Heat Transfer
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Lecture – 11
Extended surfaces III – Varying cross – section area

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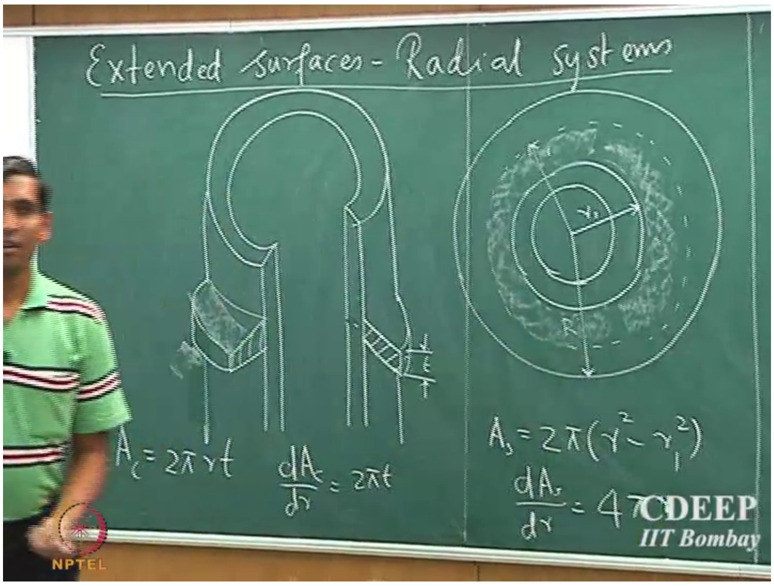

All right, till the last lecture we looked at extended surfaces on Cartesian coordinates where the cross sectional area of heat transport does not change. If you remember the discussion we had in the last lecture in order to improve the heat transport the ways by which we can do that is by increasing the heat transfer coefficient, cross sectional area, area of connective, area of heat transport and the temperature difference. So, today what we have going to see is we going to build up on that and we going to see how to quantify the increase in this conductive surface area in a radial system that is we going to look at things in radial system .

So, what you see here is essentially the cut open of a cylinder with a fin which is placed around the cylinder which is in the outer rim of the cylinder in principle you can have multiple such fins, but let us first look at one fin. Now, here is the cross sectional view or top view if supposing if the outer radius of the base cylinder that we are looking at that is r 1 and let us say the outer radius of the fin that you are considering is let us say capital R and if the location can be described as heat transfer occurring along the top and the bottom edge of the of the fin and let us say that we are trying to write a model to characterize the balance at any position R, supposing if the thickness of the fin is T then what is the cross sectional area of heat transport.

It will be at any location R the cross sectional area of heat transport will be the perimeter multiplied by the thickness of the fin, to the perimeter at that location is 2 pi r multiplied by t that is the cross sectional area of heat transport via conduction therefore, dAc by Tr is given by 2 pi into t, what is the area of heat transport via convection. Let us say I call this s the surface area of heat transport via conduction, that will be this total area around the top surface plus the shaded area on the bottom surface of the fin because we conduct a mode of heat transport is going to occur from both the top and the bottom of the fin. So, therefore, the area would be 2pir square minus r1 square, the radius of that location is r and the radius of the place of the outer rim of the base is r 1.

Therefore the net area of heat transport in the top surface is pi r square minus r 1 square, but because there is heat transport from the top and the bottom you have to multiply by 2, dAs by dr is given by 4pir, once we know these areas we know the general model equation for such of fin, we should be able to plug these in and find the model.

