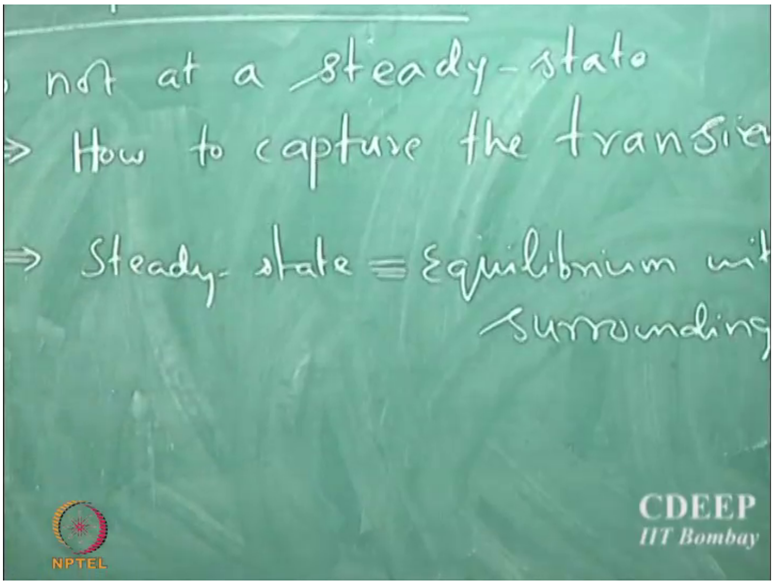
Heat Transfer
Prof. Ganesh Viswanathan
Department of Chemical Engineering
Indian Institute of Technology, Bombay

Lecture – 13
Transient analyses I: Lumped capacitance method

So far in most of the cases in conduction, we assumed that the system is maintained at a steady state.

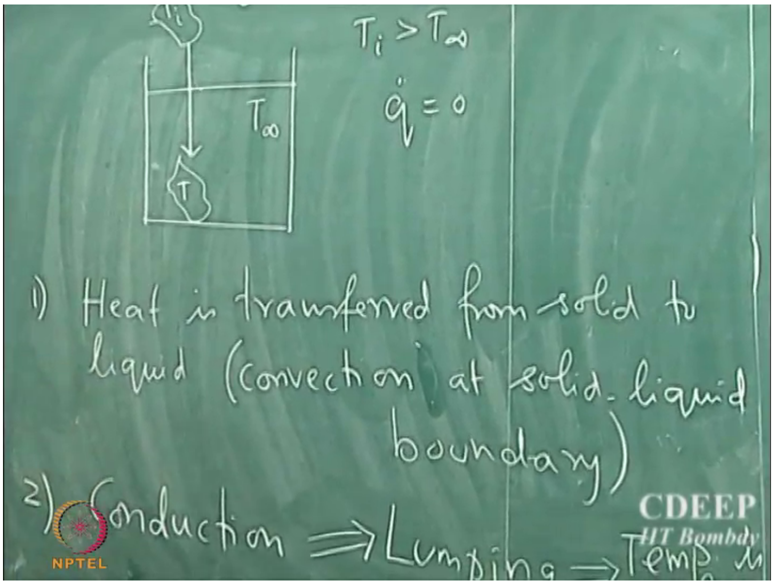
(Refer Slide Time: 00:25)



So, what happens if the system is not at it is steady state. So, this really translates into question, how to capture the transient. That is what it really translates into the question. What if the system is not maintained in the steady state, it is possible, it is not maintained a very easy example is you take a cup of coffee tea or milk or whatever and just leave it on the table. So, this really know a steady state till the system has or reaches the same temperature as that of the surroundings.

So, really the steady state of a cup of coffee or [FL] or whatever is kept on the table is the state at which the every location or every particle in that cup, actually reaches the temperature of the surroundings or the steady state really here means, steady state is equilibrium with the surroundings. So, all the cases that we have seen all the models that we have written so far, except for the general model; that we wrote they are all for system where the where it has already reached an equilibrium with the surroundings. So, the question is that is not always true, there are systems where they may not be under equilibrium. So, how do we characterize them, how do we quantify them and what kind of analysis and understanding we can get, that is what we are going to see in most of today's lecture ok.

