NPTEL

NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING

IIT BOMBAY

CDEEP
IIT BOMBAY

ADVANCE PROCESS CONTROL

Prof. Sachin Patwardhan

Department of Chemical Engineering,
IIT Bombay

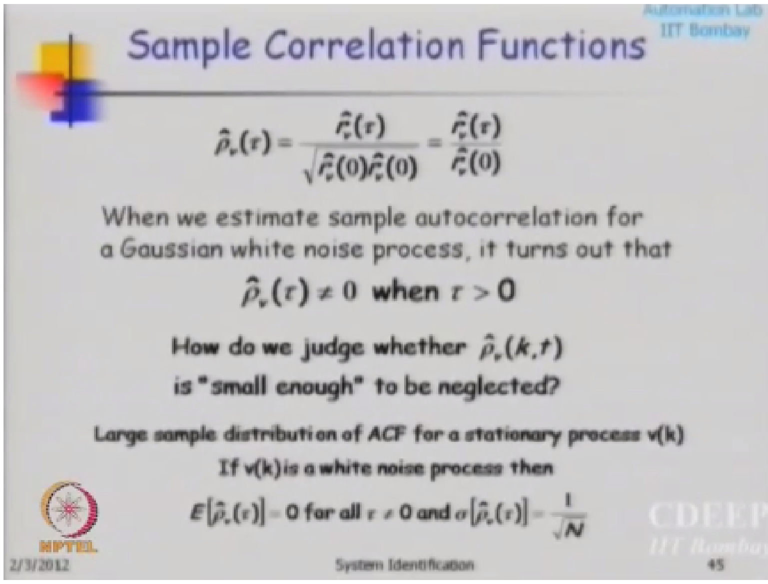
Lecture No. 08

Topic: Development of Control Relevant Linear Perturbation Models (Part 2)

Sub-topics:
Development of ARX models'

Okay, so let us take a very quick review at what we did last time. So we looked at this auto correlation function.

(Refer Slide Time: 00:24)

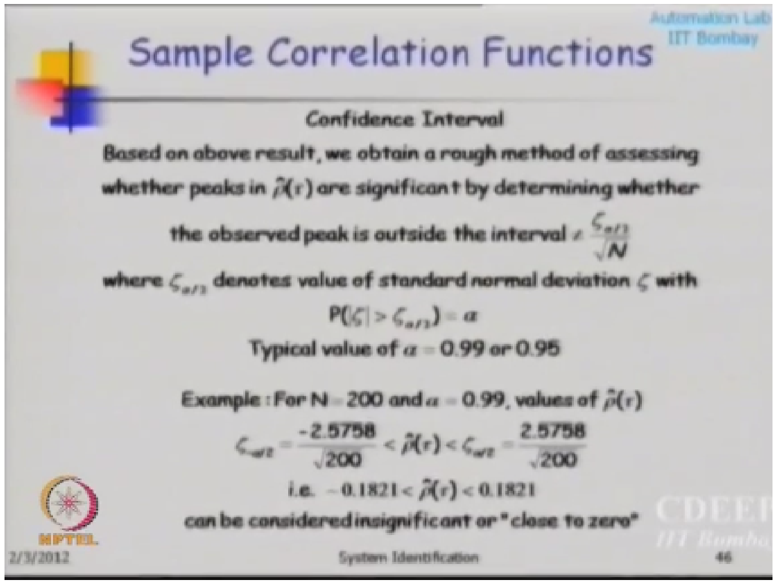


And in auto correlation we also found out how to find out auto correlation from sample auto c0orrelation function. And from sample auto correlation function you could estimate whether the random variables which are separated in time, in the same time series okay, in the same random process or they correlated. So this is a normalized as you can see it is a normalized function, you normalize it with covariance at 0, and so the maximum value of the auto correlation function is always 1.

You can never get, it is equal to 1 at lag 0 okay. And at all other lags typically it is less than 1, it can be negative, it can be positive, but maximum value cannot exceed 1 okay. Now the question that we ask was is this correlations small enough to may neglected, because even for a white noise process, when you take a specific realization and compute auto correlation, you will never get equal to 0.

So you have to take a call and something that is close to 0 and for that we found out confidence intervals. The idea for that was the weight was done was making another fact that the parameter estimate are themselves random variables. The estimates of auto correlation, because we are generating them for the samples okay. So estimates this ρhat vτ themselves are random variables. And you can show that their distribution is 0 mean with variance 1/√n for large samples. And the distribution is Gaussian, so from that we were able to take, come up with confidence interval.

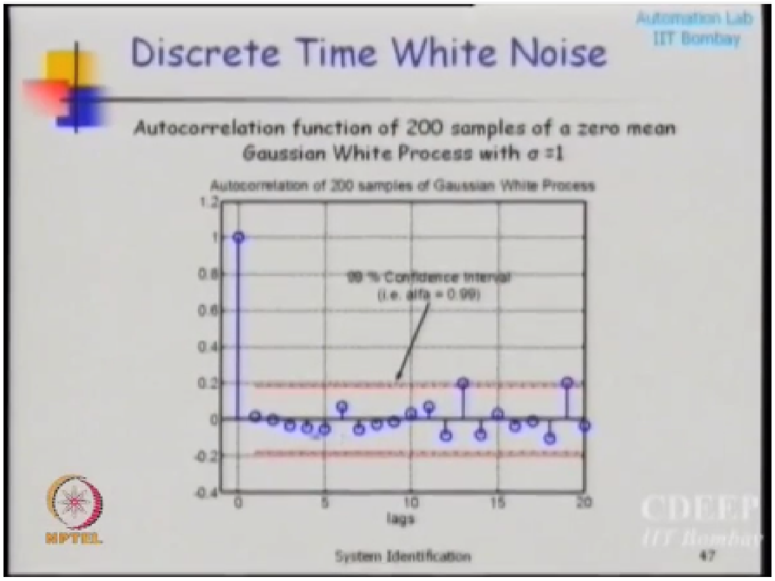
(Refer Slide Time: 02:22)



And now I am not going to explain the confidence interval, you have to go back and revise with this products some point in under graduate. So I have just shown here one calculation for 99% confidence interval, it will take 200 samples and then how will you get estimate of this confidence interval. So typically 99% confidence interval or 95% confidence interval is used. MATLAB, demonstrate the MATLAB program, it will typically give you 99% confidence interval.

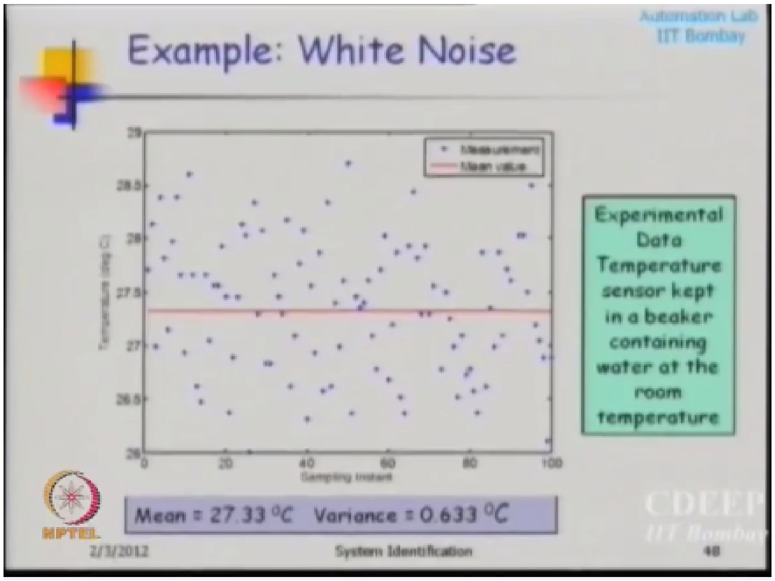
And I have shown here for a white noise, you know how do I compute this bounds, I have taken a white noise which is Gaussian, variance one in a mean 0, taken 200 samples in MATLAB and found out realization.

(Refer Slide Time: 03:24)



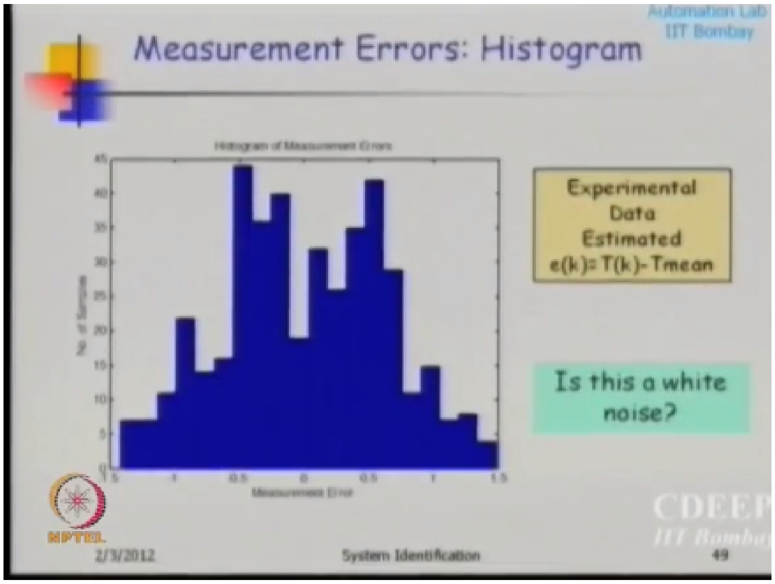
For that realization I want to see if there is auto correlation and auto correlation as we have seen here, it does not come out to be exactly equal to 0, it comes close to 0, and to take a call that this is close to 0, I have to use this confidence interval. Everything that is within the confidence interval is negligible, I do not have to – so this is white noise, why it is white noise? Because all these auto correlation values are small. Small in the sense that they are within the confidence bounds okay. So you can view this, it will say white noise.

(Refer Slide Time: 03:56)



Then I just showed you an example from my lab, this is experimental data. Sensor kept in a beaker for water at room temperature and then the data which I get is all over, you know nit is difficult to find out what is the true value, and you can appreciate that actual temperature in the beaker is not 0, it is constant okay.

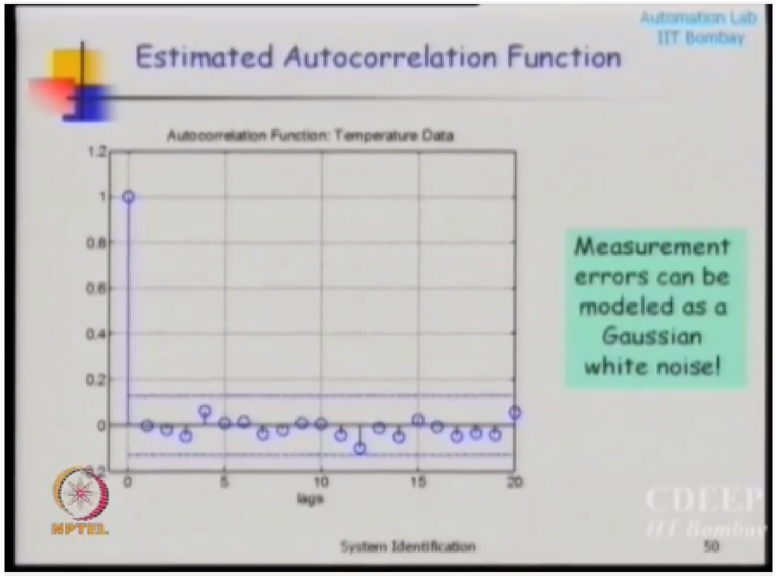
(Refer Slide Time: 04:18)



So this is just the histogram of errors, what I have done is I have taken mean value subtracted it from each observation, and I got the errors in the measurement. Suppose we take mean as the estimate okay, now you can estimate, you can appreciate why the estimate of the mean is a random variable. If I take some more samples, I will get a different mean, I will not get the same mean okay.

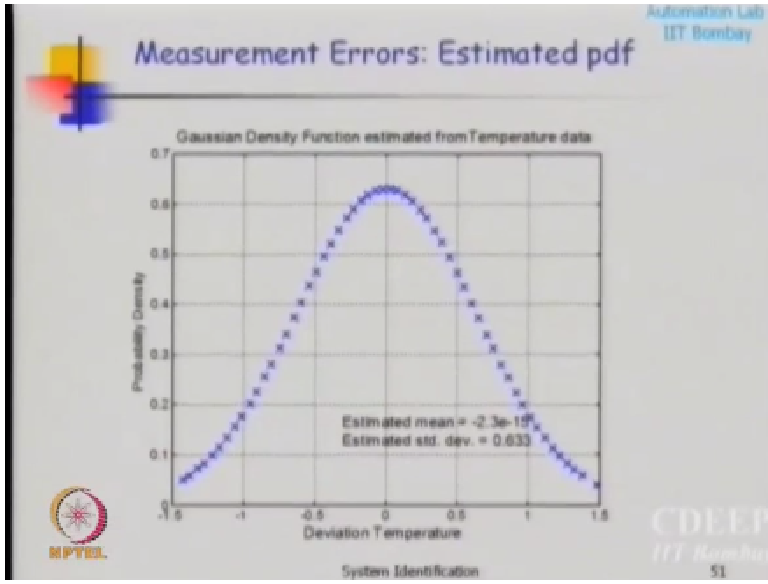
Or if I take, you know if I divide this into two parts 100 and 100 I think a 100 points. If I take first 50 as another mean, each one of them is an estimate and estimate only when you take all possible realizations, you will get the true value okay. Otherwise, what you get is an estimate okay. And then our estimate itself is a random variable that estimate has a distribution and then you can talk of confidence interval and so on okay. So now question is, is this measurement sequence is the noise error in the measurement, is it a white noise okay.