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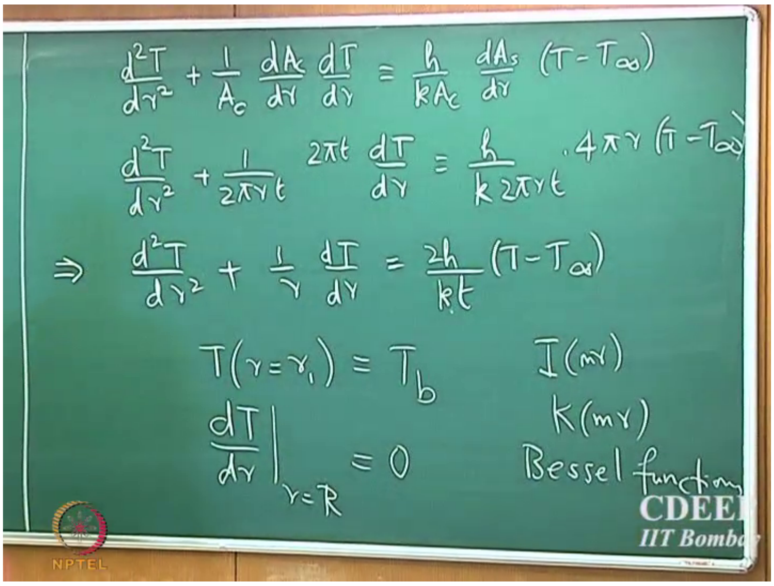

So, what is the general model d square T by dr square plus 1 by Ac which is a function of position into dAc by dr into dT by dr equal to h by kAc into dAs by dr into T minus T infinity, where T infinity is the temperature of the surrounding fluid. Now, we can plug in these quantities it will be d square T by dr square plus 1 by 2pirt, dAc by dr is 2pit into dT by dr that is equal to h divided by k into 2pirt into 4pir into T minus T infinity. If we can rewrite this as d square, I can just cancel all the terms and it will be d square T by dr square plus 1 by r dT by dr that is equal to 2h by kt into T minus T infinity.

This model equation completely describes the heat transport via from diffusion in the fin and the convection from the top and the bottom surfaces. The first term the left hand side term captures the thermal diffusion of heat in the fin and the right hand side captures the heat transport via convection from the top and the bottom of the fin. The boundary condition if Tb is the base temperature if T at r equal to r 1 is Tb and if I say I impose an adiabatic boundary condition I could in principle put a non adiabatic if any of those 4 boundary conditions that we listed yesterday you could in principled those any of those 4 boundary conditions. So, let us say that we put the second case of no flux for adiabatic boundary condition.

Also we can solve this equation what will be the nature of the Eigen functions of this problem it is an Eigen value problem as you can see the radial coordinates j what are the other it is not j, what are the other Eigen functions in radial coordinates, what are the distinct Eigen functions in radial coordinates.

Student: I to the power.

Student: I to the power

I to the power of something is one no it is a summation it has to be a series summation we have 4 types there is J, there is Y, there is I, and there is K. These are the 4 distinct types, depending upon the coursing function that you have you get different types of Eigen functions. So, for this type of problem the Eigen functions are basically I and K well defined in a moment what m is, these are the 2 Eigen functions for this equation and these are called as Bessel functions if you know.

These are called as Bessel functions, you can write this in the form of Bessel equations and that will be the Eigen functions for the Bessel equation, how to get these we make a substitution very similar to what we did in the last lecture.

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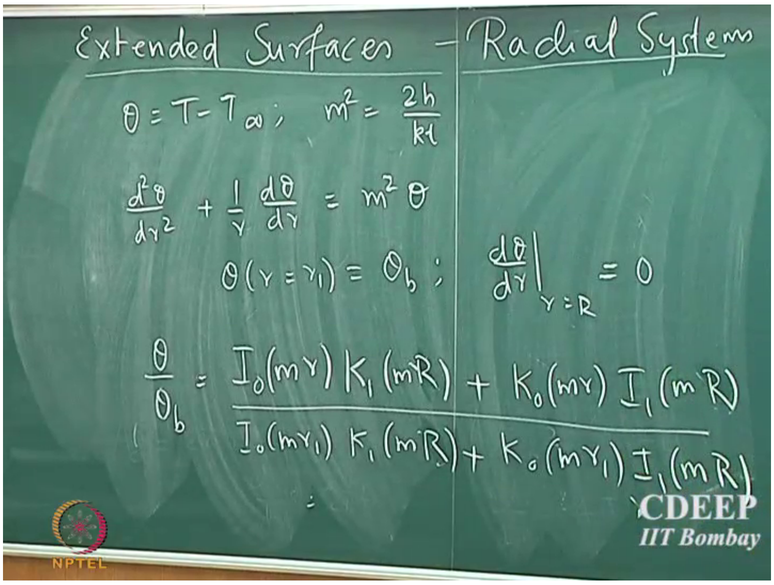