(Refer Slide Time: 02:44)



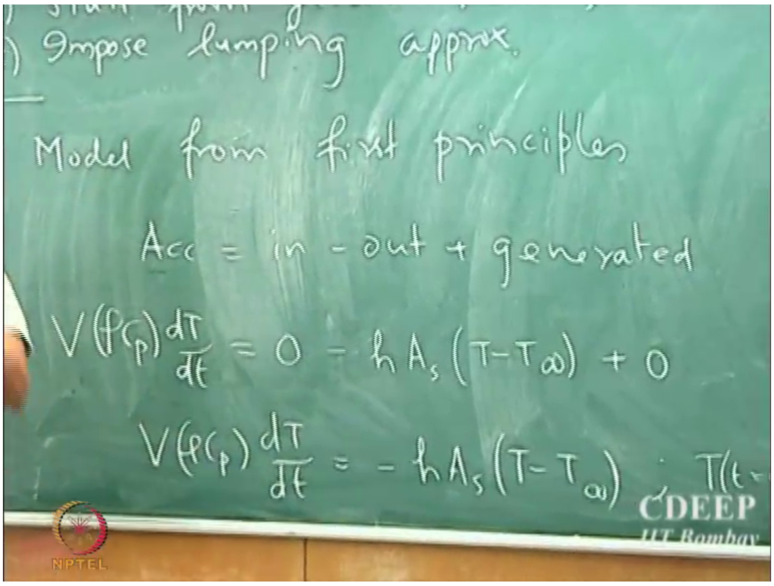
So, supposing we take a container, which is filled with a certain fluid and let us say that the temperature of the fluid is T it is filled with some temperature, and let us say that there is some solid material, whose temperature is T I, and if I assume that T equal to 0 that is time equal to 0 is essentially when the when this body is actually outside the fluid and at T equal to 0, I drop it inside. So, I drop the solid body into the liquid. So, supposing if T i is greater than T infinity. So, as soon as you drop it, there is going to be exchange of heat. So, the solid because it is at a higher energy state, it is at a higher temperature it will lose heat to the surrounding fluid, and then it will attempt to equilibrate itself with the surroundings right so, that is the normal process.

So, the first observation is that, transferred from solid to liquid. What is the mode of heat transfer? Convected right. So, what really occurs it is the convection at the solid liquid boundary. So, what occurs is the convection at the solid liquid boundary, what happens inside the solid? So, there is conduction. So, there are two ways to go about it. So, supposing we say that the conduction is important inside then we know how to write the model. We have done that we said accumulation equal to input minus output plus generation etcetera. So, if I assume that generation q dot is 0, we know how to write. Now supposing if the dimensions of the object are very small, is the dimensions of the object in all three dimensions are significantly small, then one can do what is called the lumping of the one can do a lumping approximation. What it really means is as we have seen before is we assume that the temperature is uniform in the solid. So, that is what we mean by lumping here.

Now how do we model this? You want to try how should we approach, I mean the objective is we need to quantify this process where do we start. So, the whatever heat that is lost if you are assume that the temperature is uniform, whatever internal energy which is lost to the fluid is equal to the rate at which the temperature is changing somehow right there will be some proportionality constant etcetera. So, that has to be somehow related to the rate at which the temperature is changing inside the solid right.

So, there are two ways to go about it, there are two ways to write the model, one is certainly you could start from general model and then impose lumping approximation, it is not very difficult to do in fact, it is very very easy to do this.

(Refer Slide Time: 07:19)



All you have to do is just integrate and substitute the corresponding boundary conditions now a simpler way to do this. So, that involves some math of course, a simpler way to do this is one can write model from first principles. A simpler way to do is one could write model from first principles. So, we know that common mantra for all of these we said accumulation equal to what comes in minus out plus what is generated. In fact, the original general model also we derived by using the mantra, but if you use this this root it is just that you will whatever lumping approximation you would actually do by integrating etcetera, would already be taken into account by writing the model from first principles.

