Chemical Reaction Engineering II
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Lecture - 32
Residence time distribution function

Friends, it is a quick time to summarize what we have learnt in residence time distribution so far. So, you looked at what is what is an non ideal reactor and what is the residence time distribution function, what are its definitions and we had looked at what is the ways to measure it experimentally, that is looking at the pulse and the step input. And we have also came out we also looked at what are the RTD of the residence time distribution functions e curve. And the cumulative distribution function f curve in last lecture.

So, today let us start with this lecture. Let us start looking at the properties of different functions and also proceed further. So, suppose if I look at the an important property of the residence time distribution is the mean residence time.

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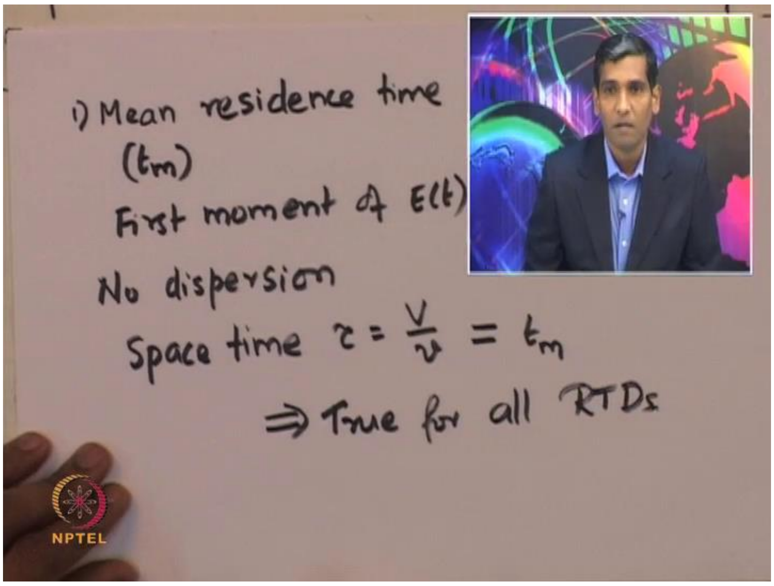

So, the mean residence time is actually given by the first moment. So, if I tm it is the symbol that will I use for mean residence time, it actually given by d first moment of the of e of t that is the RTD function. So, e of t is actually a distribution and that distribution can actually be used to decipher some of the properties of the distribution itself and some of the properties of the reactor system. For example, mean residence time is an important property that is actually used to control various things in the system; when there is no dispersion across boundaries that is between the point of injection and the entrance of the reactor.

Then in these situations the a space time that is: tau which is equal to v by d volumetric flow rate with which the fluid is actually flowing through the reactor that is equal to the mean residence time itself. Now, this is independent of any RTD function that is actually representing the non ideal behavior of the reactor under no dispersion conditions irrespective of the RTD function, the mean residence time that we obtain, would be exactly equal to the space time of the reactor itself.

So, this is true; true for all RTD's this is true for all RTD all residence time distributions irrespective of what type of reactor as long as the dispersion is actually absent. So, now let us look at how to calculate the mean residence time from the residence time distribution function e of t.

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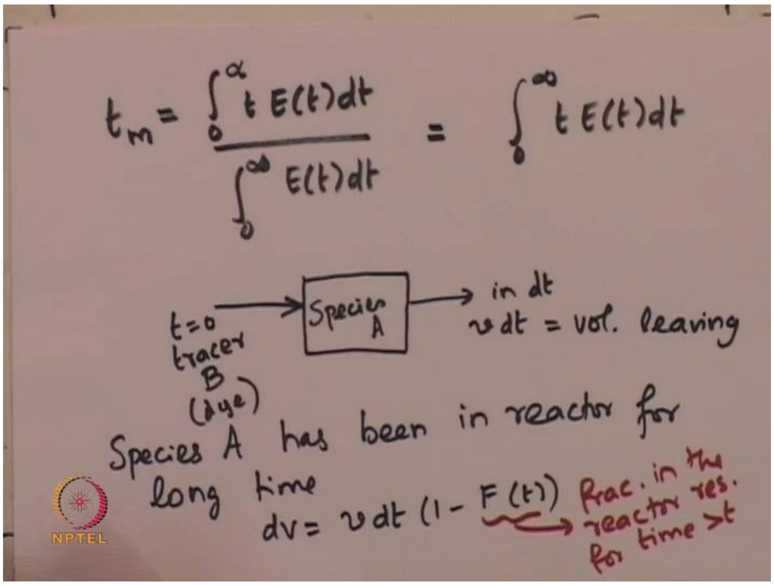

So, t m, which is the mean residence time is actually, given by the first moment as we observed, as I mention in the previous note few moments ago that is 0 to infinity t into e of td t divided by integral 0 to infinity e of t d t. So, that is the that is the residence time distribution. And because, the integral of the e curve which is the RTD function between the 0 to infinity that is equal to 1 the this expression can further be simplified as integral between 0 to infinity t into e of t d t. So, that is the that is the expression for the mean residence time if the RTD function e of t is known.

So, if the residence time distribution function is known 1 can simply plug it in this expression and find out what is the mean residence time. Now, suppose let us look at suppose let us consider the reactor, and let us assume that it is filled with species a it is fitting that species and. Let us say that at time t equal to 0 a tracer molecule tracer species b is injected into the reactor. Let us say it is a dye and the sometime d t.

So, let us say that the amount of tracer which is actually leaving the reactor, in this time delta t whose age is actually lies between that time is actually given by v times d t where v is the volumetric flow rate with, which the fluid actually leaves the reactor and that is equal to the volume of the tracer; which is actually leaving the that is actually the volume of the a fluent stream which is actually leaving the reactor not the tracer ok. So, now suppose if you want to know that the species has been there for a long time suppose.

So, species a, has being in the reactor for a long time. So, remember v d t is the volume of the a fluent is actually leaving the reactor in this time d t and if you want to know what is the volume of species a which is actually leaving in that time delta t. So, then that will be given by d v which is equal to the total volume of the fluid that is actually leaving reactor multiplied by 1 minus f of t. So, f of t is basically the fraction that has been in the reactor for time which is greater than t.

So, this is the fraction which is actually. So, that is the fraction in the reactor residing for time larger than t. So, so 1 minus ft multiplied by the volume of the effluent stream will actually tell us what is the amount of species a, which is actively leaving the reactor in that small time d t. So now, if the sum this overall the molecules of a then; that will tell us what is the net volume of the species which is actually leaving the reactor. So, if sum overall a molecules.

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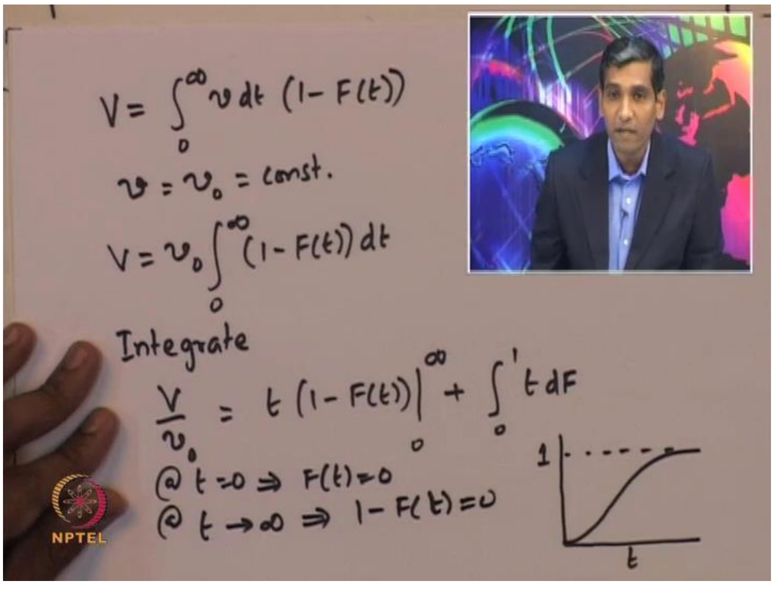

So, if the total volume that is leaving is given by 0 to infinity v d t into 1 minus f of t. So, from here, if you assume that the volumetric flow rate with which the fluid stream leaves; the reactor if that remains constant. If that remains constant and this is generally not true for gas stream, but, it is normally true for liquid streams that is actually leaving the reactor if it is a gas stream. Suppose, if it a operated under constant pressure and under isothermal conditions that is constant temperature and if the number of molecules or number of moles that is not change because of the reaction.

