Heat Transfer
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Lecture – 14
Transient analysis II: Full method

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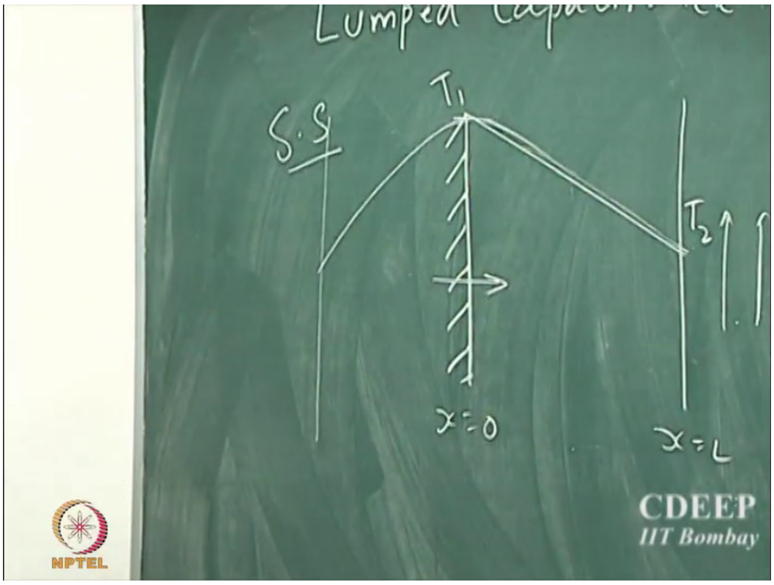

What is the temperature profile at steady state? What is the proper temperature profile? Quadratic? Linear?

Student: (Refer Time: 00:26).

Yeah.

Student: (Refer Time: 00:28).

A 1 so, you want to try? What is the temperature profile? Linear right? So, you will have a linear temperature profile. So, supposing I say that T 1 and T 2 are the temperatures at the either side of the boundary. It is a 1D problem, where you have heat conduction happening in this direction.

Student: (Refer Time: 00:50).

Yeah, but I can still have a higher temperature.

Student: (Refer Time: 00:55).

Slope is supposed to be right. So, supposing if I have a symmetric condition. So, this is the kind of profile I will have, right.

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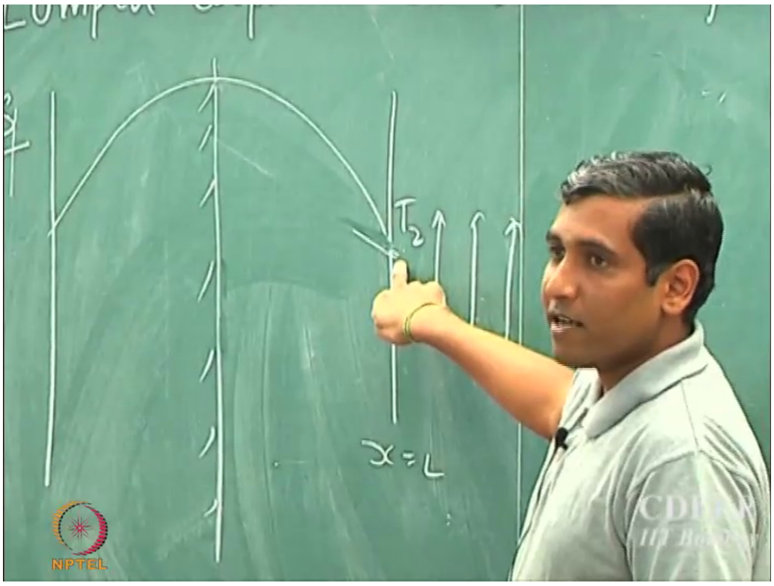

So, supposing if you have a (Refer Time: 01:20) then what you will have is a quadratic profile right. So, it is not linear. So, how many of you said linear? You should understand that when you have a flux boundary condition, this is very important to understand this; when you have a flux boundary condition you should always make sure that the gradient at that location is 0, very good.

So, supposing so, what I want to look at is, what is the convective rate at the boundary, right. So, conduction is happening only at that location. So, I want to look at what is the conductive rate, and I want to know when can I lock the conduction process inside the solid. So, what should I do? I need to compare the rate of conduction, and the rate of convection at the boundary, right. That is what I need to do. Because these are 2 competing processes. I want to know when can I ignore the conduction process.

