51).

But if it if it is not continuous right.

Student: Sir if (Refer Time: 03:54).

If you assume the fluid to be non viscous.

Student: But is there.

But is there a fluid which is not viscous.

Student: (Refer Time: 04:09).

It does not make sense because at the boundary viscous forces are the ones, which play dominant role even if the viscosity is very small.

Student: At the boundary.

No at the interface excuse me.

Student: at the interface it, it not the (Refer Time: 04:22).

Why in the boundary layer it is not viscous forces?

Student: Viscous forces are the forces (Refer Time: 04:29) molecule.

Yeah. So, supposing I take a section here.

Student: Not there the out.

Here it is 0.

Student: That is what.

At the boundary at the interface it is 0.

Student: (Refer Time: 04:41).

But we are talking about retardation in the boundary layer right. So, so we are talking about the element which is present here, I mean it could be here this element could influencibly be very close to surface it does not matter only that monolayer of that fluid particle which is in touch with the surface has 0 velocity, and because the velocity is different the viscosity is now going to viscous forces are going to retard the fluid from flowing in the same velocity above it the velocity cannot be the same because the viscous forces are playing a role. But if there is no continuity then that is not going to propagate up to the boundary layer. So, because the fluid stream is continuous, that is why the gradient is propagating all the way up to the boundary layer otherwise it is not going to propagate. So, continuity is an important assumption here if the system is not continuous, then everything is going to crash right at the monolayer.

So, it is very important continuity plays an important role here and because of this gradient, you also going to have a gradient in the x direction right because of the continuity equation you going to see that in a short while. So, now, this thickness is what is called the boundary layer thickness and this is obviously, a function of position this is; obviously, a function of position and. So, the boundary layer thickness is defined note that it is a definition. So, boundary layer thickness is defined as. So, the location where boundary layer thickness is defined as that location, where the u of y u at delta y delta x. So, delta is the height of the boundary layer at any x location. So, u at delta y is equal to 0.99 times u infinity ok.

So, that is the 99 percent of energy transport is already accounted for that is what is called the boundary layer thickness, and then you have something called a fully developed regime what is a fully developed regime how is how can we identify what is fully developed regime what is it mean?

Must have been taught you in fluid mechanics no

Student: Boundary layer (Refer Time: 07:31).

Oh I see. So, what happens is supposing this is the this is the profile that you will seen now at some location x 1, let us say this is x 1 at some location x 1 or let say at any x greater than x 1, you will see that the profile d u by d x will become 0. Now what is it mean it means that.

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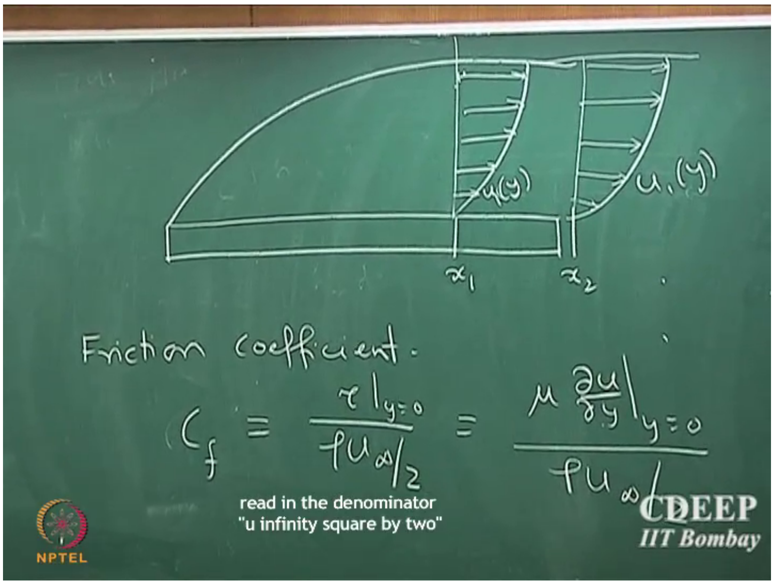

