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ADVANCE PROCESS CONTROL

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Lecture No. 04

Topic:
Development of Control Relevant Linear Perturbation Models - 1

Sub-Topic:

Introduction to Z-Transforms and Development of Grey-box Models

In our last lecture we were looking at Z-transforms, I just briefly introduced Z-transforms, so Z-transforms are defined for the sequence of vectors or a sequence of scalars, infinite sequence world from time 0 to time ∞ it is a discrete sequence.

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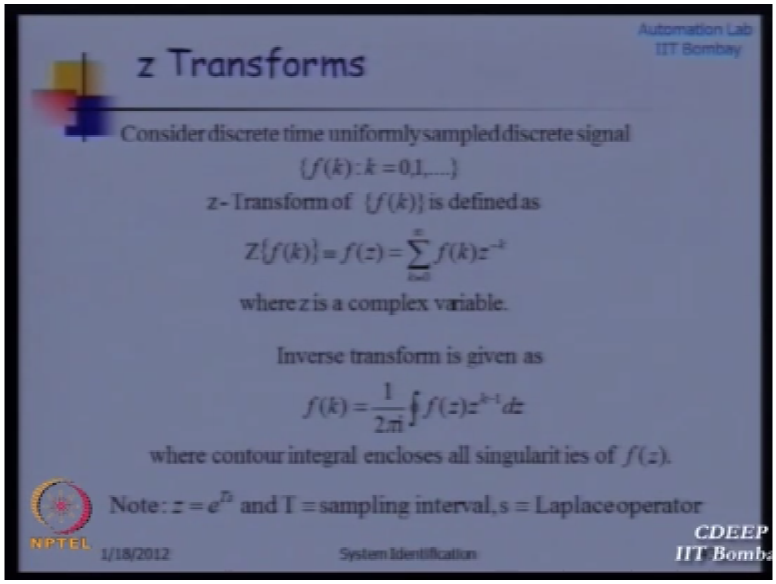

The set of values which are connected at uniform sampling interval, whatever it is could be, as I said it could be one second it could be 0.1 second, it could be 1, depending upon the application. And Z-transform of this infinite sequence is defined by what is given here, and corresponding the defined, so Z-transform converts the signal from time domain to Z domain. And it can define inverse Z-transform which converts the signal from Z domain to time domain okay.

The relationship between S and Z-transform, and I am going to derive it in this class, I am going to leave it to you to read about it, I have given two reference at the end of these notes, I represent giving notes slightly revised notes, we make our corrections in the last lecture. So I opposed those notes, so you can have a look at it.

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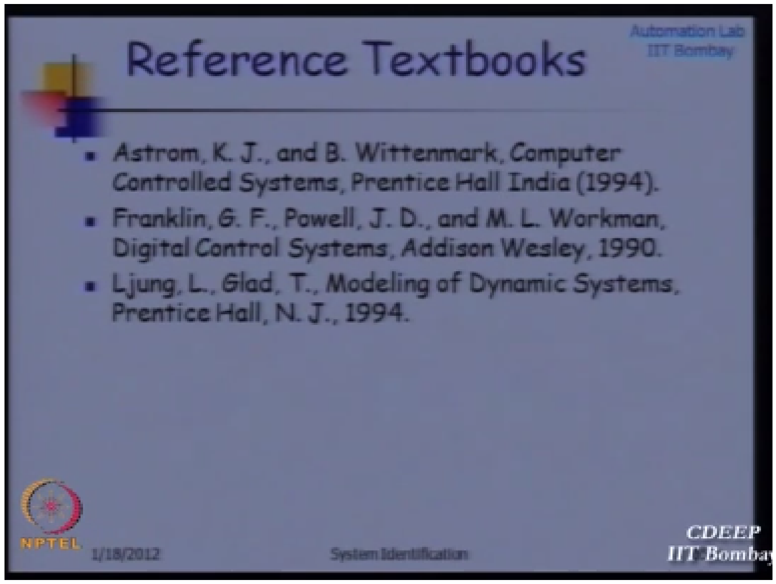

At the end if I put three references which, so at the end I give three references, the first text book here is Astram and Wittenmark. You can say this is like Bible of digital control, if you want to know, if you want to perceive career control systems, this is the book which you must have. This was, what I have given here is one which I have published by Prentice Hall India no longer Indian nation is available.

