Heat Transfer
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Lecture – 10
Extended surfaces II-Fixed cross-section area

The question is where you want to incorporate the assumption and when you want to incorporate the assumption. You could incorporate the assumption after you write the generalized model. Now this generalized model does not include the assumption that the cross sectional gradient is negligible and this is the way to do that you cannot arbitrarily neglect.

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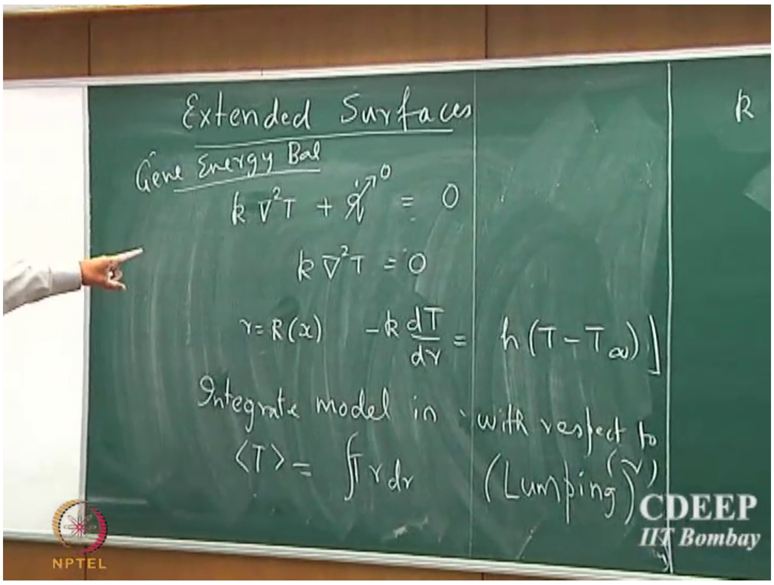

The second order derivative in the Laplacian, because the boundary condition is now going to be different and the way to see that made to appreciate that is I would probably urge you all to take a look at the solution of 2 different types of problems. 1 is you take a diffusion operator and you put 0 boundary conditions Dirichlet or 0 boundary conditions and you take the same problem put 0 flux boundary condition and you take the same differential operator and put a 2 flux boundary conditions, like something like this and you have to convince yourself that the nature of the solution you get in all these 3 are completely different.

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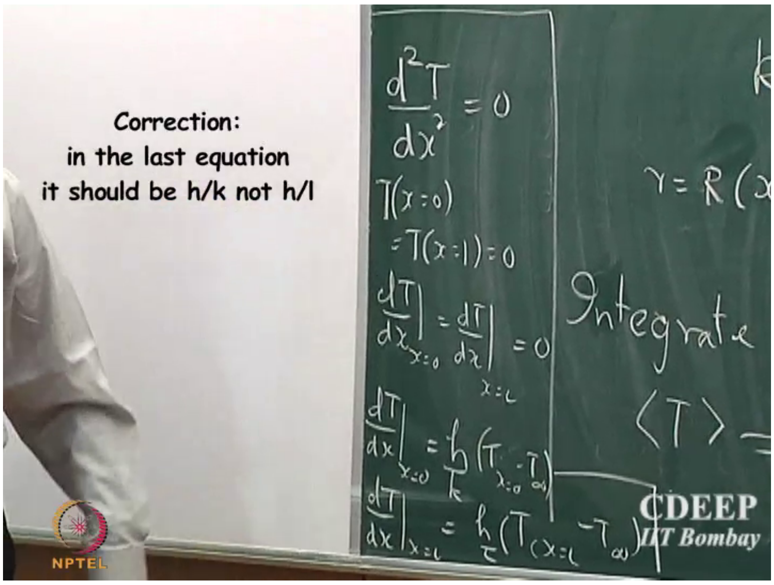

Particularly in the third case it will be completely different if you want me to state it I would like you all to convince yourself by solving this problem, you solve d square T by d x square equal to 0 with 3 types of boundary conditions, you can say T at x equal to 0 equal to T at x equal to 1 equal to 0 that is 1 problem and then you say d T by d x at x equal to 0 equal to d T by d x at x equal to l equal to 0 that is the second problem. And the third 1 is d T by d x at x equal to 0 equal to some h into h by k into T at x equal to 0 minus T infinity and d T by d x h by L into T at x equal to L minus T infinity that is the third problem. One could in principle have mixed boundary conditions you should actually even try to solve that also and you must appreciate that the these 3 classes of problems have completely different solution.

