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ADVANCE
PROCESS CONTROL

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Lecture No. 06

Development of Control Relevant Linear Perturbation Models (Part 2)

Sub-topics:
Introduction to Stochastic Processes'

Today's lecture is one of the most difficult lecture. Let us please pay attention and wherever you want me to stop just stop me okay. So we have been looking at this black box modeling and I introduced what is called as output error modeling.

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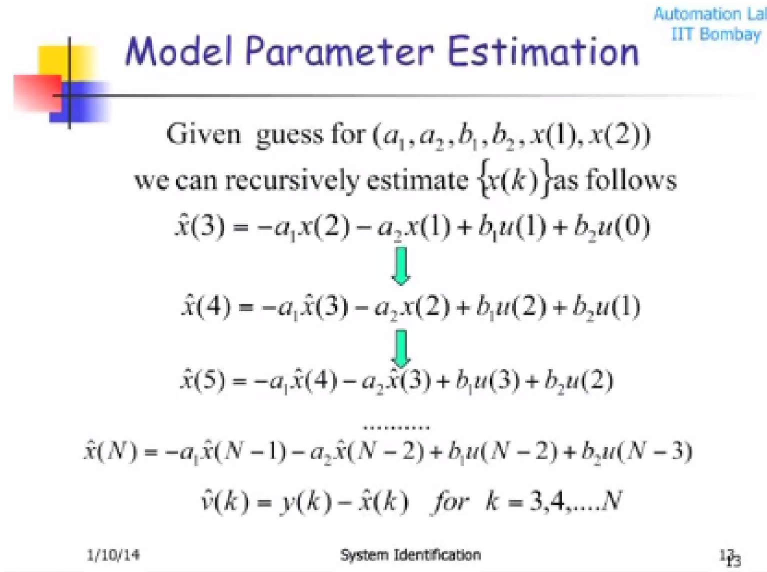

So I just showed you how to go about doing output error modeling for a second order system, we had four parameters to be estimated a1, a2, b1, b2 and whether we want to assume x1, x2 to be 0 or whether we want to estimate that was a choice.

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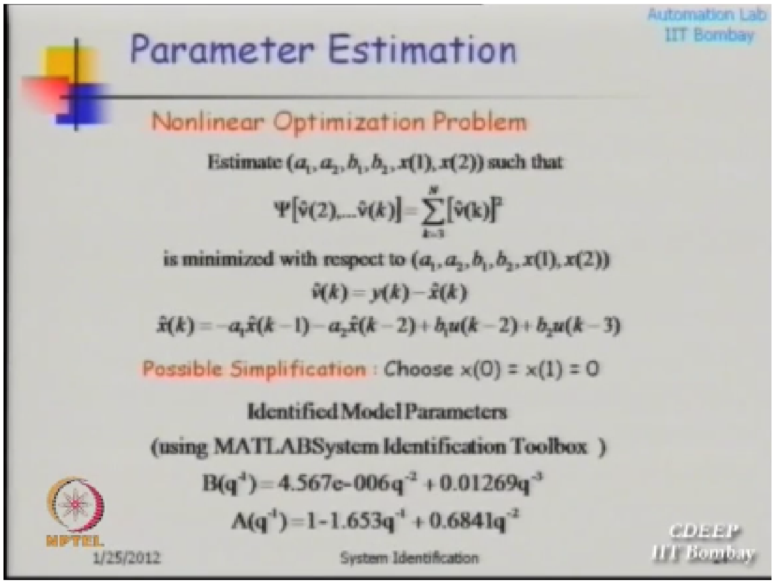

Well, the problem that was finally formulated was an optimization problem. So we said that estimate a1, a2, b1, b2 such that some norm of the residue model residue, model residue is Y measured minus Y estimated or Y predicted. Some of the square of this model residue should be as small as possible, that was the idea. So choose these parameters such that this is one way of formulating the problem, one could say YY^2, I could minimize sum of absolute values no problem.

You could minimize infinite norm which means, you could minimize maximum error no problem, it is your choice. We typically use two norm, because two norm comes with lot of associated properties, maybe if life permits I will just hinted those properties, we do not have time to get into everything. But analysis of the method is much easier using two norms, so we normally use two norm okay.

We could choose, we could simplify by saying x1, x2 is 0 and we can identify the model parameters. I am showing you here this particular problem solved for the two time system that we are considering, model parameters estimated are given here, debated polynomial and denominator polynomial. I am looking at a seashore system, single input, single output okay. we will worry about multiple input and multiple output little later what to do okay.

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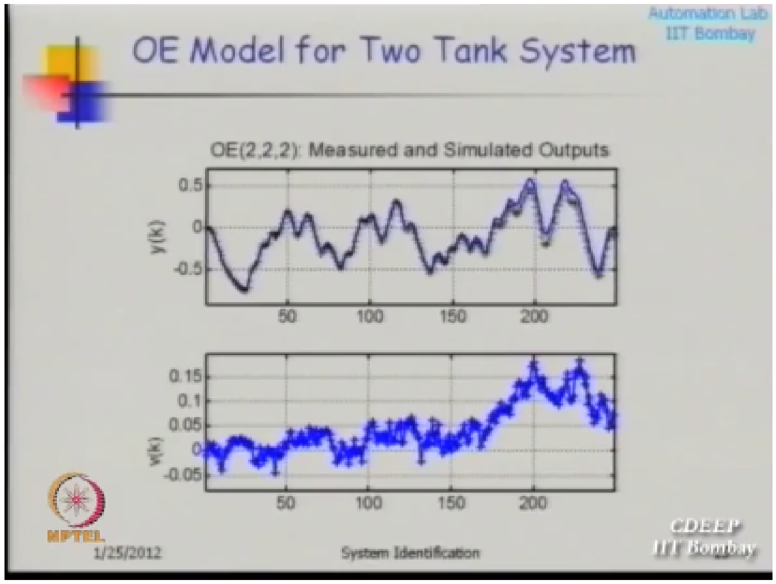

I showed you this plot 2 at the end of my last lecture, so the blue line is the model prediction and I think the plus are the measurements. Even though blue line is shown as a continuous, actually it is not continuous, it samples many times we just draw it continuous out of some kind of an habit. We should actually draw with points. The difference between these two is this VK signal, VK actually, technically I should put VhatK, not VK estimate of VK.

Estimate whatever is not explained by the known inputs, known inputs that are going outside my computer okay. Whatever is not explained by whatever is going outside my computer that is captured approximately in this VK, approximately because we have an estimate, we do not know the two value okay. Now the problem is I want to bridge this gap, I want to model, I want to develop a model for VK, this is a very, very, very tough problem.

I have a signal which is bearing like random function, it is drifting right. It is very difficult to come up with a structure for this signal. Now moreover we just have measurement of this signal okay. Let us assume that, you know we have done this output error modeling and we have this signal with us now. And now I want to develop a model for this signal, I want to uncover some kind of structure into this signal. I have a measurement of this signal, I do not know what is the cause.

The output error model be developed was a cause and effect model, you gives rise to change in X was the effect of U and then we found the model. Here what is the cause, we do not know okay. This is combined defect of everything that we do not know, it contains measurement noise okay, it contains unknown disturbances, unknown fluctuations. For example, I have a pump maybe receiving voltage supply which is fluctuating that results in flow fluctuations which results in fluctuations in my – see my input was the control wall position.

The control wall position is known, but the flow fluctuations caused by, even if my control position is steady, the flow fluctuations is caused by fluctuations in the voltage supply voltage is not measured, I am not going to measure that, I am not going to measure everything that happens okay. So this effect is there in my – now as if this two were not enough we also have errors coming because of approximation.

Where approximating a nonlinear, a dynamic system using a linear model okay. So this VK has everything okay. And so it is driven by some kind of an unknown source, I want to cover, I want to attach a model, I want to come with a model to discover this system.

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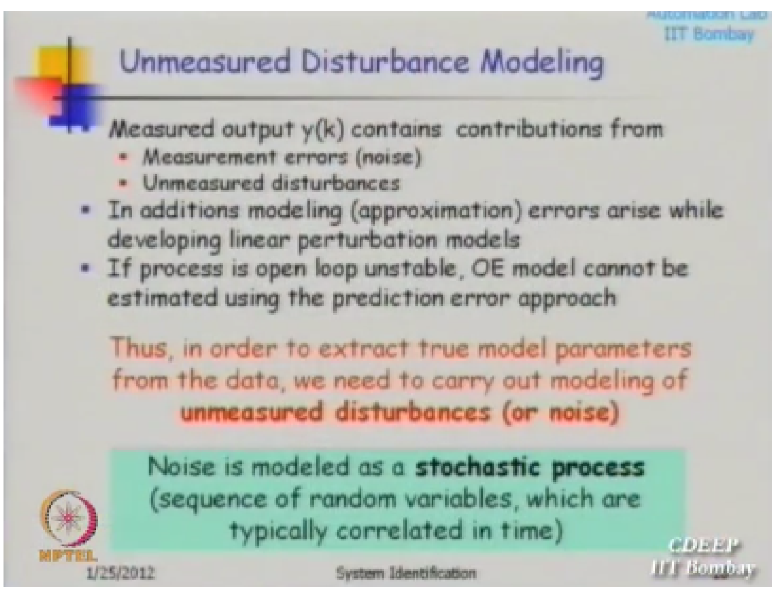

So measurements contain effects of unmeasured errors, and unmeasured disturbances, measurement errors are unmeasured disturbances. In addition you have approximation errors that arise because of approximation that you come up with okay. Well there is one problem with output error modeling, why I want to move from output error modeling to some other modeling, noise modeling.

Because output error modeling can be used only when the system dynamics is open loop stable okay, system dynamics is open loop stable, then the predictor is, if you start guessing an unstable pole, the predictor is unstable, the predictions can become unbounded and you cannot solve that problem numerically, it is not possible to solve okay. So one requirement, one fundamental requirement is that for output error model development by this method which I described just now, you have to have the model just to be open loop stable okay.

Which means pole should be inside unit circle, we have done this, what is open loop stable okay. The discrete time system, pole should be inside the unit circle okay. So now actually if you want to work with system that are unstable or if you have system which are subjected to significant amount of unmeasured disturbances which is more often that case in the real systems, then you better model these un measured disturbances.

So the tool that I am going to use is called as stochastic processes okay. Stochastic processes are sequence of random variables okay, which are typically correlated in time, what happens now even though it appears random has some relation to what has happened in the past. What is the relationship that is what you want to uncover okay.

Well what I am going to do now this lecture let us try to give a very, very brief introduction to these stochastic processes, and realistic to give a introduction to a topic which has taken centuries to develop, very, very complex business. And try to put it in nut shell, and put it in one or two hours or whatever three hour or four hours it is very difficult. But even though it has taken centuries to develop, you know even language that we learn as a kid.

Language has taken centuries to develop, but that let me write, we start syllables and word, condenses, without understanding what it means. For example, take a word responsibility, we start using this word at the age of four, or five or six, we have the understanding what is the meaning of responsibility okay. You might know spelling in the beginning, and as tiny walls as you mature you start getting the meaning.

So I hope as you start working in this area more and more meaning will appear. So this brilliant theory was, what we study today as formal theory of stochastic processes was developed by Russian mathematician Colmogado around 1930s he formulized the structure of this stochastic process theory. The other mathematician who contributed to this, I think he was a Austrian Bener was these two people stand out as jands.

And so the formal theory that we have, actually they started of formulize this theory, the efforts were on since very long time. Probability in statistics is the old topic and people have been, looking at for generations but it took a around 1930's 40's where it took a shape that we study now okay now let us began with what is a Random variable.