International nation of this book is of course available, you should buy edition 3 if you are interested. The second book is also equally important reference, a very good book Franklin and Powell on digital control systems. Throughout this course I will not going to refer to one book, I will do with multiple books, I will keep giving you references. It is not possible to come up with one single reference for this course.

Because this is your advanced control course which is my version okay. So advanced control is a notion Mendel, I am picking up a few things from it and put it and giving it to you, because what I think is relevant is, what is represented. So it is going to be collection of the new material form variety of sources. So at this part I would refer to this particular text book okay.

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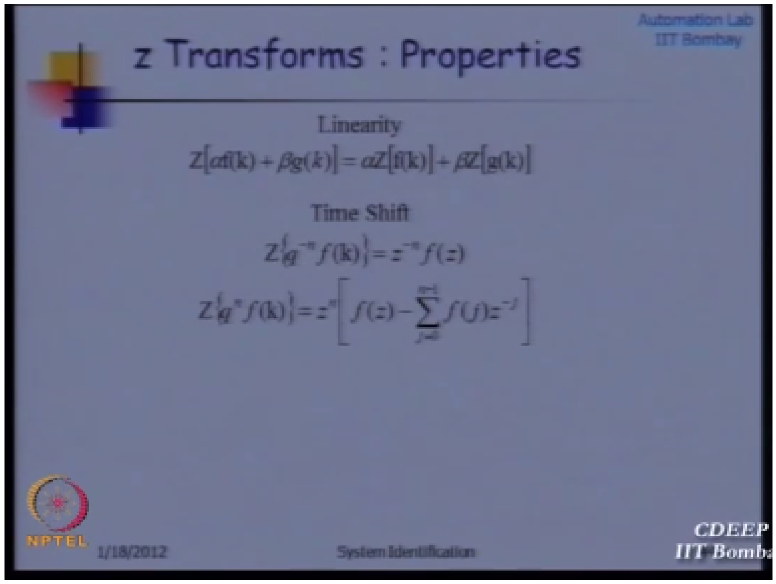

So let us go back and have a look at Z-transforms okay. So we looked at the properties just a quick review that linearity modes if you add to signals, but if you take two signals and take a linear population of two signals and take Z-transform is nothing but Z-transform of a linear combination of Z-transform will be reduces. Then time shift is one of the important property for us, this time shift property is used to convert difference equation into an algebraic equation.

Just like analogy is used, Laplace operator to convert a differential equation into an algebraic equation. Likewise, here Z-transforms are used to convert a difference equation into an algebraic equation, linear difference equation. So this is the fundamental result which is used for converting, and you can see similarities between the result in Laplace domain. In Laplace domain if you are nth derivative, you will get S^n if you remember, you will get S^n.

Here you have a difference operator which is n steps higher in future you will get Z^1, Z^1 you see that, that is quiet close resemblance between the – and you have initial value theft in mind, final value theorem which are analogous to what you have in continuous time or in Laplace domain.

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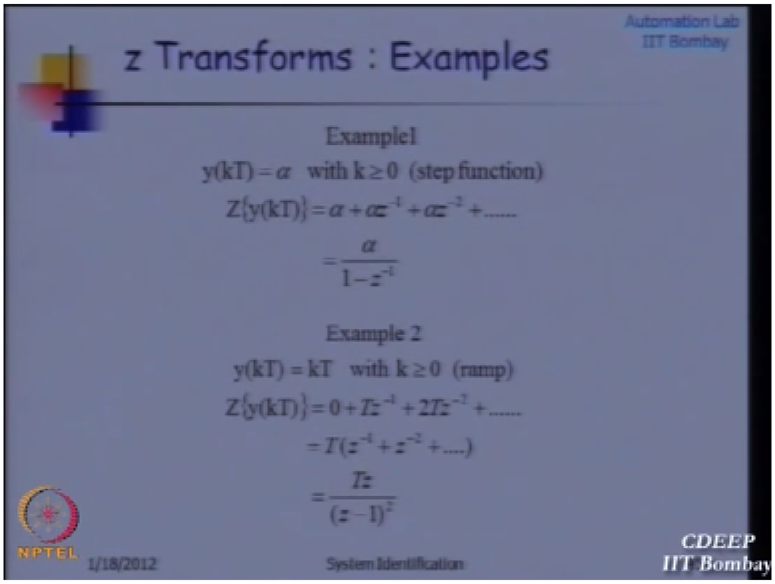

In the last class I gave some examples of Z-transforms the simple signal which is units that at 0 before time 0 and becomes value α at time 0 to ∞, such a signal if you take Z-transform we will get Z^{-1}. And likewise if you obtain this book computer control systems or Franklin and Powell give us the examples or you will see tables of Z-transforms of variety of signals.