And in fact, if you understand that that is the reason why you cannot incorporate this assumption directly into the generalized model and this is the correct way to incorporate the assumption that the gradients in the radial direction is 0 particularly when you have this kind of a convection or flux boundary condition.

If you have these 2 types of boundary condition then you would assume you can directly ignore the gradients and you would not make a mistake because of the nature of the solution that we will get for these 3 types of problem. I it is a very good exercise all of you should actually try to solve these 3 types of problems and you will see these 3 classes of problems in almost all the engineering systems that you will see in this program and also probably elsewhere.

It is a very very common type of differential equation problem that you will see in many many engineering systems and there are some very good reasons for that I will not get into the, but there are very good reasons why these classes of problems will appear in almost all the natural systems that you will try to model. Assuming cross sectional area temperature to be uniform is not an appropriate and not a realistic thing if the aspect ratio is reasonably good; if the aspect ratio is very small then you can assume that the temperature is almost uniform.

Now, of course, you do not have to assume that to be uniform it just makes your life easy to solve the problem that is all now you could in principle take the full problem and get the full solution and you will see that the solution that you will get from this equation will not be very different, if you impose the condition that the temperature is nearly uniform in the cross section, it does not matter.

You can go either way it does not matter you can start from the generalized equation and then you can integrate you will get exactly the same and you can start from you can you incorporate the assumptions while building the model you will get exactly the same 2 errors it would not matter. And in fact, some of your things about the integral balance is already incorporated here this is exactly what you do when you write an integral balance ok.

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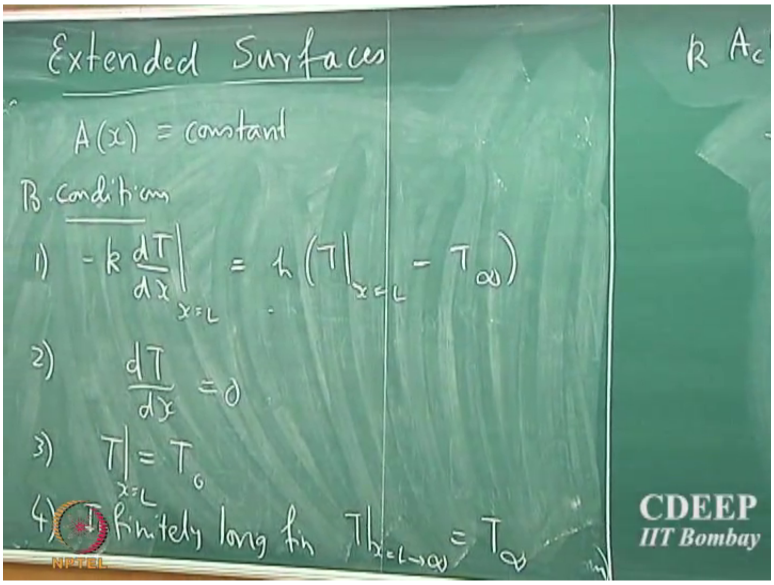

So, let us take one of the questions that he asked what happens there A x is constant before that we should actually go to the boundary conditions of this problem.

So, in principle there are 4 different possible boundary conditions that could have 1 is; obviously, you can have A flux boundary condition d T by d x at x equal to l will be h T at x equal to l minus T infinity. So, that is 1 possible boundary condition that you would; obviously, guess that just like how heat transfer is occurring from this cross sectional area you would expect that the heat transfer would occur from the other edge of the fin well.

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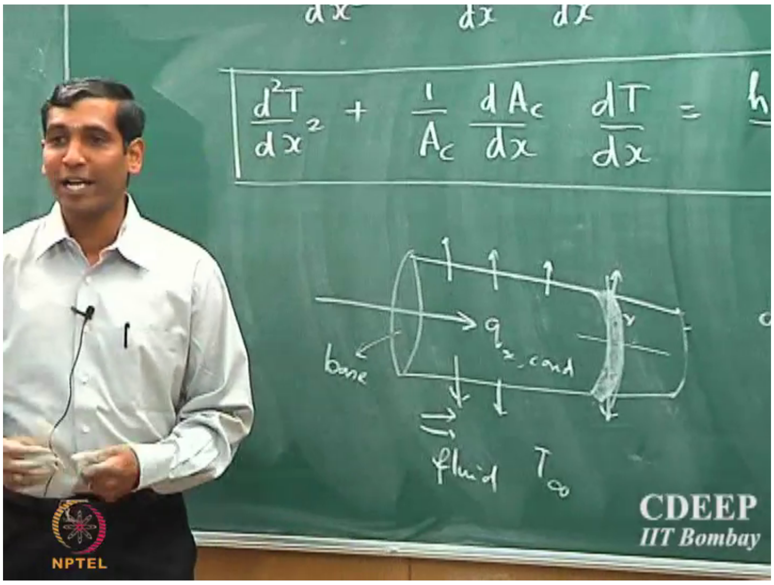