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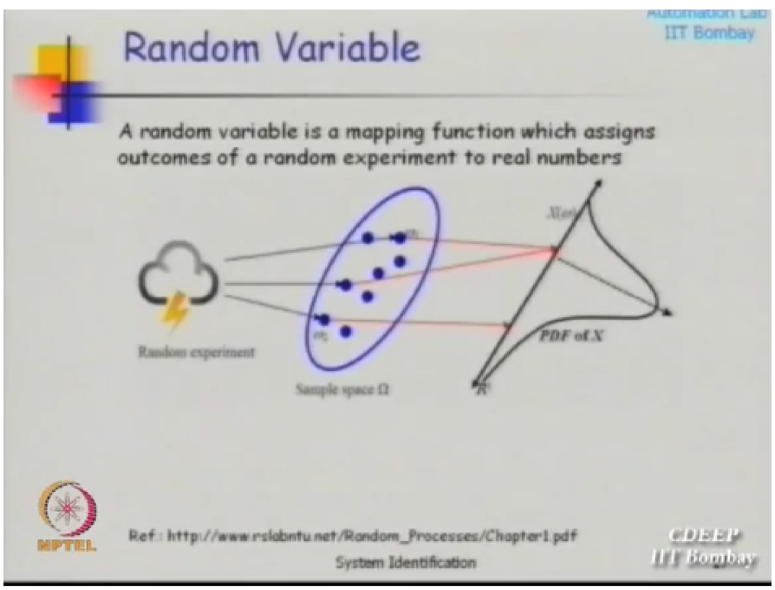

Random variable is a mapping which assigns outcome of a random experimented okay to what is called as a to a real number okay, so there is some random experiment for example the noise in the measurement okay I am going to assume is a random number the noise that occurs now the noise that will occur in that next instant or instant after that or does it happen something in the past okay.

May or may not have any really it is random okay if you open a instrument manual you will see you know something like what is a standard deviation of the measurements it will tell you what is the spread if you keep the measurement measuring instrument into your into that say I am measuring temperature if I thermocouple into some constant temperature path you expect temperature to be constant measurement to be constant but you will never get constant measurement I will show you actual data.

You will get you know data which is fluctuating all over okay well may be we can just have look we will postponed looking at a data when ever that slight comes this version of a PowerPoint seems to have trouble okay so this is a random experiment and we assume that outcome of that experiment belongs to what is called as sample space is set of all possible outcomes okay and then we you know rectify behavior of this variable through a probability density function.

Okay you know about probability density functions you have studied at some point in your undergraduate or in your 1st year of undergraduate studies are graduate studies and probably disunity function is model if you look at it very carefully it is a model okay it is a model that explains how the randomness is.

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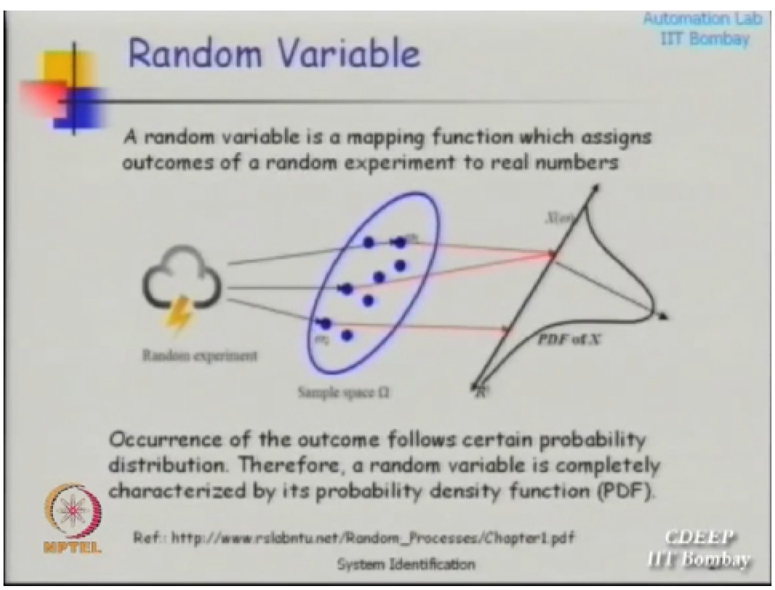

And this probably density function is a model for 1 random variable right one single random variable okay right now we are not talking of and then in books you will have these examples of coin tossing experiment and so on right coin tossing experiment the sample space will be only 2 toss it what heads and tail and so on.

Now this abstract concept called probably density function it actually is a some kind of a you know limiting notion which tries to mathematically conveniently express all possible outcomes of a random variable okay.

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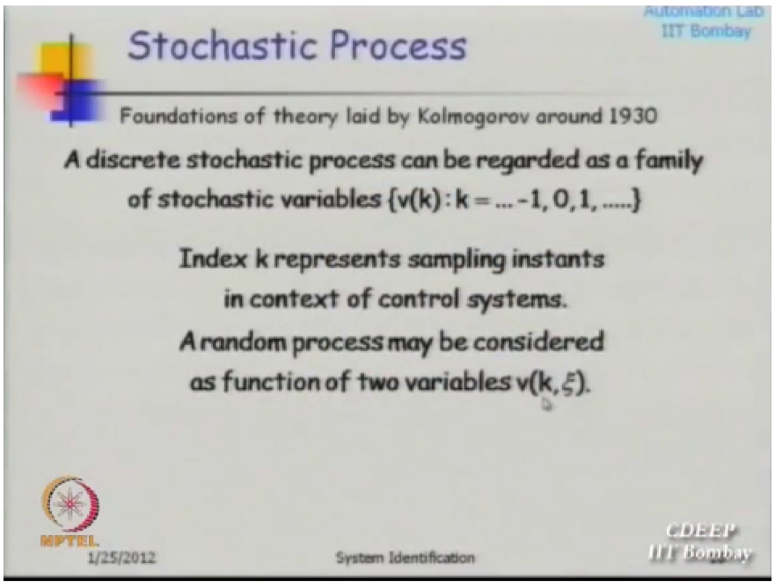

Now with this very quick review of random variables I am going to move what Kolmograove did stochastic process well this definition is hard okay so let me clear to you today you may not understand this in this course but keep it in mind you will understand it eventually a discrete times stochastic process is a family of random variables now I am going to look at family of random variables.

Not just one random variable, so v(k) where k here comes from my index set in the context of digital control index set is time okay if you call current time as 0 – 1 is 1 in the past -2 is 2 in the past +1 is 1 in further okay +2 is 2 in further okay so in context of control this is not only way to define a stochastic process where in context of control okay k represents time typically we take some initial time 0 and we define a stochastic process going from 0 to infinity okay.

So that means it is a set of random numbers okay so at each time point okay there is one random number attached okay yeah not a sequence I take back I need a collection I it is not a sequence, sequence is a specific sequence is called something else I will come to sequence little later.

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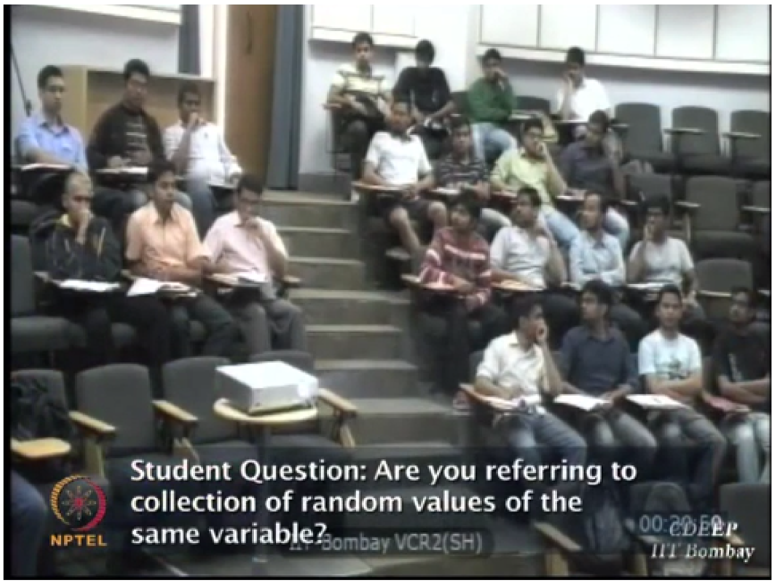

No it is not one random variable when you take it only as special dimension that is a good question see here I have 2 this random this stochastic process as 2 components one is time another is randomness okay so I am going to denote this stochastic process.

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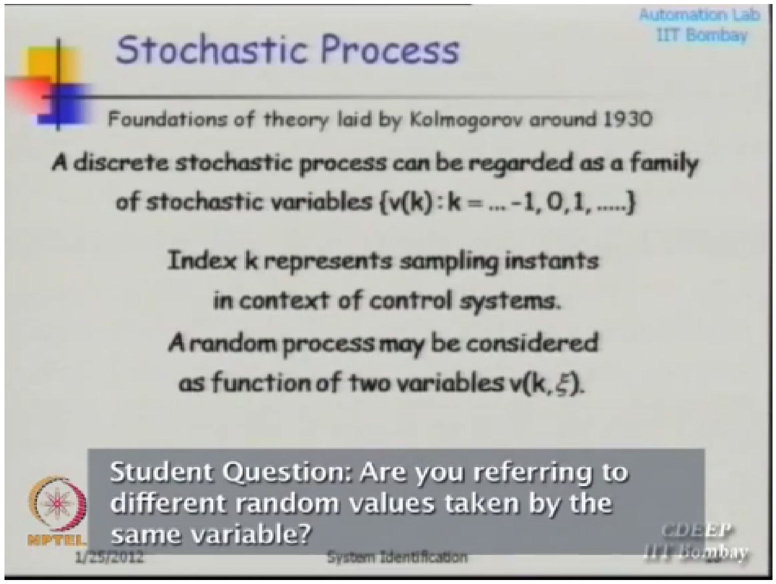

As k and is ξ, ξ is the random variable k is that time variable okay k goes from 0 to infinity this set entire set is denoted by this random process v(k) ξ okay v(k) ξ is a set which is okay. No different values taken by this called a realization let us no do not confuse that 2 okay I will show you picture I will try to show a picture and then.

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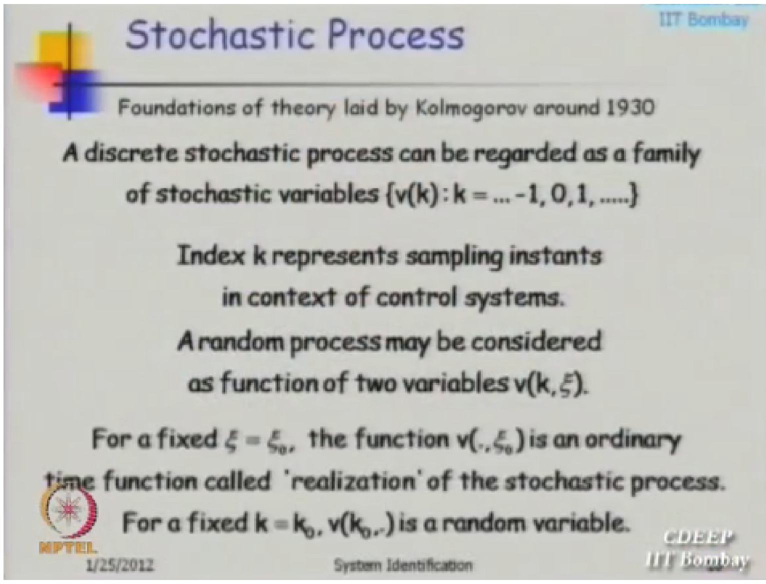

If I fix myself to ξ = ξ 0 then it is a ordinary time function sequence of random variables okay which is called realization of a stochastic process I will give examples okay then it will become clear and I fix myself in time it is a random variable at any given time k I have a random variable okay now let us look at picture that will you will understand.