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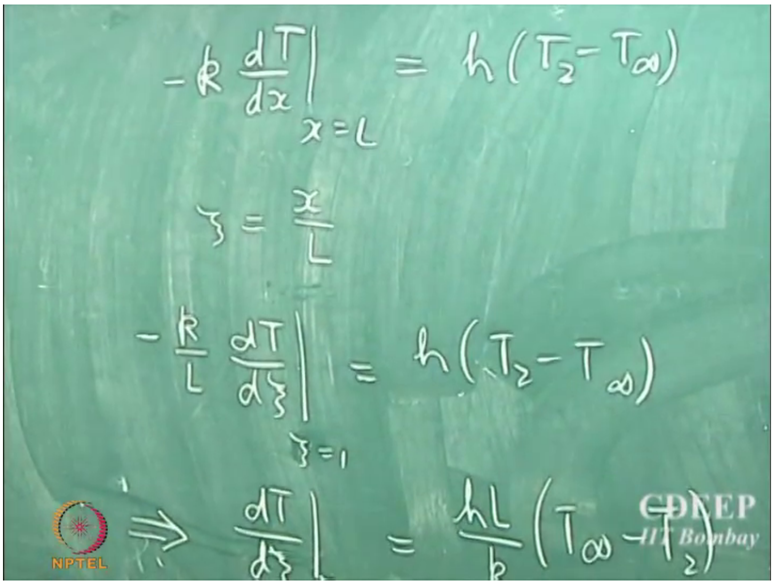

So, what I need to do is, I need to write a energy balance at x equal to L. That is what I need to do. So, minus k dT by dx at x equal to L should be equal to the amount of heat that is transported because of convection band. So, that will be T 2 minus T infinity, right. That is x equal to plus L. So, the flux of heat that is transported via conduction at the boundary it should be equal to the amount of heat that is actually transported from the solid to the fluid that is circulating around. Does everyone understand this?

So now supposing I define a variable called z, which is x by L. So now, I scale the dimensions. So, keep in mind that when I said when you lump it the dimension of the system plays an important role. So, when you want to compare the 2 rate processes, it is useful to non dimensionalize the coordinate system. So, let us now divide define a new variable z, and set x by L is now that quantity. And so, I can put that here it will be minus k divided by L into dp by dz at z equal to 1; that is equal to h into T 2 minus T infinity.

So, I can do an algebraic manipulation of this. So, dT by dz equal to 1, is given by h into L by k into T infinity minus T 2. That is clear to everyone?

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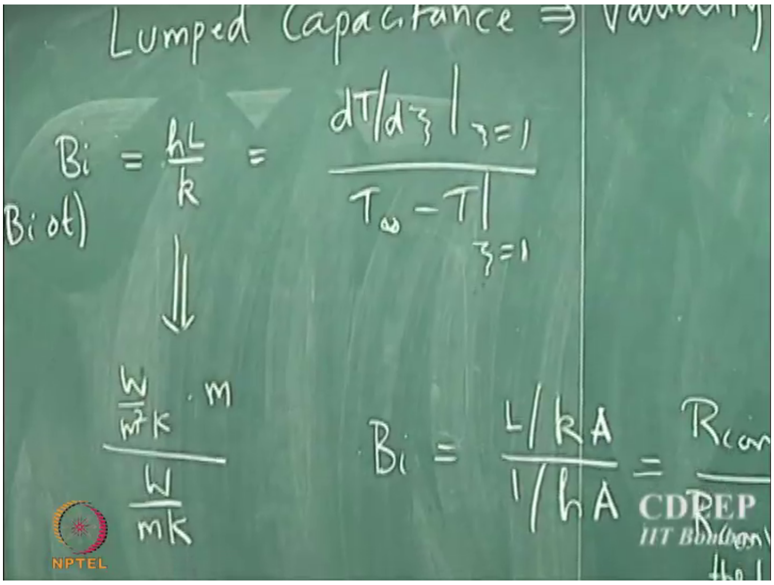

So now h L by 2 a equal to dT by dz that divided by, T 2 is nothing but temperature at z equal to 1. So, this quantity is what is called Bi number. So, this is a dimensionless quantity. What are the units of heat transfer coefficient? Units? Watt per meter square kelvin.