Now, the second possible boundary condition is that there is no flux may be that section is insulated. So, if the boundary is insulated with some insulator, then there is no heat exchange that is you impose A an adiabatic condition at the other end of the fin, and then the third 1 is you could impose a constant (Refer Time: 05:52) boundary condition. So, this is very unlikely, but just for hypothetical case you can assume that T at x equal to L is some constant temperature there are some example, but they are not that many and the fourth 1 which is generally used for some standardization purposes.

So, supposing if the fin is infinitely long infinitely long fin what will be the boundary condition is fin is infinitely long want to try, if this fin is infinitely long and if the temperature of the fluid is T infinity you what would you expect.

Student: (Refer Time: 06:35).

T equal to equal to T infinity right, so, you expect that T at x equal to l l tending to infinity will be equal to T infinity. So, that just for standardization purposes you will never achieve such an ideal situation; however, long the fin is going to be, but it is just for standardization purposes such kind of boundary condition has been prescribed all right. So, next we go to a specific case where.

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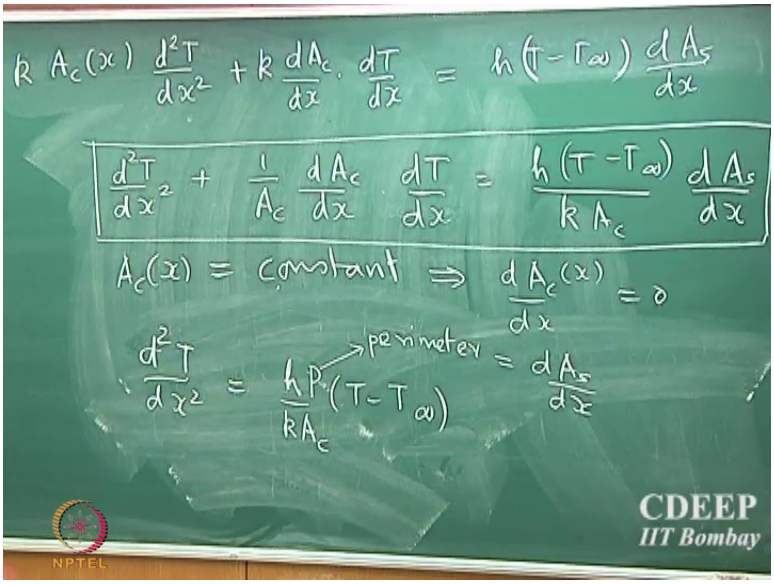

So, supposing we assume that the cross sectional area.

Student: (Refer Time: 07:11).

Is constant which means that the cross sectional area is same and therefore, the curved surface area is actually a curved surface area of a cylinder. So, this equation would simply which means that d A c x by d x is 0 which means that the gradient of the cross sectional area with respect to x is 0 because it is a constant. And so we can simply reduce this equation at v square d by d x square that is equal to h by A A c T minus T infinity, what is d A s by d x for constant cross sectional area it is the it is 2 pi r perimeter right it is the perimeter.

So, if p is the perimeter P is the perimeter of the fin that you are considering and that should be the that should be equal to d A s by d x, the change in the cross sectional surface area for heat transport is simply the perimeter of that particular fin.

How do I solve this equation is it complete do I need to know something else to solve the equations, I need to know the boundary conditions right. So, if I specify the boundary condition that x equal to 0 is the temperature of the base ok.