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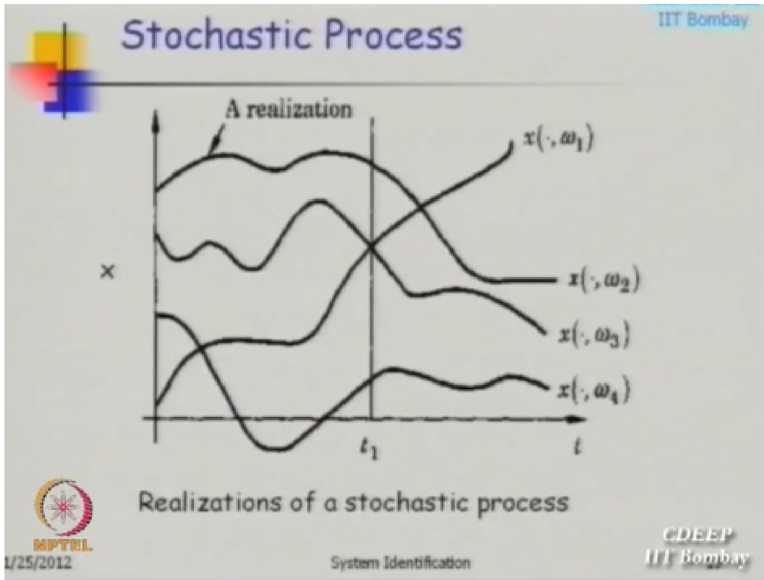

What is trying to be communicated here this is a random process a random process may have or has multiple realizations in time okay what I plotted here is multiple realizations in time okay see for example of a this random process could be you know you are measuring heat beat of a patient okay now of course in reality you do not measure it from 0 to infinity but let us assume for a long time you know measuring heat beat of a patient okay.

Let us say you have a device which is placed in the shirt and then you know it can sense the heat beat and sends you a signal everyone second you get or every 5 second so whatever interval you get a information about heat beats okay what happens today morning 8 tonight 8 okay what happens tomorrow okay I am let us say I am plotting that well are behavior today to today morning 8 to evening 8 so you know today I will get this plot okay tomorrow if I do the same thing let us say one minute at every 1 minute I am plotting heart beat information okay.

Tomorrow between morning 8 to evening 8 I am not going to get the same random sequence okay I am not going to get the same random sequence tomorrow I am going to get a different random sequence okay now if I fix myself to sometime let us I say I fix myself to time 12'o clock and decide to watch observe collect information about the heat beat of the patient at 12'o clock in the afternoon okay you will see a random distribution do you agree with me you will see a random distribution so at if I fix myself to a time I have random variable but if I look at the sequence which is generated today between morning 8 to evening 8 I get a realization of a stochastic process.

what is a stochastic process heat beat of the patient collected at every one minute interval okay another example which I many times have been giving in these lectures is let us say average temperature of you know of that day at 12'o clock in the afternoon okay I want to view these other stochastic process so collection of 365 average temperatures from January 1st to December 31st, they are 31st I take a span of a year, I look at average temperature of the day okay, you go to you know Google whether and you say that Bombay temperature for last year it will show you a plot maybe it is taken at some particular point in the days say 4'o clock or afternoon 12 I do not know what reference point they use.

But they will show you a plot okay, the random sequence that you got this year is not the random sequence you got next year, I am trying to view this 365 variables okay, as a random process they are connected the temperature they are physically connected collection of random variables I am not saying temperature and pressure and you know any arbitrary combination I am looking at temperature at 12'o clock average temperature at 12'o clock so what is k here day, today, tomorrow k+1 is tomorrow, k-1 is yesterday and so on, okay.

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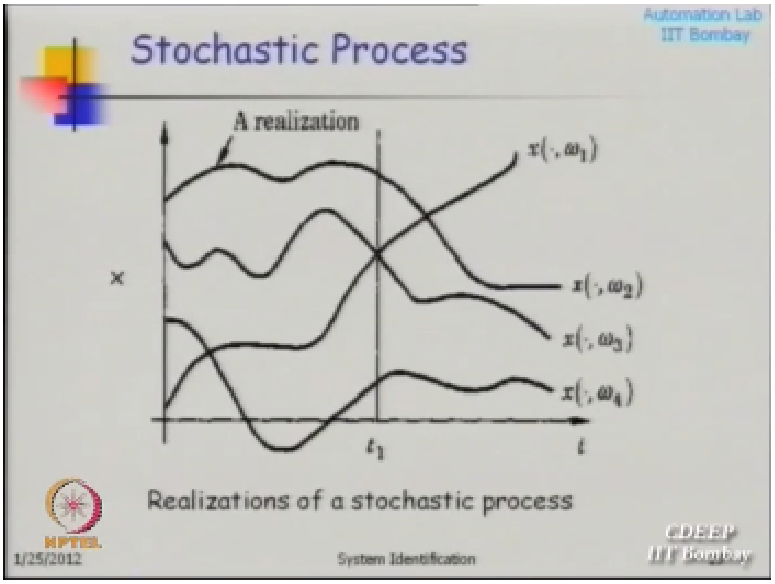

So if I plot the data for year 1991 okay, I will get one realization if I plot data for 1992 I will get another realization stochastic process is same if I fix myself to you know 24th January or 25th of January it is a random variable. I start looking at data for average temperature on 25th of January for last 100 years I will see a distribution right, if I fix myself to 25th of January, so what is time to communicate here is that if I fix myself to t1 okay, I will see a random distribution each one of them can have a different distribution, okay.

The distribution or the randomness associated with temperature, average temperature today may not be same as randomness associated with temperature tomorrow or March 15th or whatever right, so it all depends up on, it would depends up on where your.

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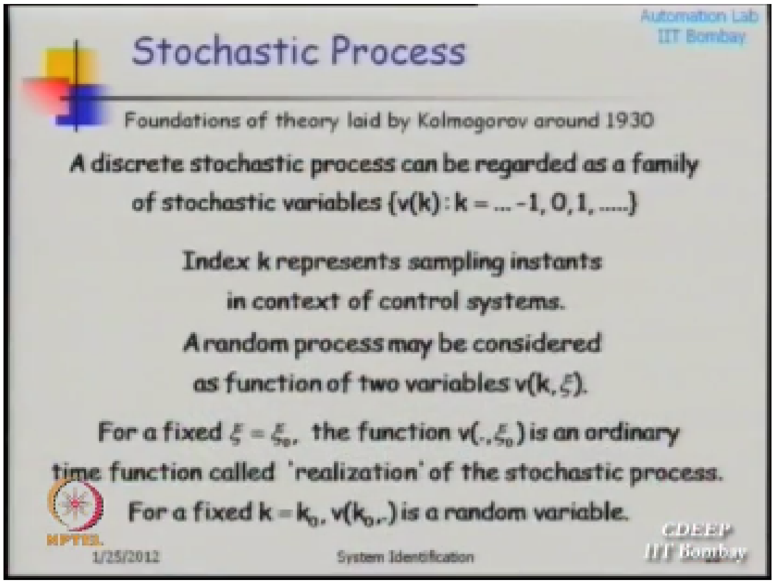

So this last sentence is important is if I fix myself to a time k=k0 okay, then it is a random variable okay, so when you define a probability density function for a process it has two things. At every k there is a separate probability density function okay at every time instant k there is a separate probability density function.

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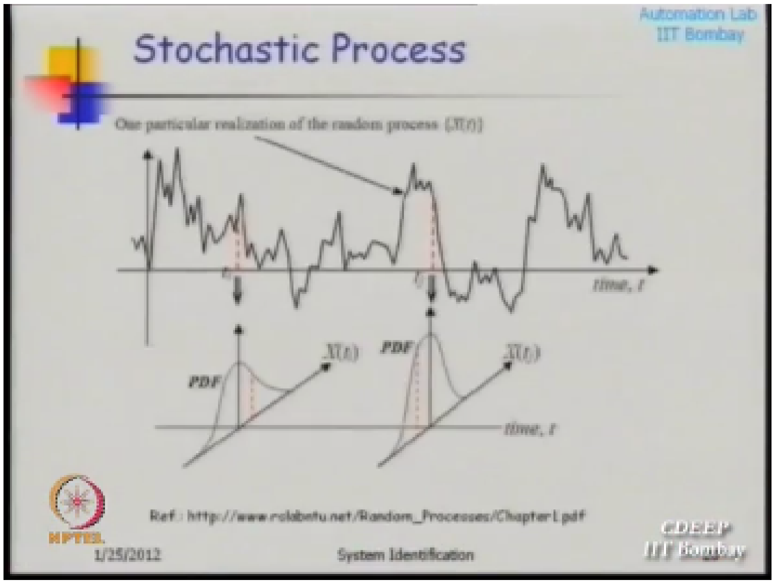

This is another picturization so this is the stochastic process okay, some stochastic process plotted here and you can view that at every time instant there is an associated probability density function so this is not probability density function at this time, this is my probability density function at this time, okay. Because they are collecting they are talking about a sequence a collection of random variables each one of them will have a different density function associated with it, okay, yeah.

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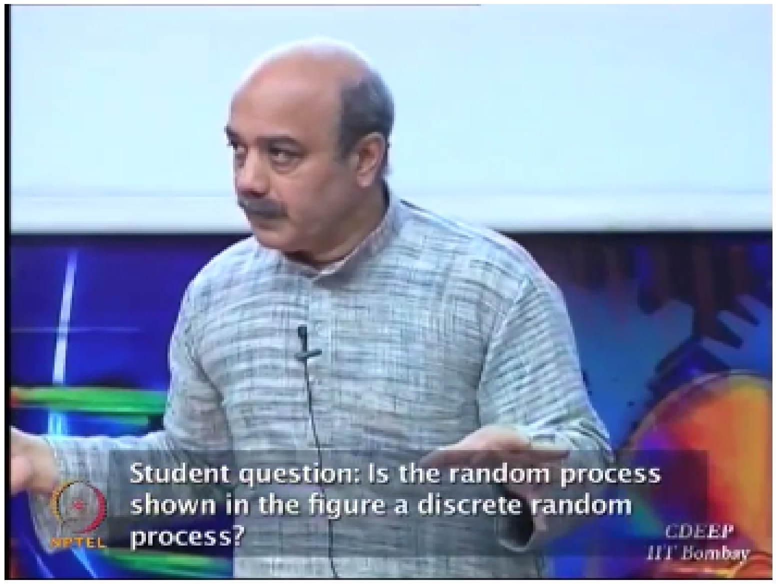

This one is a continuous stochastic process actually the so the number of random variables in a continuous stochastic process is ∞, plus the it is random variable can take infinite values, so this plot is a little you know complex plot then what I am trying to explain. This is not a s=discrete process what I shown here is a picturization is not a discrete process, the discrete process will have only value at the finite time instance.

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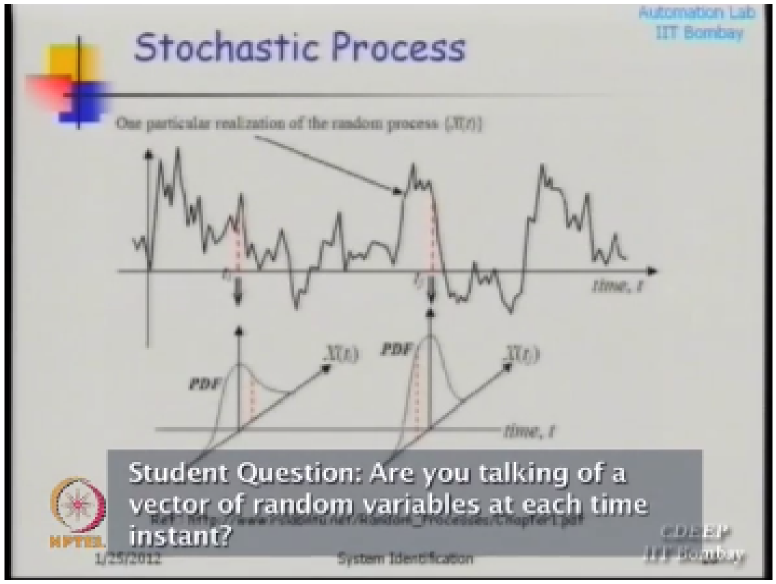

Yeah, right now I am talking about any one, once I am talk about vector of, vector stochastic processes let us not get into it right now, we will get into it at some other point later point okay, I am not talking about vector okay, see for example you might want to say that why one temperature you know morning 8’o clock afternoon 12, evening 4 and night 12 I want to you know look at this four average temperatures okay, these vector so it s a vector stochastic process.