So, remember newtons law of cooling the heat transfer rate is h times area times temperature difference. The heat transfer rate unit is watts. So, heat transfer coefficient unit is watt per meter square per kelvin. So, this the units here are watt per meter square kelvin multiplied by meter divided by watt per meter kelvin. So, this is a dimensionless quantity. It is called the Bi number. It is written as Biot. It is spelled as Biot, called the Bi number.

So, we could rewrite this Bi number as L by K A divided by 1 by h A. So, all I have done is the cross-sectional area at which the heat transport is occurring from the solid to the fluid. I am just multiplying the area in the numerator and denominator.

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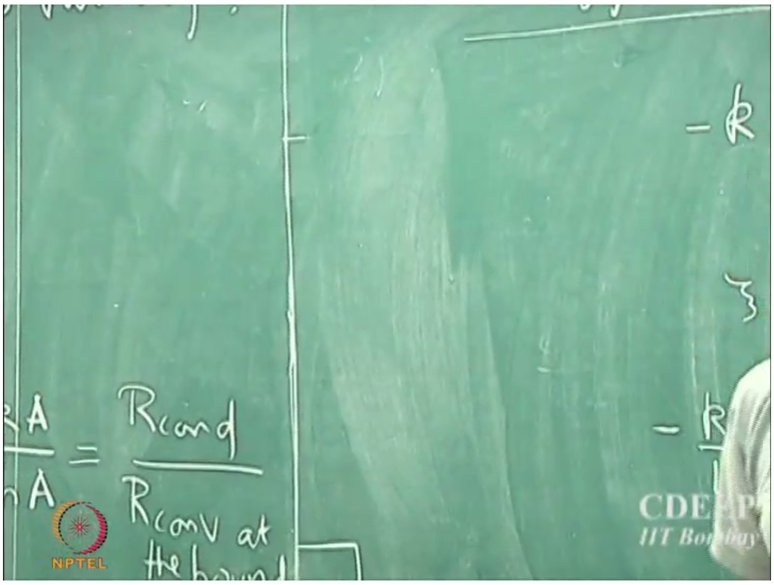

And I have just manipulated the expressions. What is L by k? Is the resistance for conduction and what is 1 by h A?

Student: Resistance.

Is a resistance for convection right. So, it is important to qualify the resistance for convection, saying that it is the resistance for convection at the boundary. It is the resistance for convection at the boundary of the solid and liquid interface. There is a reason why I am qualifying this. This must be about 8 to 9 lectures from now. You will see another dimensionless number, which looks very similar except that the definition of the resistance is slightly different. And that is why it is important to qualify it by saying that it is the resistance of convection at the boundary where the heat transport is occurring from solid to the liquid.

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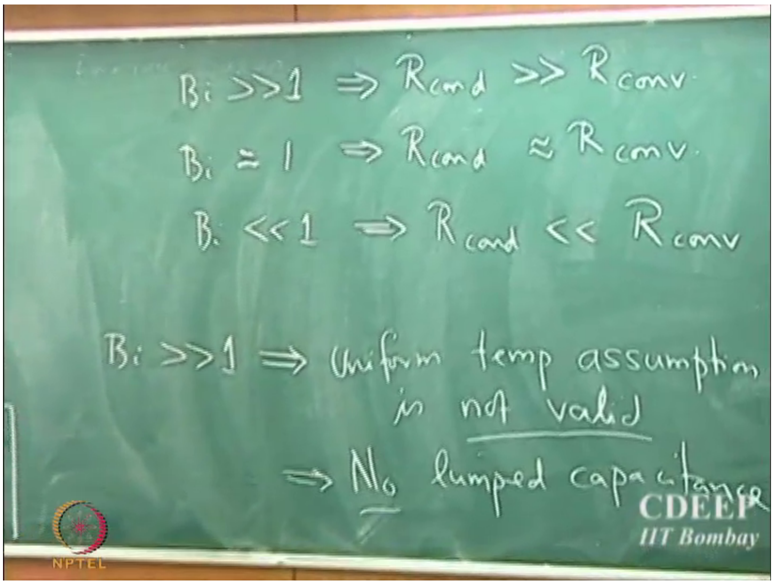

What happens when Bi number is much larger than 1. Which means that resistance to conduction is much larger than resistance to convection, right at the boundary. When Bi number is equal to 1, the resistance to conduction is almost equal to if I say almost equal to 1. Conduction and if Bi number is much smaller than 1, then you will have resistance to conduction. It is much smaller than resistance to convection. So, suddenly by defining a dimensionless quantity. We are able to use all the intrinsic properties and the dimensions of the system, in order to identify when the lumped capacitance method is important.