(Refer Slide Time: 05:24)



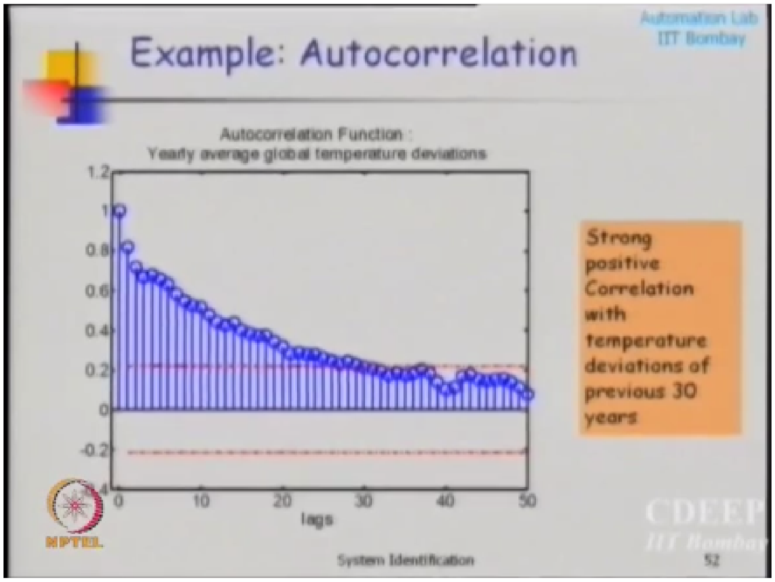
Now same thing I have done, I have found out confidence interval, and you can see that auto correlation function, you know all these values which you get for lag 1, lag 2, lag 3 up to 20 lags, it is within the confidence bounds. So this is practically a white noise okay. So if you take more and more values, this estimate will become sulphur, and sulphur will go closer and closer to 0. If I have to take 1000 samples or 2000 samples and will actually identically come to that, that why you should take more samples.

(Refer Slide Time: 06:01)



Anyway this is, I have just fitted a distribution to that data which I got.

(Refer Slide Time: 06:06)

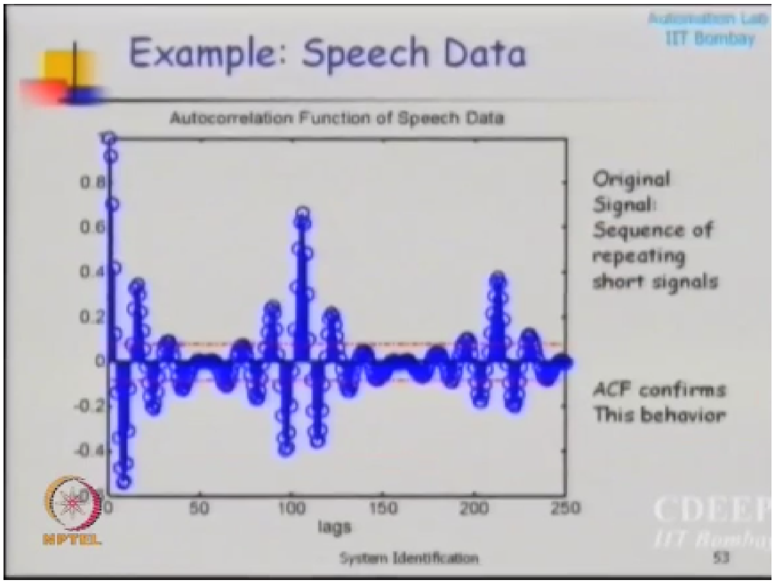


This is another data which I had shown you, the temperature, global temperature data which is here in the global temperature available data. I wanted to know whether there is a auto correlation in this theory which means what is happening now is it inherit to what has happened in the past. And this simple auto correlation estimation shows that almost there is a auto correlation for past 30 years.

What has happened now is, a cumulative function of what is happening over last 30 years, it is not just. So there is lot of memory into the system okay. Now how to uncover and get a model we will see, we will come to that, but at least we know that this is not a white noise okay. My bounds here are given here it is point, it comes out to be some 0.2 something. So it is far away from the white noise okay.

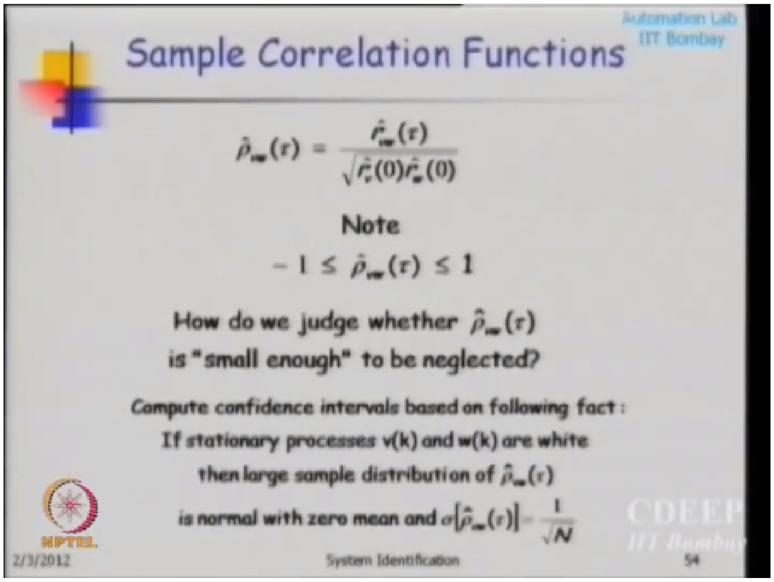
When it is not white we cause it as a colored noise, why we cause it colored I will come to that or why we call white noise as white noise.

(Refer Slide Time: 07:14)



This is the speech data okay, again you can see there is a correlation between what is happening at one time instant and what is happening with some lag, that is something in the past, lag means in the past okay. So the data is auto correlated, there is a relationship within the data okay. So the sound signal is not a white noise, if it was a white noise you will not make out any, it is a correlated signal that without even knowing signal processing you can appreciate that a white noise will be just what you probably hear when you switch on the radio you hear that carrr… noise it is probably sound realization of the white noise.

(Refer Slide Time: 08:06)



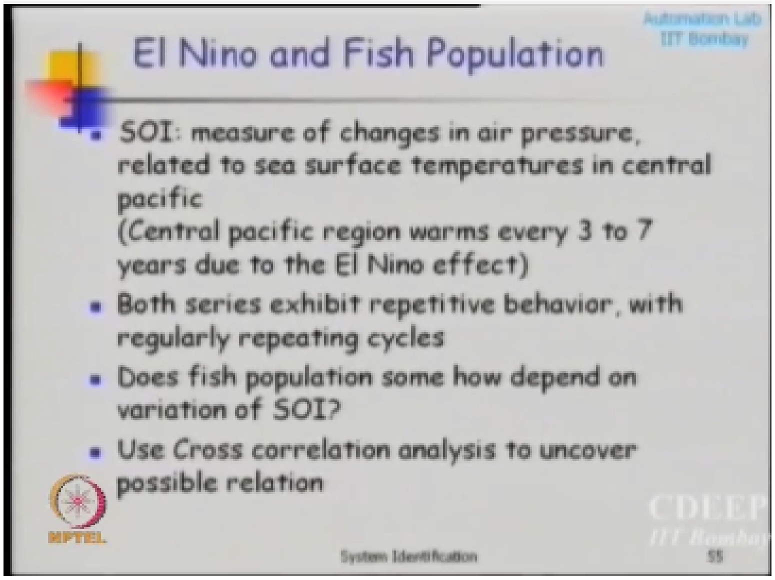
Now the next question that I want to ask is, if I have two different sequences are they correlated? Okay, if there are two different time series what is the way to do that? That is to cross correlation okay. So again I can compute sample cross correlation, sample cross correlation is using data, you have data for one series, you have data for the other series and then you want to compute cross correlation per.

Then we normally develop this normalize measure which is divided by variance of each one of them okay. So if you see here this is series B, this is series W okay, you normalize this cross correlation, cross covariance using these two similar variances that is normalization. Because it is easier to view the signal and again you can show that cross correlation can never exit plus or minus 1. So that is the fundamental result.

And I am not going to prove this, you will have to, I mean this is just a beginners introduction to this vast area. The question again you can ask is how do you judge whether cross correlation is small enough. See I have to make a call on whether two series are correlated each other or not, is this a cause and is this a effect, I want to know that. Now again you have to develop a confidence interval and same result holds that each of these correlation function is, you can view it as a random variable with distribution which is Gaussian, 0 mean and $1/\sqrt{n}$ for a white noise.

If in a white noise you will get, and so what you actually do is that you do not know what is it for the colored noise okay. You know what is it for the white noise. So you can actually find out whether this is the white noise or not okay. So by using this confidence interval, you can say that this is not a white noise okay, both of them are not white, there is some correlation.

(Refer Slide Time: 10:22)

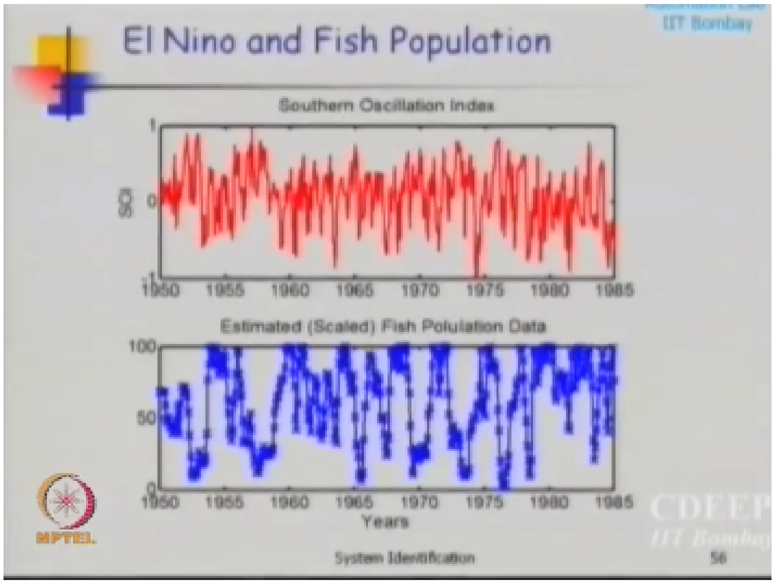


So again we, this is some data taken from the net, actually it comes, if you see this one reference I have given this Subway way and stopper at the end of the notes. Sumway and stopper actually given this data and on their book page of – so this is some index which actually is the measure of air pressure, changes in air pressure, related to sea surface, temperature in the central pacific region. Now you probably have heard this E L Nino effect that is central pacific region keeps forming because cycle 3 to 7 years.

And then that is blamed for many things you know certainly you see in newspaper that this year monsoon is not good for us this early is active or something like that so there is a also a data about fish population in central pacific region collected some government department and this is scaled data it does not tell you exactly numbers in between 0 and 100.

There is some bench mark taken which is 100 and so the question in forming of all the changes in the pressure and temperature in the pacific region does it have any correlation with the fish population okay so the way to do this is to find out cross coloration between these two series.

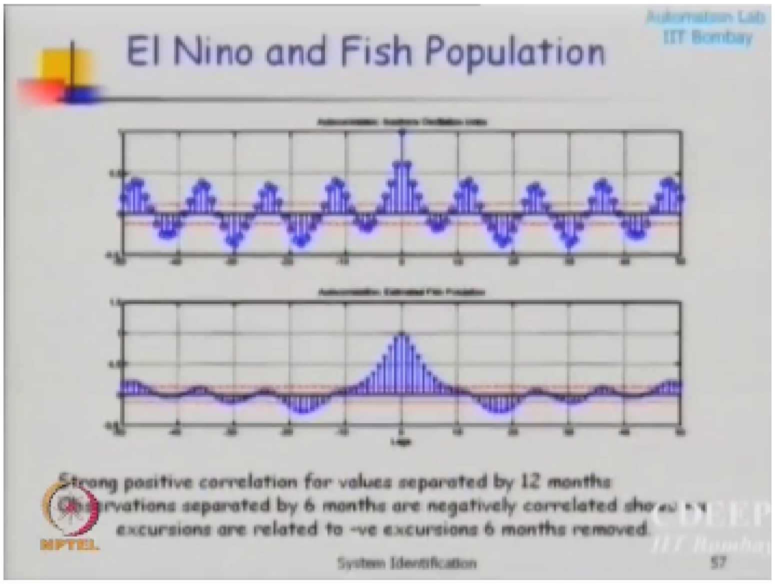
(Refer Slide Time: 11:59)



Okay now we have other 2 series this is first one is the southern oscillation index and other one is this is the data taken from 1950 to 1985 and 2nd one is also 1950 to 1985 this is monthly data okay fish population every month okay I do not know how they have done scaling this is scaled data but right now w2e can look at has a scaled data available to use the they may not go into publish the 2 numbers okay.

So typically many times data when it is made available it is made available as scale scaled data so 0 may not be in the which becomes 0 it might be it becomes low and then there is some high okay so now if I look at autocorrelation see this is period data and you want to see what is the repeating you know what is correlated in what in time within the data itself so first I am going to look at SOI and also I am going to look at the southern oscillation index and also the fish population data.

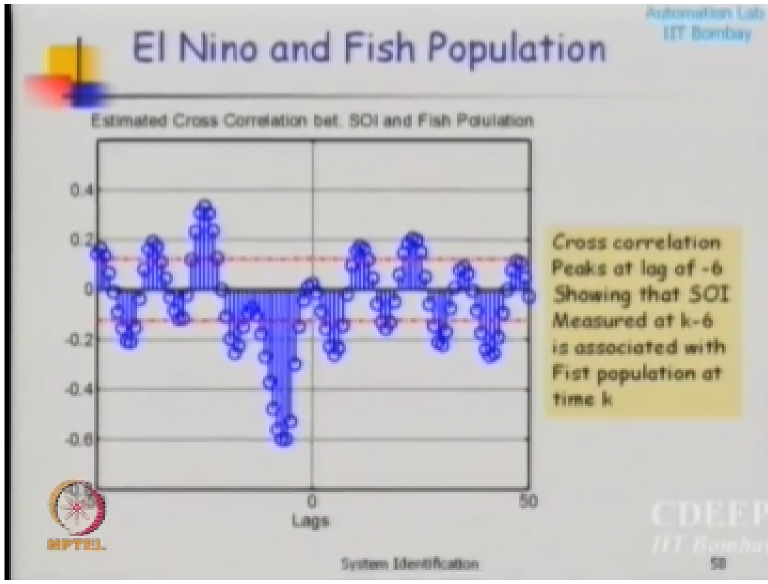
(Refer Slide Time: 13:07)



You can see that the first one shows a nice period behavior okay with a period of 12 so there is positive correlation with for time points which are 12 months apart and there is a negative correlation between time point which are 6 months apart so this is these 2 points are 6 months parts they are negatively correlated while these 2 points are positively correlate that 12 months apart okay so that is what so what is happening now as a correlation with what will happen after 12 months.