You can view that way it is possible okay, or there are four patients attached to you know four ECG and then you know you are getting four random sequences possible okay, so.

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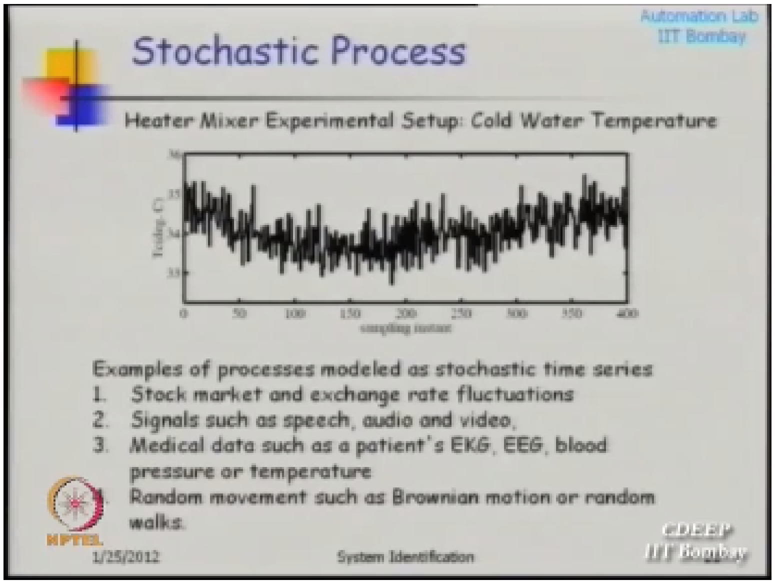

This stochastic variable is used everywhere okay, if you know about this you can enter into financial market and do you know stock markets that is exchange you can model exchange rate fluctuations, conversation currency conversations you can model audio, videos speed signals you can I mean just imagine when you are receiving data okay, in your mobile let us say it has signal, signal is the speech okay plus noise which has picked up and you better model a noise if you want to clean it right, you do not have to be electronic engineer to understand appreciate that the signal which you get here will have data you listen to it you will get you know funny sounds which is noise.

And it is not able to reject the noise so you get those funny sounds, okay. So medical data you know ECG and EG not EKG ECG, blood pressure and temperature data or Brownian movement okay, all these things can be model as stochastic process I am just showing you here measurement of a temperature recorded in a experimental setup that I have well there is no heating nothing there is a some I am recording temperature of water in the some.

Look at this date here okay, it is going all over the mean temperature you can talk about can you say what is the temperature inside this when you talk about some kind of mean okay, when you talk about you know standard deviation around the mean all those things you can talk about, okay. But now I collected this data for 400 samples next 400 samples if you collect data will you get the same sequence you will not, okay. Now how do I model this, so the way I am model it is by imaging that at every time point there is random variable attached and I try to model each one of those random variables, how do I model random variables probably density function, okay.

So if there are 400 samples and if I can find out those 400 probability density functions I am done, well looks very simple while talking, very difficult task, okay. And coming with an idea that one can model such a random behavior with a sequence of or a stochastic process to generations it is not at all easy.

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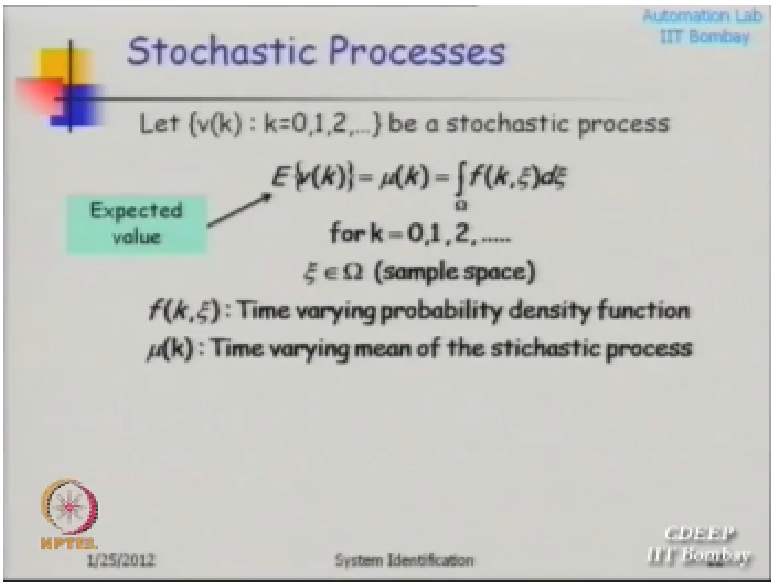

Okay, so let us see whether we understand this how do you characterize this I am going to sort of jump and go in terms of concepts what are the good books well, Astronomy books which are I have studied in my last lecture notes end of my lecture notes are Astronomy Victor marx gives a good introduction very brief introduction the kind of introduction I going to give because if you do not have time to get into stochastic processes full blown. There is a book by Astronomy on stochastic control systems which is also kind of a bench mark book.

If you want to know more about stochastic processes much more in detail in depth you should refer to Astronomy's book on stochastic processes, stochastic controls. Well, we have to talk about mean variance all the usual stuff and this operator e is going to hunt you for next few lectures. What is e operator expectation, okay expectation of a random variable, random sequence you can appreciate.

Now for a random process there will not be one mean value will be there one mean value, because we are looking at you know collection of random variables for each random variable there is a mean value okay, so that is why this density function here if you look here it has two attributes one is time attribute other is this you know probability space attribute let us call it ζ.

So I am only integrating this over ζ, okay so I get this μk, which is actually sequence of means okay which is sequence of means very important why d we plot mean temperature if I want tot study you know what is the behavior of you know temperature at 12o clock in Bombay what I will do is you know I will record data for lat 100years okay, collect data for lat 100 years on 25th of you know January at 12o clock what was the temperature I try to find a means today is mean to be different form tomorrows mean we can appreciate that okay.

I will try to find out a standard deviation today standard deviation is same a towards standard deviation okay. I might attach I might say that well that behaves like a Gaussian distribution this is module which I am proposing okay, it may explain my data it may not explain my data but you know you can propose some kind of a model and find out it's parameters estimated parameters but the mean for each day is going to be different okay, mean for each they are going to be different.

The standard deviation is going to be different if you take Gaussian if you assume that each one of them is a Gaussian random variable okay simple model with Gaussian random variables requires only two things bean and standard deviation okay the simple way of modeling and then I will have a stochastic process which is Gaussians stochastic process each one of them is a Gaussian variable and each one of them is characterize by is own mean and own standard deviation okay.

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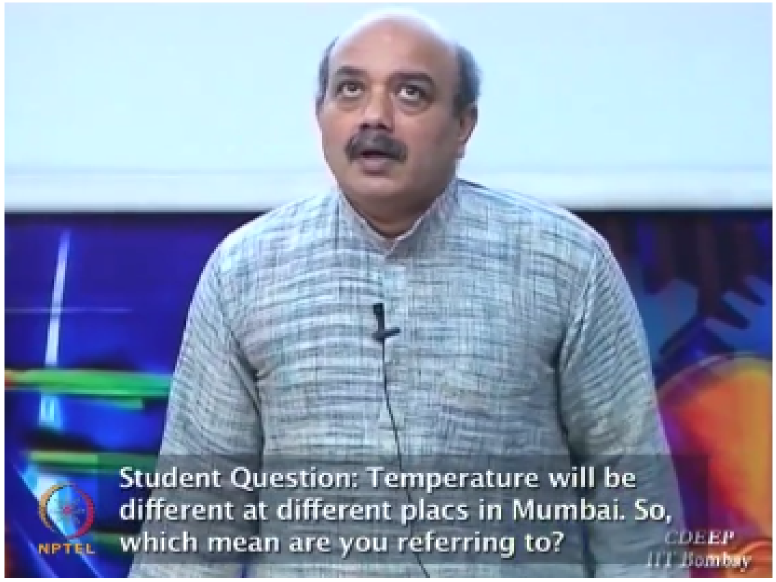

No this is bean of bean I am talking about you take mean temperature okay mean temperature in sense what I mean here is okay I should explain in the difference average temperature has in specially average temperature which temperature in this room or outside okay or in the tree or in the sun which temperature so I am talking about a average temperature not in time but physically you know what I will do if I will find out a mean temperature in Bombay I will put temperature measurement at some 100 locations in Bombay at 12 o clock I will find in the average.

So now I am talking of bean of mean that means I have every day you know I am doing this business okay finding a mean temperature in that in this area okay because what mean temperature in Bombay is is also can fresh and further, is it in IIT or outside IIT temperatures allover right. So that is what I mean here by and then the next thing is of course to look at distribution of this mean temperature in time okay what are all possible outcomes all possible outcomes are all possible values this mean can take okay.

I can collect the sample of mean temperature over last 100 years okay, and then, so mean here is first mean which I meant was special mean in the second mean which I am talking about is the temporal mean okay, now temporal mean will have a distribution right temporal mean will have a distribution.

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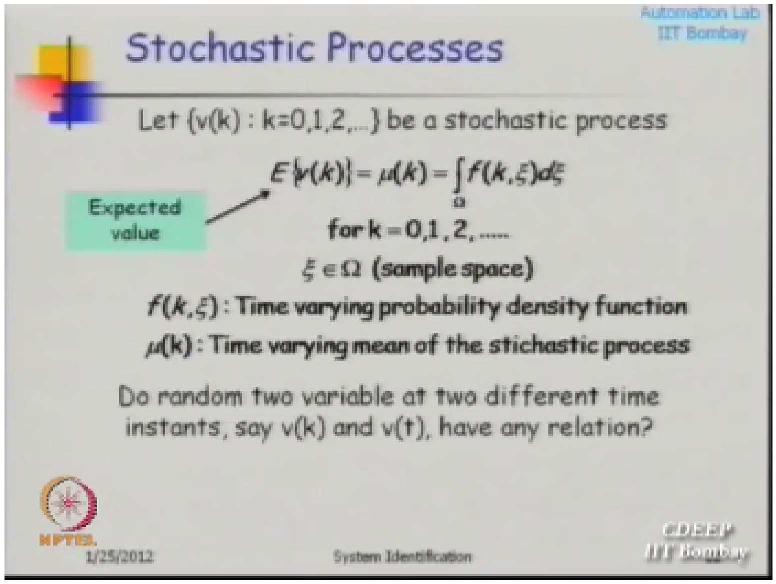

Let us not take into that right, it can be let some here to take let us look at now simple well example I get trying out to be more complex in what I am talk, heart beat example is better you know heart beat example is not distributed it is one person one heart beat right yeah.

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Yeah, at a particular time system I integrating over Ω sample space all possible values heart beat can take it cannot take negative values of course that is why see typically when you go to and put – infinity to + infinity there is no – infinity heart beat right so whatever 200 so the real sample space will be some were between it is the discrete space of course in this case heart beat yeah.