So, suppose I zoom up this flat plate here, at some location what happens is that you will have a will have a velocity profile, but then the profile will be maintained. So, this arrow distance tells you what is the velocity of the fluid at that location and. So, there at the intense or there velocity of each of the fluid particles across y direction, would actually be maintained if you actually go in any x direction. So, which means that if let us say that the velocity profile is u of y. So, if I call this as u 1 of y that is the velocity profile at x 1 direction. So, x 1 location and let us say this is x 2 then the velocity profile will continue to remain the same which means that the boundary layer thickness will remind this thing after the location x 1. So, that is what this regime is what is called as a fully developed regime. So, the regime before is called developing regime and after x 1 is called the fully developed regime.

So, now the properties in this boundary layer is actually characterized by what is called the friction coefficient. So, it is the friction which is causing these kind of velocity profiles. So, it is characterized by friction coefficient, C f is the symbol that is given for friction coefficient and that is given by the shear stress at wall shear stress at y equal to 0 divided by infinity by 2.

So, that is the friction coefficient and so, this characterize the shear stress which is present at the interface between the flat plate and the fluid. So, that will be mu into d u by d y at y equal to 0 divided by infinity by 2. So, that is the friction coefficient shear stress is given by viscosity multiplied by the velocity gradient x component velocity gradient at y equal to 0. So, now, similarly very similar to the momentum boundary layer, if we want to characterize heat and mass transport in the boundary layer, note that at the near the inter near the interface of these two phases, it is the boundary layer which is controlling all the transport processes.

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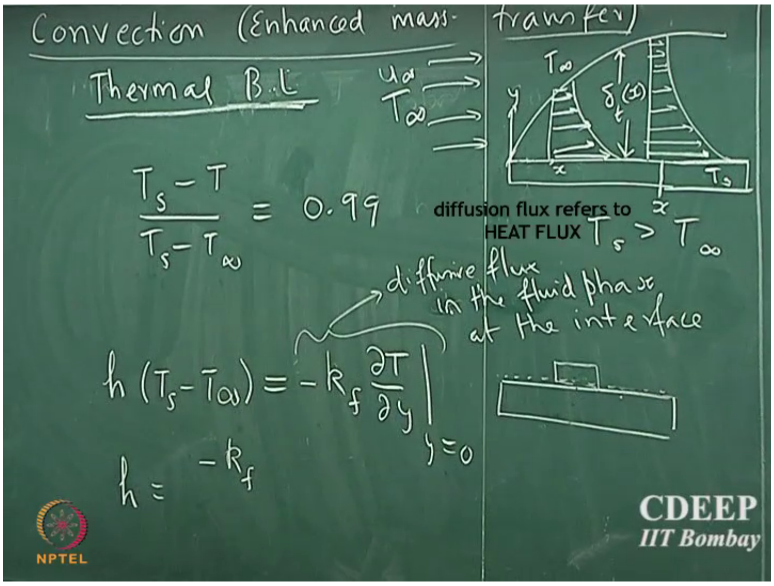

So, therefore, the there has to be something called thermal boundary layer, very close to the interface there will be a thermal boundary layer. So, supposing if the temperature of the flat plate is let us say T s and the temperature of the free stream fluid is T infinity, now there is going to be a boundary layer. So, that is the thermal boundary layer. If I assume that T s is greater than T infinity what will be the temperature profile.

Student: (Refer Time: 12:03).

In the.

Yes it will.

Student: (Refer Time: 12:06).

So T a T infinity is the temperature of the free stream fluid which is above the boundary layer and. So, the temperature profile will be asked fluid particles which is close to this surface, would actually quickly reach equilibrium with that flat plate and the temperature of that will be very close to or equal to the temperature of the flat plate itself. And therefore, there will be a gradual decrease in the temperature till the boundary layer. So, that is the temperature profile that you would expect and similarly you can draw this at different location. So, one could define thermal boundary layer thickness. So, this height is called thermal boundary layer thickness, which is again a function of the x direction and. So, this is defined as is very similar to how the momentum boundary layer was defined, we said 99 percent velocity, similarly here we can definebT s minus T by T s minus T infinity should be equal to 0.99.