Okay or vice versa what is happening now is correlate to happened 12 months back okay so more detail models you have to use something else we will come to that this at least tell you that truth there is a auto correlation within the data okay this is not a with noise this is not complete kachada in the data it as some signal the same is true about the fish population data again it shows the periodicity. The periodicity is slightly different than this seems to be some time lag between the 2 okay.

(Refer Slide Time: 14:25)



So now if you take cross correlation the cross correlation the cross correlation peaks at every 6 months interval which shows that the index measure that time 6 months back is related to fish population now okay so if index is changing so you know this I can use to do some predications okay I can use this idea to or I can use to develop a prediction model for this the way to do it is use in transfer function models that we will learn later.

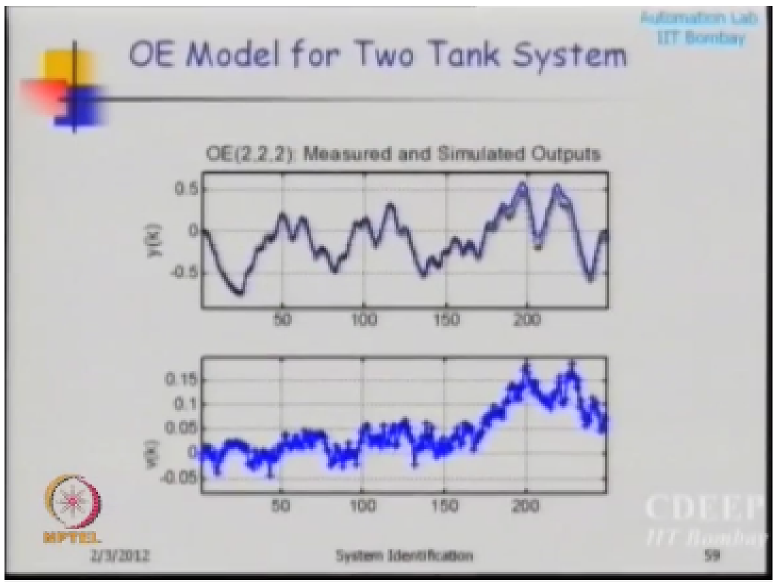
But right now we are just analyzing the 2 series is that I am saying that there is some correlation there is some correlation between the fish population and is index okay and it seems to be lacked by 6 months that is all we know from this correlation analysis right now okay what is it correlation how to predict okay.

What will happen see you are interested in knowing what is going to be the fish population after 6 months because in some industry might depend on it so how to develop prediction models will come to that little later okay but this at least tell you that there is prima facie case for building a model that relates index with the fish population okay if this thing had come everything within these red bounds.

Here everything had come within the red bounds then we do not know we cannot say whether there is correlation or not and then we cannot develop a model between these 2 variables okay see these are blood box models you know seemingly you know fish population and temperature are we do not know the physics that correlates this with the fish population we are not getting into the physics we are going to developed a black box model which just correlates you know SOI with fish population.

Okay and this model is these kind of models we keep with our control time okay a transfer function models that we are going to develop are as going to be this time okay we know that this is a cost and this is the effect what is the difference equation that relate with that question we will ask later.

(Refer Slide Time: 16:34)

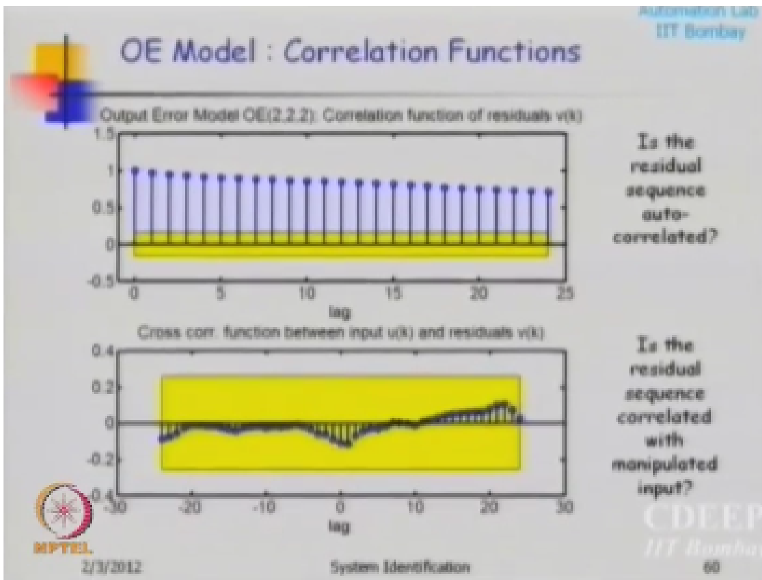


Okay let us get back to our data that we were looking at okay this was model for the 2 tank system which we developed output error model we developed and then we had this question why did we start all these business of scholastic process because we got this we got this series okay this was v(k) was everything that is not explained by inputs okay we got a model this blue is the model and +++ are data points.

And there is a difference and the difference is plotted here with this blue line okay and then we wanted to check whether there is any signal left there is something relevant left in this okay so what is to be done first check weather auto correlated if this auto correlated that is the first question I should ask okay the second question I can ask is that see this model which I have developed in between y and u suppose some effect of u is left in this some effect of u is left in the residuals okay.

Then the residual and u will be correlated my order I have chosen second order model second order could be wrong maybe I should go for 3rd order some effective is not getting captured and that signal is left into this I want to extract everything that is okay so I am going to do 2 things 1 is auto correlation for v(K) and across correlation between input u and v(k) okay.

(Refer Slide Time: 18:15)



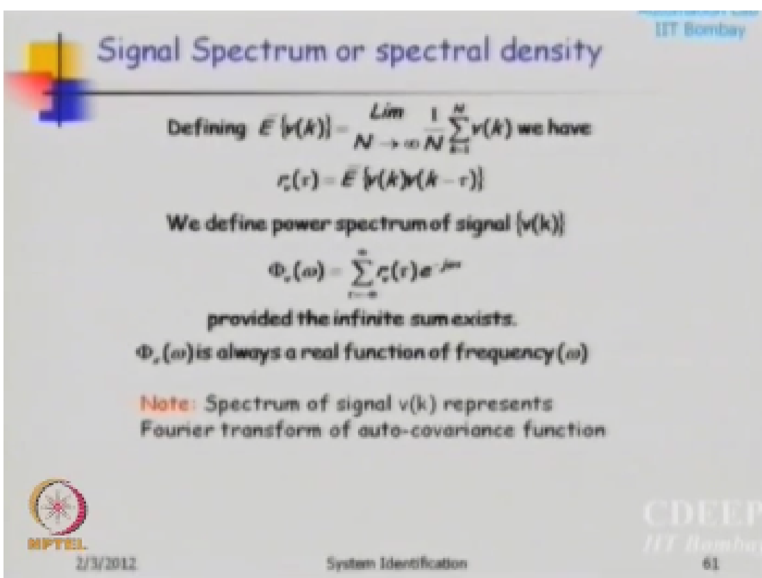
Everyone is clear about this okay so this what you get if you use a math lab as a function called auto correlation function and you can use that ACF at after I finish this set of + I will give you demo of the tool box I think butlery procure butlery as given you some demo but I will complete that task well all the tools that we are talking about are there in math lab or scie lab if you do not have math lab can use science lab in a public domain tool.

All the tools are available you should know how to use that that is the main thing you should know the fundamental is the theory the theory is finally complex you should try to develop an understanding otherwise you know you end up using it without a knowing what is inside and then there is a big problem so the question is if there is something this week A now you can see here clearly this signal is auto correlated.

A this a very strong auto correlation with past in fact even with last 20 samples it is not over it might be there for last many samples so there is auto correlation which is storing positive auto correlation cross correlation does not seemed to be there so 2nd order model seems to be okay I am not going to gain too much by going to 3rd order model okay.

I will come to an example where actually I will choose 2nd order model and it will not be sufficient you will see cross correlation between the residuals and you and then you will say my model order is wrong I will come to example in this case somehow it as worked out second order see we started by saying I wanted to develop up 2nd order model or 3rd order model or 4th order model. I will first develop the simplest model 2nd order model if it works why should, I go to the 3rd order okay.

(Refer Slide Time: 20:06)



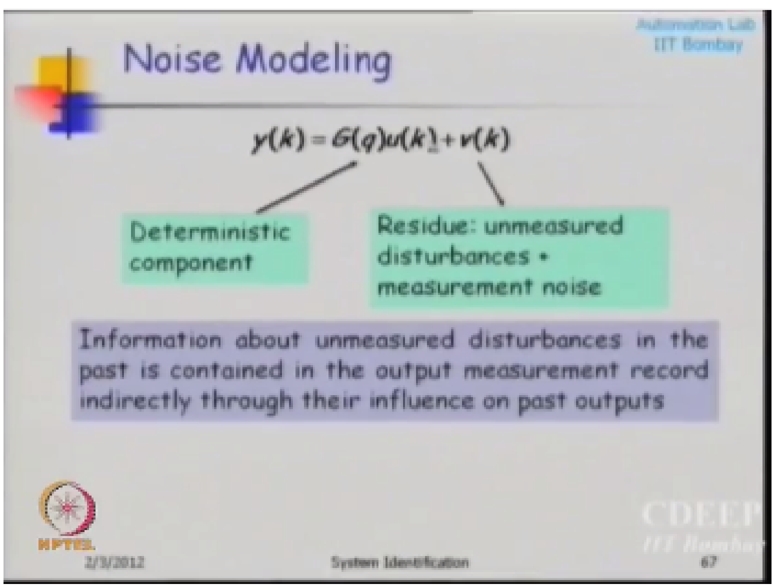
Well again I am going to give you a very sketch introduction to fairly complex idea of spectrum of a signal okay it is fine in this course if you just get some kind of a working knowledge and not deep understanding deeper understanding of this will take time a so well we are going to defined what is called a powers spectrum of the signal week A okay power spectrum is defined as a function of frequency ω.

Okay right now just aspect this okay and try to get working idea as to because if I give a signal to mat lab it will give you spectrum try to understand how to interpret the spectrum okay how it is a transform and then the way it work both of you how are electrical engineers and have worked with signal conditioning they would be comfortable with the spectrum idea so others who are from mechanical chemical might find it little difficult but doing time series analysis and modeling we cannot escape this idea of spectrum.

So we defined power spectrum of a signal as see look here this is infinite sum going from τ-infinity to infinity or v τ are nothing but auto correlations okay with the signal and this is e^{-j} ωt okay so this s actually a Fourier transform it actually represents a Fourier transform or the auto covariance function okay that is the interpretation of this it helps you to transform view the signal in terms of the you know signal power as a function of frequency we are actually transforming from time domain to frequency domain.

So you start suing the words and slowly the meaning will percolate okay, so now after this abstract background which is needed language which I need now I am going to get in to the practical problem, so now I want to develop a model we started with this right.

(Refer Slide Time: 33:14)



Y = gq x uk + bk you develop an output error model with never attended to model vk now I want to model vk suing this ideal of stochastic processes auto correlation cross co relation process okay. So this is going to my deterministic component okay deterministic in a sense u is known to me okay everything that is known to known to me will be you know in this elusory okay. So this residue will contain two things on measure disturbances measurement noise in fact it is also have approximation errors, because the real process will not be linear you are developing a linear model so this is combination of everything that is not known not known to you.

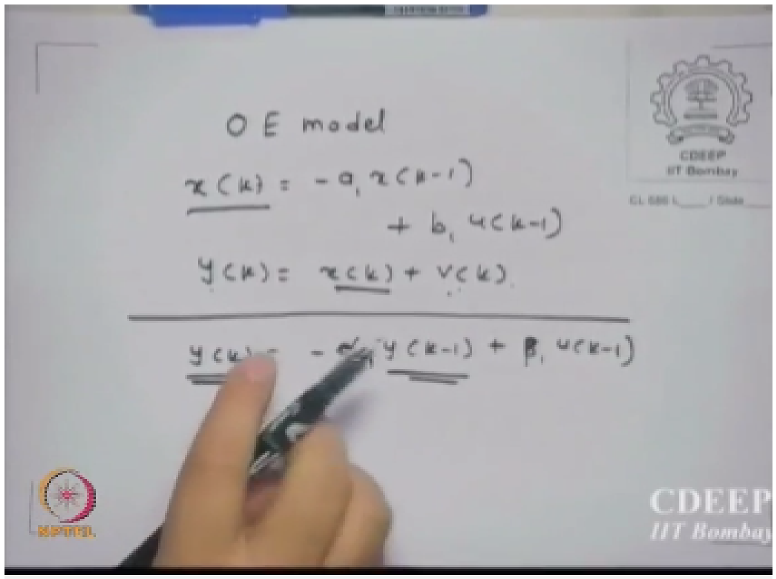
Now information even though you have not measuring a disturbance it is effective present in y right is effect is present in y and if you develop this kind of model it is effective present in b we saw that vk is out of correlated, so now there is hope to uncover a model from this vk okay. so what is an obvious type of structure okay and obvious types of structure is yk is some function of pass u and also past y because where is the information about disturbances hidden it is there in y itself right effect of disturbances is present in y.

So I want to use that information develop a model uncover it and then you leads for control okay, now I have this little term coming here ek okay, I am going to tell about this model f is some function initially I take the linear function and in this course we are going to stick to linear functions non linear functions would be part of an advance course not really in this course when will I know my model is correct I will stop only when this ek is a white noise why white noise? In the white noise there is no auto correlation white noise is like compute kachara okay it is complete dirt you can thorough it, there is nothing left no signal left in white noise okay.