So, when conduction resistance is very large. So, can we use lump capacitance method? Why? Why cannot we use?

Student: (Refer Time: 08:48).

Right so, when the resistance to conduction is very high. So, you should always translate resistances to the temperature profile. Whenever the resistance is very large, you cannot assume uniform temperature. So, this means so, when Bi number is larger than 1, uniform temperature assumption is not valid. So, when Bi number is much larger than 1, you cannot assume a uniform temperature inside the system. Which means that you cannot use lumped capacitance method. You cannot use the lumped capacitance method to approximate or to characterize the heat transport process.

So now, it is almost easy to read what happens when Bi is almost equal to 1, what happens when Bi is almost equal to 1?

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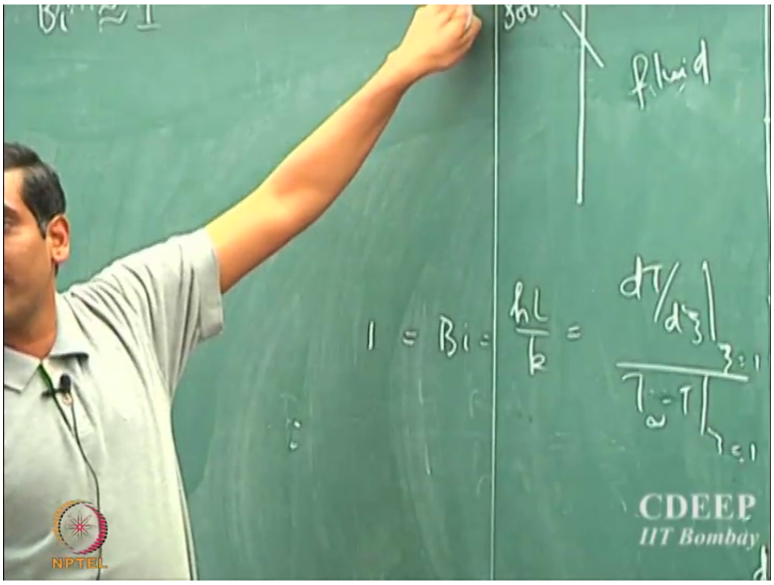

Can we use lump capacitance? Yes? No? You cannot use? Because you still cannot ensure uniform temperature. Keep in mind that the lumped capacitance method works only when you can assume a uniform temperature distribution. So, when Bi number is almost equal to 1. So, note that so, supposing if this is the boundary, where you have solid here, and a fluid here.

So, we said that Bi number if h L by k which is dT by dz at z equal to 1, Divided by T minus T infinity minus T infinity minus T at z is equal to 1. So, this essentially signifies the ratio of the temperature gradient at the boundary in the solid, and the temperature gradient for convection outside the boundary. So, it is not just that it captures a ratio of resistances, it is also equivalent to the ratio of the gradient temperature gradient inside and temperature gradient outside.

So, when these 2 resistance to conduction is equal to resistance to convection, which means that there will be significant amount of gradient, and these 2 gradients will be exactly equal to each other, right. When Bi is equal to 1, it means that the gradient temperature gradient in the solid at the boundary and the temperature gradient outside the boundary will be equal to each other.