Then 1 may also assume that the volumetric flow rate with this the fluid leaves reactor the flow in stream volumetric flow rate is probably perhaps remains constant. So, by using this, we can say that v equal to v naught into integral 1 minus f t d t. So, now, we can integrate this by parts. So, if we integrate you will find that v by v naught that is equal to t into 1 minus f of t limits 0 to infinity plus integral 0 to 1 t d f. So, that is the integral. This is basically, when we do an integration by parts, we can see that we can split the integral into 2 sections is t into 1 minus f t evaluated between 0 and infinity and 0 to 1 t times d f.

Now, if a look at the f curve; the f curve typically looks like this. So, this is with respect to time and this is 1. So, at time equal to 0f of t is 0. And when t goes to infinity1 minus f of t is 0. So, that can actually be easily seen from the ft curve or the f curve.

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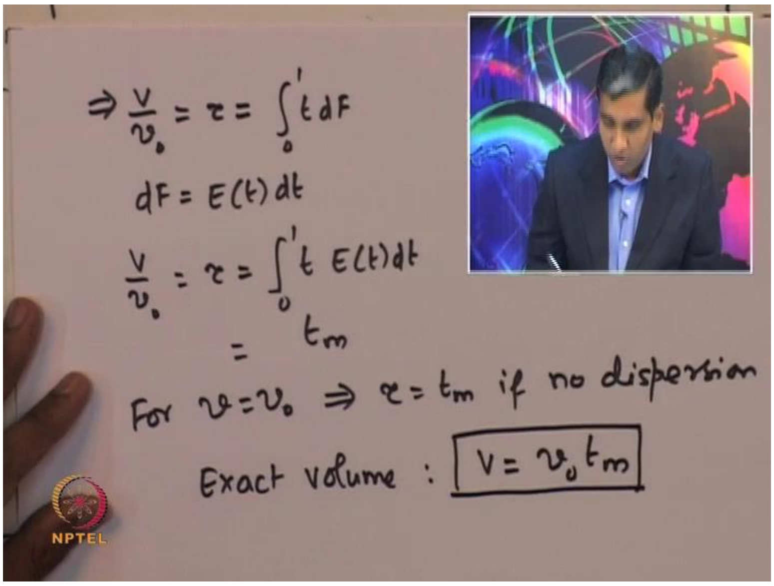

So, now, substituting these expressions you will find that v by v naught. That is equal to tau which is the space time of the reactor. And that is 0 2 1 t times d f. So, what is d f; d f is nothing, but, the residence time distribution itself, d e t into d t gives the differential of the f curve and therefore, v by v naught that is equal to tau that is equal to integral 0 to 1 t times e of t d t and that is nothing, but, mean residence time itself.

So, this shows that for any RTD if there is no dispersion between the point of injection and the entrance of the reactor 1 can show that, the mean residence time is actually equal to the space time of the reactor itself, irrespective of what is the RTD function e of t. So, so clearly for v equal to v naught for constant volumetric flow rate then tau equal to t m if no dispersion. And remember that this v equal to v naught is true for gases only if the reactor is operated under constant pressure drop and the temperature is maintained constant at its isothermal conditions.

If the number of moles does not change, because of the reaction only under those conditions the a fluent stream volumetric flow rate may be assumed as a constant ok. So, therefore, the exact volume of the reactor exact volume of the reactor, if there is no dispersion is actually given by v naught multiplied by the average residence time. So, if the average residence time is known then we can actually calculate what is the exact volume of the reactor in which the fluid is actually flowing.

So, are there other properties, we looked at mean residence time and we also shows that the mean residence time should be equal to the space time irrespective of the RTD function, as long as the dispersion is a is negligible or 0 and also if the volumetric flow rate at which the fluid stream leaves remains nearly constant. So, are there other properties? The answer is yes there are other properties. So, the other properties is we can also estimate what is the variance of the distribution and that can be obtained using the second moment obtained using the second moment.

So, the sigma square, which is the variance is given by 0 to infinity t minus t m square into e of t d t and. So, now if we expand this square term quadric this product here. So, we can expand this is 0 to infinity t square plus t m square minus 2 into t t into t m into e of t d t. So, that is the that is the integral and this is nothing, but, 0 to infinity t square e of td t minus t m square. So, this essentially the variance it is essentially quantifies the is quantifies the spread in the distribution of the RTD function. So, that is another property that is actually very commonly used in the real systems.

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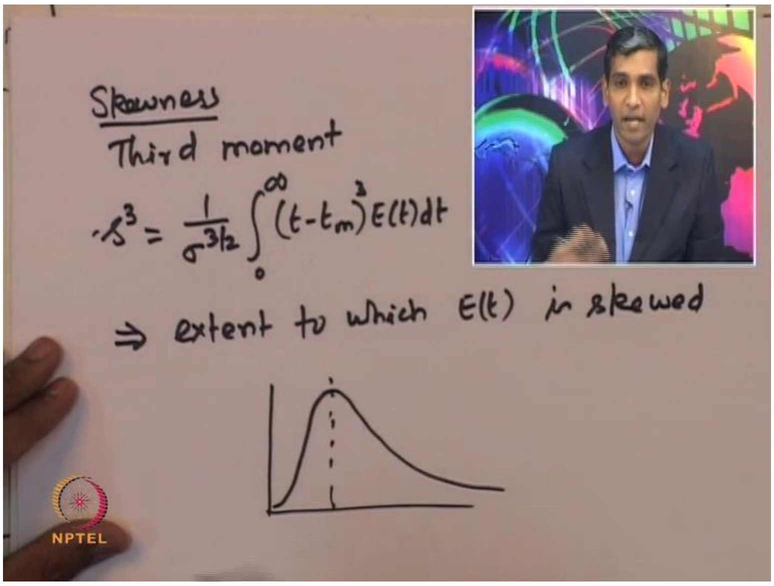

The third property is not very commonly used is the is the skewness property. It is called the skewness and that is obtained using the third moment of the distribution. And that is given by if s cube is the skewness parameter that'll be 1 by sigma to the power 3 by 2 and sigma is the standard deviation that is square root of variance 0 to infinity t minus t m the whole cube into e of t d t. So, that is the skewness and this is basically reflects the extent to which the distribution residence time distribution function is skewed.

So, remember that it may be skewed in either directions. So, for example, if the is a residence time distribution looks like this then it is out of skewed to the right hand side of the mean. So, the s cube essentially says how skewed is the distribution with respect to mean of the distribution itself. So, now, once we know these properties next to question is from real reactor data suppose if there is a tracer that goes inside and from the real data is it possible to estimate some of these parameters and what are the steps that is the no want.