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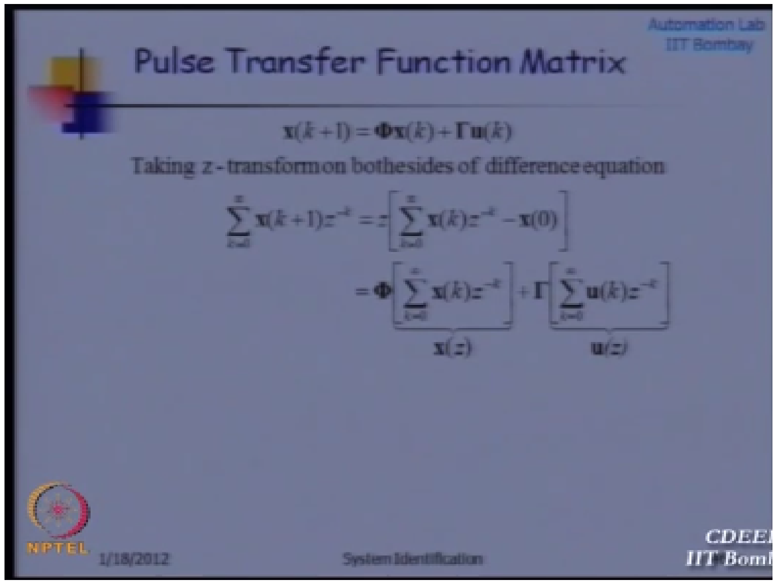

What we are interested in these pulse transfer function. And this is where we were in the last lecture, this is what I wanted to. So I said the reason why we wanted to introduce the Z-transforms is because we want to convert difference equations into algebraic operators or transfer function matrix, that is what it would be okay. here instead of Laplace transfer function matrix it will be a Z-transfer function matrix.

What I have done here is I am starting with this difference equation and I am taking Z-transform on both the sides. If you see here, I am taking Z-transform on both the sides okay.

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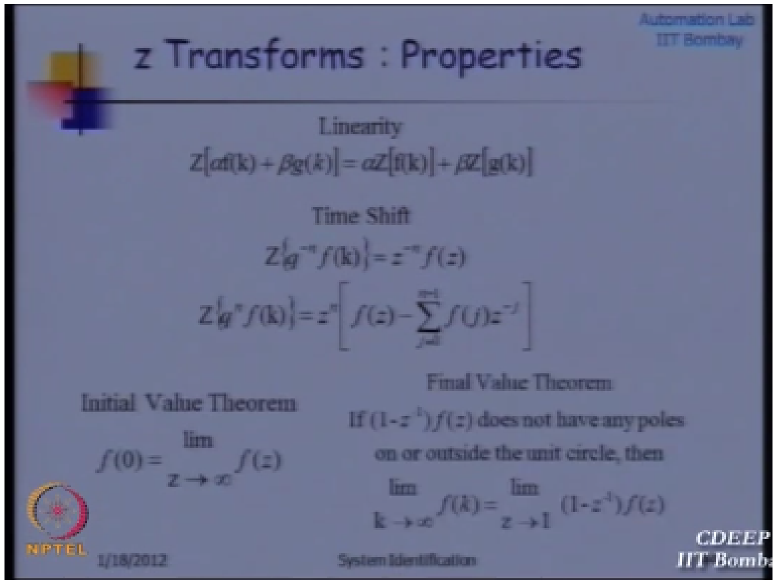

I am just going to use this time shift formula, the one which is the second one, I am going to use this formula here. In this particular case I am taking Laplace, I am taking Z-transform of Xk+1 , Xk+1 would involve here X0 okay.