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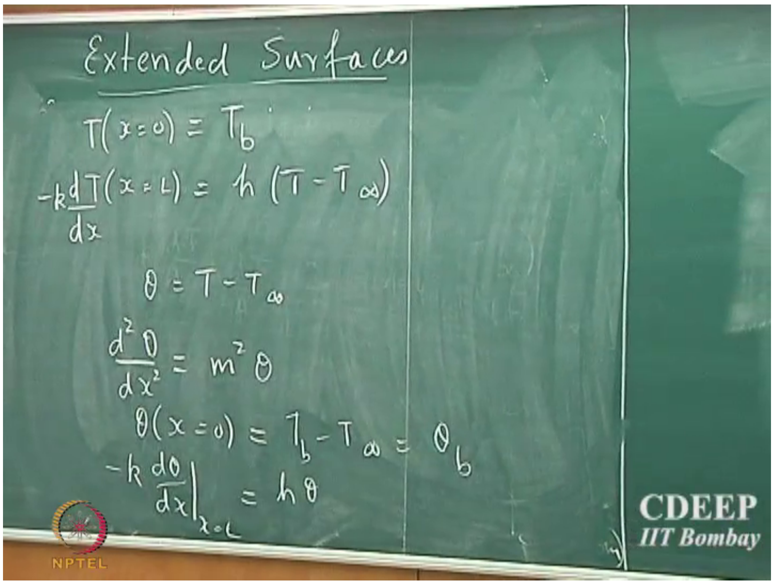

So, note that the fin is attached to a base surface and. So, if the temperature of the base surface is specified. So, that is going to be the boundary condition at x equal to 0 and I could say that T my if I assume that there is a convection boundary condition. So, if I specify this boundary condition then we can solve the equation how do we solve this equation how do we solve this yeah it is not that hard some exponential. So, you say theta is T minus T infinity you make a substitution and then this will become d square theta by d x square equal to if I call this m square m square theta and this will be theta at x equal to 0 is T b minus T infinity which I can call it as theta b right. So, I get it right.

So, T is theta is T minus T infinity right. So, and then minus k d theta by d x at x equal to l that is equal to h into theta with a pretty easy and you can solve this is an exponential solution, it is like an Eigen value problem what are the Eigen values of this differential equation Eigen functions Eigen functions are e power yeah plus m x and minus m x. So, you can solve the Eigen value problem and substitute the boundary condition I will just give you the final form.

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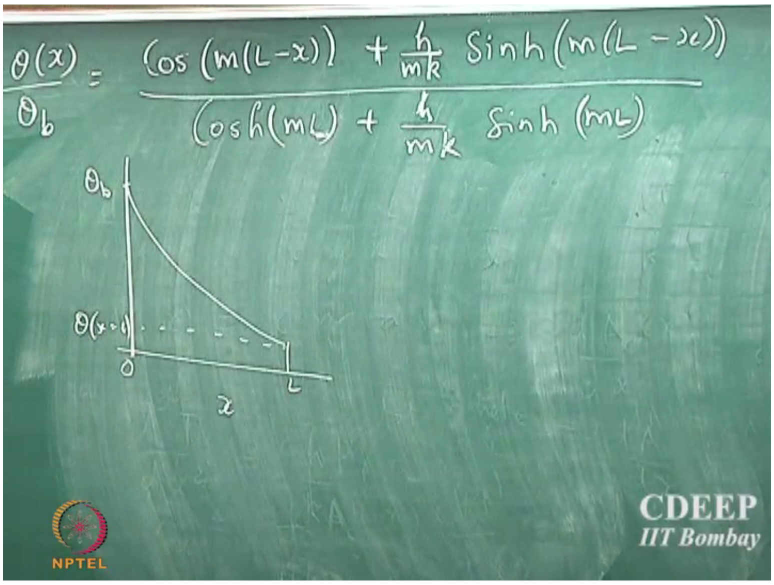

So, theta x pi theta b this is the base temperature is given by the ratio cos m L minus x plus h by m into k into sin hyperbolic m L minus x divided by cos hyperbolic m L plus h by m k sin hyperbolic m L ok.

So, that is the solution and the profile will be. So, if theta b that is the profile that you will expect. So, that is the profile that you can expect from this solution. And In fact, you should go and plot this and see it is sort of obvious to read it out if you know how cosine hyperbolic and sin hyperbolic functions look like, if you do not know I think it is a good exercise to go on plot cosine and sin hyperbolic and convince yourself that this is the solution for this kind of a problem. So, now, we said that when we started this extended surfaces problem, we said that the purpose of extended surface is essentially to increase the heat transfer rate right. So, therefore, we need to find out what is the total heat that is transferred by the fin ok.