So, note that here the temperature of the surface plays an important role unlike in the case of momentum boundary layer, the flat plate is not moving. So, the velocity of that surface is 0 right. So, here everything depends upon the temperature of the flat plate and therefore, everything has to be scaled with respect to the measurable quantities T s minus T infinity. So, note that the measurable quantity has already copped in here, and we going to see a lot more of these when we discuss all kinds of aspects of convection alright. So, now, we need to simulate a friction coefficient, we need to define what is called the heat transport coefficient. How do we find this can we make a comparison. So, I said momentum heat and mass transport they are all similar processes, how do we get the friction coefficient. The shear stress divided by what is rho u u infinity by 2.

Yeah.

What is rho u square by 2.

Student: Inertia.

It inertia right. So, it is shear stress divided by inertia what should be the equivalent here flux divided by. So, the way to see that is suppose I assume the interface. So, this is my interface. So, I have a fluid particle which is on top of that surface, now I write a an energy balance in the fluid phase at the interface. So, whatever heat that is actually transported from the flat plate to the fluid is transported via conduction right we said that the velocity of these particles as 0. So, essentially the transport is occurring because of conduction. So, therefore, the flux at that location this is Newton's law of cooling h into T s minus T infinity should be equal to what is the diffusive flux. Whatever flux of heat that be leaves the surface and goes to the fluid should be equal to the amount of heat that is actually diffusing at that location right what is that.

Student: (Refer Time: 00:00).

Be careful minus k into sign is very important here, minus k into d T by d y at y equal to 0.

Student: (Refer Time: 16:09).

That is if you assume that there are things are not moving right because now it is a two dimensional problem, where the gradients of temperature in both direction. So, it is not just that it is simply being heating the fluid, but the temp the fluid is also moving.

Student: (Refer Time:16:36).

So, it is also convicted. So, it is not only leading to just heating the fluid and increasing the temperature, unlike what we considered for a case of a solid, where it is pure conduction here the temp the fluid is also moving. So, therefore, in the fluid stream there is a temperature gradient in both directions.

So, you cannot assume that whatever the heat that is being transported is simply being used for heating of the fluid that is not a correct assumption because there is also a convection which is playing a role here. It is not pure increase in the capacity of the of the fluid. So, therefore, the local flux h into T s minus this balance is only at the boundary where the velocity is 0 only at the interface, at the interface the velocity is 0 we are able to do this simply because of the observation that there is no slip between the particles which is sitting right at the interface of the solid and the fluid. So, we are not neglecting convection note that I said it y equal to 0 right. So, not neglecting convection here only saying that the particles are at rest therefore, convection is not playing any role here. So, therefore,

Student: (Refer Time: 17:38).

Correct because of the pumping of the fluid there is a bulk motion and that is actually leading to convective mode of heat transport and. In fact, that is leading to the temperature gradient. So, therefore, from here heat transport coefficient can be written as k f. So, note that this is the conductivity of the fluid not the solid we are looking at heat transport from the solid at the boundary, and it is carried because of diffusion in the fluid phase at the interface. So, this is diffusive flux at the interface. So, note that is very important to appreciate this. So, this is the diffusive flux in the fluid phase at the interface.

Student: (Refer Time: 18:38).

Yeah if I write a balance for the flux inside the solid that is correct, because you will say minus k solid d T by d y should be equal to the heat transport coefficient times the temperature gradient.

Student: (Refer Time: 19:06).

Right, but that is if you assume that it is not uniform I know how to maintain a uniform temperature inside the flat plate that is not arise.

Student: (Refer Time: 19:18).