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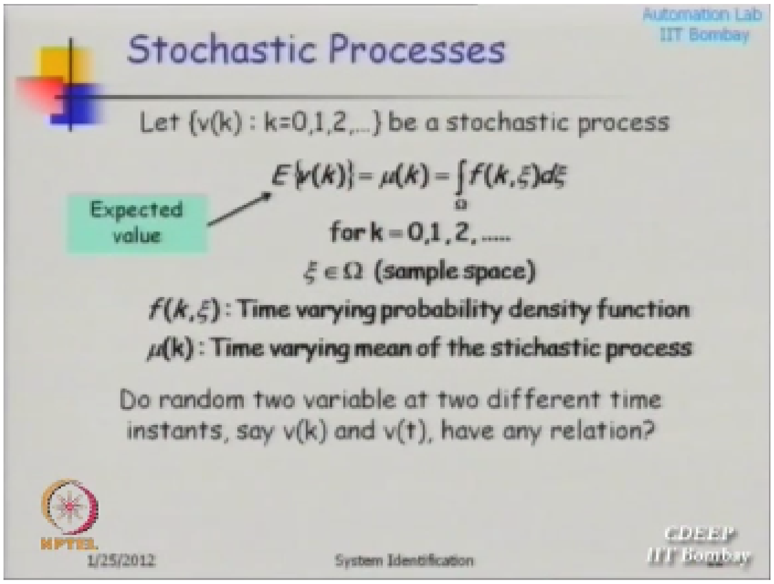

Now comes that cases okay I have a random variable now I have a stochastic process not a random variable I have4 collection of random variables I want to find out where randomness today and randomness tomorrow are they related okay or randomness today and randomness yesterday if I am looking at this average temperature business or if I am looking at heart beat is the heart beat now and heart beat 5 samples before is it related valid question.

If it is related how do I uncover this how do I understand you know whether temperature today and temperature five days back well you know from intuition that temperature today is relate to what happened yesterday and what happened day before yesterday you also know that where it is not quite related to what happened on first of January.

It is too much in past but last three four days they should be some connection we do not know what it is okay but if want to find out so the next question that I want to answer is within random process are they connections are they correlations between the randomness at instant k and instant t where the k and t I am just abstracting what I said okay.

I said that temperature today and temperature five days back are is it related abstract mathematical terms is vk and vt two different time instances k and t are they related okay that is what I want to do well what we do here is that we use second movements to characterize I do answer this question.

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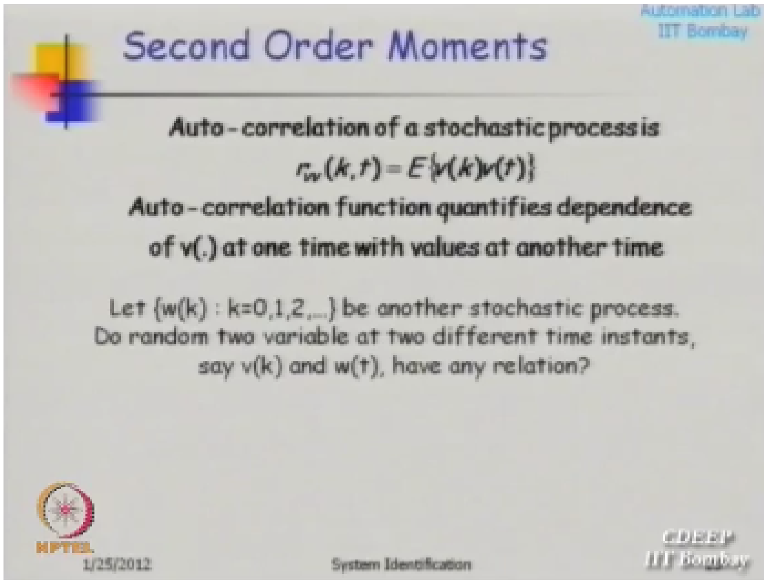

Okay I am going to define something called as auto correlation of a function auto correlation of a function what is correlation? Just leave a side what is correlation between two when you talk about correlation there are two random variables right and you can talk of expectation of jointly both of them concurrent together when they two physical like you know you talk of one coin and two coin.

Two coins being tossed simultaneously okay and then you can talk about joint distribution and so on now here I am trying to look at you know correlation between instant k and instant t okay what I am going to do is I am going to define something called as auto correlation auto co relation is expected value of vk vt right now. So expected value means it would need joint distribution function of k and t to be integrated with I am not writing all those definitions okay.

Because I am going to come with a simplified form little later this is more general definition which I gave you just now is expected value of vk vt so I need joined distribution of vk vt okay integrated over Ω 1 Ω 2 or Ω k Ω t okay then you will get this auto co relation which relates between k and t okay. So auto co relation function actually quantifies dependence of random variables within a stochastic process say I can ask this question I have measurement noise I just how your data temperature data inner some okay.

I can ask this question whether the temperature measurement at 200 instant does it have any relation with what happen three instances before? Valid question okay, so within a stochastic process are there relationships that is answer through this auto co relation function and then you can also ask the question whether two random processes are they correlated okay, let me take immunity average immunity average special average immunity let us say in IIT Bombay and average temperature in IIT Bombay okay if you start looking at this data okay as a sequence of random variables collected for 365 days valid question to ask he is the temperature stochastic process related to humidity stochastic process there.

We know that composite I know that humidity and temperature is core related right so if that works in physics it should work in Maths the stochastic process which is coming out or define for temperature should have some relationships to the humidity data right the two different stochastic processes are the core related I can ask this question okay.

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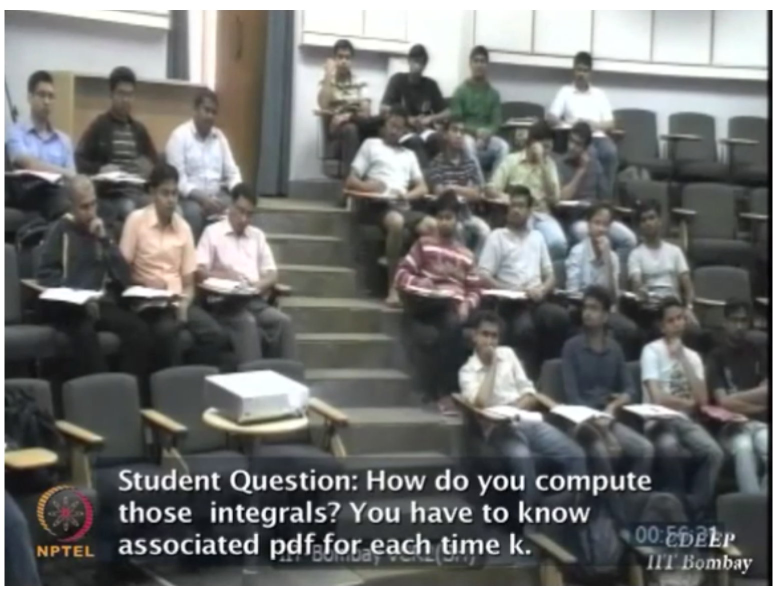

Yeah, no, no actually I am going to further simplify ideal these are the integrals and avoiding those definitions right now okay. I am not going to evaluate the integrals I am going to come up with simplifications which only requires some summations okay I am going to define a idealized stochastic process soon okay.

Just wait for a while okay you can ask this question again what you say here is the difficult it is knowing distribution function for each and then it is not possible but it is difficult okay in modeling for control the use some signified version see for example I given an analogy in the real systems you have you know the signals which are in a periodic in nature right.

But when you study those systems you study sin and cos why you study sin and cos those are some kind of idealized signals which help you to do understand mathematics or doing the mathematics you know is a very simplified way and then you develop you know understanding the system so likewise okay.

You know instead of modeling a signal which has multiple frequencies this is right, right now I am talking about signal which is completely general okay each random variables as a different distribution after some time I want to come up with the simplifications called I just stationary stochastic process I am going to say a sub classes simplified class of this is a stochastic process where random variables are equally distributed.

All of them has same distribution okay once I have that then you know I will I have some ways of handling the data which is easier than this but if you write more general definitions yes, what you say is true but to write the integral okay and then find out.

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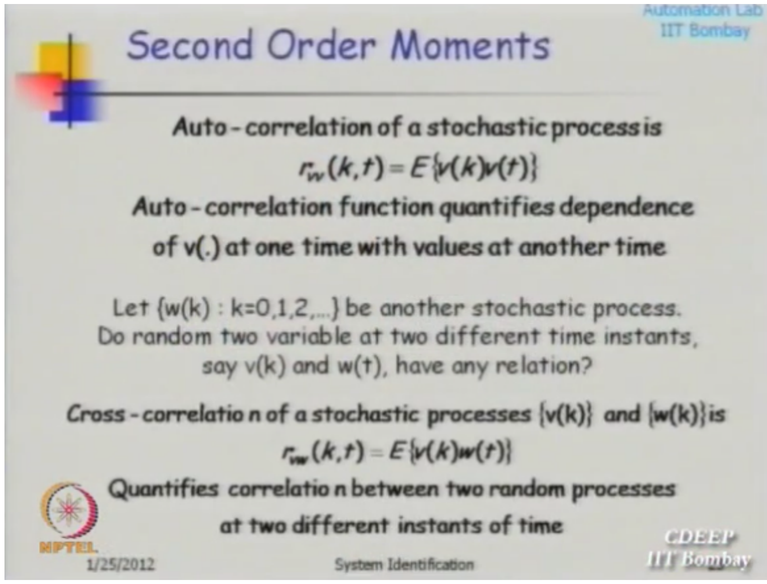

So cross correlation is relationship between random variables to different stochastic processes I example I gave you humidity and temperature okay and then I could talk about humidity three days back and temperature today are they related okay so that is what I mean here by wk or vk and wt two different time instances okay.

Is there correlation I can ask this question if I suppose by some means I have a model for these two stochastic processes I know density functions I have some way calculated okay then actually I could compute this integrals and find out the value of cross correlation okay I could find out the value of cross correlation between humidity and temperature which are separated in time right.
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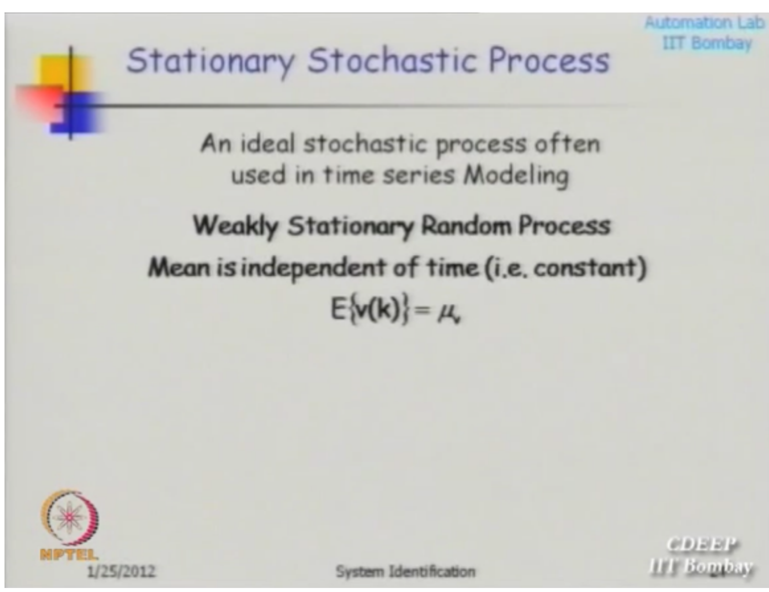

I could do that now it comes to the simplified version of this stochastic process which we use for controller in control systems particularly linear control systems okay so analogy is same instead of working with periodic signals with multi frequencies you know we like to work with sin at one frequency right sinusoidal signal at one frequency or you know real system probably there is never something like a step change.

There might be you know sudden rise in some input but you never have an ideal step change you cannot implement a ideal step change but the idealized signal we define a idealized signal call step input right we define idealized signal cross statement why we do that Maths become easy we can understand the system behavior and then extend ideas to know understand the complex processes.

So here to in the same analogy I am going to define an idealized stochastic process okay I am going to define a idealized stochastic process well the first thing is that the mean of this idealized stochastic process is constant mean does not change with time all of them has same mean, okay all of them has same mean value can you say this about heart beat of patient normal person you take normal person okay.

And I monitoring his heart beat after every one minute sampling okay but a normal person what do you expect will the mean change mean change and all will be locker for a you know patient or somebody you has under gone the surgery and post surgery might find some fluctuations and all that, that is different like a normal person okay.