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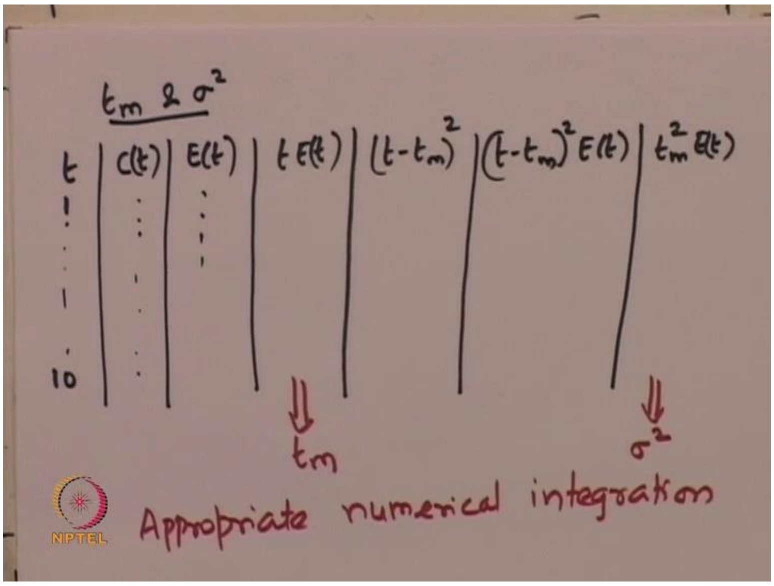

So, let us look at how to calculate the mean residence time and sigma square from the actual data. So, normally the actually data that 1 would get is basically the measurement of concentration as a function of time. So, let us say that there are several concentrations that has been measured. Let us say, from time 1 to 10 and there is been concentration regime measured. So then 1 needs to create a table whereas, a first step 1 calculates the e of t.

So, we know the formula for e oft which is essentially given by c t divided by the integral of c over their whole time domain and then the next thing 1 needs to estimate is t into e of t. So, this provides this column provides an estimate of the first moment, which is the mean residence time can be used to find the mean residence time. And the next step is to estimate t minus t m the whole square. And then find out t e minus t m the square into e of t and then from here 1 can actually find out what is t m square into e of t.

So, 1 can make such a table moment the experimental data off time versus concentration is available of the tracer is available than 1 can actually fill up this table and from this from this column1 can estimate the mean residence time and from this column 1 can actually estimate what is the a sigma square. So, and 1 needs to use an appropriate numerical integration scheme remember that the concentration is actually discrete values at different time points and.

So, 1 has to use appropriate numerical integration appropriate numerical integration in order to complete this table once this table is complete you'll actually be able to estimate what is the mean residence time. And the variance for the distribution that represents the RTD function for the reactor. Now the suppose, if we change the suppose if there is a reactor and we know the RTD function.

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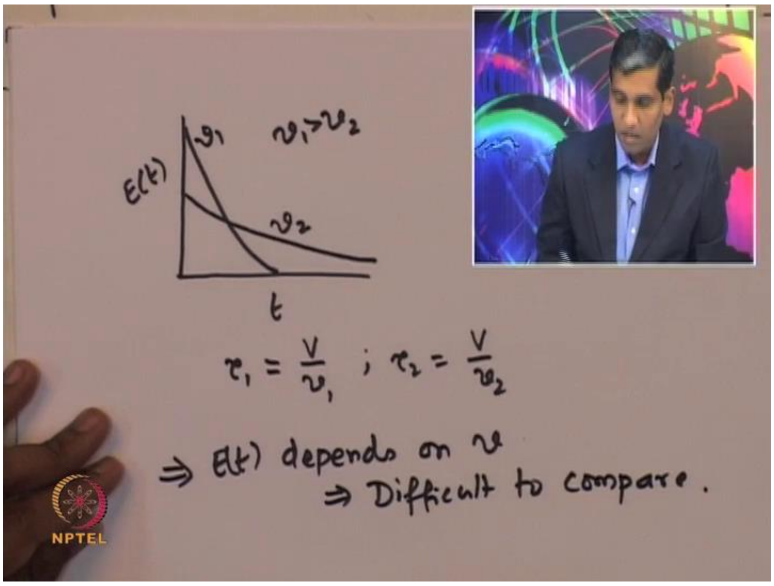

Suppose we know the RTD function suppose we know the e curve for a given volumetric flow rate v1. Now, if you want to find out what is the e curve or the RTD function for a different volumetric flow rate. Now let us consider the situation where we are actually feeding the reactor with a fluid of volumetric flow rate which is less than v1 ok. So, then the amount of time that fluid streams spend inside the reactor is going to be larger because the volumetric flow rate is actually lesser than v1 and as a result the e curve would actually look like this the slope of the e curve will correspondingly change.

So, now, because of this problem for this corresponds to volumetric flow rate v2 and because of this issue it is very difficult to now, compare the e curves at different conditions, because the e curve is now going to be dependent on the volume the reactor and also on the volumetric flow rate with which the fluid is actually being fed into the reactor even for a fixed volume the e curve is now going to be a function of the volumetric flow rate. Because the volumetric flow rate decides the residence time of the fluid stream inside the reactor.

So, therefore, the tau which is the space time when the volumetric flow rate is v1 is given by v by v1 and tau is given by v by v2. So, clearly the amount of time that is spend by the second says second in the second case that is when the fluid is being feed at a volumetric flow rate of v2 that is going to clearly be larger than the that of the time that is actually spend by the fluid elements inside the reactor when the volumetric flow rate is v1 because v2 is actually smaller than v1. So, because the e t dependence on property such as volumetric flow rate is difficult to compare. So, as a result it is useful to actually define and normalized RTD function in order to facilitate the ability to compare different RTD curves.

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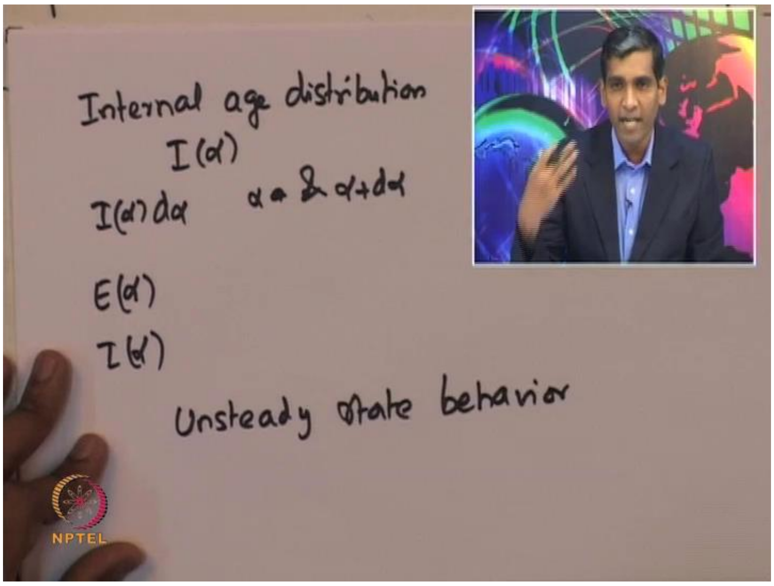

So, so let us look at what is the normalized RTD function. So, suppose if we define theta as the ratio of the t divided by tau, where tau is the space time of the reactor. If you define theta as the ratio of time versus the space time of the reactor then we can now rewrite the RTD function e theta as basically tau multiplied by e t. So, that is tau is the space time multiplied by the corresponding RTD function gives the normalized RTD function e of theta and…

So, now theta here which is the ratio of time to tau essentially represents the number of reactor volumes of fluid based, on the entrance condition that actually flow through the reactor in that particular time. So, now, this normalized RTD function e theta provides a facilitates a way by which the performance of the reactor or the RTD function itself can be compared when the sizes are different. So, therefore, if we look at the RTD curve of the normalized RTD function, then the curve looks like this where.