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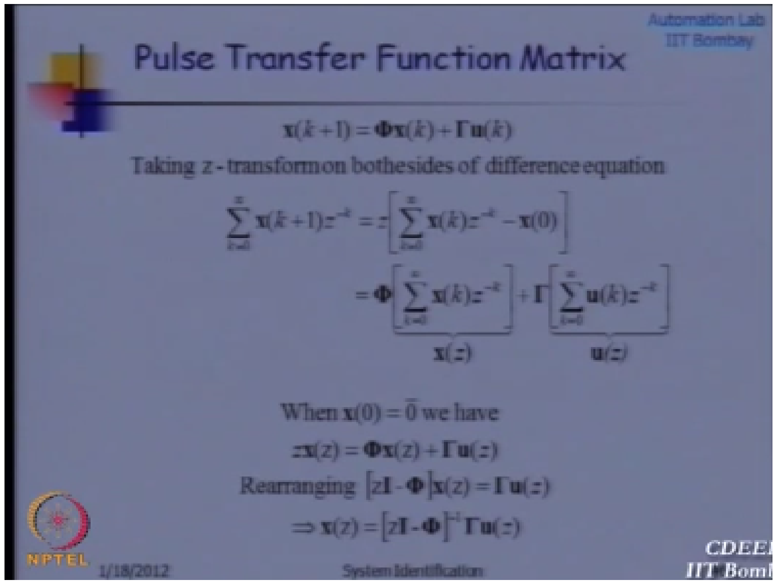

If I just use that formula and come up with the Z-transform I get summation K going from Z to ∞, I take the ∞ x(k)Z^{-k}, so this is nothing but xz and you have this term X0 which is appearing, and I am taking Z-transform with the right hand side. So if I actually combine this now, if I combine this and make one more assumption that initial state, initial perturbation at time 0 is 0 which is usually the case when you find out a transfer function matrix.

Whether it is in continuous domain or in Laplace domain, or in Z domain. A usual assumption is that initial state is 0 with that you will get this transfer function matrix. Here all the competency here this term is nothing but xz, I am taking x0=0. So this term here collapses to z xz, right hand side is five times xz, and γ times uz okay. So this is just the next step just using the definition of Z-transforms okay.

Then I will just rearrange, remember this ϕ is a matrix, so zi-ϕ times x(z) γ(z) and this gives me a relationship between the space and u(z). When you derive a transfer function matrix your purpose is to come up with relationship between input and output, I want to eliminate the state okay.

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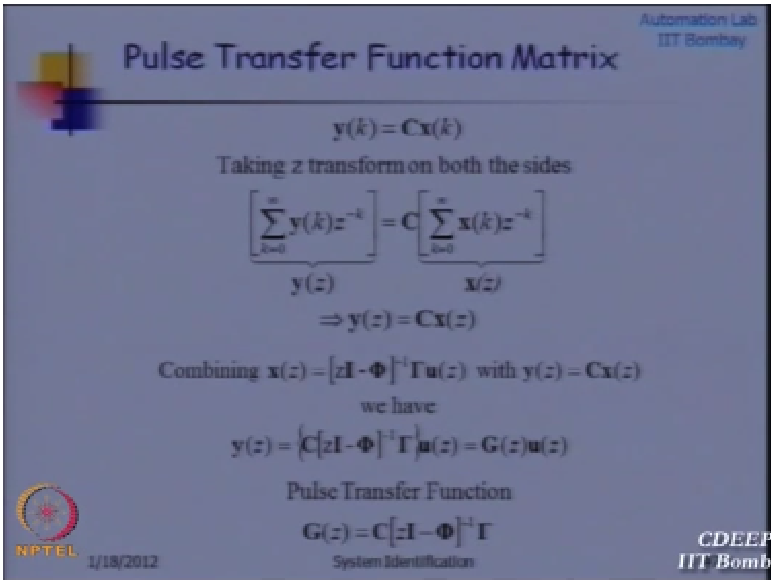

So I am going to look at this relationship Y are the measured outputs and X are the states, C is the matrix that relates measured outputs with the states, or perturbations in the measure of could be the perturbations in the states. And then if I just get Z-transform of both the sides I get y(z)=cx(z) it is a straightforward, anyone hear about this, I am just applying a definition of Z-transform.

Now I am going to combine these two results, I am going to eliminate X, I want to have a relationship between measured outputs and manipulated inputs okay. So what you get here is this matrix $C(zI-\phi)^{-1}\gamma$ okay, again you will notice that this is very, very similar to what we have for U transforms okay. So shift and get a transform, this is very, very similar and you get g(z) u(z), this is called as a pulse transfer function again same thing.