You have you are monitoring his heart beat mean now mean after one minute, mean after two minutes, mean after three minutes, mean before five minutes from now we say mean is not going to change right so a good model a practical model for this scenario would be constant mean model all of them connection of random variable all of them has same mean absolute fine very nice signal okay.

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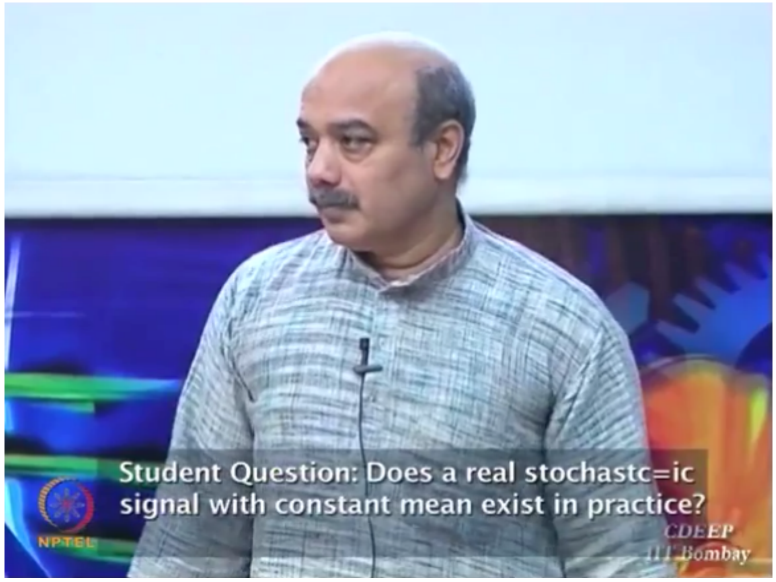

All of them have same way no, no do not confuse with the reality and a model this is a proposed model.

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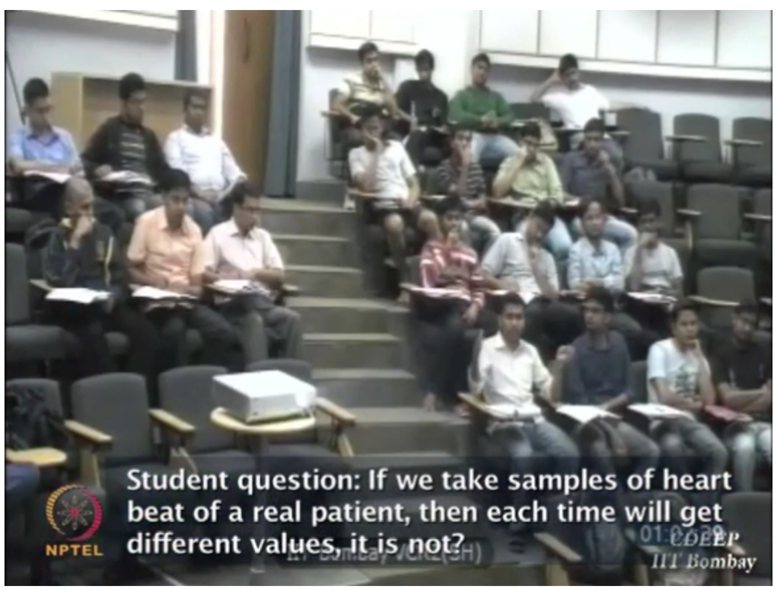

No, no one minute you are confusing between the estimate of the mean and a mean true mean I am talking about true mean of the distribution estimate of the mean might change if I take data for an hour I am talking about patient who is not you know doing too much for activities which are normal activities is doing for that patient of human being who is not you know joking for an hour and after that sitting and writing not like that.

He has some normal activity and then you are monitoring his heart beat okay so you are probably confusing between you know if I take data for one hour and if I take five hours the mean will be different the mean is not different estimated of the mean might be different I will come to the difference between the estimate and a mean.

I am talking about the true mean okay why do you find in the medical book or heart beat of human beings is 84, what does it mean? 84 per minute are a fine 84 minute or 72 per beats per minute why? Why? Why? Where do you this number you take any sample it will be 72 no, you take all possible you know measurements for at all possible times for human beings one or two might come up with exactly two.

But it will not be 72 right okay so this as this limiting value this is the you know distribution mean which is 72 yeah, if you take they know all possible not sufficiently high if you take all possible values that can occur then you know you get 72 that is what it means okay so here I am saying a stochastic process who is extend to mean for all possible values is constant.

But constant means at every take any time instant you have a stochastic process which has constant mean and which is same which is not change in time okay this is an idealized signal stochastic process okay in the same sense we do is idealized deterministic in sin ωt cos ωt okay.

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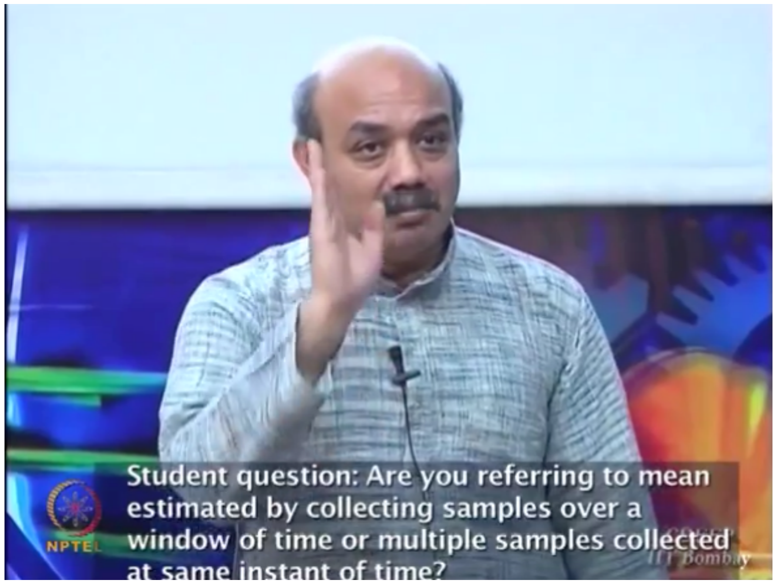

No, no it is a mean of that random variable not a whole type because see what we are going to do now, now it is a very good question okay because you know it is constant for all the variables we are going to mix the true we will take mean over time and say this is same as mean in the space so because is make this assumption I will be able to use data in time to estimate being in the space actually. I am saying that I am proposing a model that there is the constant mean okay, so if I take a sample now if I want to estimate mean from the samples, if all of them as the same distribution if I take a sample now upto next instant that does not matter, I collect all of them and take a mean.

Because all of them are coming from the same distribution same mean okay so that actually helps me to simplify computations. So this is an idealized model okay there is one more thing required for this idealized process okay.

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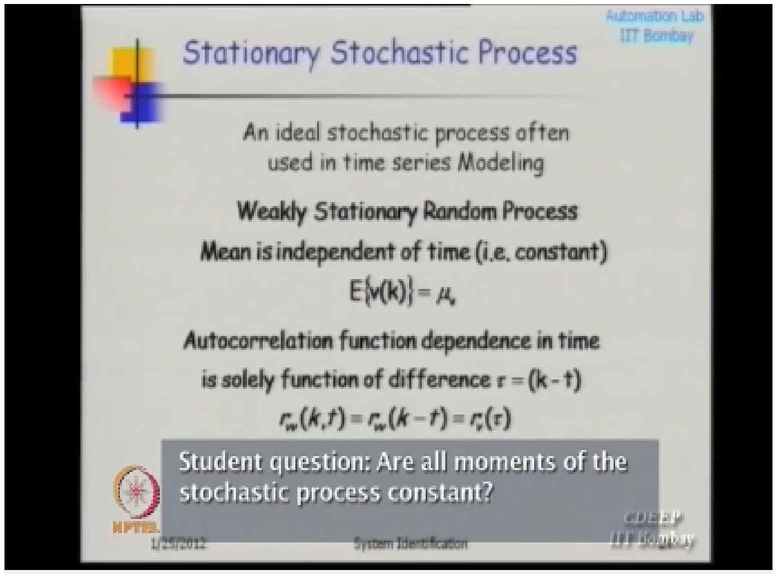

Well you have good question, so we have two distinctions here, strongly process and stochastic process, so weekly stochastic process only 1st 2 , moments are constant okay for the strongest stochastic process all moments are constant which means same distributions, so for practically purposes work with weekly stochastic process.

I am not getting into all these in the 1st lecture okay so, you have to jump and ask great 10 questions when I am right now grade 1oaky. Again important thing for this weekly stochastic process idealized process is that if I take auto co relation between you knows what happened now and what happened say 10min before okay that should remain in variant.

If I do it now in the afternoon, in the morning in the night if the patience you know for the model for the patience heart beat is a stationery stochastic process model okay. Then what should matter the time gap between the 2 samples, if I have to say what is the relationship between the randomness at 12 o clock and 11.50 okay that should remain as relationship between the randomness at 4o clock and 3.560 okay.

Only what matters is the Δt k – t matters okay, it should not matter where I am in time okay, such a idealized process okay it is called as weekly stationery stochastic process well I already thanks to you I have already qualified you strongly stochastic process, and weekly stationery stochastic process. Weekly stationery is only 2 moments are satisfying these conditions we cannot anything about the higher moments.

The 1st two moments satisfy this condition then it is called as the weekly stationery process, if you take all of them have Gaussian distribution with identical mean and variants and you know auto co relation then it follows that identical distribution what we work in practice you know I am starting with more general thing.

What we work in practice is a Gaussian random variables which are identically distributed that stochastic process is very simple to handle and most useful in modeling, so what we use in practice is which are identically distributed all of them have same mean same distribution, whatever I am assure them is that all of them are Gaussian. Life is simple after making this, probably when people started looking at this simple process by the time it came he came up with very general theory you know.

Which is stochastic process where the randomness can change from so if you start reading a book it will start with more general thing and then it becomes and understand why you need such a you know complex math's but any way we will be suing stationery process mostly, so that what you asked smith asked is very good question because of this particular thing, because all of them as same distribution.

I can just pick samples in time and then estimate mean, estimate is that okay everyone with me on this, you want me stop anywhere here no okay. Now so what is the advantage I can estimate the statistical properties using samples collected in time, see what will happen can I find out average temperature of today by taking samples in time, I cannot right.

A stochastic process which is you know the 1st one which we are talking about you will have to collect average temperature on 25th of January for last 100, 300, 400 whatever those many years, you cannot take time samples because this is not the stationery stochastic process okay. but heart beat model is the stationery stochastic process, I can take heart beat 10min back now after 10min, after 15min I can collect.

Mean is constant not changing with time okay so for that model I can use time data to here I have to do time data means I have to collect data you know in a different way right you understand the difference.

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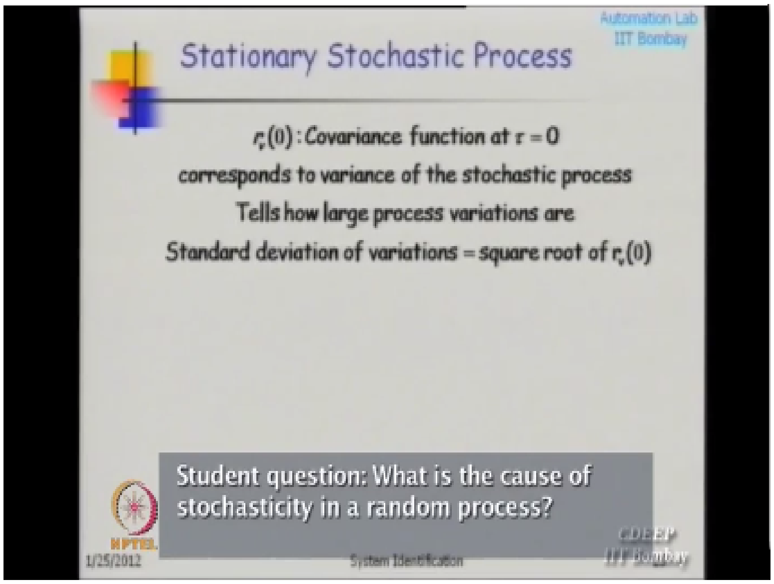

Well right now we have not talking about input modeling we are just talking about random variable or a sequence of random variables all of which as same mean. We have no clue there is no cause and effect modeling right now okay. There is no attempt to assign any cause it is just when I am saying if I develop a model for that temperature you know as a function of loose of time in a day.