So, irrespective of whatever is the volumetric flow rate for a for a given the reactor volume the RTD function essentially looks like this. So, now and there is another definition that 1 needs to know is the internal age distribution. And the symbol that is commonly used is i of alpha where, i of alpha is d alpha that essentially represents the fraction of the material that this present inside the reactor, in a time span of alpha in a time span for a period between that is between alpha and alpha plus d alpha.

So, that represents the faction of the material that is actually residing inside the reactor, whose period of residing inside lies between this lies between alpha and alpha plus d alpha in that small interval. So, e alpha essentially presents the age of the fluid that actually is leaving the reactor and i alpha represents the age of the fluid that is actually, present inside the reactor. So, these 2 have its own utility and particularly, the internal the age of the fluid elements that is actually present inside the reactor has significant importance when 1 looks at when 1 wants to study the unsteady state behavior of a particular reactor.

In particular a good example of that would be that suppose, if there is a catalytic reaction and the catalyst is actually decaying with time then it is important to know what is the internal age distribution and it is important to actually consider the age distribution in modeling the performance of such kind of a reactor. So, i alpha the internal age distribution. Is essentially given by 1 by tau into 1 minus f of alpha and e of alpha has we know is actually given by minus d by d alpha tau into alpha because of the connection between the e curve and f curve.

So, the relationship between the e curve and the i curve is nothing but e of alpha is minus d by d alpha into tau into i alpha. Now, for a CSTR for an ideal CSTR i alpha is essentially given by 1 by tau into exponential of minus alpha by tau. So, that is the internal age distribution for a CSTR after all these definitions that we have seen that is the e curve, f curve and the i curve and the mean residence time variance and skewness.

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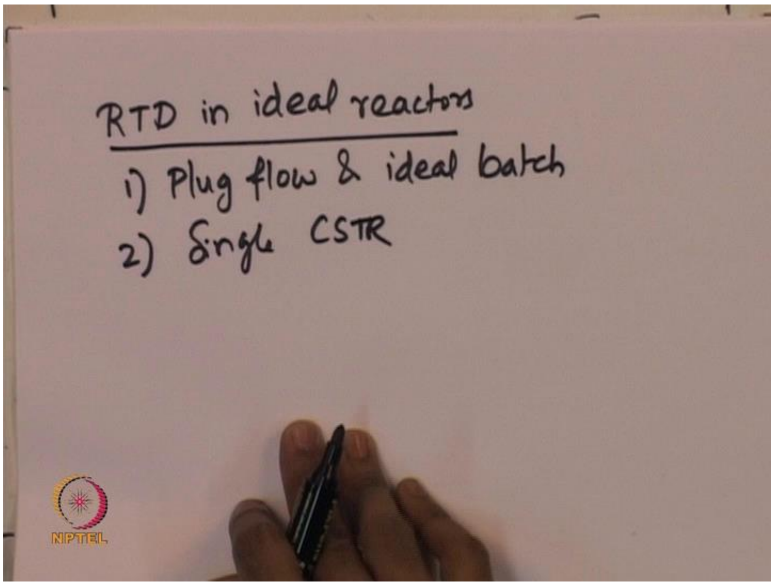

Let us look at the residence time distribution in ideal reactors. So, particularly we will consider 2 cases 1 is a plug flow. And ideal batch reactor plug flow and ideal batch reactor and second 1 is we look at the single CSTR case. So, these 2 we look at and will attempt to find out how to get the RTD for RTD curves for these 2 types of ideal reactors.

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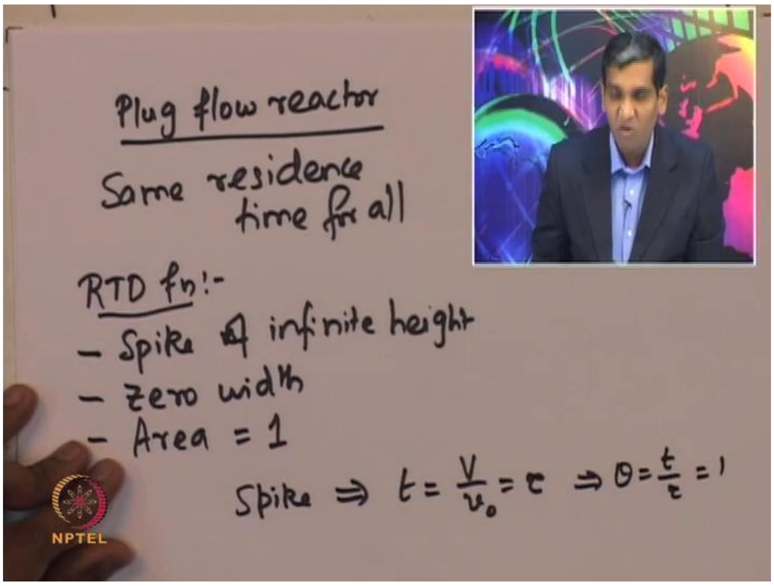

So, let us first consider the plug flow reactor. Let us consider the plug flow reactor. So, what is the property of the plug flow reactor all atoms or all molecules of the material, it is actually entering the reactor, will spend exactly the same amount of time before they leave the reactor, which means; that all the elements or all molecules of the material will have exactly the same residence time.

So, same residence time for all fluid elements that is actually entering and leaving the reactor. So, therefore, the RTD function must have the following properties. So, first thing is it must have a spike of infinite height because all of them will have same residence time therefore or they will all leave like a plug. So, therefore, the e curve must have a spike of infinite height and also it must have 0 width and not just that the area under the curve should be equal to 1.

The spike will be exactly at the mean residence time and that is very important because that is that is the property, which actually captures the nature of the plug flow reactor. So, therefore, the spike will be exactly at t equal v by v naught that is equal to tau, which is the space time of the reactor and because there is no dispersion the space time of the reactor will also be equal to mean residence time of the reactor or in the non-dimensional terms theta equal to t by tau that is equal to 1.

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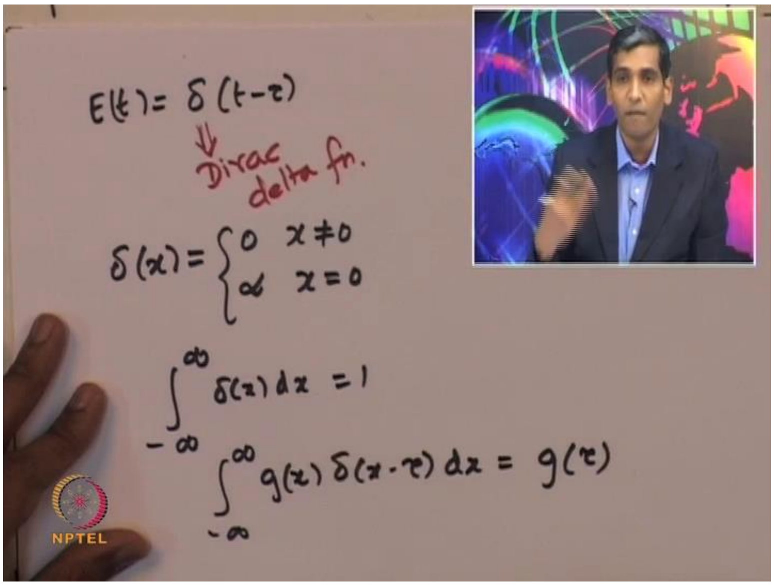

So, therefore, the corresponding e curve because of these properties of the RTD function for the plug flow reactor the e curve should simply be presented by the Dirac delta of function centered at the space time of the reactor. So, this is the Dirac delta function that is a Dirac delta function is defined as flows. So, Dirac delta function delta x that is equal to 0; if x is not equal to 0 and its equal to infinity, when x is exactly equal to 0. And the property of this e curve is actually given by minus infinity 2 plus infinity delta x d x should be equal to 1 that is property of the Dirac delta function.