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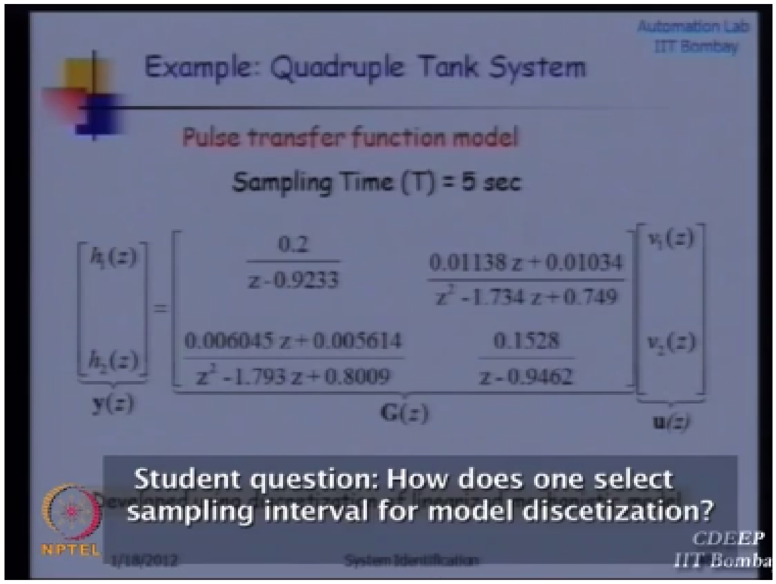

While if I do it for quadruple tank system, I will get this transfer function matrix between two level perturbations H1 and H2 and 0.2 voltage input perturbations okay. So these are my two, now just remember right now what we are doing is going through developing a control relevant model starting from a mechanistic model, starting from a first principle model or a group of model.

You have a good model which is coming from physics or mixture of physics and correlations. And then we are deriving that model to come up with this control relevant model, this model is control relevant model, I can use this for controller synthesis okay. yeah, sampling time typically you choose based on what is called as, you know turbulent frequency or the fastest frequency in a signal okay that you have.

Now what is the fastest frequency in a signal time, you have to have some oplary idea about how fast your particular system behaves okay. So one, there are multiple ways of choosing a sampling time one way is that you find out the smallest time constant, you have to send what a time constant is yeah. So find the smallest time constant, take one tenth of that or you find out the settling time, you know what is a settling time, you do a step change and find the settling time and take 100^{th} of the settling time.

So here I have chosen five seconds as one tenth of your shortest sampling, the shortest smallest time constant okay. Actually the tendency in the industry is very, very small sampling time. Well it is not required okay, it will tend to sample temperature for this at every one second. Where the thing is going to happen for a minute okay. So actually you should chosen sampling time based on the response time of the system itself okay.

So here five seconds is because the smallest time constant in this particular system is about 50, so I have taken one tenth of that probably okay. You can find out good discussion about the choice of sampling time in Franklin and Powell or in Astram and Wettenmark okay.

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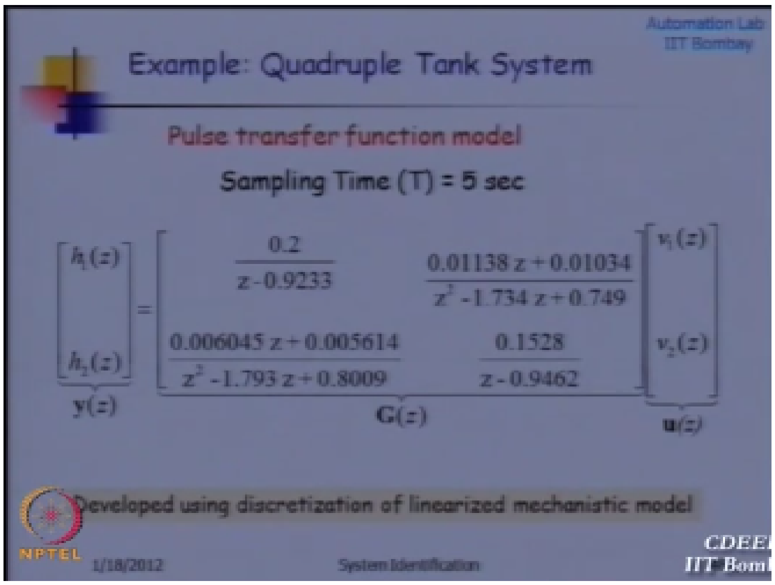

So what we are dropped here that we come back what we have got here we have to eliminated the state variables there are four step and there are four levels, now this transfer function matrix relates only measure that is with manipulated inputs okay it does not say anything about unmeasured levels okay.