Using some data over 200 years it is not what I am saying is, first of all it is a model that you have to understand it is the proposed model okay and can you repeat your question.

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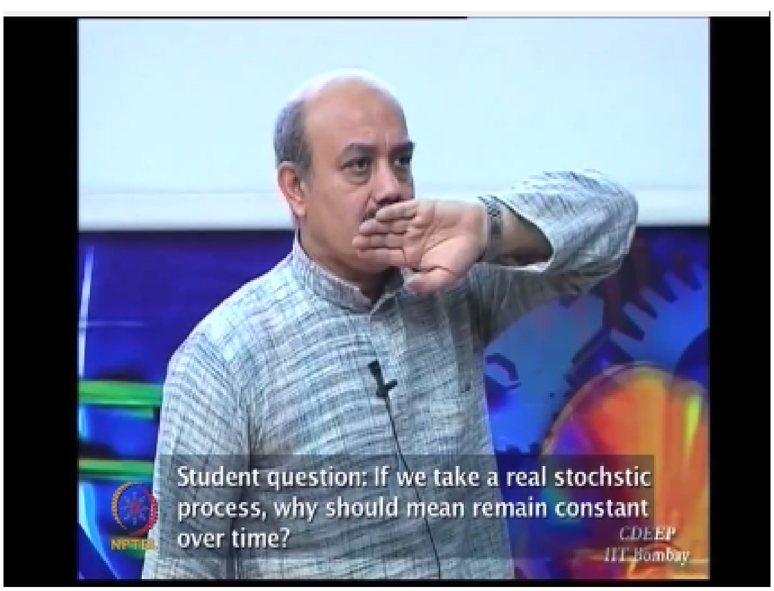

So mean is constant means what? It only says that expected value what is the best value that you can expect next time is constant okay every time if you ask me to say what is the best guess you can give you will say 72 because that mean is not changing with time okay, so but you have to understand it that this is the proposed model okay, this is the model which fits see in chemical engineering we study CSTR.

If you go to a real plant you will never find a CSTR I mean you will find something which is reality CSTR and PFR it might be close to this idealized thing CSTR and PFR but they may not be you know a perfect. So perfect mixing base reactor is an idealization okay so constant mean over time okay, set of random variables whose all of which as same mean is a idealization. We can see as we go further you can probably ask me again this question yeah.

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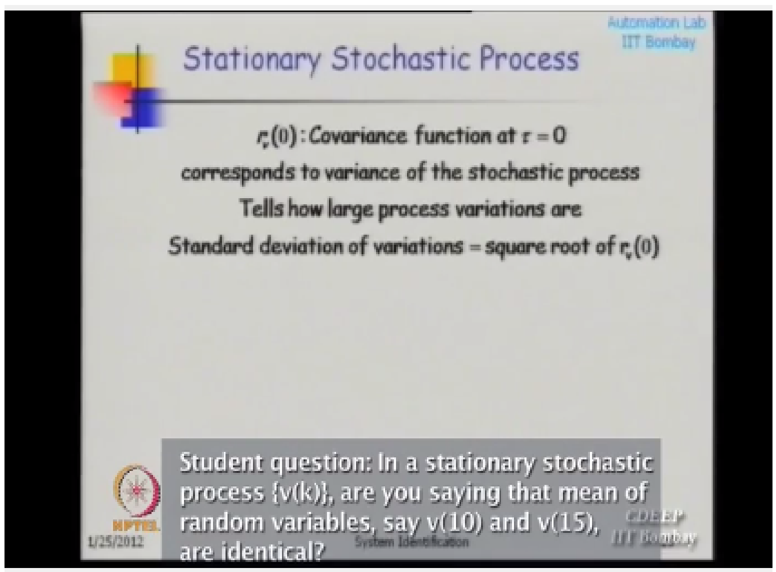

All of them as same mean, every minute see I have one random variable at each minute, I am proposing a model understand this that all of them as same mean okay. it does not change with time, so because of that you know what happens next instant is from a realization of a random variable which as the same mean okay, it is assumed as the distribution if that as same distribution then I can use a sample at this time, sample at next time.

This is not possible for the stochastic process where the temperature is taken over a year; I cannot use the data of yesterday because yesterday mean is not constant.

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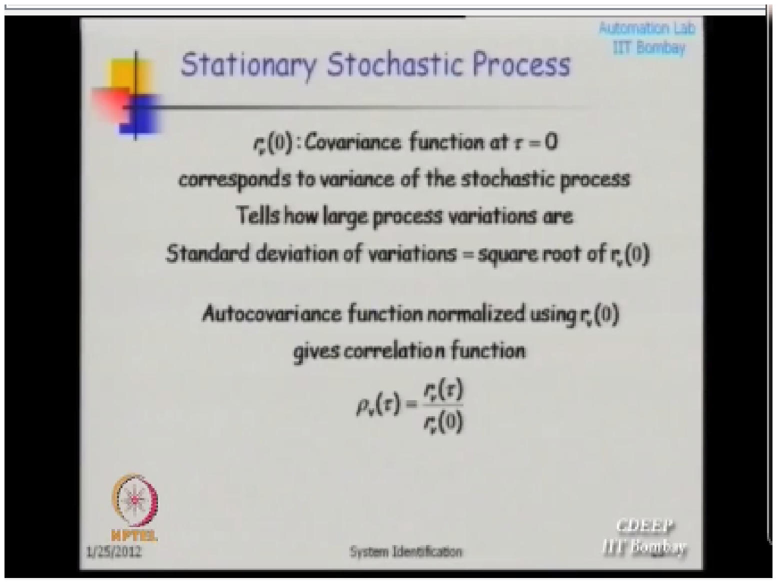

So one can of course propose a constant mean model for the temperature variation but that will be a bad model that will not fit the reality okay.

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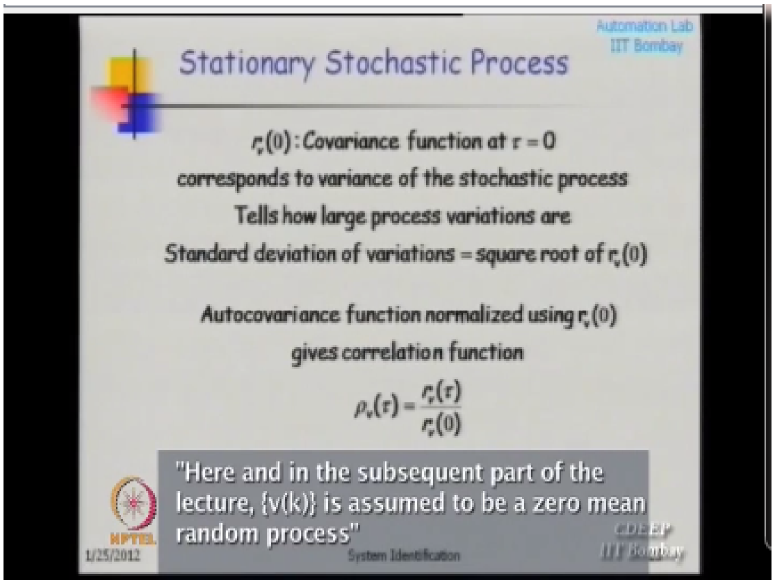

Real data when you will understand this now I have a problem of chicken and egg I will first define the language and then start using it okay so I am defining all these term which I am going to use subsequently auto correlation function and you know so what is expected value at when k and t both are equal to 0 what will you get? See what expected value is of?

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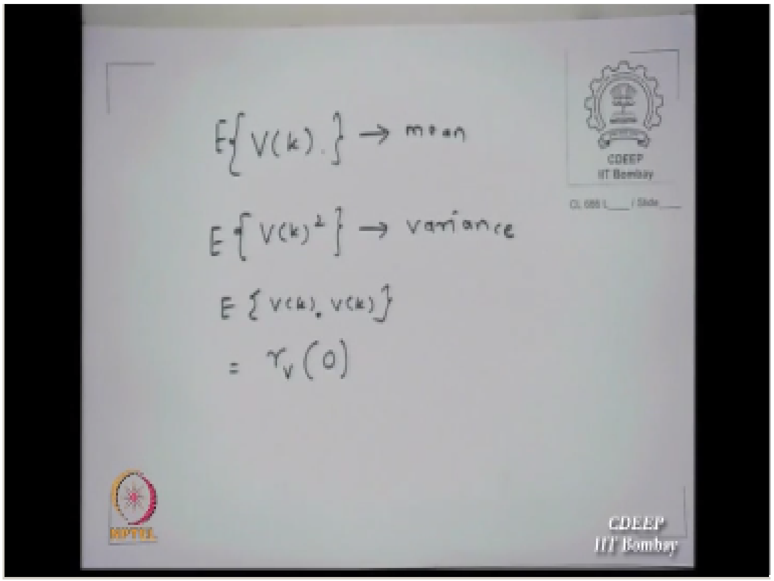

We have this random variable vk okay for the time been okay let us consider expected value of vk will be mean right okay what is expected value of vk^2 is variance right this is nothing but expected value of vk,vk or vk.vk so which is same as see how did we define correlation function okay to define the correlation function here which is difference between k-t okay just a minute let us look at here yeah

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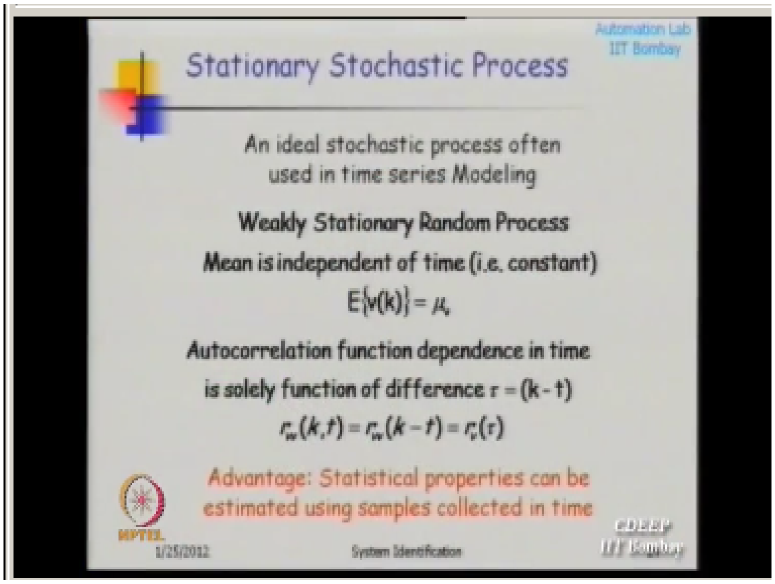

So see what is auto correlation of bk with bk I can ask this see we are talking about vk with vk-1 okay vk with vk-2 what about auto correlation of vk with vk will give you what will it give you variances is a single variable it gives you variances okay one we are not talking of vector of random variables we are talking of one random variables yeah so it will be v^2 know just go back here.