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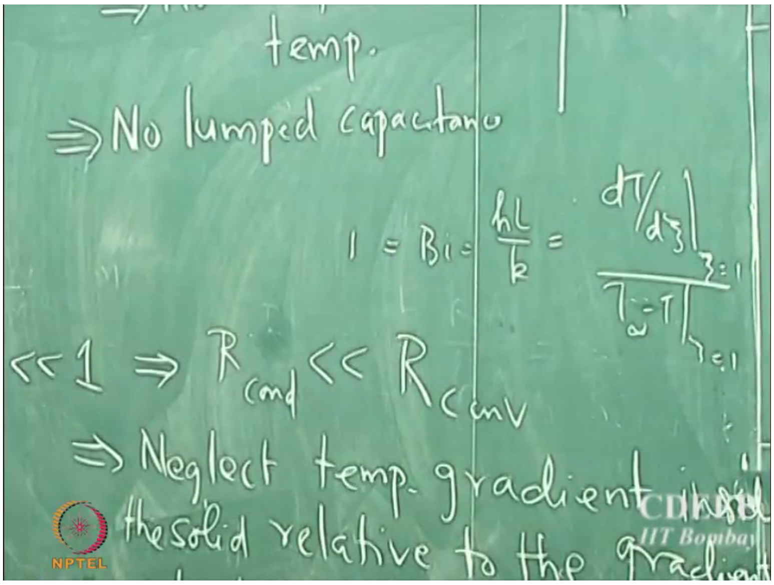

Which simply means that no uniform temperature can be assumed. You cannot assume uniform temperature inside the solid.

So, which means you cannot use the lumped capacitance method, no lumped capacitance method is permitted when Bi number is almost equal to 1. So, the third case is where Bi number is much smaller than 1; this means that the resistance of conduction is much smaller than resistance to convection. So, what does it mean? That the temperature gradients inside the solid is going to be very, very small. So, you could neglect. It does not mean that there is no temperature gradient. All you can assume is that you can neglect the temperature gradients. It is very important to qualify this. When you say neglect, it is actually in comparison with something.

So, you neglect temperature gradient, relative to inside the solid relative to radiant outside.

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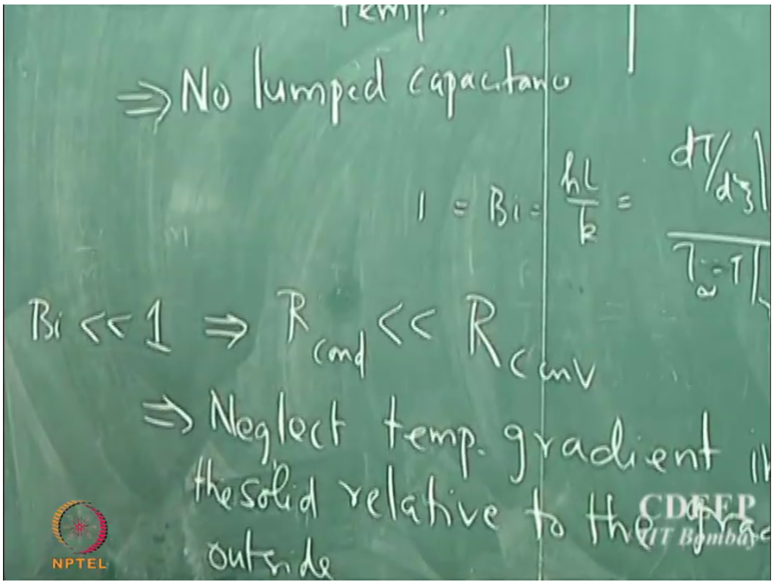

So, this is the justification for using the lumped capacitance method. It does not mean that the temperature gradient is 0. It is an approximation or it is an assumption and it is valid only when the Bi number is much smaller than 1. So, it is the purpose of dimensionless quantities, as you will see in down this course; many dimensionless quantities that will be defined in this course. And also, many other courses in, one of the main purposes is basically to characterize the different ranges of the process.

Characterize what is it going to be the different features of the process are different values of these constants. So, instead of looking at the heat transport coefficient, length, and the effect of conductivity individually. It is enough to look at the effect or the variations of the features or properties of the temperature with respect to Bi number. Which actually captures all the effects that is included in heat transfer, coefficient, length and the conductivity. So, this is the elegance that you get by using a dimensionless quantity and defining a dimensionless quantity.

Any questions so far? So, let us try to sketch the temperature profile. So, supposing if Bi number is much larger than 1, which means that the resistance for conduction is going to be much larger than the resistance for convection, which means that, this means that there will be sharp temperature profile in the solid.