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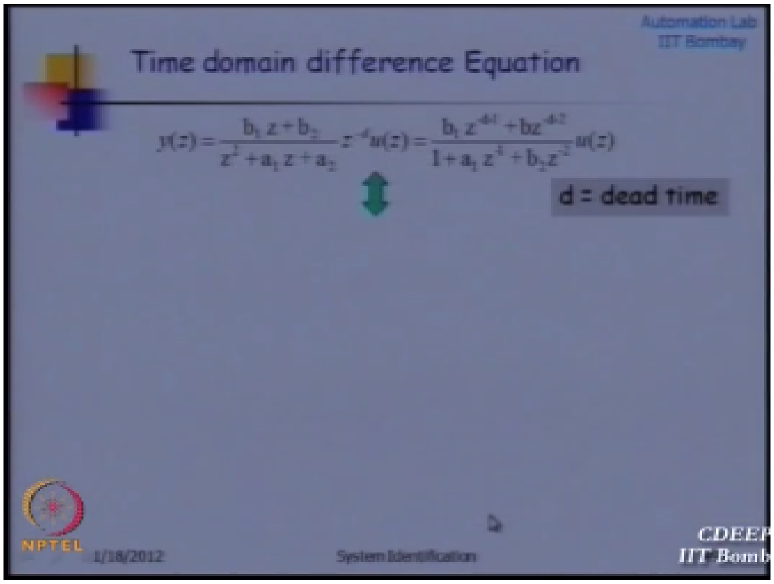

Now I would go back and relate to time domain okay, so you should be very, very comfortable going back from time domain to Laplace domain or Laplace domain to time domain or zee domain to time domain to z domain and so on, okay you should go back and these are only representations it is only one model which has been represented through move on which being used through different filters or different you know different inspects it specific thing ultimately so I want to show that this model.

This is a model between some wide and some I am picking up copy this right now we will say that say H, H_1 B_2 or H_2 V_1 and so on I am just saying this that one general transfer function between some how to write okay if you get some how to write and some into let us assume that there is a transfer function is given by B1z + B^2 upon z^2 + a1 z+ a^2 or else 1 more time of here and here okay z – b okay now this z – b actually represents time delayed or dead time or other what is a dead time.

How many of you know about what is a dead time everyone knows dead time is the time required for you know it has different interpretations in different context it could be time with requires of actual terms to start doing up will give a signal it could also mean that I replied for the measurement to reach the computer that d when the measurement then have relevant temptations, so a time delay of d samples the model for that this tells out to be z^{-d} okay if I have a dead time of g samples.

The d samples then the model is z $^{-2}$ so I have combined this model which has a time delay of d samples and then I get this model is z $^{-1}$ I just want to convert it that into time domain so I am cross multiplying and then you take z inverse to z inverse to write the z inverse we were going to compute to integrate there you just go back using the formula that time shift if you see this formula you will see that it is very easy to go back to time domain difference equation okay, so here z $^{-1}$ simply differs yk – u + 1 then z – t you operating on z or z $^{-d-1}$ operating of u.

Simply becomes k – d – 1 okay k is the current I know that k is not parent time k – while in k – 2 and so on, okay so I get this linear difference equation in model starting from the original transfer function model is it clear when you do this difference equation model, I have written it as current equal to.

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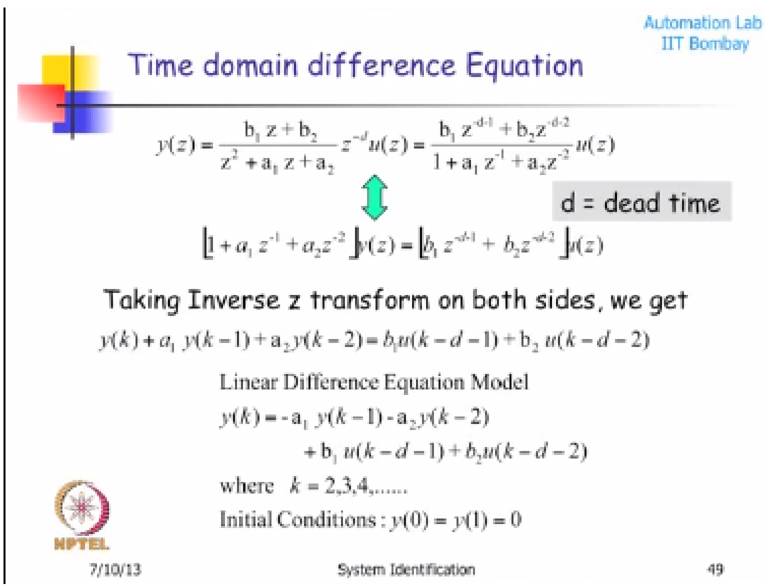