We say theta equal to T minus T infinity and you say m square is 2 h by kt, we can transform these equations into d square theta by dr square plus 1 by r d theta by dr that is equal to m square theta and the boundary conditions will be theta at r equal to r 1 will be theta b and d theta by dr at r equal to capital r will be 0, I am not doing to derive this solution you should derive this and convince yourself.

I will give you the solution mr K1 mR 2 plus K naught mr into I1 mR 2 divided by I naught mr 1 K1 mR 2 plus K naught mr1 into I 1 mR. So, R is the outer radius of the fin, that is the solution and just like what we did in the last lecture one could derive what is the net heat transfer total heat that is transferred by the fin, note that when you want to quantify the process is occurring in the fin you want to find, what is the total heat transfer rate.

Student: What is I n Bessel.

Ing are the Bessel functions.

Student: So, where is the bessel.

There are 4 Bessel functions these are called the first kind and J is the 0 kind there are 4 Bessel functions J and Y supposing if you have a modified, there are 2 types of Bessel equations one is the regular equation the other one is the modified. Supposing if you have m square minus lambda square you have 2 Eigen value problems then the solution you get is J, J and Y and when you have this kind of problem what you get is I naught and k. So, you should go back and read the Bessel equation solutions you will find this for those who are interested to know more about Bessel functions you should actually read the book.

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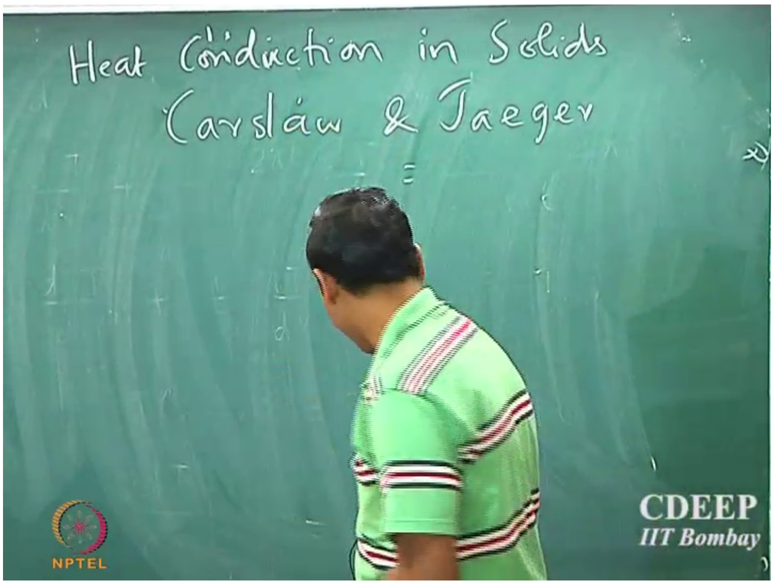

I will give you the reference it is called Heat Conduction in Solids by Carslaw and Jaeger I think it was published in the mid 50's or early 60's. This is by far like a golden standard for most of the problems that has dealt with in relationship with conduction and you will see like explicit presentation of all these Bessel functions, various kinds of solutions that will come about and what are the different nature what is the nature of different kinds of problems and what is the nature of different kinds of solutions. So, some of you are interested you should actually go and read this book it is a really very well written book and I think it is and also quite lucidly presented.