In addition to that the important properties by a that the satisfies the convolution integral that is equal to g of tau. So, integral of g x if g x is some function of x multiplied by the delta function to d x that is equal to g evaluated at that value of tau itself where x minus tau is actually equal to 0 that is where this spike is actually present. So, now, let us calculate the mean residence time for this RTD curve.

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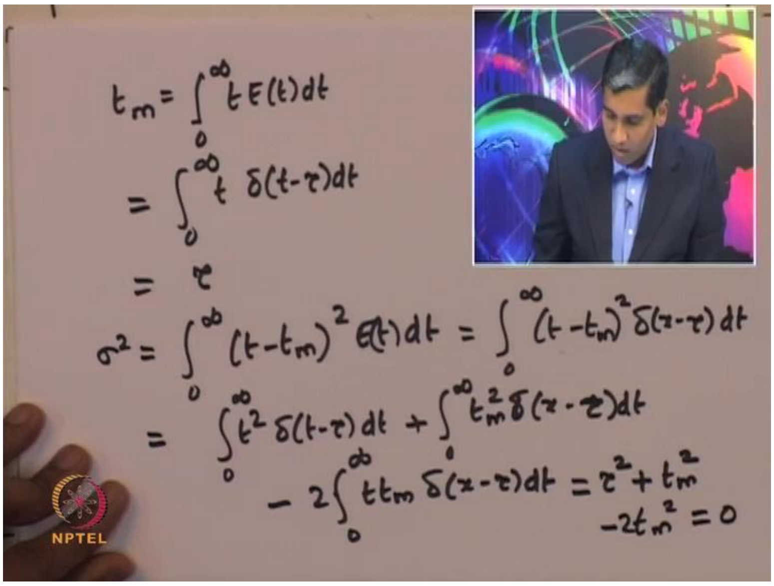

So, the mean residence time t m is actually given by integral 0 to infinity t into e of t into d t that is the that is definition for the mean residence time in terms of the RTD function. So, that is equal to plugging in the e curve for plug flow reactor will find that it 0 to infinity t into delta of t minus tau into d t. And that is nothing, but, tau itself. So, therefore, the mean residence time is exactly equal to the space time and this actually 1 would easily guess.

Because we said that the an important property of the plug flow reactor is that all material that is actually entering the reactor and leaving the reactor. We will actually have exactly the same residence time and that the e curve is actually going to be centered at a at the space time. So, therefore, the mean residence time must be exactly equal to the space time of the plug flow reactor itself, which is which 1 would actually, guess and is also clearly shown by the RTD function also. So, now, let us look at the second moment that is the variance of the of the distribution.

So, that is given by 0 to infinity t minus t m the whole square into e of t into dt. So, that is equal to t m square into delta function into d t and. So, now, we open up this the t minus t m whole square and then 1 if 1 integrates you'll find that this essentially reduces to reduces to t square into delta of t minus tau into d t 0 to infinity plus integral 0 to infinity t m square delta of x minus x minus tau d t minus 2 integral t into t min to delta of x minus tau d t. And that is essentially. So, the first term here because, of the property of the delta function is it is simply be equal to tau square and the second property will simply be equal top lust m square.

So, that will be the second 1 and third 1 will simply be2 into t into t m t into delta function integral t m is constant. So, that will come out of the integral and t into d into delta function will essentially be equal to the mean residence time. So, that will be equal to 2 t m square and that is equal to 0 because the mean residence time in the space time are exactly equal.

So, therefore, the variance is actually equal to 0. And that reflects the property of the RTD function that actually be intuitively guess that is the if there has to be a spike at a exactly equal to tau with an area under the curve is equal to 1 and the height of the spike is equal to infinity which means that the variant should be equal to 0 for the distribution. So, let us look at the f curve for the plug flow reactor.

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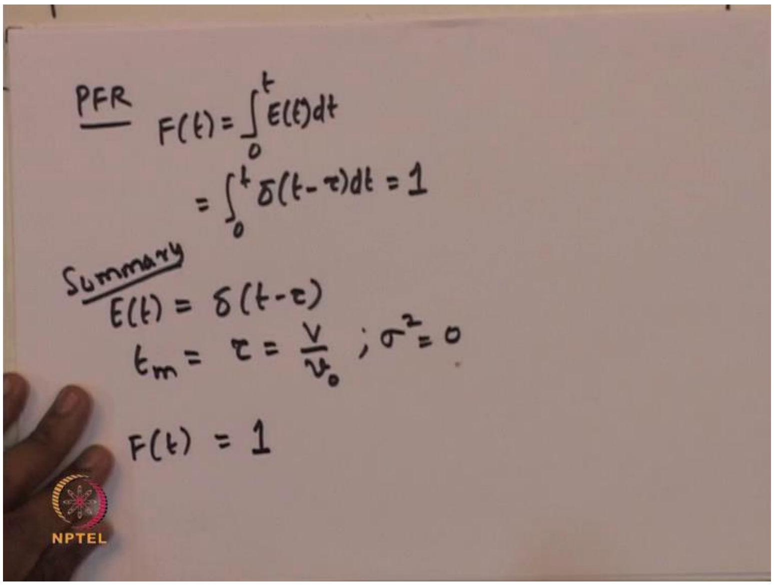

So, for a plug flow reactor the f curve f of t is essentially given by 0 to t e of t by d t that is by definition and. So, that is equal to integral 0 to t delta of t minus tau d t that is equal to 1 by we know that this integral is equal to 1 and therefore, the f of t curve is nothing, but, 1. So, as a result the as a result, so the properties or the RTD function for plug flow reactor is essentially given by e of t to summarize is equal to delta function of t minus tau.

So, that is summary for plug flow reactor summary for plug flow reactor where the residence time distribution function is essentially given by delta t minus tau. And the mean residence time is equal to the space time of the reactor, which is the volume divided by the volumetric flow rate and the sigma square is essentially 0 the variance is actually 0 and the f t is essentially equal to 1. So, therefore, if we actually attempt to sketch the e curve and f curve we will find that.

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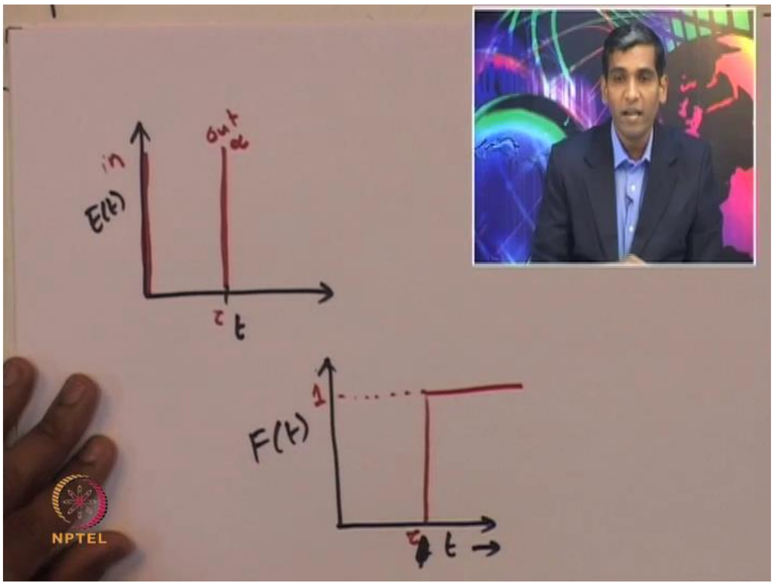

So, that is time and suppose, if this is tau here at t equal to 0 if there is a spike tracer that is actually put into the plug flow reactor. So, that is the spike then exactly after a delay of tau time which is the space time of the plug flow reactor, the tracer will actually come out and the same amount same quantity of tracer will actually come out of the reactor. So, that is the out steam and the height will be infinity. Now, suppose if a look at the f curve.