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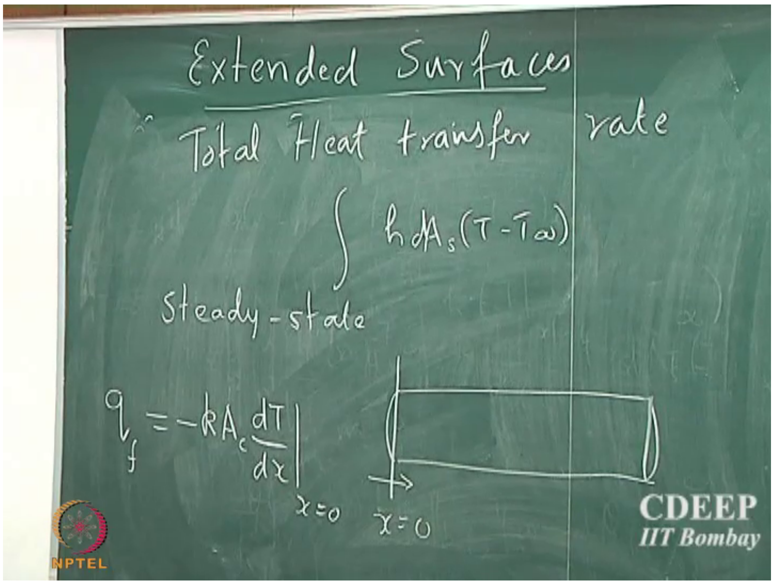

So, that is what that is the quantity of interest which is going to quantify the problem that you are looking at. So, we need to find out what is the total heat transfer rate.

How do we find this how do we find the total heat transfer rate that is the total amount of heat that is transported by the fin. So, note that there are 2 processes 1 is it is transferring by conduction and the heat is being lost from the other end of the fin and while doing that it is also simultaneously losing heat way of convection. So, how do we find the total heat transfer rate?

Student: (Refer Time: 13:25).

Yeah.

Student: (Refer Time: 13:28).

They are supposed to be.

Student: (Refer Time: 13:31).

Added how. So, 1 could integrate h into d A s into T minus T infinity across the whole fin. So, that will give you the total amount of heat that is transferred. So, that is one way to do that, but. So, note that I always told you that you should use your intuition. So, there is an intuitively simple way to do this. So, we assume that this system is under steady state right is that the system is under steady state conditions.

So, if it is under steady state conditions it means that the temperature profile remains constant right it does not change the time. So, whatever heat. So, supposing if this is my fin this is my fin, because I we impose a steady state condition and there is no heat generation inside. So, whatever heat that is actually transferred to the fin at the base should be equal to whatever total heat that is being transferred right, it is a very simple (Refer Time: 14:56) simple thing there are is it clear to everyone.

Whatever is the total amount of heat that is transferred from the base to the fin should be the total amount of heat that the fin is able to transfer, because I said it is a steady state condition the temperature profile has to be maintained. So, whatever heat that comes in should actually go out otherwise the temperature profile is going to change our assumption is not valid. So, in order to satisfy the assumption that we made that it is under steady state condition an intuitive way to find the total amount of heat transfer is, what is the amount of heat that is actually transferred from the base to the fin at x equal to 0.

In fact, if you integrate you will find that it will be exactly the same and that will be give will give you the expression. So, that will be minus k. So, the total amount of heat that is transferred by the fin q f is minus k into cross sectional area into d T by d x at x equal to 0. So, that is the amount of heat that is transferred to the fin at x equal to 0 and that should be equal to the total amount of heat that is transferred.

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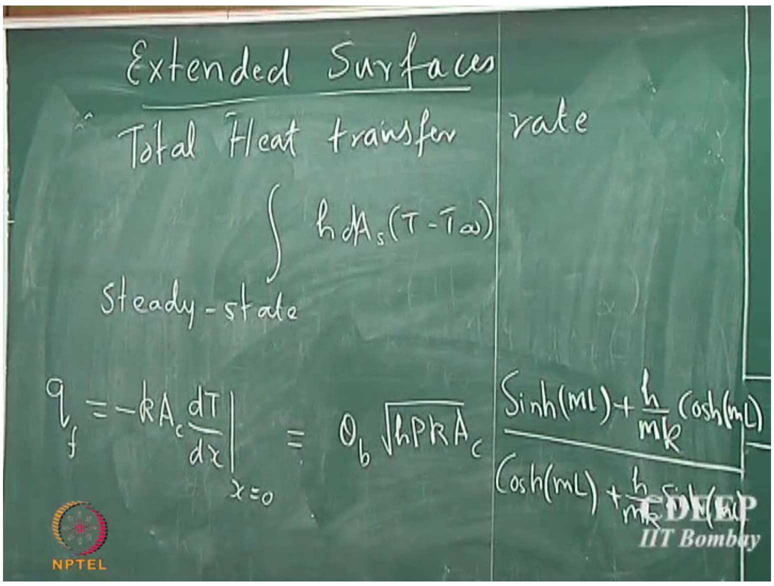