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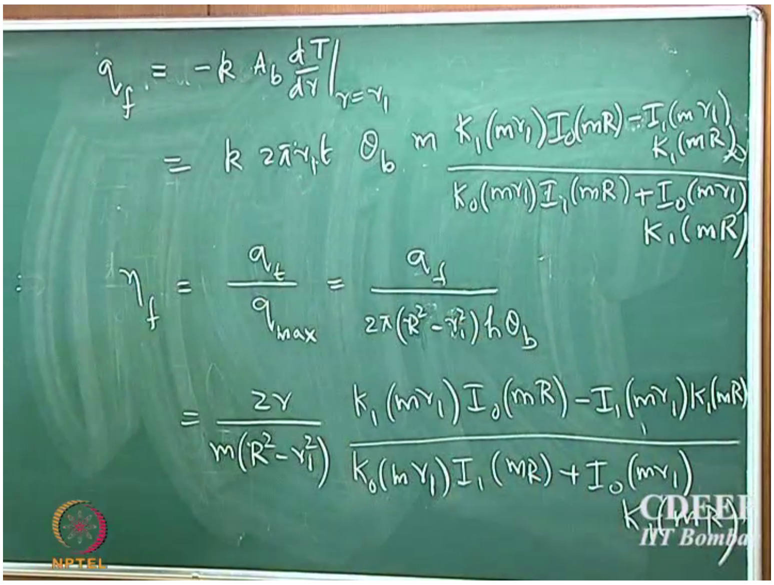

We said that we need to find the overall heat transport rate and that is very simply given by just like the way we did intuitively we said that it given by b dT by dr at r equal to r 1 we said whatever heat that is transferred from the base if the total amount of heat that is transported through the fin. So, based on that intuitive argument we could find out what is the total heat transfer rate and that is given by k 2pi r1 t into theta b into m into K1 mr1 into I naught m capital R plus minus I1 mr1 K1 mR divided by K naught mr1 I naught mRI plus I naught mr1 into K1 m capital R.

And similarly one could find the fin efficiency which is essentially defined as q t by q max, q max is given by what is the maximum possible heat transfer rate when can the fin achieve maximum possible heat transfer rate.

Student: (Refer Time: 16:22).

When all of the fin is maintained at a constant temperature and that constant equal to the temperature of base itself, therefore that will be q f divided by 2 pi r square minus r1 square multiplied by h into theta b and plugin all the expressions will be 2 r divided by m R square minus r1 square that into K1 this should be capital R it is the total heat transfer rate note that it is a function of the total radius of the fin, that will be K1 mr1 2 m capital R minus I 1 it is the area at.

Student: Sir (Refer Time: 17:39)

Excuse me thanks, it is the area at the base, it is r 1 excuse me minus mr 1 into K1 m capital R divided by K naught mr1 I1 m capital R plus I naught mr 1 into K1 m capital R. So, that is the efficiency of the fin which is in the radial direction, so far we looked at only one fin in fact all the examples I told you whether car radiator or the humidifying power we all have many fins life.

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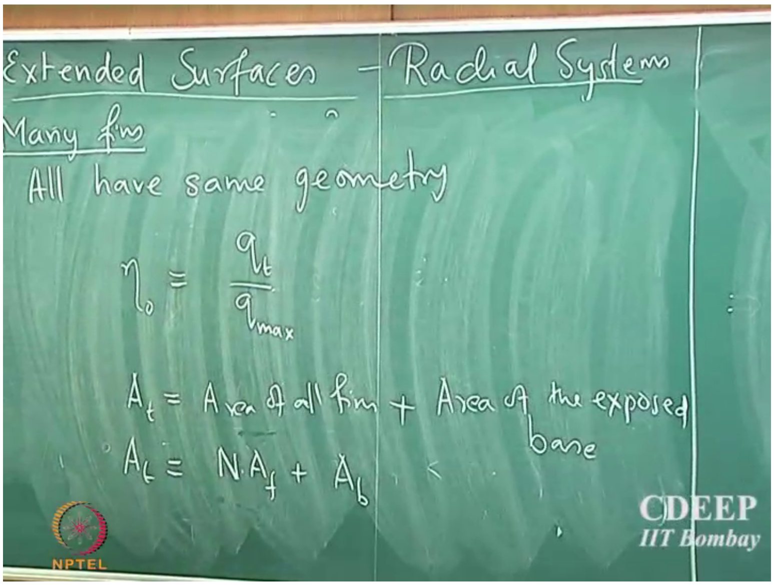

So, what happens when there are many fins, how do you find out the total heat transfer rate, that is the question says ultimately what I want is, I want to find the total heat transfer rate. If I have many fins and if I know what is the total heat transfer rate from 1 fin? And if I assume that all these fins are identical which is typically the case the geometry of all these fins are typically identical.