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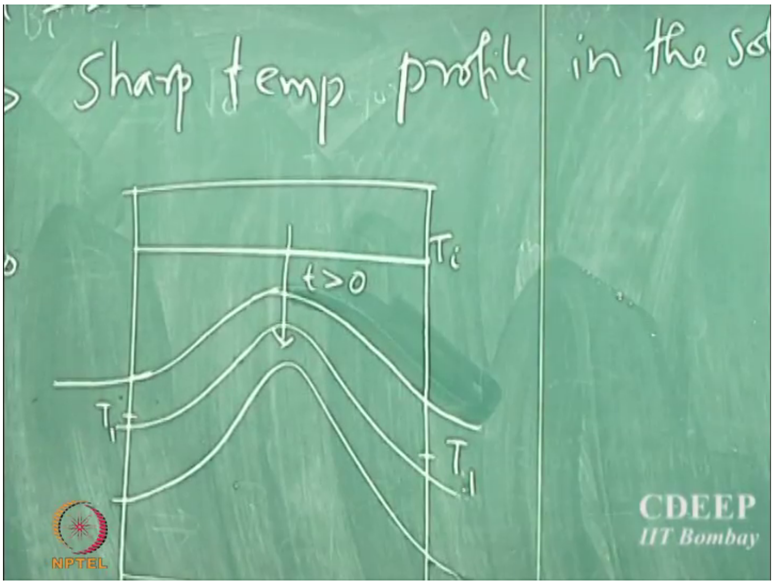

So, what I am going to sketch now is the time profile of the temp time profile of the temperature distribution in the solid slab.

So, supposing if I maintain it at same temperature on both sides of the slab. So, I will have a asymmetric temperature profile. And so, supposing if Ti is the initial temperature at which the solid is being exposed to the fluid which is surrounding. And let us say the temperature here is T infinity, then as time passes by sorry. So, because the Bi number is much larger than 1, and the resistance to conduction is much larger than the resistance to convection at the boundary.

So, you would expect that the gradient outside the solid is going to be much smaller than the gradient inside the solid. So, as time passes by. So, this is T greater than 0. So, you will start seeing profiles which will look like this. So, you will have sharp gradients inside the solid, while you will have not so sharp gradients outside the solid. So, this is we have not solved the equation. Without solving the equations, simply based on the intuitive understanding of how the temperature distribution is going to be based on the Bi number and resistances, we are able to actually sketch the temperature profile. So, this is very important of course, you will have to write model equations and solve them.

But if you do not know how the or what the model equation is going to give, you do not know whether the solution is right. So, it is very important to be able to get an approximate temperature profile or sketch of the temperature profiles even before solving the equations. And using dimensionless quantities and such resistance arguments, it actually helps you to come up with the what is going to be the approximate temperature profile.

So, similarly I could draw for Bi much smaller than 1, if Bi is much smaller than 1, what would you expect it will be what will be the temperature profile inside the solid.

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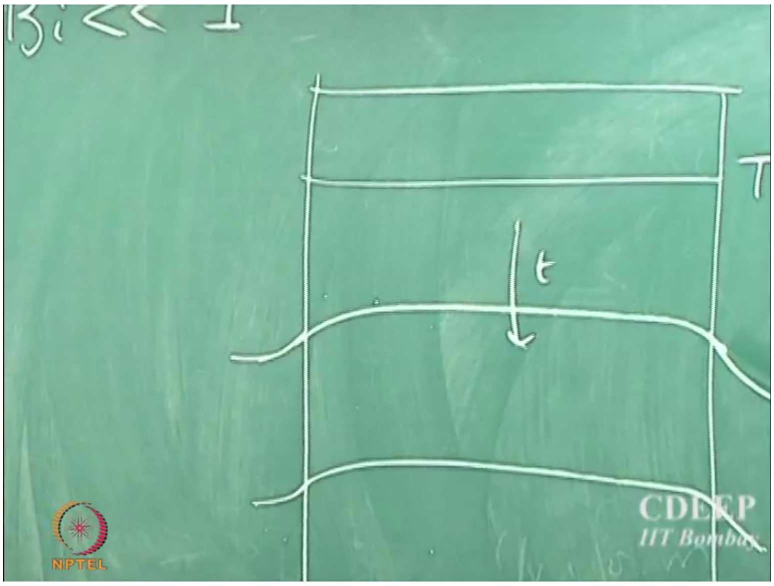

Student: (Refer Time: 18:08).