So, that will be equal to and that should be the theta b square root of h P k c k A c into sin hyperbolic m L plus h by m k into cos hyperbolic m L divided by cos hyperbolic m L plus h by m k sin hyperbolic m L. So, that is the total amount. So, note that it is the total amount of heat that is transferred the local heat transfer rate is not constant.

So, remember the resis when we discussed about the resistances, that the whenever there is heat generation or heat loss at local location from this from the solid that you are looking at the total heat there the local heat transfer rate is now going to be a function of position. In fact, that is part of alluded to in this expression. So, note that the total heat transport rate is now a function of the length of the fin that you are considering.

If, it well to be constant then irrespective of the length the heat transfer rate should be exactly the same that is not true so it is a function of the length I mean sort of it is reflected in the total heat transport rate. So, if you know the length of the fin then you should be able to calculate what is the total amount of heat that particular fin can, actually transfer from the base and this is an important design parameter. So, you have if you want if you know what is the total amount of heat that you want to transfer then you could use this expression in order to design the length of the fin. So, this is often an important parameter that you have to find, what is the length of the fin that you have to use in order to achieve a certain amount of heat transport rate is that clear to everyone ok.

So, I will just give you the expressions for the other type of boundary conditions, but I would strongly encourage you to derive them and convince yourself.

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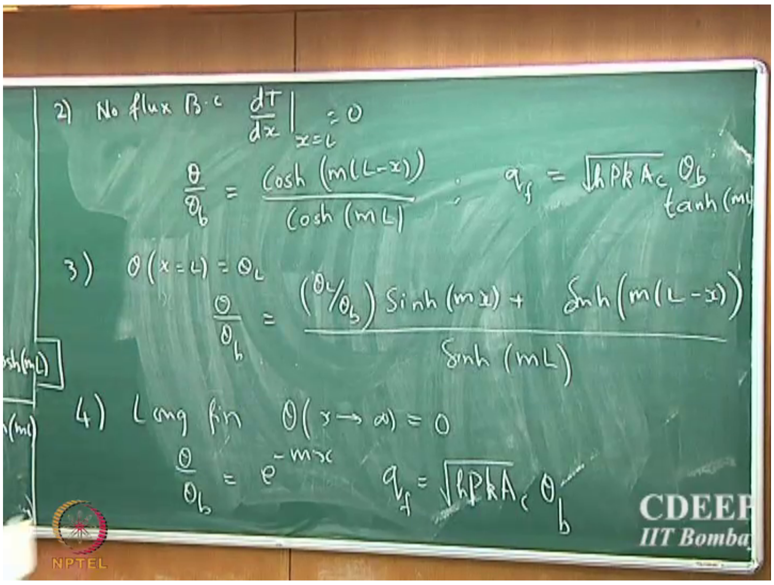

So, supposing if you use no flux boundary condition. So, that is the second type of boundary condition d T by d x at x equal to L is 0 and the solution will be theta by theta b is cos h m L minus x divided by cos hyperbolic m L and the total heat transfer rate q f is given by square root of h p k times A c into theta b into tan hyperbolic m L. And for the third boundary condition where you have a fixed temperature that is theta at x equal to l is theta l, the solution will be theta by theta b equal to theta l by theta b multiplied by sin hyperbolic m x plus sin hyperbolic m L minus x divided by sin hyperbolic m L and for the fourth type of boundary condition where you have long fin infinitely long fin.

So, theta at x tending to T infinity that is equal to 0 that is the temperature and the other end is equal to the temperature of the fluid itself. So, that case theta by theta b is simply given by e power minus m x and q f is given by square root of h P k A c into d theta b. So, those are the 4 different solutions and I would encourage all of you to actually derive these expressions and convince yourself that this is the solution. So, there is another definition and some expression for the definition with that we will finish todays lecture what is called the efficiency.