So, many fins and if I say all have same geometry same geometry if I assume that all the fins I have with us of same geometry then I could define something called an overall efficiency the eta naught which is the total heat that is transferred that is the total amount of heat that is transferred divided by the maximum that is transferred by all the fins together.

So, supposing Af is the total area, when I say total area here I mean that area of all the fins plus area of the exposed base that might be useful to draw picture now, supposing, this is the picture I drew when I when we discussed about the when we initiated discussion about the fins.

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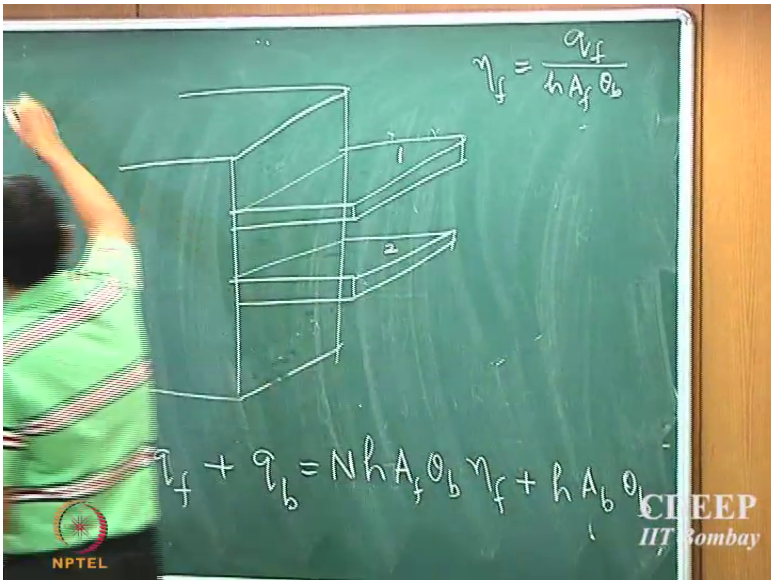

Supposing there are 2 fins, in principle the heat transfer is going to occur from the top bottom surface of the first fin, if I mark this as 1, if I mark this as fin number 2 from top bottom surface of the second fin and from the all the exposed area of the base itself right. The total area that is available for heat transport available from a composite system like this where you have many fins along with the base would be area of all the fins which is available plus the area of the exposed base.

So, area of all the fins will be supposing there are N fins which are present then area of all the fins is N into area of individual fin plus the let us say Ab is the area of the base which is available for heat transport. That is the area which is available in between and for those that is the area of the base, the total area which is available for heat transport is now given by N into Af plus Ab.

What is qt, qt is the total amount of heat so, that will be N into qf if qf is the total amount of heat that is transported by one fin to the fluid around plus whatever is transported by the exposed base. That we can write as N into h into area of the fin multiplied by theta b into eta n, note that the efficiency of a single fin at let us mark it here, efficiency of a single fin is simply given by qf divided by h into area of fin into theta b right.

That is the definition of the efficiency of a single fin, based on that definition N into h Af theta b into eta f will be the total amount of heat that is transported by N fins together plus whatever is the amount of heat that is transported by base which is given by h into area intro theta b according to Newton's law of cooling. So, note that here theta I already account for the difference in the local temperature and the fluid temperature.