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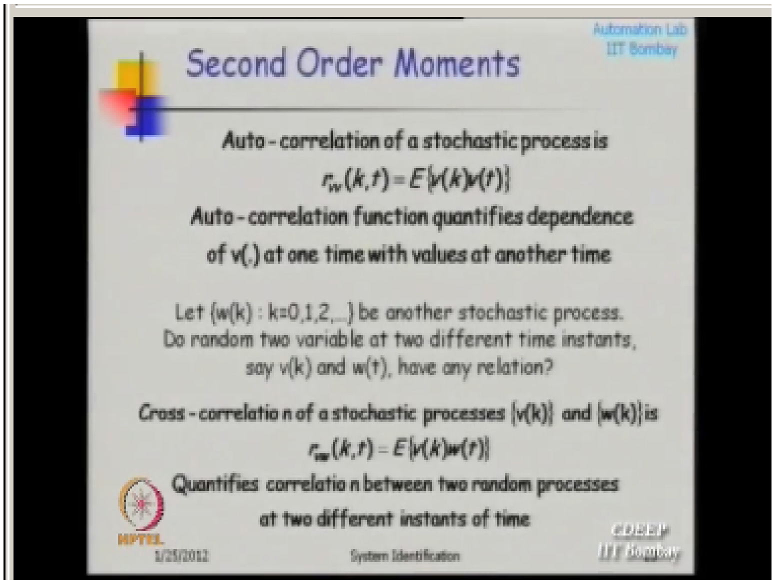

Now correlation variances will be b-u yeah correlation yeah correlation.

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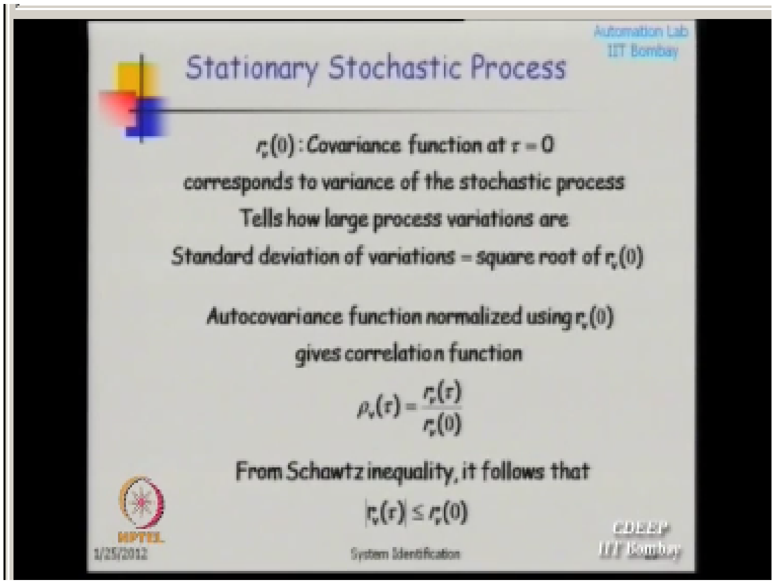

Yeah it will be co variance correlation function this is here I do not know to modify here this machine is not a so correlation function at τ=0 will be same as expected value of bk^2 and then it tells you how about the variability if the variable is 0 mean it will be variances right the variable is 0 it will be variances same as variance and a standard deviation will be square of that will give you standard deviation what we normally do is that we to study you know correlation in time we normalize this r0 and then plot okay I will show you this plots and normally what we know is that r0 is the maximum value there can be no value greeter than r0 okay so within a process to cystic process you can show this I am not going to do this derivation we can prefer to books on.

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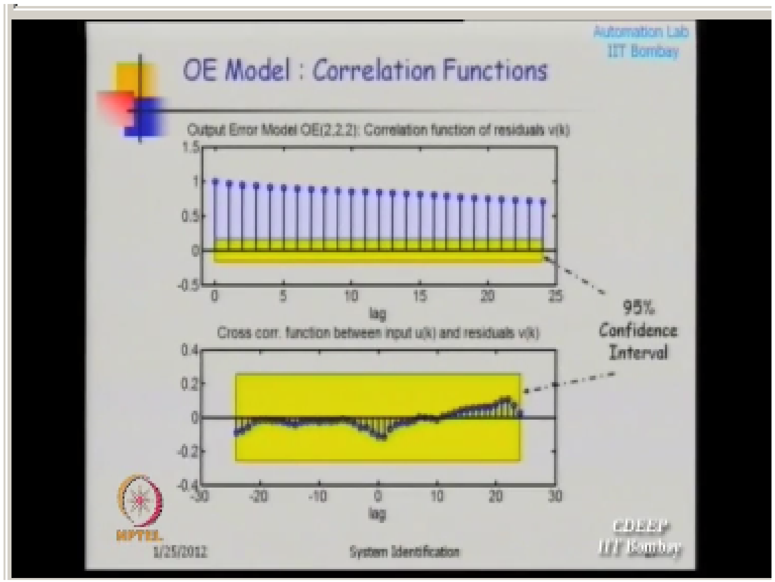

So I will just show you, this is okay let us go back and here okay this is the signal you remember this signal that we have the okay now I am going to apply this ideas this signal I have this signal here okay I am looking at this as a simulated process there are 250 samples I think there are 250 samples yeah okay so this is a process I want to ask question whether vk and vk-5 vk and vk-6 vk and vk-1 are they correlated okay I am going to estimate this using the sample data I have not yet come to the point how to estimate these numbers from data samples but let us say there is I want to talk about its soon.

But before that I want to show you how this correlations look like okay so a model see I am trying to construct a model now by saying is there a relationship between a randomness at time k and k-1 ka and k-2 and k, k-3 does not matter if I go at 150 you know 150 and 145 okay or you know 100 and 95 or 95 and 90 are they related okay I can estimate this using this auto correlation function okay auto correlation function can be estimated directly from this data.

Okay now I am going to go you know for the way I am going to show you the plot of auto correlation first and then I will come to how to compute it okay so let us get first physical inside into what is auto correlation so what I want to uncover here is there relationship between you know take any point here let us say this point and five samples before or ten samples before or 20 sample before are they related okay what I am going to do is to look at auto correlation within this process okay.

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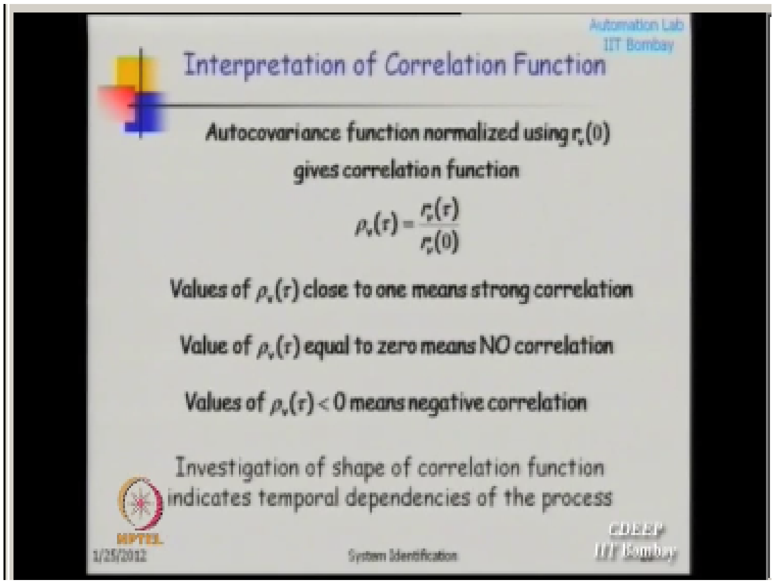

That will tell me how does the auto correlation plot look like you see this graph here I have plotted two things here one is auto correlation that is vk-1 so this is what is this first point? Vk, vk lack 0 okay what is this second point this is r0 the first point here the first point here is r0 what is this r1 what is r1? Correlation between vk-vk-1 this has been normalized using a variances at 0 okay so that is why maximum value it takes is at instead one instead 0 lack 0 is at lack 0 and it is 1okay.

But you can see there is a relationship between randomness at k and k-1 k and k-2 and k and k-3 okay so this is a so this simple calculation of auto correlation tells me well I am assuming that this is a stationary process okay this is a stationary process I model it at a stationary process and then I am trying to find out correlation within this randomness okay so that gives me this particular plot so tells me that this is not completely random what is happening at k as relation to what is happening to k-1 lack 2, k-2, k-3 in fact you can see even if you go 25 instead in the past there is still relationship and very strong relationship okay what about relationship between u and what are the two sophisticated process input which is given to the explication as an signal.

And trying to find correlation between the vk and uk that is the second plot will not interpret it right now just look at this if this correlation coefficient okay if it is closed to one that means there is a strong correlation okay to this correlation coefficient is closed to 0 which means there is no correlation okay if this correlation coefficient is positive then there is a positive correlation if the correlation is negative it is correlation okay and auto correlation can be of different shapes.

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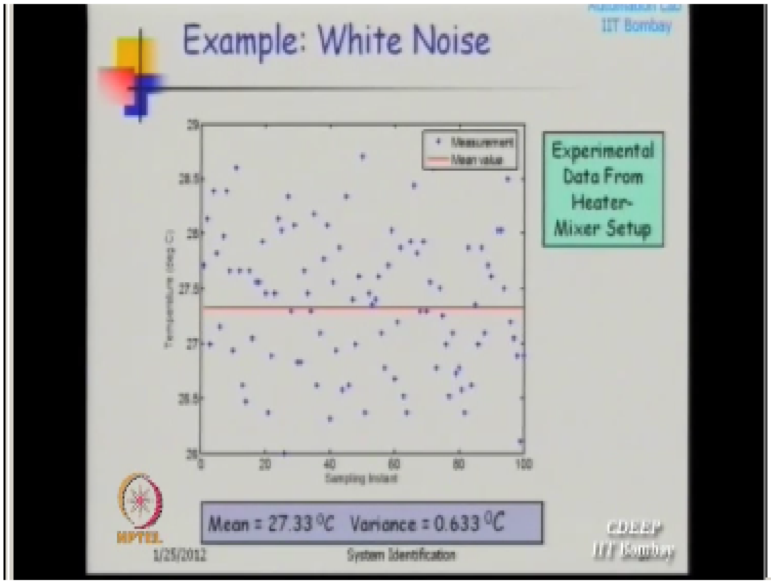

I will give you another okay this is example I am going to talk about this white noise after some time so this white noise is sometime in which there is no time correlation what happens now has no relations to what happens after sometime this is data collected from a real temperature bath a constant temperature bath you can see samples are all over right they are not constant.

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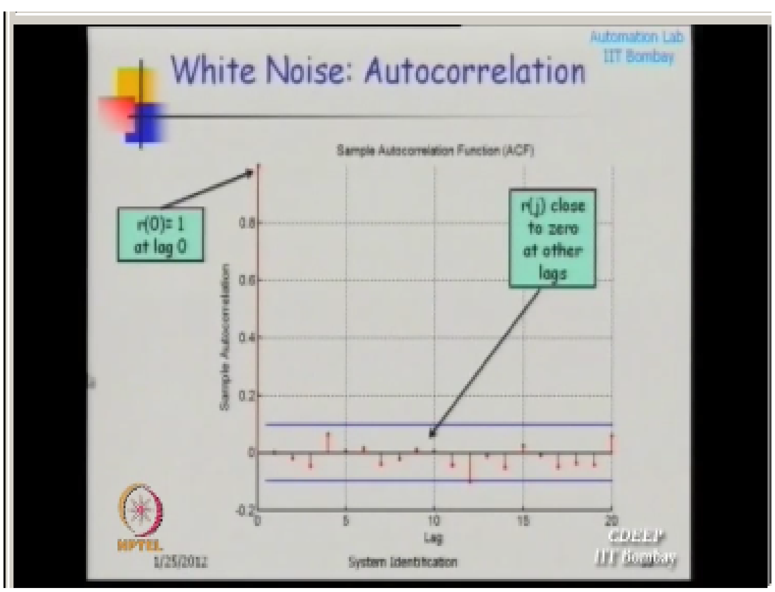

And if I find auto correlation function it is very close to 0 for any other lack it is closed to 1 for lack 0 yeah for lack 0 it is 1 and for all other lacks it is closed to 0 okay so this simple calculation of auto correlation tells me whether randomness now and randomness sometime in the past are they related this is the model that you are processing okay.

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