You could do either way it does not matter and in fact, you should get exactly the same model. So, it is very easy to write this rho Cp dT by dt, that is the accumulation term what is the input what is the input? So, the first thing I want you all to realize is that when we wrote the this first principal root to find a general model we wrote a differential balance now when we introduced the lumping approximation there is no differential element here. So, what we do yeah we need to multiply this by y y area.

Student: (Refer Time: 09:53).

That is not right. So, the first term here corresponds to accumulation. We should remember that rho Cp reflects or quantifies the amount of heat that is stored inside the system right inside the solid.

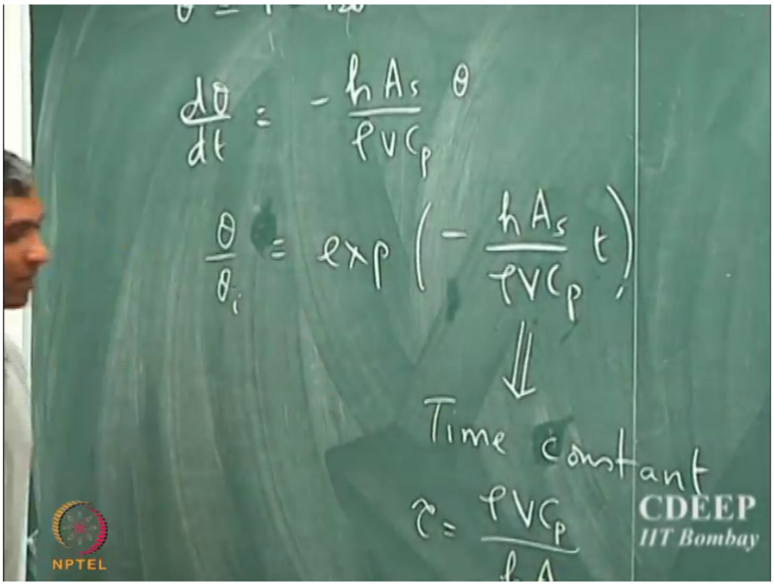
So, that is a volumetric process. The amount of the capacity of the system or the solid to store heat is actually a volumetric process and not a surface area process. So, therefore, you have to multiply this by the volume of the system that you are looking at. If you do it from the first principle, if you do it from the generalized equation if you integrate etcetera, the volume scaling will actually come from the integration.

In fact, it is a good exercise to try this; start from the first principle there generalized model, introduced the lumping by introducing the integration over the whole volume and you will see that the volume term will actually come as a scaling factor in the accumulation term. So, what comes into the solid nothing because you said that it is its heated and suddenly you have dropped it inside. So, there is no heat that is coming into the solid also input is 0 what is going out? H into. So, that is a; the convective mode of heat transport is actually a surface area process. So, it is h into the total area which is available for heat transfer multiplied by the temperature difference right.

So, that is the amount of heat that is going out plus we said generation is 0. So, the model is very simple. So, it will be V times rho Cp is equal to minus h s T minus T. So, that is the model what is the initial condition? A is Ti that is it. So, the initial condition is T at p equal to 0, h into Ti that is the initial condition for the model they are clear to everyone. So, you must appreciate. So, far we did not get into this because it was not required to understand this this issue because you are looking at differential balances. So, you must understand that the capability of a system to store heat is actually a volumetric process and the capability of it to transport heat is actually a surface area process or a cross sectional area process.

So, whenever there is conduction it is the heat that is flowing across the cross section, and whenever there is convection from the solid to the fluid around it is the area the core the surface area which is the surface area process. So, you must understand; what is the correct set of dimensions, which are associated with each of these terms to the model alright. So, the solution is very trivial. So, if I assume theta is T minus T infinity.