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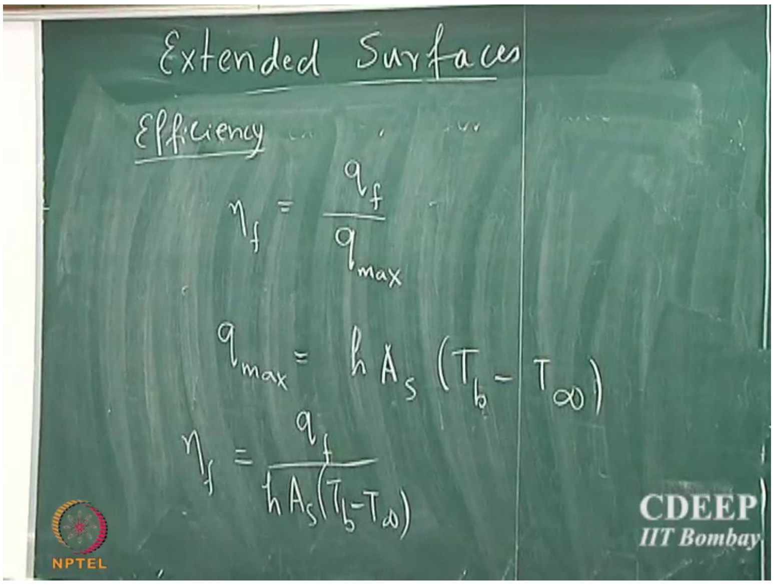

So, note that when we said the purpose of a fin is basically to enhance the heat transfer rate. So, therefore, from design point of view it is useful to define a quantity called efficiency, how efficient is the fin that I have designed can I calculate the efficiency and find out what. So, if I want to compare 2 different fins in instead of comparing what is the actual total heat transfer rate it is useful to compare the efficiency of the fin that has been designed.

So, therefore, the efficiency of a fin is basically defined as the total heat transport rate that is the total amount of heat that is transferred by the fin divided by the maximum possible heat that can be transferred divided by the maximum possible heat that can be transferred by the same fin, what is q max how can we find the maximum possible. So, let us pose a slightly different question when can we achieve maximum heat transfer from a fin of a given geometry.

Student: (Refer Time: 22:02).

When can we assume when can when can we get maximum heat transport rate under what conditions.

Student: (Refer Time: 22:14).

Temperature.

Student: (Refer Time: 22:18).

But that is that is like infinitily long fin you never going to achieve it. In fact, if the temperature the right end is T infinity then the temperature gradient or driving force for convection is almost 0 at the end of the fin. So, which means that the end of the fin are almost of no use the any heat transport rate is governed by the temperature gradient which is the driving force.

So, if the temperature at the end of the fin is equal to that of the fluid temperature it is no use. So, you do not want to design a system where some parts of your fin is going to have a 0 driving force that is useless it is a waste of money. So, how do we find maximum when do you achieve maximum heat transfer rate it is hypothetical you are not going to achieve it temperature is.

Student: Constant.

Constant where.

Student: (Refer Time: 23:20).

Throughout the length what is that constant.

Student: That is equal to the (Refer Time: 23:26).

That is equal to the base temperature. So, what is the purpose of a fin the purpose of a fin is to transfer heat from the base to the fluid around when can you achieve maximum heat transfer rate, when the temperature in the fin is constant and not just at any constant it should be equal to the temperature of the base itself because that is the maximum possible heat that you can transfer. So, therefore, q max is simply given by h into the overall conductive heat transport area multiplied by T b is the base temperature so the maximum possible rate.

So, note that this is maximum possible rate, but what you will actually achieve in reality will; obviously, be lesser than this. So, this is the maximum possible and therefore, 1 can define efficiency as q f divided by h A s into T b minus T infinity.

So, that is the efficiency of A fin. In fact, for this problem the efficiency would be so, for the second type of boundary condition where it is an adiabatic case.