So, this is the e curve and suppose if I look at the f curve of the reactor an exactly tau equal to exactly t equal to tau that is the space time or the mean residence time of the plug flow reactor the f off value will be exactly equal t 1. So, that is the e curve and the f curve for a plug flow reactor.

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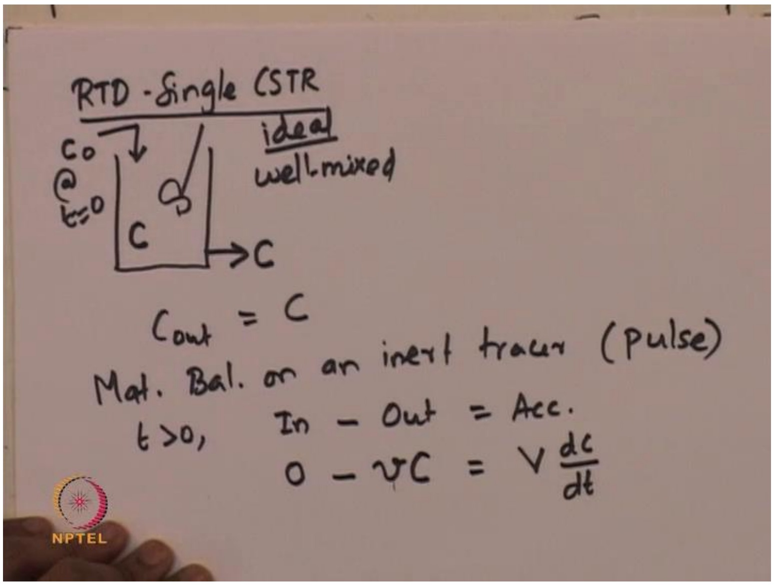

Now, let us look at the CSTR case what is the RTD function for a single CSTR. Now suppose, if has a CSTR and is some this is the inlet stream and this is the outlet stream of the CSTR and it CSTR is well mixed. And it is assume that it ideal CSTR and therefore, the it is a completely well mixed system. And let us now because, it is completely well mixed system the concentration of the species, which inside the reactor should be equal to the concentration of this species and the fluent stream as well. So, which means; that the outlet concentration is equal to the concentration of the species in the reactor and.

Let us now write a material balance on a inner tracer suppose there is any inner tracer which is actually fed into the reactor. So, let us say that am inner tracer fed into the reactor and if the concentration of the inner tracer is actually c naught at t equal to 0. So, at time t equal to 0 some t naught c naught quantity of tracer is actually feed into the reactor and now, we can write a material balance in order to find out what is the RTD function. So, for any time greater than whatever fluid is actually whatever, the tracer is entering the reactor that should minus whatever, is actually leaving that should be equal to the accumulation of the tracer inside the reactor.

Now, if you assume that it is a pulse tracer, if it is actually a pulse tracer which means; that the time at which the tracer is actually feed into the CSTR which is exactly t equal to 0 and nothing before and nothing after t equal to 0. So, therefore, at any time greater than 0 no tracer is actually entering the reactor. So, therefore, the inlet is 0 minus what leaves is the volumetric flow rate v of the a fluent stream multiplied by the concentration of the c and that should be equal to v into d c by d t which is the accumulation of the tracer in the CSTR.

Now, because the concentration of the species inside the reactor is equal to the concentration at which the species is actually leaving the reactor. The c is here essentially represents the outlet concentration of the species from the reactor they are reflects the concentration of these species, with which it actually leaves the reactor in the a fluent stream.

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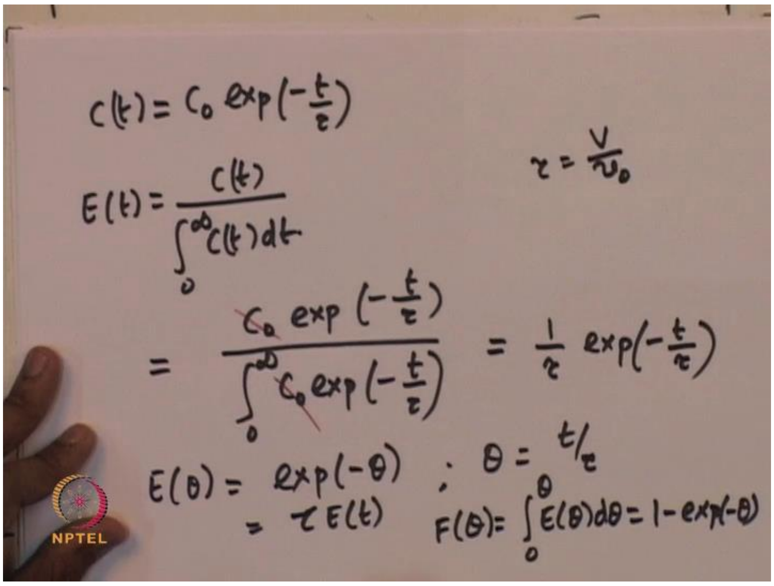

So, now, 1 can actually integrate this expression to find out that c of t is equal to c naught into exponential of minus t by tau, where c naught is the initial tracer concentration initial pulse tracer concentration of the initial tracer that is actually fed as a pulse tracer and from this we can find out that e of t is given by c t divided by integral 0 to infinity c of t d t. So, now we know the expression for c t the dependence of c on time and other properties. So, we can plug that in here we will see this exponential of minus t by tau divided by integral over 0 to infinity minus t by tau.

So, performing the integration, we will find that because c naught is constant 1 can actually cancel out c naught from the nominator and denominator and. So, we will find that this we will be equal to 1 by tau into exponential of minus t by tau. So, that will be the residence time distribution function for a single CSTR now in terms of the dimensionless in terms of the normalized RTD function a e of t is essentially given by exponential of minus theta where theta is actually t by tau. And e of theta is nothing, but, tau into e of t.

So, that is the normalized residence time distribution function, And now we can actually find out what is the f curve. So, f of theta is nothing, but, integral 0 to theta e theta into d theta that is actually 1 minus exponential of minus theta. So, that is the f curve that is the expression for x f curve which is 1 minus exponential of minus theta, where theta is t by tau and tau is the space time of the reactor where tau is the nothing but v by v naught that is the space time of the reactor. So, let us attempt to sketch thee curve and the f curve.

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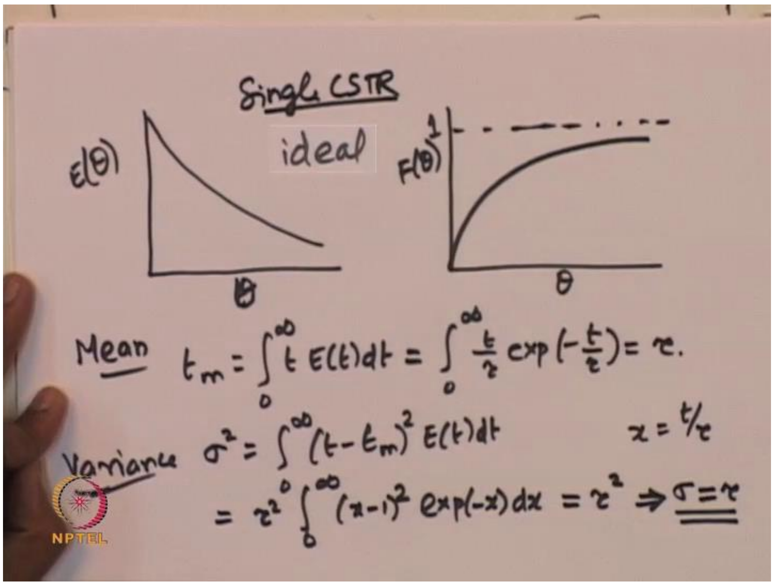

So, the e curve. So, the normalized RTD function. So, the e curve essentially looks like this: is an exponential decay and then the corresponding f curve is actually looks like this. So, this is 1 and this is theta. So, it actually essentially looks like this and. So, the mean; mean can actually estimated as t m that is equal to integral 0 to infinity these are different properties of the distribution t into e of t d t that is should be equal to 0 to infinity t by tau into exponential of minus t by tau and that should be equal to tau.