(Refer Slide Time: 13:24)

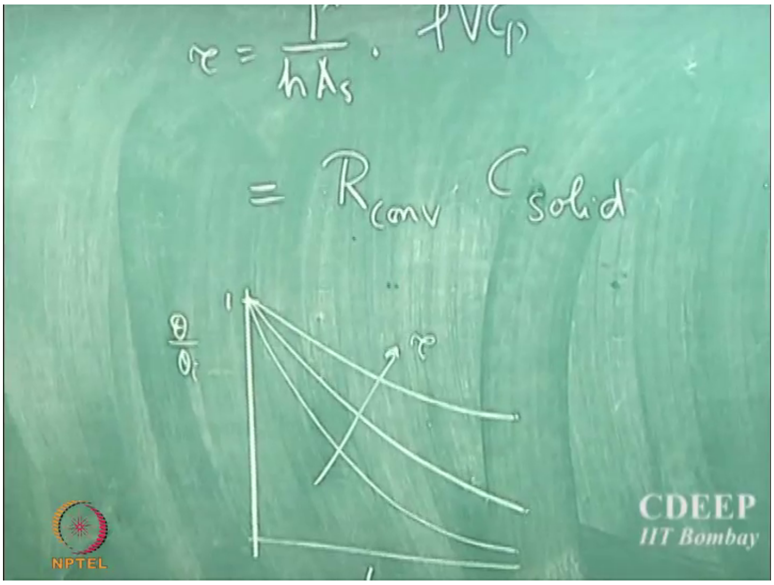


I can simply rewrite the model as d theta by dt equal to minus h A s by rho V Cp into theta. And the solution is theta by theta i equal to exponential minus that is the solution what can we say about the coefficient in front of time yeah what should be the units of this coefficient in front of time? It should be one by second right because the quantity inside the exponential should be dimensionless.

So, if T time is seconds then the units of the coefficient should be one by second. So, it really signifies at time constant. So, based on the coefficient one could define a time constant for this process at tau equal to rho V Cp by h into a s. In fact, if you put down the units plug in the units you will see that this will exactly have the units of seconds, dimensions of time. So, one can define time constant for the process. So, what it signifies is what is the time that is taken what is the dimensionless I mean what is the time that is taken in order for the heat to be lost from the solid to the fluid, and note that you can actually see two terms here. So, there is a once again a competing process here one is the ability of the solid to store heat, the other one is the ability of the system to lose heat to it is surroundings.

So, one could actually rewrite the time constant as something like this.

(Refer Slide Time: 15:43)



So, you can write tau as 1 by h into A s into rho V into Cp what is 1 by h into A s? It is the resistance for convective mode of heat transport. So, you can define this as resistance for convection what is rho V Cp in terms of electrical engineering terms or is it resistors and yeah capacitors. So, it is the C of the solid. So, it is the capacitance or the capacity of the solid to store heat. So, a time constant is simply the convective resistance that is offered by the system for convection of heat from the solid to the fluid multiplied by the capacitance that is the ability of the solid to store heat in it is volume is that clear.

So, it is very important to understand time constants. In fact, this probably the first time you are seeing in this course a definition of time constant and you will see a lot more very soon, mostly you will see one more in today's lecture and you will see many more in the future lectures why time constants are important. So, time constants are the one which captures the; which tells you what is the time that is taken for a certain process to occur based on the properties of the system.

So, note that these are all the properties of the system that you are considering. So, if you know what the properties are I do not need to know whether density is increasing decreasing independent of the others, As long as I know what the time constant is as long as I am able to relate the temperature change with respect to time constant I am done I can actually play with many different parameters in order to achieve the same feature that I want to have. For example, I could increase the capacity of the solid by choosing a different material, and if I know how to change the heat transfer coefficient, which you will see when we discuss convection then I could actually make sure that these two are simultaneously changed in such a way that the time constant does not change.