It will be almost flat, it will be almost uniform. So, one could sketch the temperature profile. If Ti is the initial temperature, we will have sharp gradients outside, but you will have lots of sharp gradients inside. Because you Bi number is much smaller than 1, the temperature difference for conduct convection outside the solid is expected to be much larger than the gradient of temperature at the inside of the solid near the boundary. And so, as time goes by so you will start seeing temperature profiles which are almost flat inside.

So, this is the 2-different profile, but Bi number is one what will be the profile it will be somewhere in between these 2 right. So, this is the profile for one extreme, and this is the profile for other extreme condition. And Bi number is equal to 1. You will see a temperature profile where the gradients are not very steep, but at the same time the gradients outside the solid are also not very steep.

So, they will be equal to each other at the boundary. And so, you will see as temperature profile which is somewhere in between these 2. So, supposing we next take the actual system, right. We try to sketch the temperature profile in a slab without writing the model equation. So, let us now write the model equation. So, supposing you have a slab, and this is plus L minus L.

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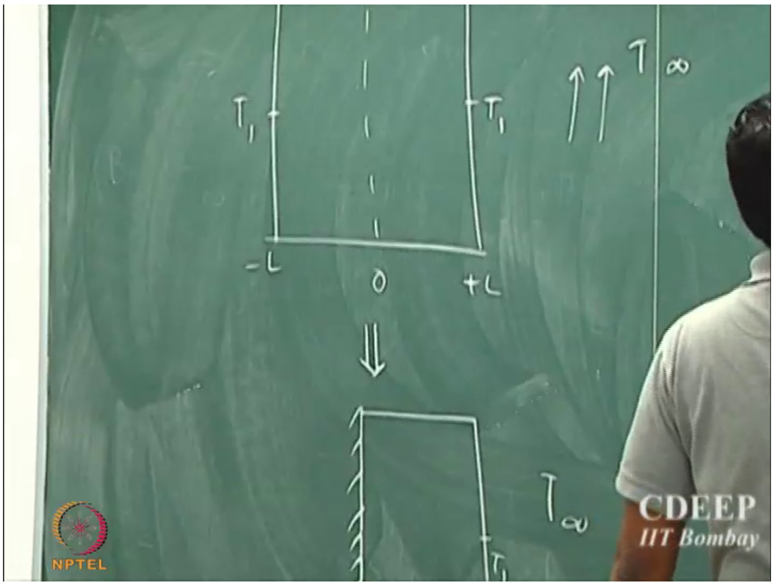

And let us say the temperature here is T 1, and if the fluid which is flowing around the temperature is T infinity.

So, let us write the model and in fact, we can check whether the temperature profiles that we actually intuitively guessed what is going to be the profile, that will actually be the same as what we would predict from the model. So, this is equivalent to a half slab with adiabatic conditions in one boundary. And the temperature here is T 1. And we have T infinity is the temperature of the fluid which is flowing past it.

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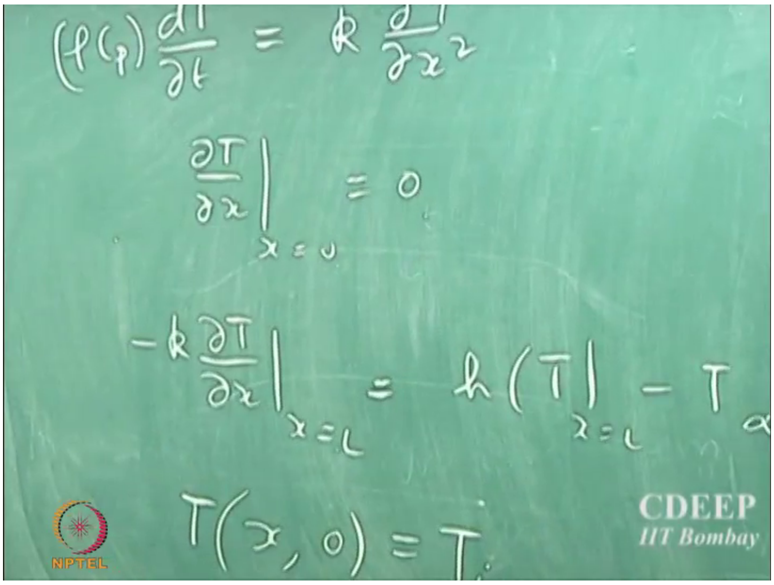

And so, we can write a model there will be rho C p it will be equal to, note that it is the partial derivative now.