So, that is exactly what we have observed before if there is no dispersion then irrespective of whatever is RTD then the mean distribution time should be equal to the space time of the reactor itself. And now the next the variance sigma square is given by 0 to infinity t minus t m square into e of t d t and that should be equal to tau square integral

0 to infinity x minus 1 the whole square into exponential of minus x d x. So, where the change of variable is done by setting x equal to t by alpha and. So, integrating this is standard expression.

So, by integrating this expression 1 can find that is equal to tau square, which means; that the standard deviation of the distribution is actually equal to the space time of the reactor itself. So, for a single CSTR for a single CSTR the mean residence time is equal to the space time of the reactor. And the standard deviation of the residence time function is also equal to the mean residence time of the reactor itself. So, now, if we compare the various compare the RTD function and the various properties of CSTR we can find that.

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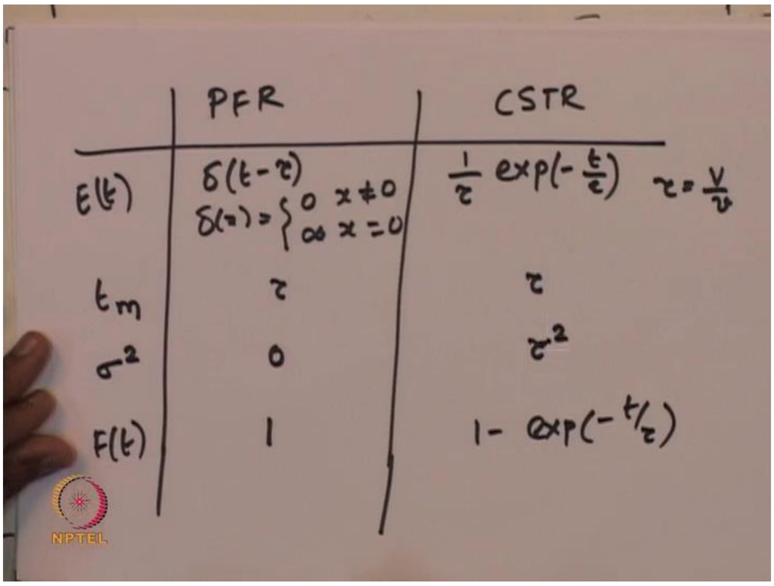

So, suppose if we make a comparison. We can summarize the function and the properties that we found. So, far for a plug flow reactor is CSTR. So, the residence time distribution function e of t is essentially the delta function for a plug flow reactor, which means; that a there is just a delay and whatever, is fed into the reactor is going to come out of the reactor exactly after a certain delay and the delay is given by the space time of the reactor. And here delta x is actually defined as 0 for x naught equal to 0 and infinity for x equal to 0.

The corresponding RTD function for CSTR is 1 by tau exponential of minus t by tau where tau is given by the v by v tau is the space time, which is given by a volume of the reactor divided by the corresponding volumetric flow rate. And then the mean residence time for a plug flow reactor is given by tau and it is a same for the CSTR because there is no dispersion and. So, the mean residence time should be equal to the space time of the reactor itself and the variance for a plug flow reactor is 0 while for the CSTR it is actually equal to the square of space time of the reactor itself and then the f curve is actually is 1 for a plug flow reactor. And it is 1 minus exponential of minus t by tau for a CSTR.

So, that is summarizes the various properties of the RTD that summarizes the RTD function and the various properties of the function for the plug flow reactor and a CSTR.

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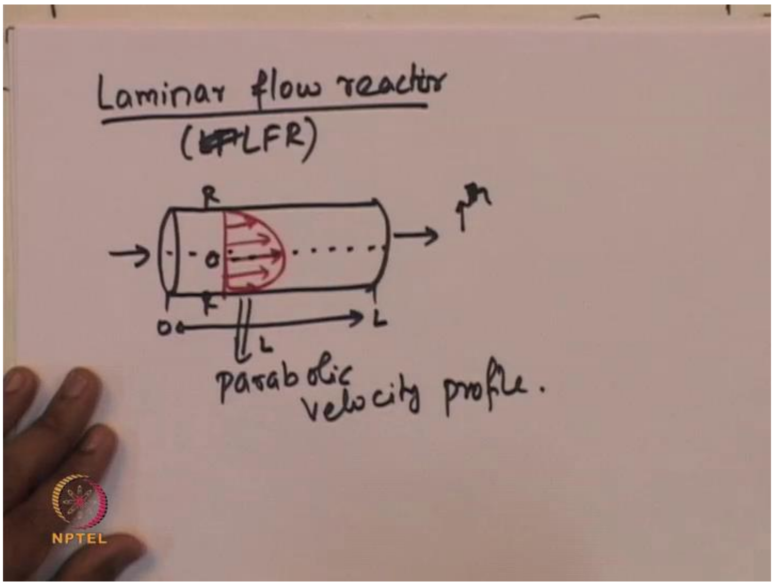

So, next let us look at the rather reactor tor laminar flow graph. Let us try to estimate the RTD function for the laminar flow reactor LFR will be referred to as LFR here after. So, suppose if there is a tank and this is the fluid stream which is actually entering at 0 and leaving at l. So, that is the length of the reactor that is that l. And the fluid is actually entering under laminar conditions and it is expected that there will be a parabolic velocity profile, there will be a parabolic velocity profile with maximum at the center and 0 near the walls maximum at the center and 0 near the walls.

So, suppose if the center of the reactor is. So, that is the center of the reactor and that is r equal to 0. So, if I label this coordinates as r this coordinates as r and at r equal to 0 it will be maximum velocity and at r equal to r, which is the periphery the velocity will be

0. So, that is a parabolic velocity profile that is a parabolic velocity with, which the velocity profile which is the fluid is actually flowing through the reactor we. Now, clearly this suggests that the fluid particles which are fluid elements, which are actually at the center they will actually have the shortest residence time because they have the maximum velocity. So, they will leave the reactor much faster than they; will leave the reactor faster than the other fluid elements which are actually present in other radial locations other than 0.