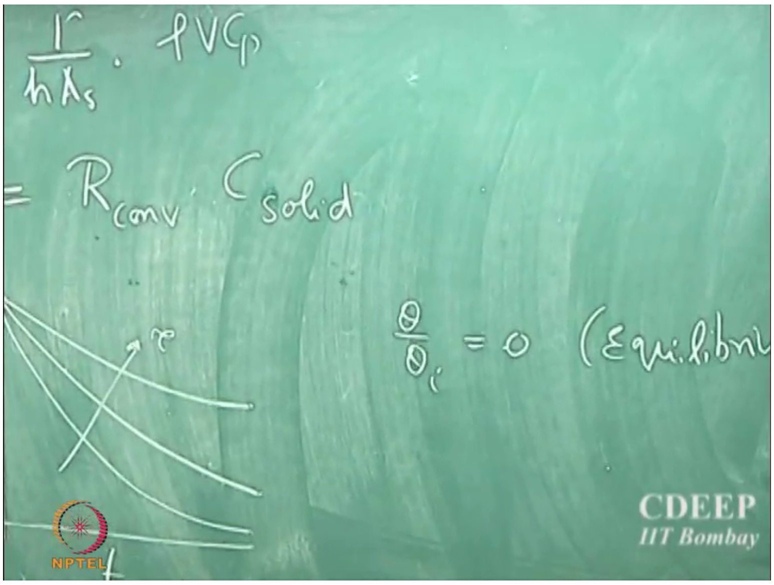
So, this provides an elegant method to compare the heat transport processes across different materials. So, note that these properties are different for different material density, capacity etcetera; they are all intrinsic properties of a solid. So, they will be different for different materials. So, defining such kind of a time constant provides you an elegant method to compare the rate of heat transport, when you use different materials in the same system. So, it is very important to understand this and you will see many more of such constants and dimensionless quantities in the future lectures ok.

So, I could draw row 5, simple exponential form. So, this is time and this is theta by theta i. So, that is a equal to 0 theta is theta i. So, it should start from 1. So, as I increase the time constant. So, note that time constant quantifies or it tells you gives you an idea as to rate at which the heat is going to be transferred and. So, as you increase the time constant the slower will be the heat transport. It will take a much longer time to be T equilibrium, what is the equilibrium?

Student: (Refer Time: 19:31).

T equal to T infinity. So, equilibrium is theta by theta i equal to 0, that is the equilibrium.

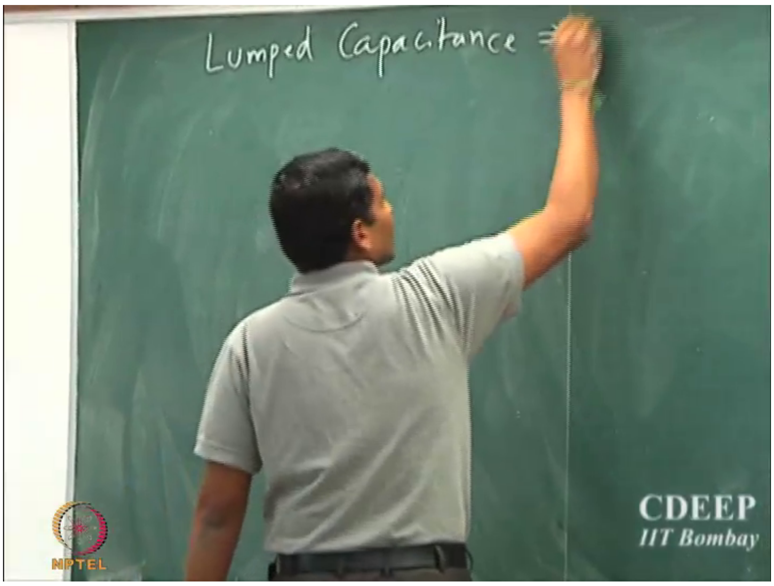
(Refer Slide Time: 19:34)



So, when the time constant is large for a given system, it takes much longer for it to equilibrate.

So, depending upon the system what kind of properties you want to have for a given system, it has to be designed so that you get the correct time constant. So, that you have to you should be able to predict when the system is going to reach an equilibrium. So, this is a very simplistic example, now this leads to an important question actually. We said that we made a lumping assumption or lumped capacitance.

(Refer Slide Time: 20:24)



So, the important question is when is it valid, what is it is validity, when is it valid, can we quantify the validity of this method? It is not enough to say that the dimensions are small and therefore, I can approximate it to be uniform temperature. If the conductivity of the solid is high, not necessarily not always it is not just the conductivity it also depends upon the dimensions right. So, it is both property of conduction and the dimensions of the system that you are looking at. So, what we are going to see now is; how can we incorporate all the properties which are related to the conduction and convection process and come up with a quantitative quantity which can be used to justify when this method is valid.