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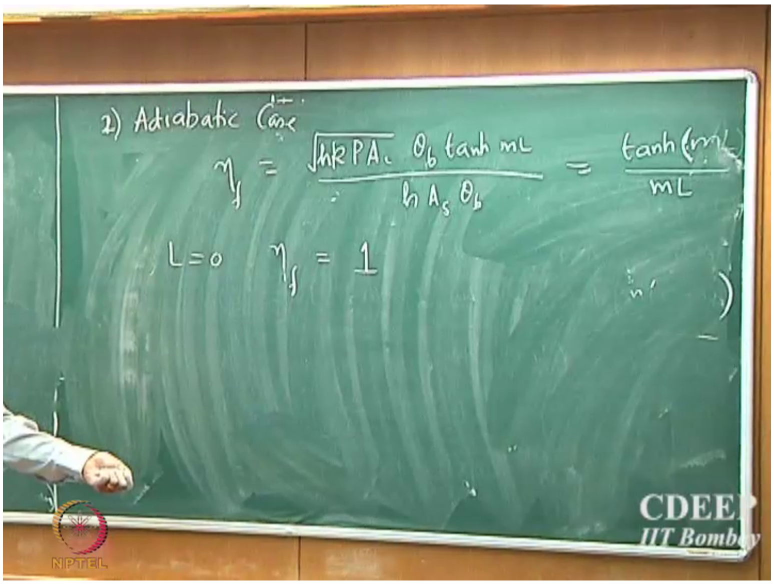

So, eta f simply given by square root of h k perimeter into A c into theta b tan hyperbolic m L divided by h into a s into theta b theta b is nothing, but T b minus T infinity. So, that is the substitution that we made when we solved the differential equation. So, that is nothing, but tan hyperbolic m L divided by m L what happens when l equal to 0.

Student: (Refer Time: 25:30).

What does tan m L by m L when L equal to 0.

Student: (Refer Time: 25:46).

What is tan hyperbolic when L is 0?

Student: (Refer Time: 25:53).

Yeah.

Student: (Refer Time: 25:55).

0 so it is 0 by 0. So, what is it oh it is not that simple how do you know that it is 1.

Student: (Refer Time: 26:03).

When you have an indeterminate system what do you do?

Student: (Refer Time: 26:07).

You do an l hospitals rule l hospitals rule. So, you will find out that eta f is equal to 1 that is sort of obvious to intuit and the reason why you can intuit is you said that the maximum temperature in the fin is achievable only when the fin size is 0, you can never achieve a constant temperature which is equal to the base temperature.

So, therefore, the efficiency if at all you have the base cross sectional area which is sufficient to transport heat as much as whatever is required to be transported then the efficiency is going to be 1, but this is not a realistic case it is just hypothetical yes.

Student: (Refer Time: 26:50) number you designed.

Will be 0.

Student: (Refer Time: 26:52).

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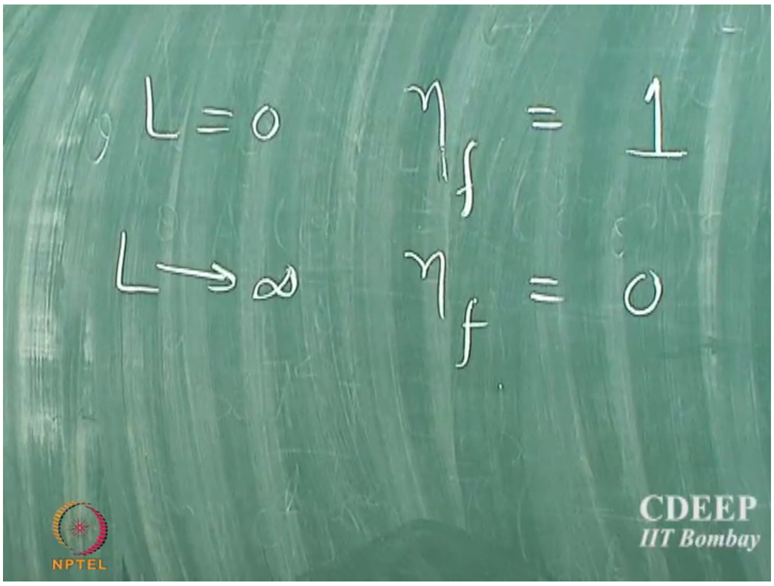

That is right. So, which means that as L goes to infinity eta f goes to 0 that is right. So, it is not that you design a long fin and you are going to have very high heat transport. So, it is completely counter intuitive. So, although intuitively you would guess that I design a very very long fin and I will be able to achieve as much as heat transport that I want that is not really true. And the reason why the efficiency is very poor is that the as you as the length increases, the driving force is constantly decreasing because you have 2 processes which are occurring simultaneously one is the conduction and the other one is the convection. So, the driving force which is actually forcing the efficiency to be 0.