So I develop this model till ek becomes a white noise okay I will do this for a particular case and I am show you that you know you have to go on increasing the model order till ek becomes white noise okay we will see one this specific case I am going to go back to the same data I am going to call this okay I can propose a linear model which is of this form can you everyone see this model?

It says that yk is function of a 1 yk – 1 linear difference equation model simple linear difference equation model okay, this is difference from output error model why it is different? In output error model you had here xk xk – 1we said that yk is escape with the ek here I am directly using yk, I am saying current measurement okay output error model we had two things we had xk and let me just go back here and remind you okay.

(Refer Slide Time: 36:54)

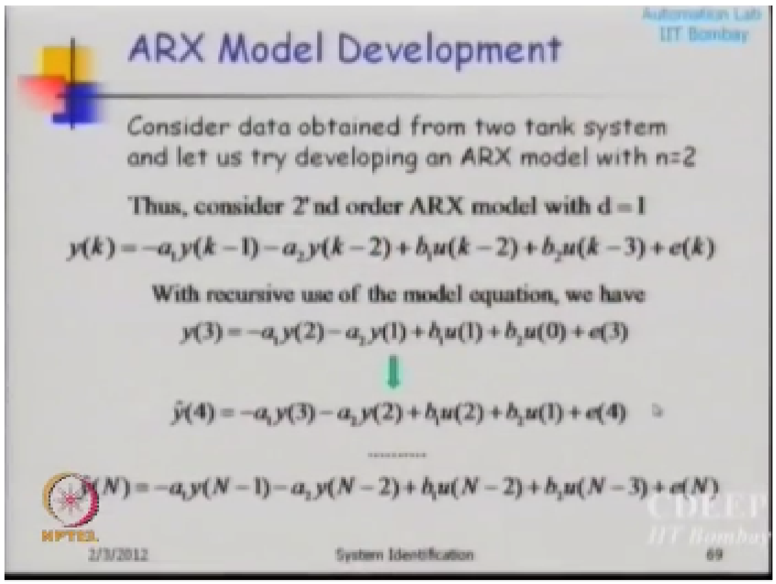


See my model output error model was x k = -a1 x k – 1 + b1 uk – 1 and yk = xk + vk now this model is fundamentally different from writing yk as – a1 or let call it α 1 y k – 1 + v 1or β 1 u k – 1 these two models are fundamentally different because in one case you are working with x which is effect of u alone okay and in this case we are working with y directly y is measured okay x can never be measured okay x is a combine effect of vk and sorry y is a combined effect of xk and vk okay.

Y can be measured so this model is fundamentally different they look similar by there is a big big difference okay, now how many you know here you see that I am taking y k -1 y k – 2 y k – m3 how many such passed values I should take yeah. No, so I want to develop this model in such a way that efk will become white noise I do not know what order chose, no but it depends upon how do you chose the model order it is not obvious that obvious becomes white noise I will give you an example.

So how many pass outputs I should include? Okay now here because your modeling noise together with deterministic component you do not know how many passed y's are important what is the auto correlation we did not know right now, okay so how much how many pass we should include we will chose model order in such a way that ek becomes a white noise okay, so now I am show you an exercise in which I can start with model order 2, 3, 4, 5, 6, you see that till you go to 6 model order you did not get white noise okay that is because now you are trying to capture deterministic dynamics together with stochastic dynamics both are capture together.

(Refer Slide Time: 39:57)



How do you develop this model given the data this model development is very easy you can even write a simple program in mat lap to do this have you done linear d2 well we have done in our course and numerical methods but others formally we have done v2 methods at some point if you not done I am not going to repeat it I going to do it here in the notes, so hope you have taken printout in the new notes because I have done lot of real element.

Let us look at a second order model okay what it is dare with me I have data of y with me or data of u with me okay so I have collected y data and u data perturbation data let us see how to develop a second order model okay, now for this time system we know that either unit time delay so that I have included here so my model becomes yk is a1 x yk – 1 previous value the measurement previous two values previous and then uk – 2 and uk -3 that is because of 1 unit time delay in addition to the basic time delay.

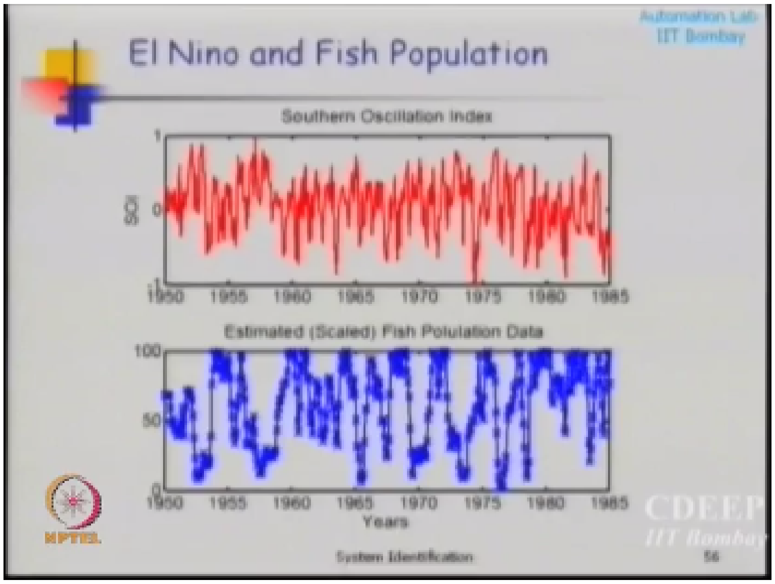
So this is my model, second order model I want to estimate a1a2 b1 b2 from data I have data for y and u okay great thing about this model is that y is known u is known estimating a 1 a2 b1 b2 is going to be very easy how many parameters are there four parameters okay four are known how many equations you need? Four equations how many unknowns are there in this? Four unknowns really, or five unknowns which is the fifth unknown? Ek you did not know what is the ek so there is a trouble you cannot just use four equations okay.

So now let us start writing the equation my first equation will be y3 I have started data from I have data which is index from u0 u1 u2 u3 y0 y1 y2 y3 okay so my first equation that is can write is for time 3 you agree with this, okay because my data in u it is start with u0 I do not have data for u – 1 there is some 0 point so my first equation is going to be this okay there are unknowns a1 a2 b1 b2 e3 these are five unknowns okay.

My next equation is y4 okay this as a1 a2 b1 bb2 but e4 as prompt out okay and likewise if I go on writing this equations you know how many equations I will get I have n capital n data points I will get sorry I have capital n + 1 data points because 0 is consider so will get n – 3 equations you will get how many unknowns are there all these are unknowns so how many unknowns are there n – 3 + 4 there are n- 3 equations and there are n – 3 + 4 unknowns, so number of unknowns is much more or is four unknowns are more than a number of equations okay so you have to do some trick to come up with the solution okay.

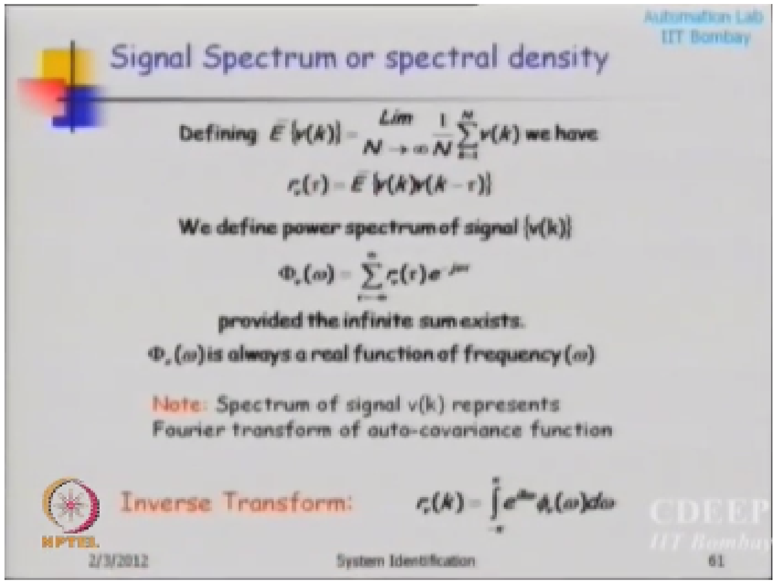
Those are estimate no, because no what you say is right, I think there is a typo here moment I put in e this should not be y hat this should be y4 I agree there is a typo here this should be y4 and it should be sorry it should be y4 not y hat 4 if I remove e it will be y hat. Why we do Maxwell slot and you know because we can give a different perspective of the signals or the system in frequency domain and you can do some analysis in the frequency domain, okay. So here given a signal okay, what is the see.

(Refer Slide Time: 22:18)



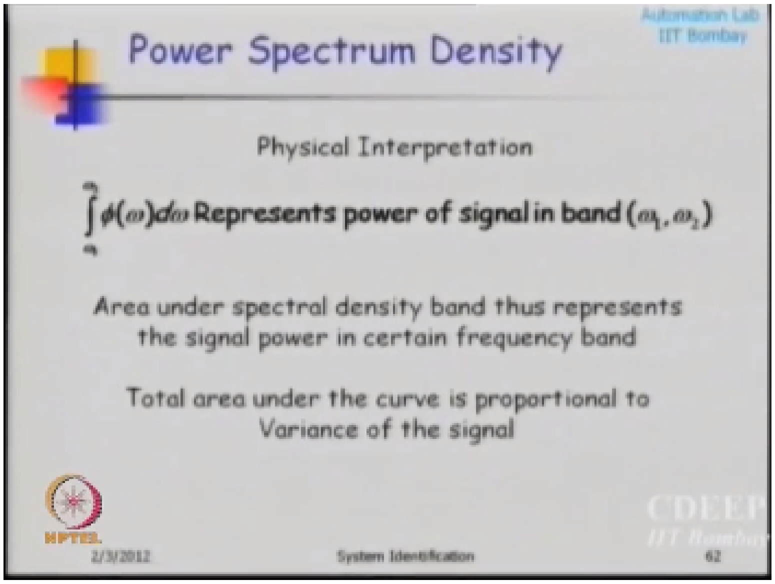
We saw the signal here this signal you know I would like to know at different frequencies what is the power of the, what is the you know what is the power of the signal at different frequencies okay, what is the is this does this make sense power of the signal and you can frequencies, frequency as in the way you define frequency per month not per second it will be per month yeah.

(Refer Slide Time: 23:04)



So I can take a transform and then you can define the inverse transform right now you know let us keep the transform understanding at this one slide I am move on to interpreting the transform okay, if I show you transform of a certain signals then you will get some idea.

(Refer Slide Time: 23:24)

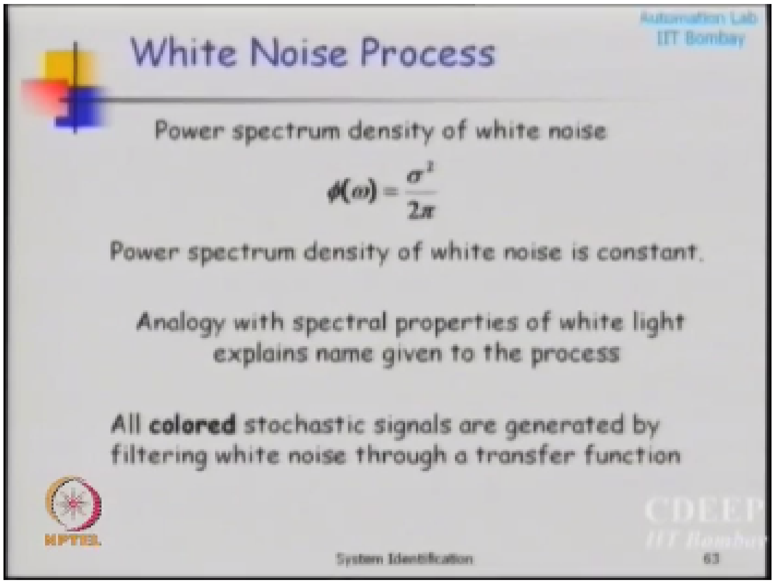


So the physical interpretation is analog I can give you is to probably density a function, what happens in probability density function, how do you find out probability between two points you are given a density you taken integral between those two points and you will get probability of event occurring between those two points, right the same thing is here. if you are given power spectrum density you can take an integral between band ω1 and ω2, you can find out power of the signal in that particular.

See if you give me a raw signal I am not able to find out, I am not able to analyze the frequency contained of that signal, what range of frequencies exist right this is of interest from a you know modular point of view that what is the range of frequency in which. The area under the spectral density band represents the signal power in the certain frequency band and total area is proportional to the variance of the signal, okay.

This cured interpretation right now the simple interpretation right now is what you try to keep in mind and we move on from this.

(Refer Slide Time: 24:32)

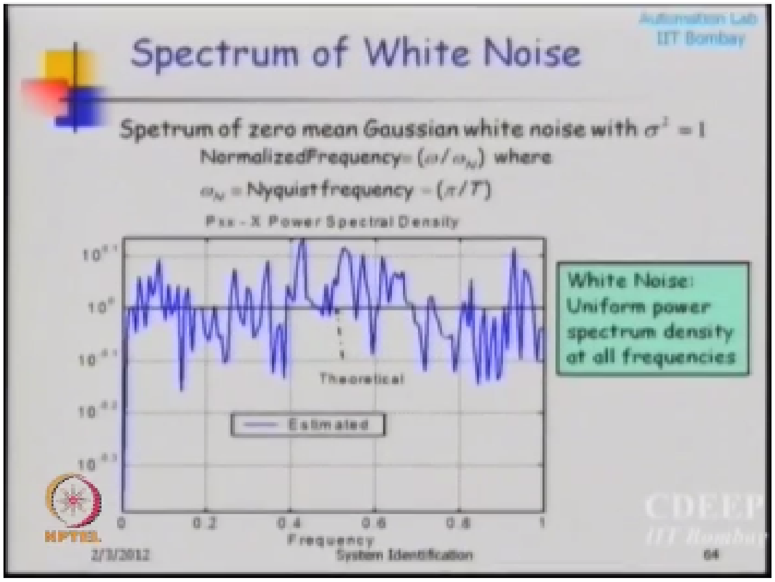


So let us look at the white noise spectrum okay, what will be the white noise spectrum see what is the variance of white noise, what is the auto correlation function of white noise it is σ^2 at Lange 0 and it is 0 at all other lags okay. So power spectrum of white noise it turns out to be just σ2/2π okay, because it is 0 at all other lags it is 0 only at 0 it is non zero only at 0 lag it is, so you are sending from -∞ to ∞ you will get only this particular term, okay.