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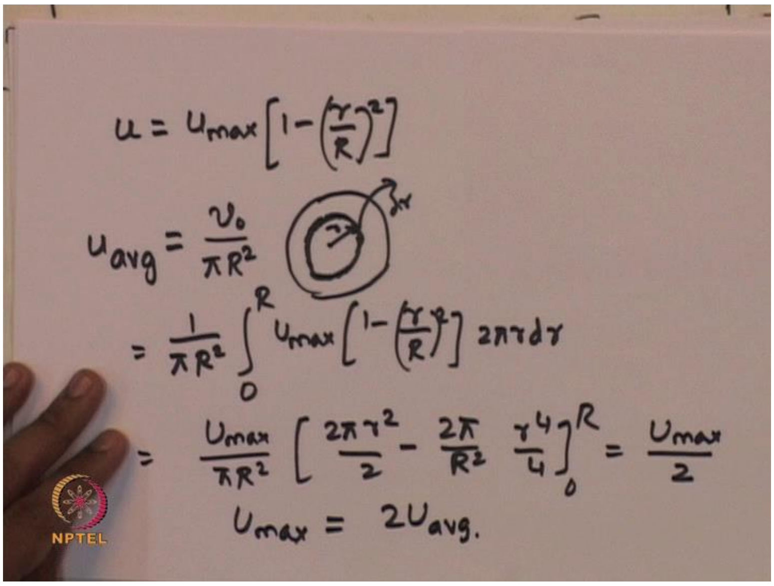

So, now so therefore, the velocity profile u is actually given by u max which is the maximum velocity at the center multiplied by 1 minus r by r the whole square now often this maximum velocity may not be known. So, instead what may be known is the average velocity that is the velocity of the fluid stream average across the whole cross section and that can actually be estimated from the velocity profile from the local velocity expression.

So, u average which is the average velocity at a given cross section is given by volumetric flow rate divided by the area of the reactor at that cross section and that is given by 1 by pi r square into integral 0 to r u max into 1 minus r by r the whole square into 2 pi r d r. So, here I have assume that if this is the cross section of the reactor if that is the cross section of the reactor then let us assume that there is a small element, which is present here from the center and that is located at the distance r and the thickness of the system this actually d r.

So, therefore, the volumetric flow rate of the fluid and in at any cross section is given by the local velocity multiplied by 2 pi r d r into integrated over the integrated between 0 and r. So, that is gives the volumetric flow rate at that cross section and pi r square is the corresponding area at that cross section. So, from this integrating this expression we will find that will be equal to u max by pi r square multiplied by 2 pi r square by 2 minus 2 pi by r square into r power 4 by 4. And the limits are 0 to r and that is equal to u max by 2.

So, the maximum velocity is simply twice the average velocity that is the averaged over the cross section of a of the reactor and. In fact, the average velocity is also called as the cup mixing average and. So, u max is equal to 2 times u average, so substituting this in the expression for the velocity.

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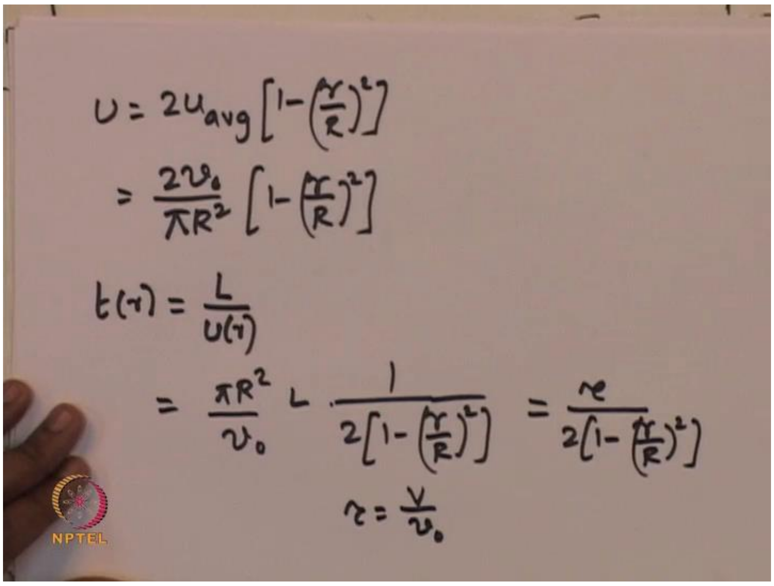

We can actually rewrite the velocity expression as u equal to 2 times u average multiplied by 1 minus r by r the whole square and that is equal to 2 v naught by pi r square v naught is the volumetric flow rate with, which the fluid is actually flowing at that cross section into 1 minus r by r square the whole square. So, that is the expression for the velocity with, which the fluid is actually flowing as function of the radial position

Now, we can now estimate what is the time that is actually spend by the fluid particles at a that is entering at a given location r. So, that is actually given by the length of the reactor l divided by the velocity with which the fluid is actually flowing in that radial location r which is actually u r and that is given by pi r square by v naught into l into 1 by 2 times 1 minus r by r the whole square. So, that is the time that is taken by different fluid elements that is actually entering the reactor at any r location ok.

So, that is actually equal to tau divided by 2 into 1 minus r by r the whole square where, tau is given tau is the space time of the reactor, which is given by v divided by v naught. So, now we need to relate the we need to now relate what is the is the we need find out what is the RTD function e t. So, in order to find that we need to know what the fraction of this fluid is that is leaving and what is the age of that particular fluid.

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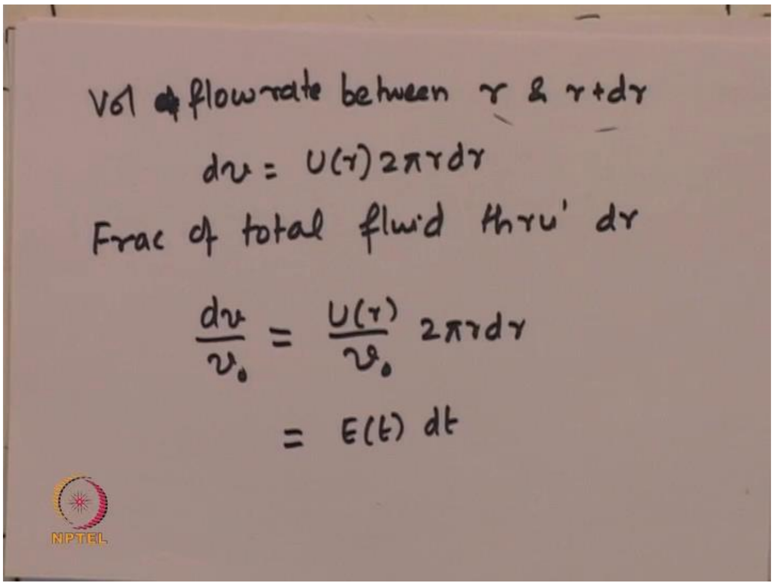

So, now the volume of the fluid; the volumetric flow rate between r and r plus d r. So, that is the volumetric flow rate of the fluid ,which is actually flowing between r and r plus d r and that is given by d v that is equal to u r into 2 pi r d r. So, that is the volumetric flow rate of the fluid, which is actually flowing in this element d r that is which means r and r plus d r.

So, now, the fraction of the total that actually that is actually flowing through this small element d r is actually given by d v divided by v naught where v naught is the total volumetric flow rate d v by v naught gives the fraction of the fluid that is actually flowing through this element d r. So, that is given by u r divided by v naught into 2 pi rd r.

So, that is the fraction of the fluid that is actually flowing through this element d r and. In fact, that is that is nothing but thee of t into d t because, the fraction of the fluid, that is actually flowing through the this small element d r and also the fluid which is actually between v and d v, which is spending the time t and t plus delta t is what is given by this RTD function e of t d t and that should be equal to d v by v naught, which is actually the fluid which is flowing between v and v plus d v whose residence time is actually between t and t plus delta t.

So, what we have seen. So, far in this lecture is an essentially different property of the residence time distribution which is the mean; we have looked at the variance. And we have looked at the skewness and then we went on moved on to the residence time distribution of the ideal reactors, but, particularly we considered the plug flow reactor and then we found what is the residence time distribution for the particular reactor and what are the properties of the residence time distribution.

In specifically, we found out what is the e curve and the f curve as related to the time as a function of time. And next we looked at the residence time distribution function for a single CSTR. We found the e curve and the f curve and the corresponding properties and then initiated discussion on the laminar flow reactor.

Thank you.