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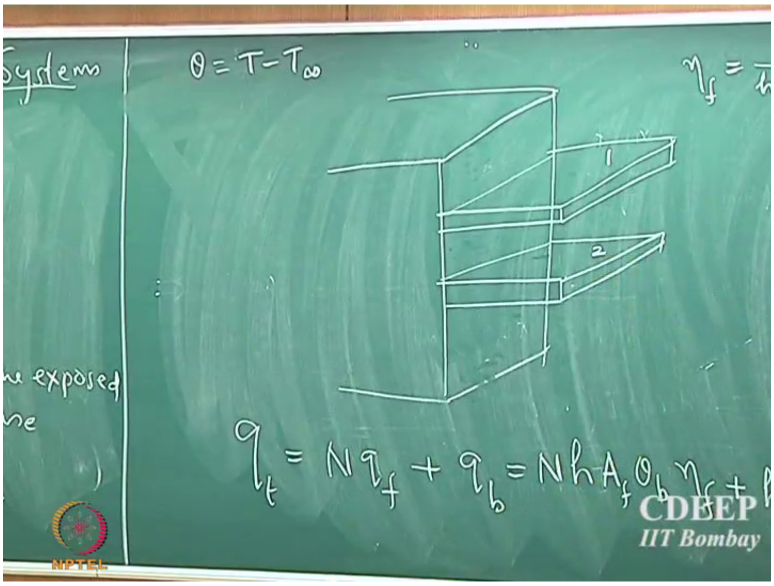

That is the nomenclature that I have been using theta is essentially T minus T infinity to the T infinity is the temperature of the fluid which is flowing around and theta is T minus T infinity. The total amount of the heat that is transported by this composite system is N into h into Af theta b into eta f plus h Ab into theta b.

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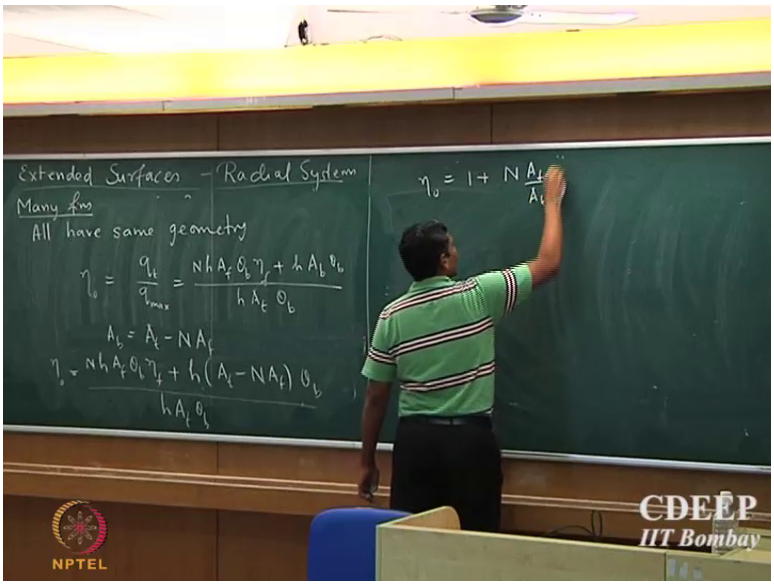

Now I plug all these here the N into h into Af into theta b into eta f plus h into Ab into theta b divided by q max is h into the total area which is available that is the composite area which is available and if all the every location in the composite is maintained at a certain temperature which is equal to the base temperature and constant, that will be the maximum heat that is transported by that composite system right. Ab is given by A total minus N times the area of f simply from the conservation of the total amount of area. So, substituting this will get eta naught is equal to N h Af theta b eta f plus h into A total minus f into theta b divided by h into A total into theta b, we can do a simple algebraic rearrangement.

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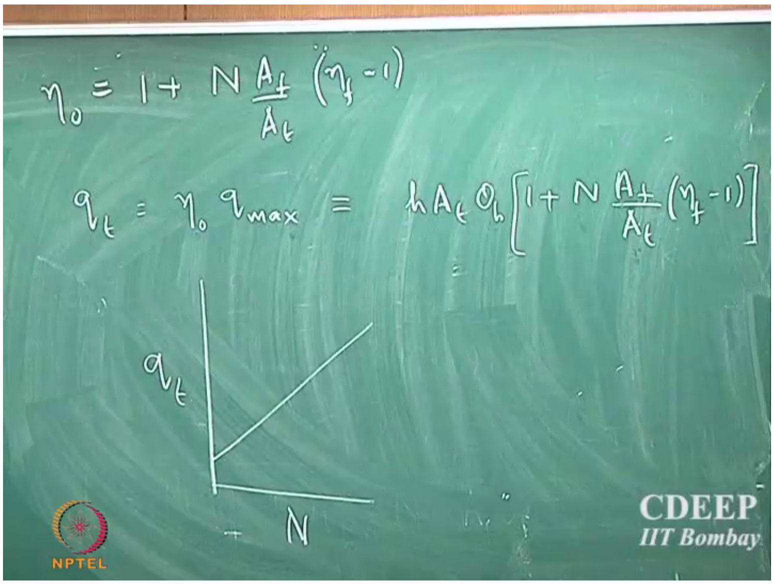