So actually power spectrum of white noise is constant okay, so that is the reason why we call it has all frequencies white noise is a signal which has all the frequencies present, okay. When you the analogies with the white light see white light with all the frequencies present, what is there in the colored light, if you knock off some frequencies okay you will get colored light, okay. so how do you a colored stochastic signal you take a white noise pass it through a filter what does, what do you do in when you create say a green light from a white light, what you do?

You have a filter okay, you have a filter you pass white light through the filter you get a green light on the other side okay, here you take white noise signal pass it through a difference equation which will act as a filter you know that a difference equation or a differential equation can be viewed as a low pass filter, high pass filter you have seen this when you studied your first course in control, right when you draw like blow a plot right, you know that this you know a four sorted α function will be like a low pass filter okay, and then PID controller is like all pass filter and depending up on how you choose D. So you have you know the same way here.

(Refer Slide Time: 26:55)

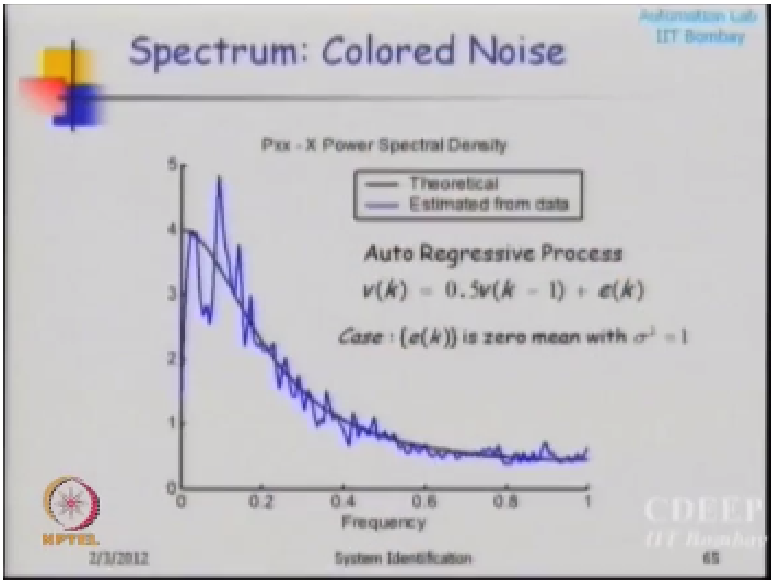


This is the I am plotting here the spectrum of white noise okay, spectrum of white noise can be estimated from data if you give data to medullar and ask to compute spectrum, it will compute spectrum and plotted for you, you can see that white noise ideal spectrum should be this okay, I have taken a white noise with variance 1 okay, so the ideal spectrum should be this is polynomial log, log plot okay.

So it is showing you 0 here and estimate is hovering around 1 see this blue line is the estimate of the spectrum okay, it is hovering around 1 okay, so the realization which I got is almost like a white noise. Because we have all the frequencies it has power at all the frequencies see this is amplitude, this is power and this is frequency. Frequency is between 0 to 1 okay, and you will wonder what is this one in discrete time systems we always plot between 0 to 1 where 1 is the normalize frequency okay, I need to normalize with respect to this ωn, ωn is called as a Nuquist frequency and this is defined as π/t okay, t here is the sampling period okay.
So here this 1 means frequency 2 π/ω/ π/t okay, it is normalized with these reference to it is ω/ωn what is ωn, π/t okay, so the spectrum if you ask mat-lap to plot spectrum it will always give you this normalize frequency spectrum okay, and it tells you what is the power at different frequencies. If I show you a colored noise signal then you will realize that what is the difference, is just keep this figure in mind okay.

No, the in digital control these spectrum will repeat okay, since it repeats we only plot the first normalization comes I think in an integral let me go back and check.

(Refer Slide Time: 29:26)

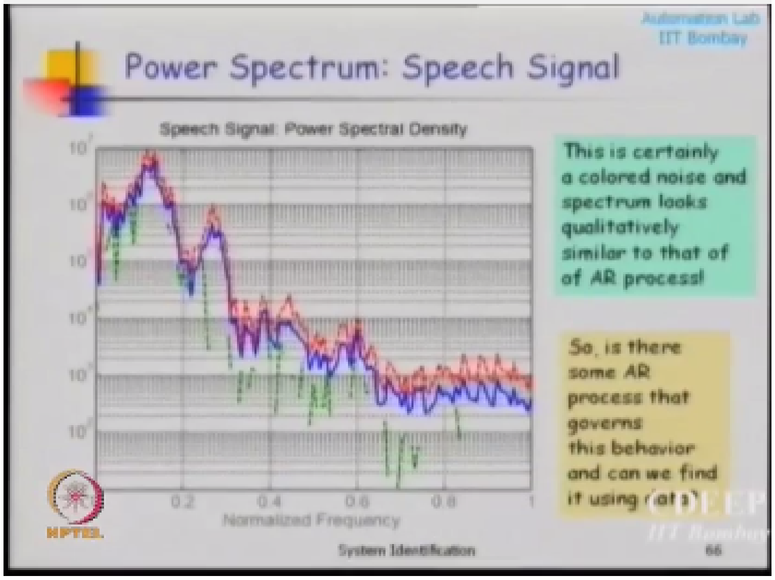


Now this is power spectrum of auto regressive process I have created an auto regressive process vk=0.5, vk-1 and ek is the white noise okay, what is our white noise I took here same white noise I have used here it is 0 mean white noise with σ^2=1, okay and this is the auto regressive process this is like the first order filter, first order difference equation it is a low pass filter it only allows go frequencies to pass through and higher frequencies are the signal that higher frequency is cut off, okay.

You can see here and the power spectrum that the spectrum is high at low frequencies spectrum becomes small at high frequencies okay, the spectrum becomes small at high frequencies so this is how a colored noise will look as against the white noise, white noise has all the frequencies okay, has almost equal power at all the frequencies where as the colored noise has high power at same frequencies low power at some frequencies.

So analogy is with these terminology I have to introduce because we keep using this okay, we are not again as I said in this course we are not going to by do hand calculation of a spectrum but you know this is a difficult idea and then we have to at least have some idea of what is the spectrum, okay. Well I am trying to compress a huge course into a few lectures and that is why.

(Refer Slide Time: 31:03)



I have to go little fast well power spectrum you know you can think about finding of a power spectrum for the sped signal at which frequencies there is more power which frequencies there low power okay, maybe when you are transmitting the signal you can decide to transmit only that part which is at you know which has significant power you can knock of the part which is low power and then you will get almost the signal that you know that you here, so that is an important aspect in signal processing.

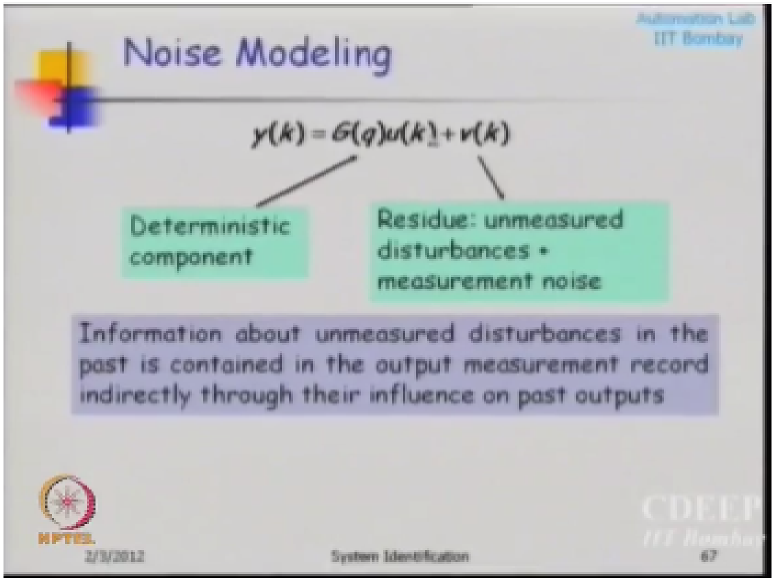
And so right now this get this correlative understanding of white noise and colored noise that is enough we will move to now modeling.

So this brings to end the prelude or back ground material which I want to do teach on stochastic processes okay, it is not possible that those two three of lectures it will give you understanding of this area. For me personally it has taken years to understand what is thing means okay, so but then you know you can start using this terms you can start using mat-lap programs keeping these slides at in the back ground slowly we learn.

We should not too much worry about you know in the beginning you do not get full meaning of what it is you should have attitude of a child where the child also language without being scared of then use some seven year child will come and say what is your responsibility he does not understand what is the meaning of responsibility, but he will use the word anyway so you start using the words and slowly the meaning will percolated, okay.

So now after this abstract background which is needed language which I need now I am going to get in to the practical problem, so now I want to develop a model we started with this right.

(Refer Slide Time: 33:14)



Y = gq x uk + bk you develop an output error model with never attended to model vk now I want to model vk suing this ideal of stochastic processes auto correlation cross co relation process okay. So this is going to my deterministic component okay deterministic in a sense u is known to me okay everything that is known to known to me will be you know in this elusory okay. So this residue will contain two things on measure disturbances measurement noise in fact it is also have approximation errors, because the real process will not be linear you are developing a linear model so this is combination of everything that is not known not known to you.

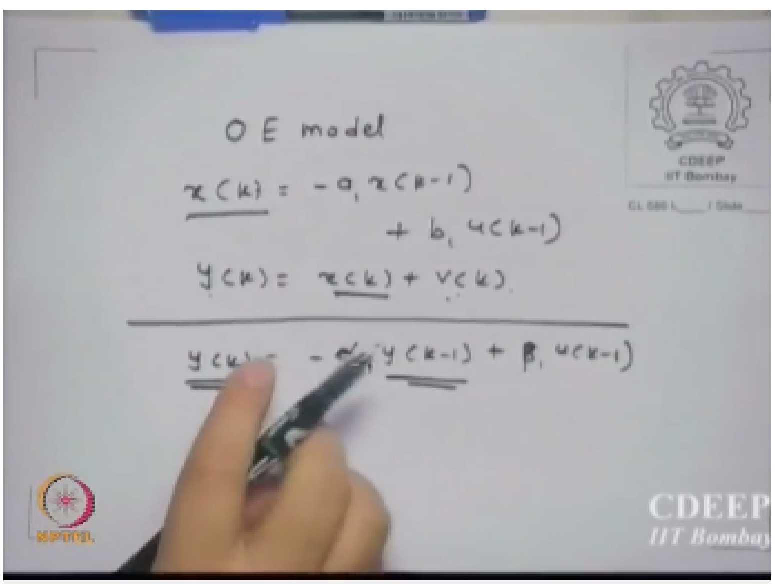
Now information even though you have not measuring a disturbance it is effective present in y right is effect is present in y and if you develop this kind of model it is effective present in b we saw that vk is out of correlated, so now there is hope to uncover a model from this vk okay. so what is an obvious type of structure okay and obvious types of structure is yk is some function of pass u and also past y because where is the information about disturbances hidden it is there in y itself right effect of disturbances is present in y.

So I want to use that information develop a model uncover it and then you leads for control okay, now I have this little term coming here ek okay, I am going to tell about this model f is some function initially I take the linear function and in this course we are going to stick to linear functions non linear functions would be part of an advance course not really in this course when will I know my model is correct I will stop only when this ek is a white noise why white noise? In the white noise there is no auto correlation white noise is like compute kachara okay it is complete dirt you can thorough it, there is nothing left no signal left in white noise okay.

So I develop this model till ek becomes a white noise okay I will do this for a particular case and I am show you that you know you have to go on increasing the model order till ek becomes white noise okay we will see one this specific case I am going to go back to the same data I am going to call this okay I can propose a linear model which is of this form can you everyone see this model?

It says that yk is function of a 1 yk – 1 linear difference equation model simple linear difference equation model okay, this is difference from output error model why it is different? In output error model you had here xk xk – 1we said that yk is escape with the ek here I am directly using yk, I am saying current measurement okay output error model we had two things we had xk and let me just go back here and remind you okay.

(Refer Slide Time: 36:54)

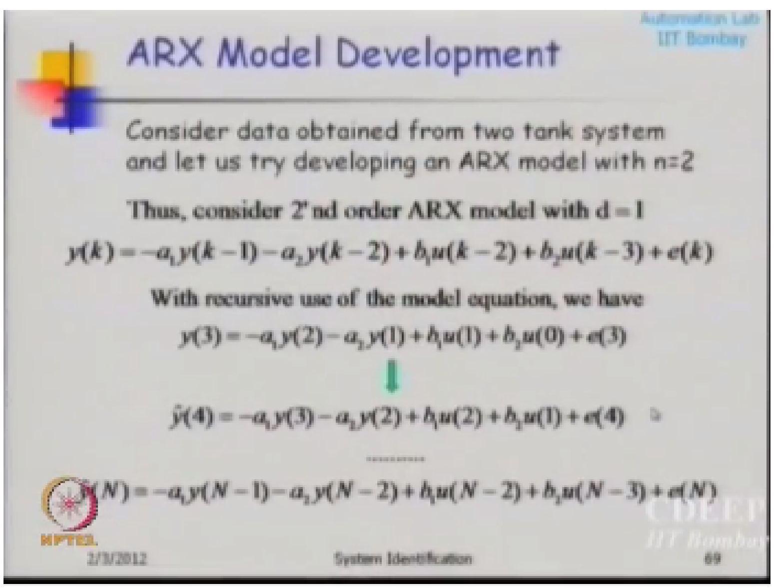


See my model output error model was x k = -a1 x k – 1 + b1 uk – 1 and yk = xk + vk now this model is fundamentally different from writing yk as – a1 or let call it α 1 y k – 1 + v 1or β 1 u k – 1 these two models are fundamentally different because in one case you are working with x which is effect of u alone okay and in this case we are working with y directly y is measured okay x can never be measured okay x is a combine effect of vk and sorry y is a combined effect of xk and vk okay.