But if it is not uniform yes you should consider the gradients inside

Student: (Refer Time: 19:19) how was the how was the flux (Refer Time: 19:20).

They are equal this will be equal to minus k into d T by d y in the solid phase it will be equal.

Student: The gradient,

The gradients will be different be careful the conductivities are different.

Student: (Refer Time: 19:25).

So, the gradients will be different, but the flux will be equal.

Student: So, I (Refer Time: 19:29).

No no gradiance will not be equal because these are completely different object if I said that sorry. So, gradients are not equal in two different phases because their properties are different the conductivities are different. So, the gradients cannot be equal if they are in contact with each other their properties are different and. So, the gradients cannot be equal it will be the flux which will be equal. So, whatever is the difference in the conductivities will be offset by the gradient for the flux to be equal to each other. So, therefore, based on this heat balance at the interface, we can write the heat transport coefficient as.

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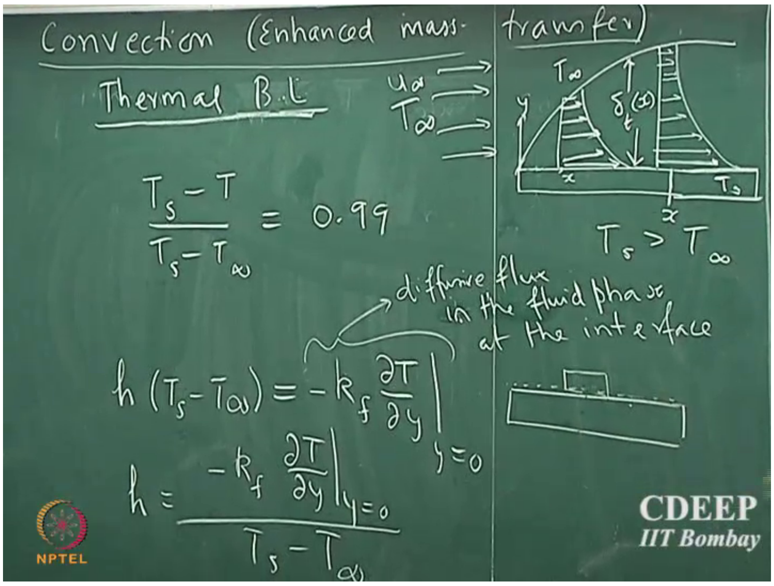

T s minus T infinity. So, that is the definition for heat transport coefficient. So, so far we liberally use heat transport coefficient, but this is the correct way to find the heat transport coefficient. It is the conductivity of the fluid remember that it is the conductivity of the fluid and not conductivity of the solid. So, it is the we are equating the heat transport flux, flux of heat that leaves the solid and goes into the fluid phase with the flux that is actually conducted by the fluid because of diffusion at the interface because the velocity is 0.

Student: (Refer Time: 20:53).

Oh sure steady state does not mean that it is not flowing you can have a steady velocity right where the fluids are moving at a same bulk velocity. So, remember that even for a simple one d slab in a steady state we had a linear profile the temperature is not constant. So, steady state does not mean temperature is constant it is the temperature profile, which is constant. So, therefore, gradients are not necessarily 0, when the when it is a steady state of course, there can be a system where gradient is 0, but that is not always the case any other question. So, one could actually make a parallel to mass transport in a similar fashion where we define concentration boundary layer.