Something in a past and the reference side is write a as parent so what does it tell you it tells you that the new measurement is refer that to the past measurement and new inputs which are coming, so what about new inputs uk – d whether time delay okay it is a time delay which means if I make a change in the input right now is going to reflected after sometime what is happening now is the effect of something that is happened in the past, okay there is something going to happen in the past so this tell you that, that correct input uk is not going to influence by k or yk + 1.

If the input in the past is influencing that current measurement okay that is because of the d okay and then this is my difference equation model okay to solve this difference equation model is not call difficult if I go u is the input sequence in put is something which is given to you either by a controller or by operator.

If you have rotating the plant you are changing the input you know what is u okay and given u I can solve this linear difference equation matching in time if I know value at instant you know 0 and 1 I can find out value at instant 2 if I know at 1 and 2 I can find out 3 I can go often in time this is a difference equation as k increases we will get.

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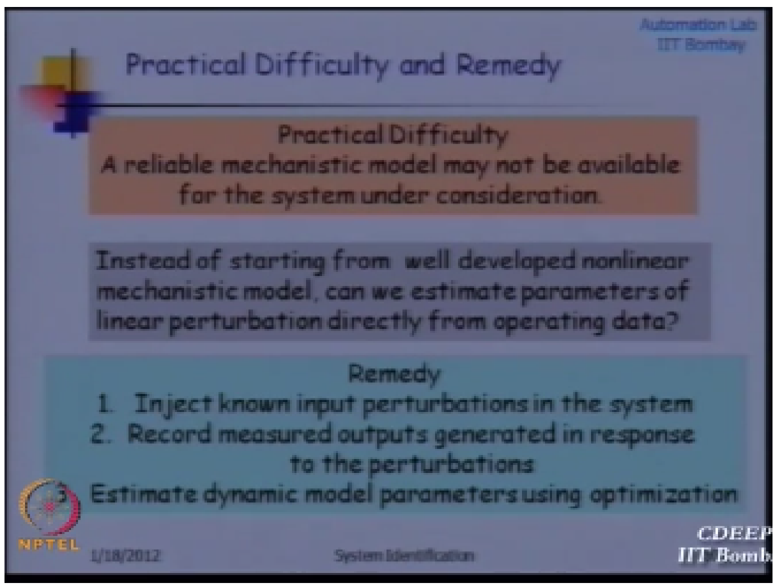

Okay so far so we got this model we started that the mechanistic model b lineralized it use tell us the expansion then we got a states specks model a discritized that if you go to and industry and so that well where is the mechanistic model you know people might laugh at you unless we are going to some variety lab there away what is happening okay, what is a mechanistic model and why it is required, so you go to a slow and or you go to some lab of you know where at walls development is going on.

They would of course know what we are talking about in that either or a general industry if you go and say what I want a mechanistic model for the system we would get everything you will not have mechanistic model, now what to do so the question is well if I go to a industry I have the system to be controlled I do not have a mechanistic model can I simply developed model from data I can do well I have to some experiment with this system okay, well I have to tweet the system you allow to make the system.

It is like the example that I keep giving this the model we developed for a car when we get we start to driving a new car or a new vehicles, so notice so you know we the manipulate variables are accelerator and the break, so we played with it know initially we slightly increase that when I see how it goes redundant, we apply the break and see after distance it stops and then we develop a model which is interrupted the output is the speed or you know breaking distance and there is input is my you know.

The pressure at to come the accelerator or the pressure at, so given my inputs and my outputs are you know + speed or in the breaking distance, so primarily these kind of models I think about to play with this system you can developed to drive my car but I do not have a model coming from physics for the car, so can I still to work with it yeah, so my approach from now one it is going to be like this well I am going to inject perturbations into the system okay I do not know with this principle model.