Y can be measured so this model is fundamentally different they look similar by there is a big big difference okay, now how many you know here you see that I am taking y k -1 y k – 2 y k – m3 how many such passed values I should take yeah. No, so I want to develop this model in such a way that efk will become white noise I do not know what order chose, no but it depends upon how do you chose the model order it is not obvious that obvious becomes white noise I will give you an example.

So how many pass outputs I should include? Okay now here because your modeling noise together with deterministic component you do not know how many passed y's are important what is the auto correlation we did not know right now, okay so how much how many pass we should include we will chose model order in such a way that ek becomes a white noise okay, so now I am show you an exercise in which I can start with model order 2, 3, 4, 5, 6, you see that till you go to 6 model order you did not get white noise okay that is because now you are trying to capture deterministic dynamics together with stochastic dynamics both are capture together.

(Refer Slide Time: 39:57)



How do you develop this model given the data this model development is very easy you can even write a simple program in mat lap to do this have you done linear d2 well we have done in our course and numerical methods but others formally we have done v2 methods at some point if you not done I am not going to repeat it I going to do it here in the notes, so hope you have taken printout in the new notes because I have done lot of real element.

Let us look at a second order model okay what it is dare with me I have data of y with me or data of u with me okay so I have collected y data and u data perturbation data let us see how to develop a second order model okay, now for this time system we know that either unit time delay so that I have included here so my model becomes yk is a1 x yk – 1 previous value the measurement previous two values previous and then uk – 2 and uk -3 that is because of 1 unit time delay in addition to the basic time delay.

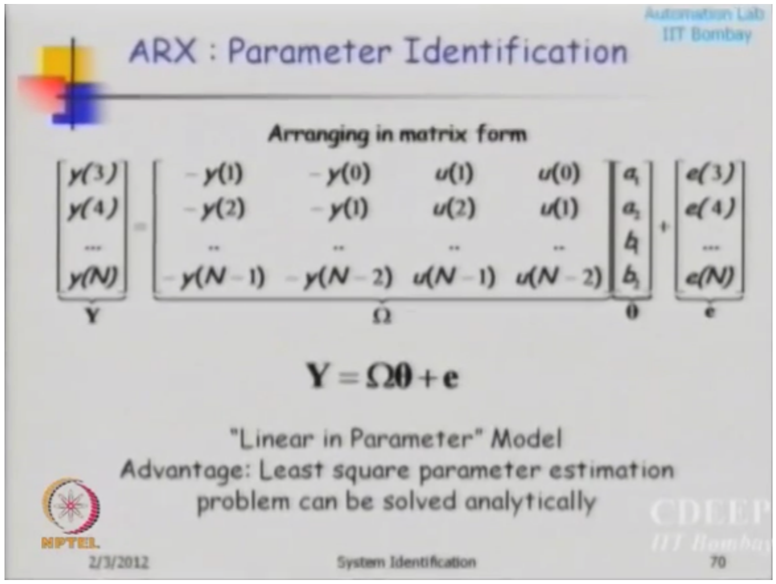
So this is my model, second order model I want to estimate a1a2 b1 b2 from data I have data for y and u okay great thing about this model is that y is known u is known estimating a 1 a2 b1 b2 is going to be very easy how many parameters are there four parameters okay four are known how many equations you need? Four equations how many unknowns are there in this? Four unknowns really, or five unknowns which is the fifth unknown? Ek you did not know what is the ek so there is a trouble you cannot just use four equations okay.

So now let us start writing the equation my first equation will be y3 I have started data from I have data which is index from u0 u1 u2 u3 y0 y1 y2 y3 okay so my first equation that is can write is for time 3 you agree with this, okay because my data in u it is start with u0 I do not have data for u – 1 there is some 0 point so my first equation is going to be this okay there are unknowns a1 a2 b1 b2 e3 these are five unknowns okay.

My next equation is y4 okay this as a1 a2 b1 bb2 but e4 as prompt out okay and likewise if I go on writing this equations you know how many equations I will get I have n capital n data points I will get sorry I have capital n + 1 data points because 0 is consider so will get n – 3 equations you will get how many unknowns are there all these are unknowns so how many unknowns are there n – 3 + 4 there are n- 3 equations and there are n – 3 + 4 unknowns, so number of unknowns is much more or is four unknowns are more than a number of equations okay so you have to do some trick to come up with the solution okay.

Those are estimate no, because no what you say is right, I think there is a typo here moment I put in e this should not be y hat this should be y4 I agree there is a typo here this should be y4 and it should be sorry it should be y4 not y hat 4 if I remove e it will be transfer the clutching the error.

(Refer Slide Time: 44:10)



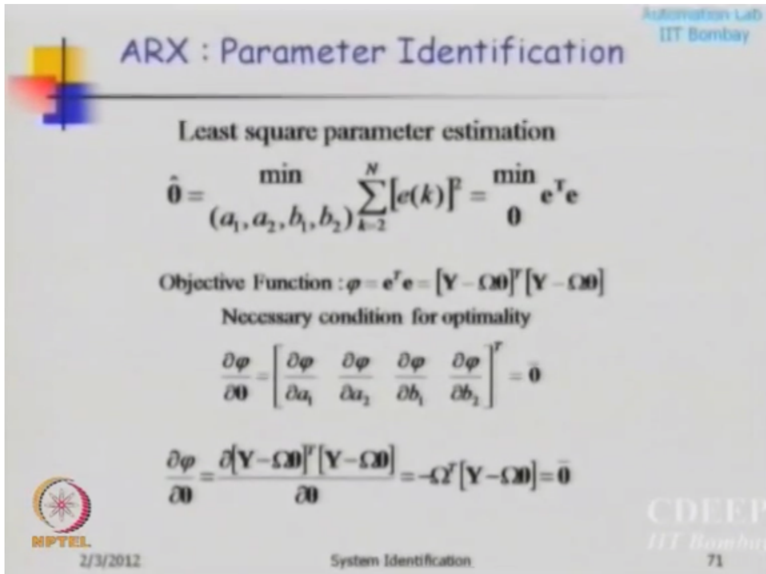
Now what I have done is I have put this equations into matrix form some of you have done these squares this form would be familiar okay I have just put this set of equations into standard matrix form Y here capital Y here is a vector of all Y's stack okay this is the matrix the first row here will be y1, or it should be y2 y1.

So these are again a side type actually y2, y1 go back here should be y2, y1 u1, u0 okay so I will correct this just correct it on your notes right now it should be y2, y1 so you have you get this it should be y2, y1 not okay so I have this four unknowns a1, a2, a3, a4 I have these additional unknowns it treat to e_n okay.

I have the additional unknowns and all of them have to be determined okay I am going to write this as a one matrix equation matrix Y okay= this Ωmatrix* θ+e , e is the vector of all errors okay now I want to estimate θ okay even so the so want to estimate e but e are the errors right, e are the errors so I am going to estimate θ such that some of the errors will minimize these square estimation right.

All of you are aware of these square estimation okay so this is the linear in parameter model so very nice model and this model I can estimate parameters unethically I can solve the optimization problem unethical how to solve optimization problem. There is an error two and three it should be two and three, I will correct this, this matrix is yeah, yeah this matrix that one index is correct yeah, yeah if you have it print out you just correct it right now I will put the corrected version on that.

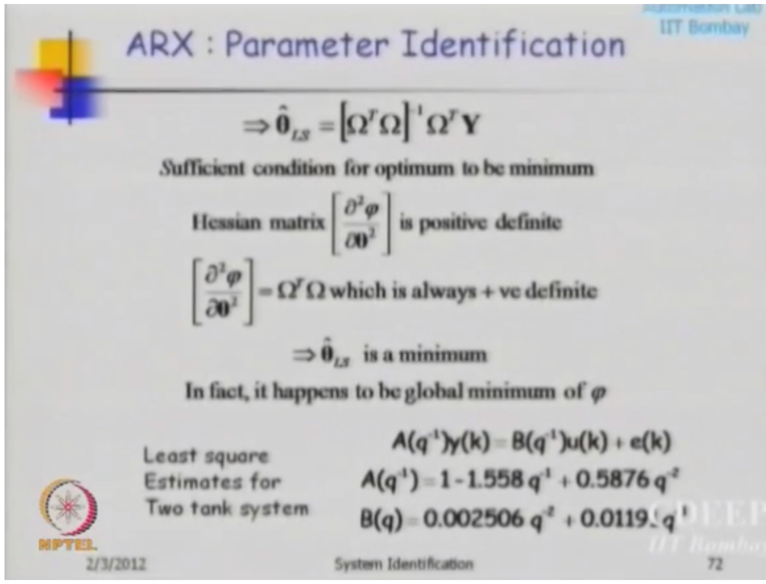
(Refer Slide Time: 46:45)



Now this square estimation problem what I want to do is I want to estimate some of the square of the errors which sensor to be the thing but e transfer e here okay and then you know e transfers e is nothing but y- Ω θ transpose y- Ω this problem can be solve unethically what is the necessary condition for optimality that derivative of the objective function, objective function is some of the square of the error that should be equal to 0 okay.

So I am finding out this, this small ψ here is my objective function e transposes e and setting derivatives of this with respect to θ, θ is nothing but a1 a2 b1 b2 0 this solution can be formed unethically so if you actually it is intermediary set you take variant and setting equal to 0.

(Refer Slide Time: 47:44)

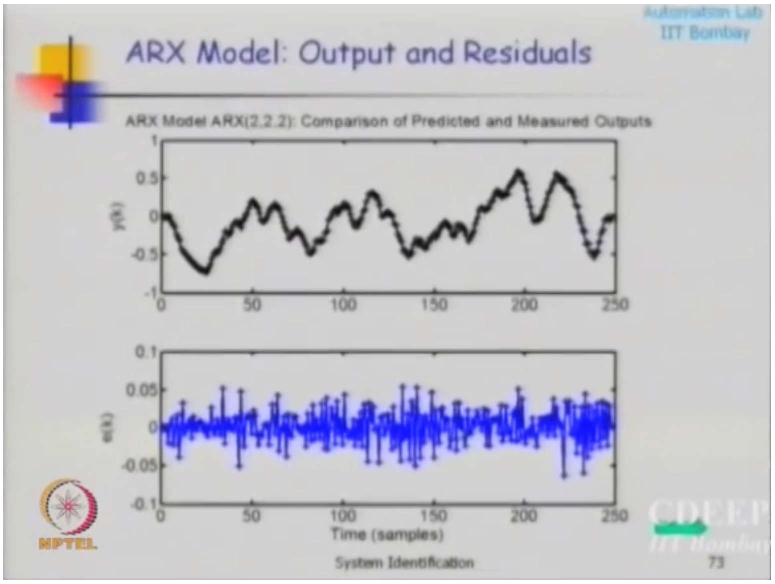


And setting equal to 0 the gradient and setting equal to 0 because set it equal to 0 you ger the least squares estimate okay this particular estimate is the least square estimate of a1 a2 b1 b2 okay obtain from data which you have collected okay well what is the sufficient condition for optimality the second derivative should be positive if I find out the LCM it turns out to be this matrix Ω transpose Ω this matrix it is always positive definite.

Why? If this matrix is it has rank equal to 4 in this particular case it is already positive definite think about it this side Ω transpose Ω will always be positive definite matrix so which means you are got the global minimum in this particular case and it reach global minimum unethically ARX model what we call it as SARX model in very, very easy to compute okay.

So if I do this on this data which I have do it I will get this model I will get this model parameters okay now the question comes is that I got this model once I got this model see I can go back and sue the definition of e to be y- Ω θ I can substitute here least square estimate I can get an estimate of e find out whether it is y transpose or not if it is not y transpose then model is not correct.

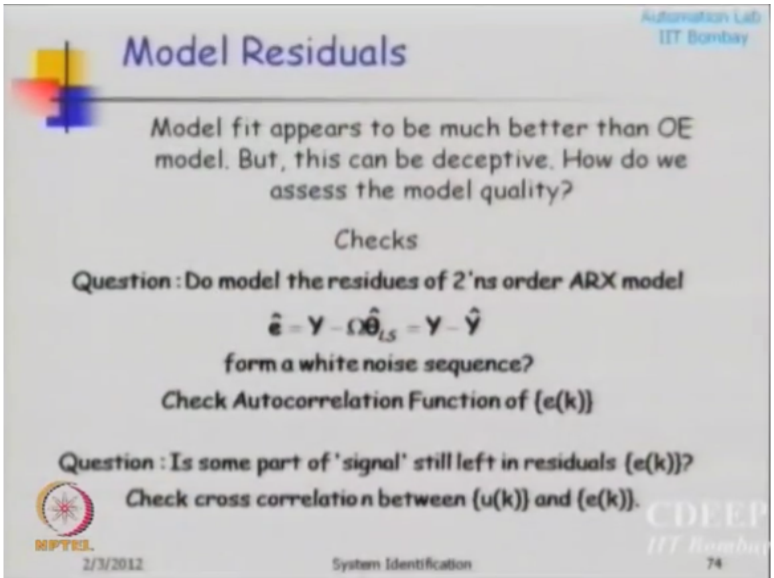
(Refer Slide Time: 49:40)



I should go back and changed all that okay so that is my next thing which I am going to do so I got this model okay I got this estimate of the model and look at the model, model looks very good you know see I just go back and check here vk was like this right you are going from 0 to . 15 now and then their was a gap between the prediction and measurement okay come back here second order ARX model okay.