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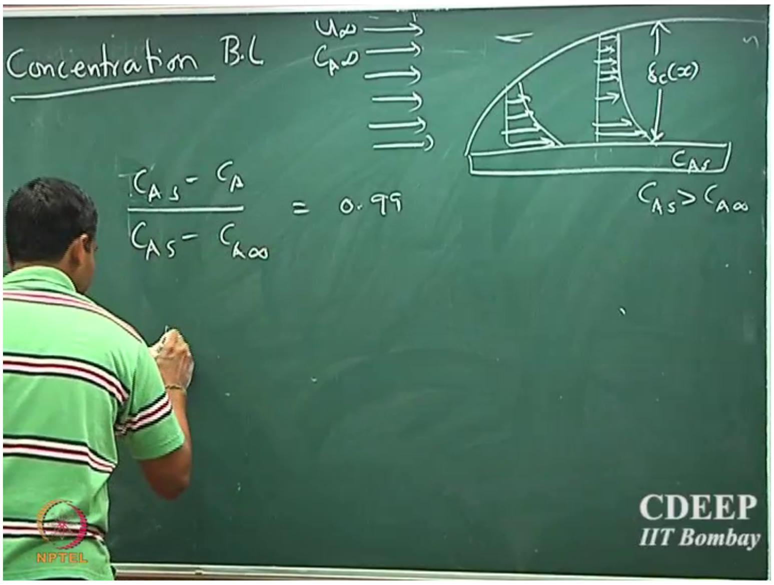

So, suppose I have a flat plate where concentration of some species is CAS and you have a bulk stream, the fluid velocity is u infinity and the concentration of the species is c a infinity and you have a concentration boundary and the profiles are very similar if I assume that CAS is greater than CA infinity. So, that is the kind of profile concentration profile that you would expect in the boundary layer that is the kind of concentration profile.

So, you can immediately see that lot of things that we talk in heat transport are very similar. Simply because the profile is similar the therefore, the gradient is similar and therefore, the process itself is similar. So, we can define boundary layer thickness if I call it as delta c. So, delta c is defined as that location where CAS minus CA, CAS CA infinity is 0.99 same definition nothing different and if I write a mass balance at the interface. So, I can define heat transport coefficient.

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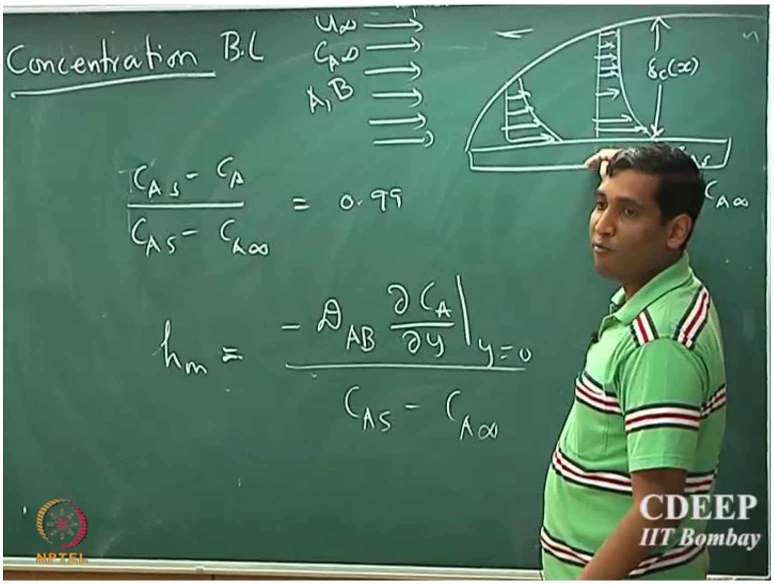

h m as what will be the equivalent?

Yeah what will be this expression?

Student: (Refer Time: 23:27).

Diffusion it is minus d. So, that is the equimolar counter diffusion. So, I said there are two species that is going and one of the species can be in excess. So, it will be the equimolar counter diffusivity of a b multiplied by d CA by d y at y equal to 0 divided by CAS minus CA infinity. So, that is the definition for mass transport coefficient. So, important message that we have learned today is that the heat transport and the mass transport processes are actually very similar if the transport is occurring across from one phase to another phase where one of the phases is moving right.

So, there is convection in one of the phases and. So, the heat transporter and mass transport process is a very similar to each other and. In fact, you will see when we write model equations in the next lecture you will see that it will see the model equations are actually very similar to each other and therefore, the characterization process also will be very similar.