And the boundary conditions will be dp by dx at x is equal to 0 is 0. And minus k dT by dx at x equal to L.

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So, if I assume that T 1 is greater than T infinity. So, that will be h into T at x equal to L minus p infinity. So, that is the boundary condition. And the initial condition will be T at x comma 0 will be equal to Ti.

So, the initial temperature of the slab is uniformly equal to the temperature Ti. And one could even as new my certain temperature profile as an initial condition, but let us not worry about that right now. So, we can solve this equation. I am going to do the math here it is very similar to the way we did the separation of variables problem in the last lecture. The solution will be so, the solution is so; if I assume theta as T minus T infinity by Ti minus T infinity.

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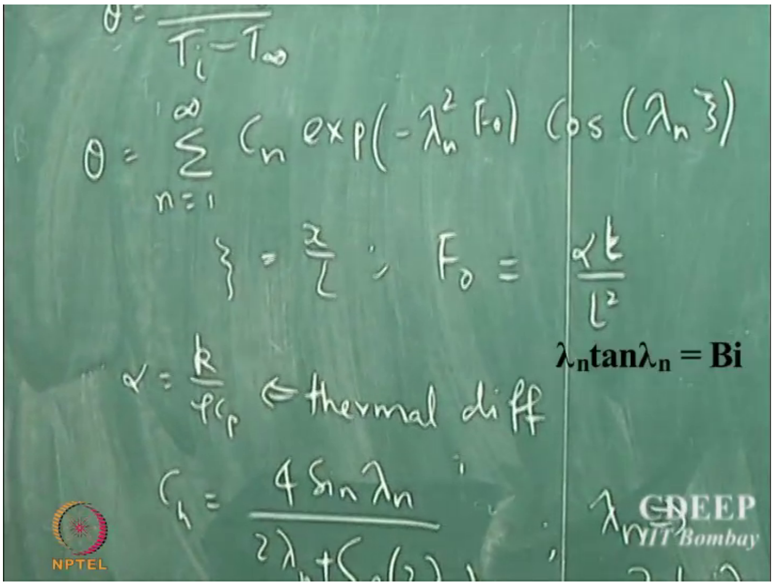

And the solution will be summation n equal to 1 to infinity C n exponential of minus lambda and square in to Fo is Fourier number. I will define in a short while.

What are this and z is x by L Fourier number is alpha L by alpha T by L square. What is the alpha? Thermal diffusivity. So, alpha is k by rho C p this thermal diffusivity. And C n is given by 4 sin lambda n divided by 2 lambda n plus sin 2 lambda n. And lambda n is the solution of this equation, lambda n tan lambda n equal to Bi number.

So, lambda n are the roots of this equation. How many solutions are there? For a given value of Bi? Lambda n tan lambda n equal to Bi how many solutions? How many solutions? What is the nature of tan? A lot of them, how many?

Student: (Refer Time: 23:43).

Yeah.

Student: (Refer Time: 23:45).

It is infinite solutions. So, tan is a transcendental function. In fact, the way to see that is you can actually express it in terms of a plot a graph is one way. You can actually see that tan is a ratio of exponentials, right is ratio of exponential of imaginary functions, right. It is ratio of sine and cos.

Student: (Refer Time: 24:06).

So, a sine and cos you can write as exponential. So, you can immediately see that it is actually infinite number of solutions. So, you have infinite roots and in fact, that is why n goes from one to infinity here. How do you solve lambda n tan lambda n equal to Bi for a given value of Bi? Is it linear? It is not linear. So, you cannot get analytical solutions for the root. So, you have to use a non-linear solver, which non-linear solver? Scilab? No, no, no. Scilab is just a software.

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So, what you use is called the newton Raphson method. You will see this in your numerical analysis course. So, you have to use the newton Raphson method to solve this equation. And in fact, there are subroutines in scilab MATLAB etcetera to do the newton Raphson method solution of such kind of algebraic equation.