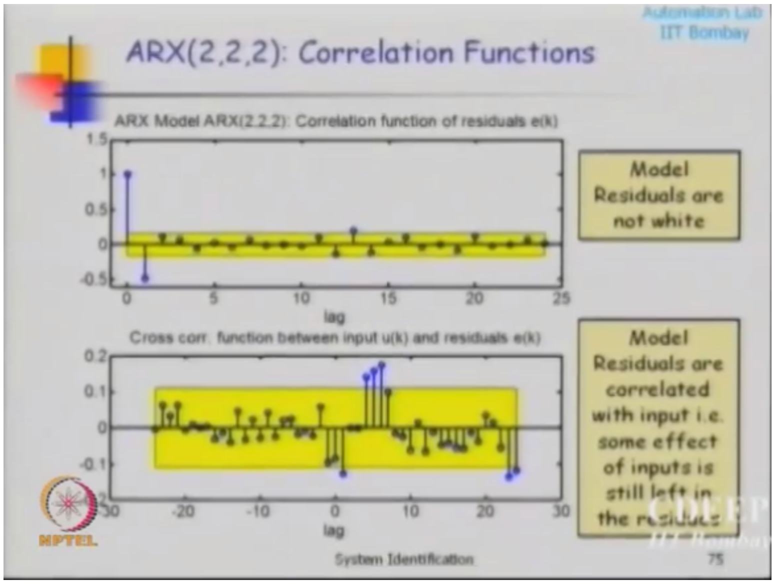
It just the model end of predictions and a measurements of setting of the top of the measure you cannot see the difference can you see here the difference the error is very, very small okay now the question is this is a white noise usually it looks like a white noise unfortunately it is not a white noise okay it turns out that this is not a white noise.

(Refer Slide Time: 50:36)



So my model residuals so this on if you just look at the two diagrams it is better than the OE model you know it has close that small gap which existed okay so first check is I find out the residuals the residuals are y measured- Ω matrix into estimate okay which is same as y-y^ what is y^? y estimated okay that I check for auto correlation function if auto correlation function show that it is not a white noise for model is not good I have to change the model okay.

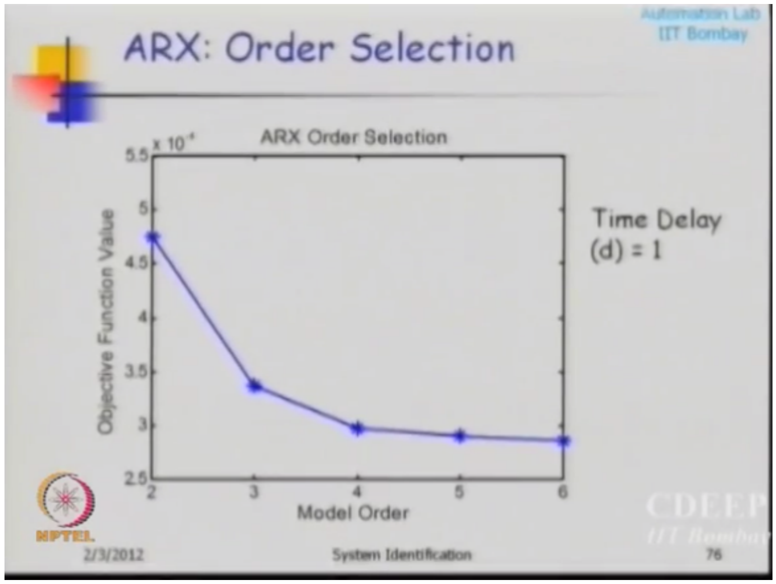
(Refer Slide Time: 51:34)



Then what should I check cross correlation between uk and ek okay so I want to check cross correlation between u and ek this is the auto correlation function you can see here this auto correlation value peaks out so this is not whiteness yeah, see the auto correlation function this guide here is the so there is a auto correlation negative auto correlation between ek and ek-1 which means ek is not a whiteness okay.

What about cross correlation ek and uk are correlated so some effect of uk is still latten in ek the model residuals is not completely white you cannot accept this model even though visually it looks very good you know the Maths looks very good but I do not accept this model okay so what I am going to do is.

(Refer Slide Time: 52:32)



I am going to on developing the models so this is I have developed a model which is second order third order fourth order it is called as sixth order and have this what is some of the square of the errors okay you can see that the objective function value of the objective function so some of the errors going on decreasing at once I if I develop the model of after 6, 6 lags yk-1 yk-2 yk-3 up to yk-6 uk-1 uk-2 up to uk-6 if I develop this model up to order 6.

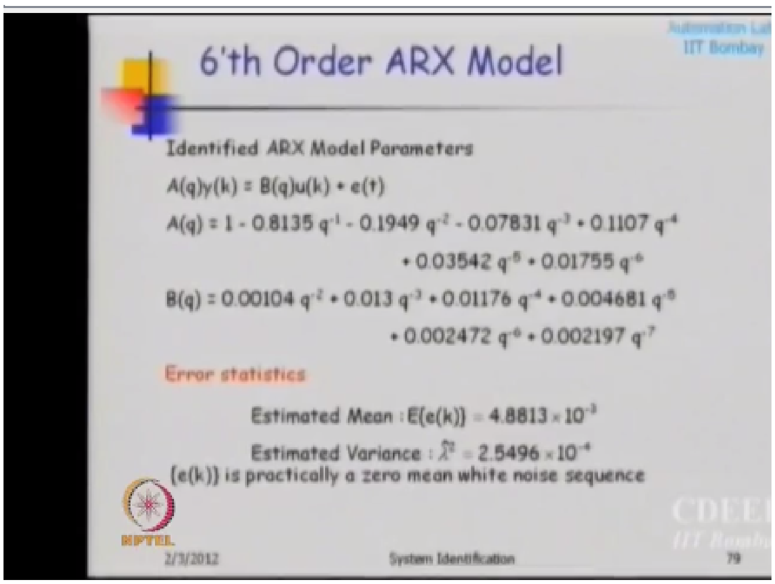
Then the noise becomes white then I have removed everything from the residuals okay so here you can see there it is almost white okay actually we should go on developing the mode with higher and higher order but there are some issues I will talk about the issues but it is not exactly almost white there is some small correlation left yk and ek sorry uk and ek.

But I am willing to leave this model yeah, no there are issues why cannot we go on developing a model of higher and higher order I ma going to talk about the issues now there are fundamental issues that too so now there is the problem you know this seems to give u a model in which the gat is closed and some of the noise is model how the noise is model I will come to that I will come that little later.

But before that I want to analyze the parameters estimate of the parameters okay and then I want to give you some insights into the behavior of the estimated model so ARX model is a very popular model in the industry it is very, very open use okay but trouble is you have to get a good ARX model you are to use large number of parameters.

And why large number of parameters is the trouble okay I will come to that now that is mat lab which means six lab in u 6 labs in y and two is the time delay time delay of 2 we starting with uk-2 so default time there is any model is 1 here it is one more lab so 2 so this actually error is 6, 6, 2 is mat lab command which I have use here okay.

(Refer Slide Time: 55:03)



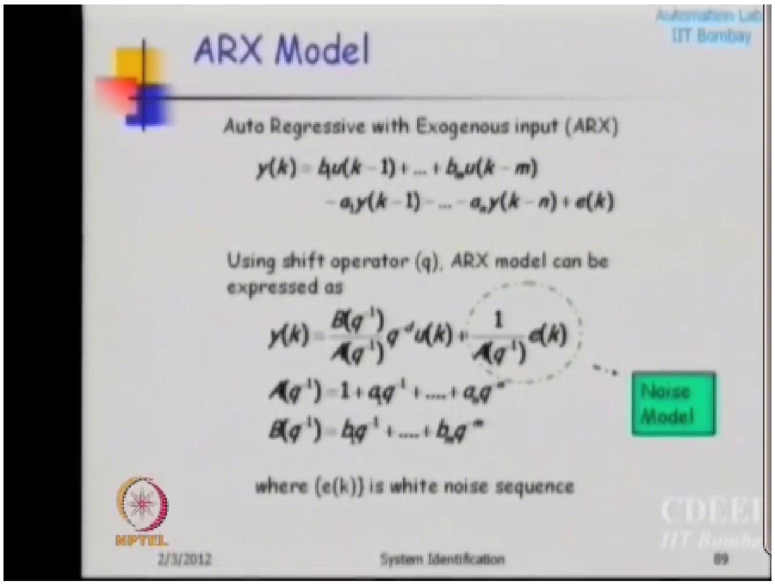
Now this CK here is void noise okay there is no auto correlation in this EK almost okay and there is no much co relation left between EK and UK so I can accept this model if you want you can go little further I stopped at 6 order okay, may be you can go upto 8 order but beyond that okay. So these are my model co efficient now model here consists of two things one is this A ploy nominal B poly nominal.

You just say ARX I think earlier I showed this you right using this, ARX model you do not remember now because all you the theory was not covered okay. So pop up this model give data it will pop up this model okay it is as easy as that but here the model consist of two things one is the transfer function A and B okay.

It also tells you about this EK and soon I will show that this EK is also as important to us as this model okay. So I have also listed here if you notice mean of EK is almost 0 not exactly 0 close to 0 10^{-3} and this is the variants. So this is like a zero mean noise wide noise almost wide noise and I know it is variants I can estimate it variants from E√n that formula we can use and variants of this equation.

So it is practical is 0 and white noise signal and what is the role that this signal is playing in terms of you know noise modeling.

(Refer Slide Time: 57:02)



I think let me go to that first and then come back to this logically I will skip few slides and come back to this slides okay. so my ARX point is like this is everyone with me on this ARX model is like this okay. where I make sure that ek is the white noise so develop the 6 further model which means you use uk – 1, uk – 2 upto uk – 6, yk – 1 yk – 2, upto yk – 6 and we got those polynomial in q aq, bq you got right.

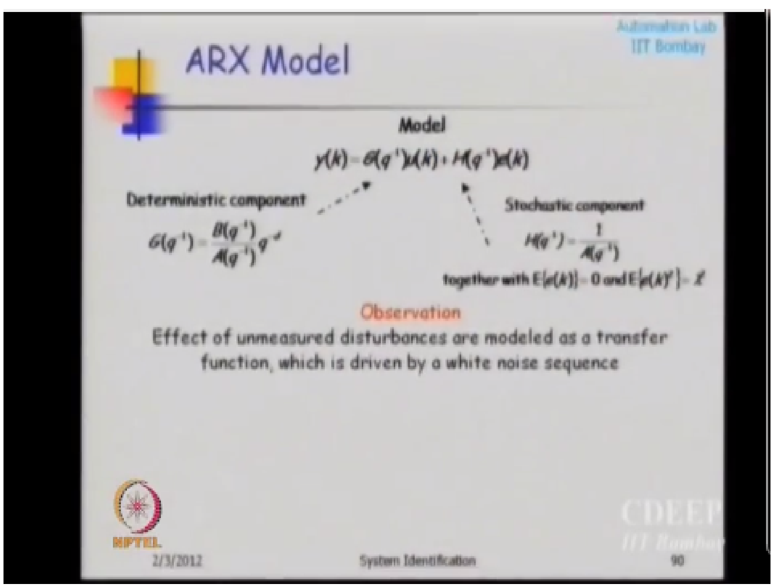
I showed you for this particular data which we have okay now I am going re write this model like this, is there any doubt with what I have done? This slide can you see the slide okay, is this clear what I have done. I have just taken q transform okay and then written it as transforms function okay.

Yk what is this part this is my g u okay this is my model for noise earlier I had written bk here okay now I am getting 1 upon A, A is the polynomial that you got from here right x ek. So this quantity 1/aq x ek is actually my noise model, so I have modeled a noise as a transfer function which is driven by a white noise, do you see this if I write this model like this my noise here, this is my noise actually model as ek passing through.

This is my noise model so this is vk okay what we made sure of ek is the white noise sequence do you agree with me okay, so without realizing you are constructed a noise model here which can be listed out only when you go to q domain. When you write it on the shift operator format you can distribute what is the noise model what is the data model.

This part Bq/ Aq is the deterministic model okay and 1/Aq x ek si the stochastic model, earlier we called it bk okay. So this is the model for unknown effects unknown components, so this model is driven by 2 inputs what is the 1st input? uk is the known signal to you, uk is the wall position that you change okay, it is driven by another hypothetical signal called ek. Ek is not a real signal; ek is a white noise source, which is driving the transfer function.

(Refer Slide Time: 1:01:37)



Let me summarize this like this, so my model can be written as 2 parts okay, yk = gq x uk what is g? Effect of deterministic component, g is my transfer function with respect to that deterministic component. What is h? For ARX model it turns out to be 1/a okay so this is my noise model okay. The noise model is consisting of a transfer function and a white noise source. White noise source is hypothetical, is driving is actually driving the issue or the model that unknown component is as if a white noise is passing through a filter.

What is this filter 1/Aq that is the filter okay, so the trick to model unmeasured disturbances is to model it as a white noise source which is passing through difference equation, that is the basic idea used in model. This particular idea as taken believe me centuries it is not at all so easy to come up with because you were trying to model something for which you do not know what real input is.

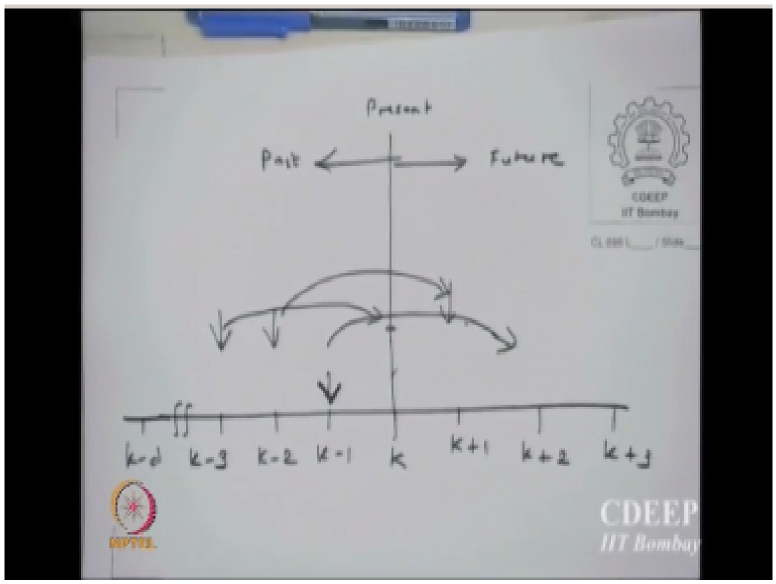
You just have it is effect present in the data and you still want to uncover a model okay very difficult, so you have constructed a hypothetical signal called white which is passing through a transfer functions which gives you the same effect as the unknown disturbances. There is no real source called white noise anywhere okay it is a model and it works in predicting the unknown disturbances.