That will be, this term will become 1 plus N into Af by At into eta f minus 1, the efficiency of the composite system can now be easily expressed in terms of the efficiency of a single fin

So, what it tells you is that if you are able to model and quantify the heat transport process in one fin you are done, you can easily find out what is the total heat transport rate from the composite system by simply multiplying. So, qt is nothing, but eta naught into q max by simply multiplying eta naught and q max you will be able to find out what is the total amount of heat that is transported by the composite system. That is h At theta b into 1 plus N into Af by At into eta f minus 1. So, what happens when n increases does the heat transport rate increase from this expression that is what you intuitively guess because that is what you want to achieve by putting many fins you want to increase the heat transfer rate that is the expression also show that by increasing N, does it increase the total amount of heat transfer rate it is very important anytime you do such quantification calculation should always check whether what you predict is exactly what you intuitively guess yes or no why.

Student: sir I want to ask (Refer Time: 26:32) or not.

Eta f also that is eta f is anyway a quantity which is less than 1 that does not matter no as long as you know eta f let us say I know what is the efficiency of 1 fin would the heat transfer rate increase if I duplicate the geometry of that fin and different locations .

Student: sir are the electric methods of the (Refer Time: 27:04) they are not worth calling (Refer Time: 27:08).

No assume that all that is true that is enough assume that the fin is always exposed with the liquid does not matter, does the expression predict that the heat transport rate increases with increasing the number of fins eta f is less than 1 that is right. So, note that this is the maximum right, as you increase N what happens.

Student: Sir it decreases.

Which one, note that the Af At is going to increase as you increase the number of fins. So, it is not very obvious to see from this expression, for various values of At, one has to actually, as you increase the number of fins keep in mind that at is also increasing, A t is not constant there. It is not very obvious to see from this expression, as you increase the number of fins keep in mind that at is also increasing, At is not constant there. It is not very obvious to see from this expression, for any specific system the first thing we should do is as you increase the heat transfer area by including multiple fins you will have to make a plot of qt versus the number of fins. So, only if the qt you is going to increase with the number of fins then it is worthwhile making such a construction otherwise it does not make any sense.

The first thing you should check is whether qt for a given system I mean cannot be linear a simplistic expression is supposing if the qt increases linearly with respect to the number of fins that you are actually constructing in this complex system only then it is worthwhile to put the number in fact that is how you set the number of fins that you should choose. What is the total number of fins that is going to maximize by my total amount of heat that is transported if it is not going to maximize then there is no point in doing that it is a waste of money to put additional fins is very expensive by the way, you must understand that machining and adding new components is not a cheap task it is very expensive to do machining and add materials to an existing system. There are lots of direct obvious cost there are lots of hidden costs which is involved in these kinds of things. It is very very important to understand what should be the maximum number of fins that should you should choose and in fact this is one of the important purposes of doing modeling.

You must get a feel of what should be the maximum or optimum amount of heat that will be transferred depending upon the number of fins that you would choose otherwise you do not want to shut the plant to change this. So, when you shut down the plant supposing if there is a big plant which is running where there is the heat transfer equipment if you want to change the design you would have to shut down the plant it cost a lot of money indirect money to shut down the plant, it hits the economy a lot economy of that particular process.

So, you really want to use these modeling to find out what is the optimal number even the before you attempt to make a construction. This is an important pair aspect of the model that you would construct not just this engineering class, but any other engineering system this one of the primary purposes of modeling.