Why do I write uk – k – d, about delay? No it will represent the value of u k d, k – d instant in the past, uk – 1 in past, uk – 2 is 2c instant in the past, no the meaning of writing q – d here or uk – d is that a value a past value of u has effect on what is happening today. See for example let us take a situation where you know suppose I teach something today okay. you will understand this upto 2 days okay.

Then how will you write the model for that? You will say that what I know today si the actually the effect of what was taught 2 days back okay, not what is taught today so what I am teaching today you will understand after two days because there is delay of two. So that is what is been quantified here through the time delay disturbed d – d. so the past what is the meaning of delay in the measurement?

The delay in the measurement what you observe now okay was something that happened in the past okay is not what is that you see, it is the delay in the measurement okay. so which means the temperature that you see now okay is actually what is happened in sometime in the past in the system, that is the meaning of delay measurement so both of them are can be represented to this model yeah then there is unit delay of one you understand one basic thing here see.

(Refer Slide Time: 01:06:29)

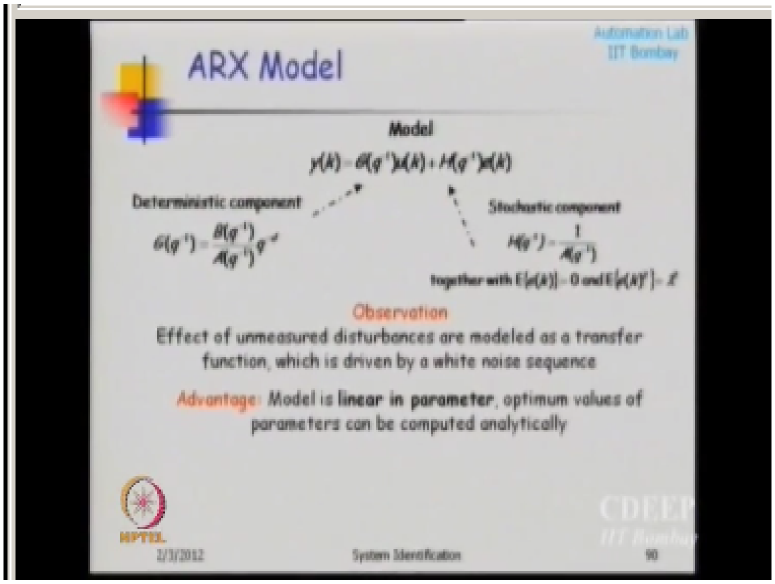


This is present so when I am writing k+1 so I am instead k k+2, k+3 is there all in future okay so k-1, k-2, k-3 so this is k-d in general these samples in the past I cannot have a model which says you will never have a model which says that what is happening now is a function of what will happen in future the delay means what is happening now, see in discrete times all you need is delay.

So if you take a input if you introduce a input here it is effect can be seen only here that minimum one delay is always there sometimes there is bold delay which means what you do here is effect might be seen here or equivalent what you do here let us say there is a delay of two what you do here its effect is seen here what you do here its effect will be seen here okay this is relate to this, this is relate to 2 in future okay so the delay is always in the past.

Okay if you talk about something into future will not have effect on present you know so the past will effect on present that model you can develop future have been effect on the present is not something as dependent so k here this remember when ever we are doing this k is also a current time k-1 is one in the past k-2 is 2 in the past am I answering your question or we can talk about it later after the class okay.

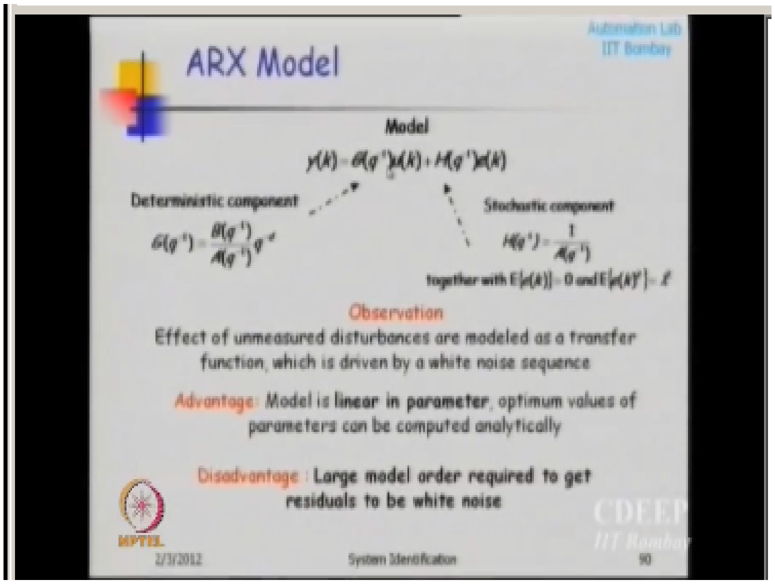
(Refer Slide Time: 01:08:39)



So basically what is happening is that you know we have got a model which is linear in parameter model and effect of one measure disturbances have got model as a transfer function which is given by whiteners we could obtain the value of this model using linear squares because model you know y and u are known in any model if you remember you have to do non linear optimization x was not known you have to guess x you did not known x you did not know v you have all kinds of problems.

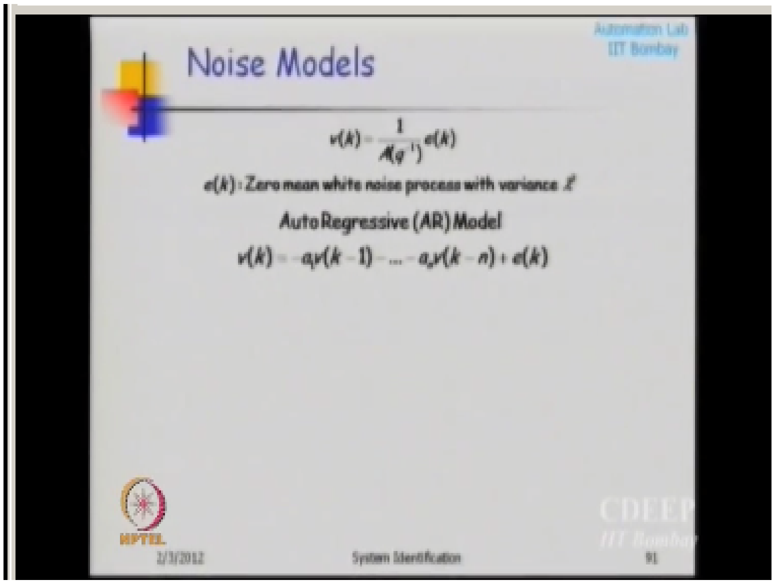
And you have to parameter the delay estimation is typically done there are methods to do delay estimation simplest way is to give the step change and see when the output starts changing because you can estimate delay for that okay so you have to do lot of studies to estimate the delays so it is not that what is the problem with this model you need lot of data in the past.

(Refer Slide Time: 01:09:54)



So let me uncover this noise model and show you what it is exactly if you turn out to be auto currency process okay without saying it actually I have a model in process how I am just multiplying this it consider a scenario where u is 0 okay if u is 0 permantely 0 what remains is only noise okay earlier is noise we have called it vk so I have chosen to call it vk here.

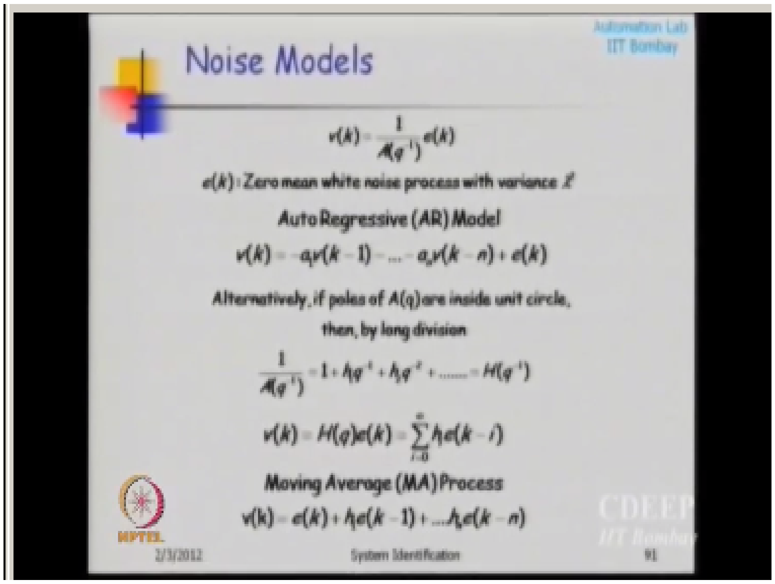
(Refer Slide Time: 01:10:24)



Okay vk=this is my noise model vk=1/aq.ek what is this model it is an auto regressive model if you just expand this you get this expand this what actually I have got is an auto regressive model okay so now, auto regression is able to help me construct a model for auto regression is nice it is a stationary process okay I can develop a model for a random signal which is frustrating okay which is auto correlated to this auto regressive model okay and I was able to get a1,a2,a3,a6 I was able to uncover the coefficient okay.

So now I can talk about that signal just using this model what actually you are doing you are representing a you do not know what a source of it is through a model which is the difference equation model this is a difference equation model okay it is diverged by whiteners so this white noise which is very it is equally important here model construct of a1 to an+ white noise and its variance you cannot separate this two okay so even though so even though this white noise does not have anything left in it is useful actually it is used in modeling the noise okay thus understand this.

(Refer Slide Time: 01:12:02)



While I can play with it now once I have model in q domain if aq you know if it is poles are inside the unit circle and then you can do along division and then get this infinite some and then I can write this model okay so what I want to show here is that we talked about two processes right moving average process and auto regressive process I did not introduce them without purpose I talked about them because they are useful in modeling signals okay very simple models now I am there inter convertible all that I am trying to tell you is that one.

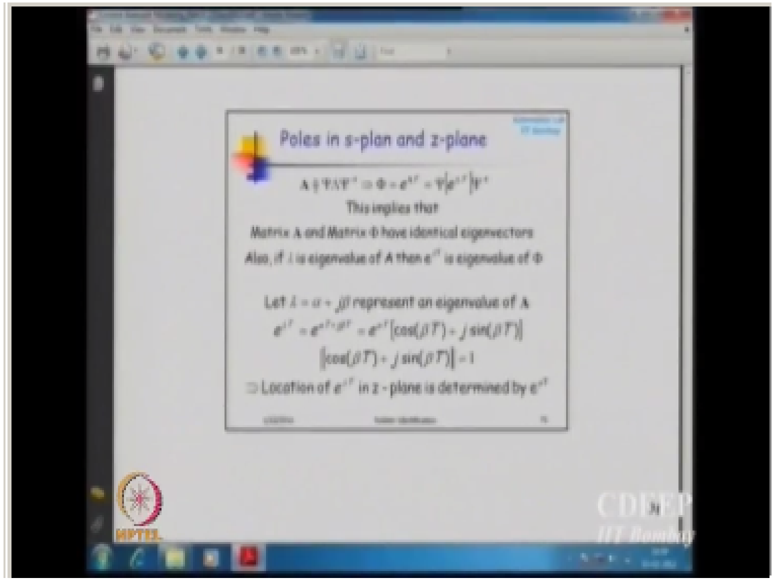
And other are inter convertible see here once upon a can be expanded as a long division you just do long division okay you can do polynomial division one upon you know maybe I can give you as exercise you can do this division and see so I can write one upon a3-1+hq1+hq2… and then that means vk is some function of past ek alone okay this h here you will see that it will go on reducing in values h will go to 0 okay if the poles are inside.

The unit circle and then you can truncate this series so another way of constructing a model moving average process okay I can develop a auto regressive model I can develop an AR model okay that is auto regressive model I can do MA model moving average model okay we have two different ways of capturing or developing a model for noise models okay where I can combine these two and develop what is called as ARMA model I will come to that little later.

But is everyone clear about the abstraction that I have come up with for noise modeling okay is this transition I started with this okay I said thi9s is my ARX model I am going to abstract as deterministic compounds or compound this one upon a bk as one q.bk and showed you that this is nothing but auto regressive process is given by white noise this signal we have seen earlier right so the in fact y it is a color noise or a white noise it is color noise I showed you an example of what ever which is color noise spectrum was you know.

Yeah it was high at low frequencies low at high frequencies this is the color noise okay now I hope model for the color noise obtained directly from data I have ordered for the color noise obtained and the mixing which I am going to say here is that it can be converted to moving average process those two model forms auto regressive and moving average are just inter convertible forms you can go from one form to the other form this form just like you can go from discrete to continuous, same thing holds here okay just a very, very quite a thing about a one problem I was told we have difficulty in solving from going from discrete types to continuous type system role number 6 okay some of you have difficulties I just want to point out one thing here just look at this.

(Refer Slide Time: 01:16:01)



Point matrix and a matrix same Eigen vectors this is a clue to the problem and Eigen values of 5 and Eigen values of a are written by $e^{\lambda T}$ if you know sampling time you can see use start with 5 you first find out the Eigen vectors then you will get sign matrix okay you will get sign matrix once you have sign matrix you can write inverse okay you will also get this matrix $e^{\lambda T}$ from this getting matrix is not difficult.

Because the relation is $e^{\lambda T}$ this is the relationship that is if λ is the eigen value $e^{\lambda T}$ of this matrix look at this lines 72 okay line 72 is the solution for that particular problem and the other part can be uncovered from heading b matrix is yeah look at this equation γ matrix is nothing but $5\text{-i}.a^{-1}$ so if you know γ if you know ϕ if you know I if you know a you can recover b okay thus use this so this is slide what 39 and 72 and clue for problem 6 okay.

NPTEL Head IIT Bombay
Prof. R.K.Shevgaonkar

CDEEP Head
Prof.B.L.Tembe

Producer
Arun Kalwankar

Project Manager
M.Sangeeta Srivastava