And I still going to get going to the controller design okay, I cannot say that well call engineer who will model this for me and then I start, I do not know daily from model using some simple to its and you know what I am going to do is I am going to record the inputs that even and I am going to return out measurement that I have okay, and then between this input and output I want to fit the model okay I want to fit a model just like if it is a correlation I can fit a differential equation I come to this difference equation.

Okay it is now necessary that you will have a algebraic correlations between variables okay you can have you know some system is there you have some input some output you can develop you know differential equation or a curious strict model which is a differential equation have a difference equation.

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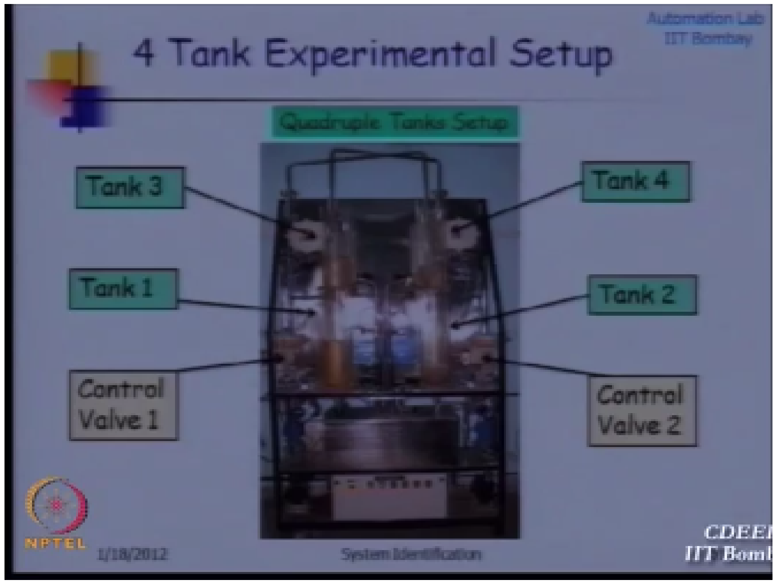

Okay so let us look at this what would the time set up and I have this straight up in my land okay, there are four times you can see them and where we saw this in the first few lectures in that two times start from each other and you can see here this is the control valve part of the flow goes to this fine remaining part of the flow goes to this, this stand, so goes to this stand so tank 1 and tank 4 get you know input that is coming from this controls there and time few more time 3 we get flow which is coming from this control work okay.

So let us assume that I do not have a model probably from physics for the system, but it is connected to my computer I can change the board okay experiment that I am going to do is I am going to change the wall I am going to protect the wall okay.

There are many questions do I part of that more a time we have both that valve simultaneously while if you want to say I would get a both the valves simultaneously we find it is difficult because if too many things are changing simultaneously how do you model then not that difficult as we will see, so okay so this is.

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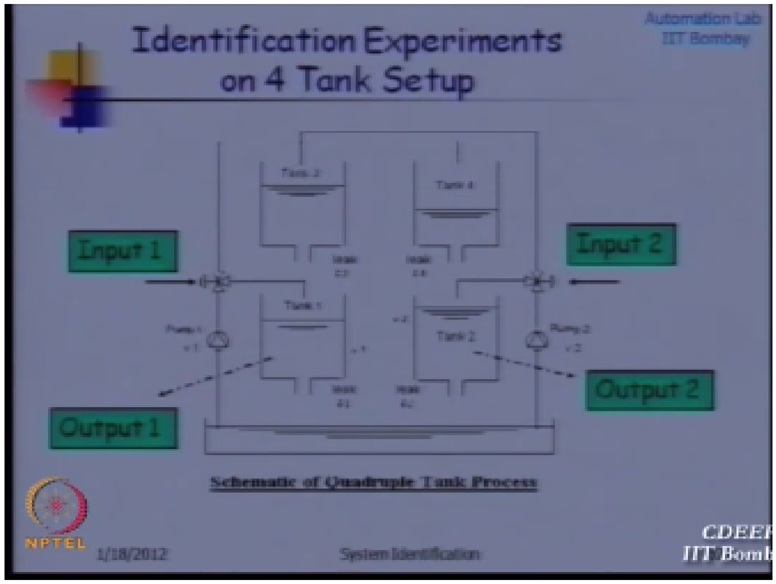

A schematic diagram of the same set up okay and full times where two inputs and I have two outputs, two outputs are still there it is right, and now going to measure all 4 levels and we want to measure only level 1 and level 2 in times fall in time to okay.

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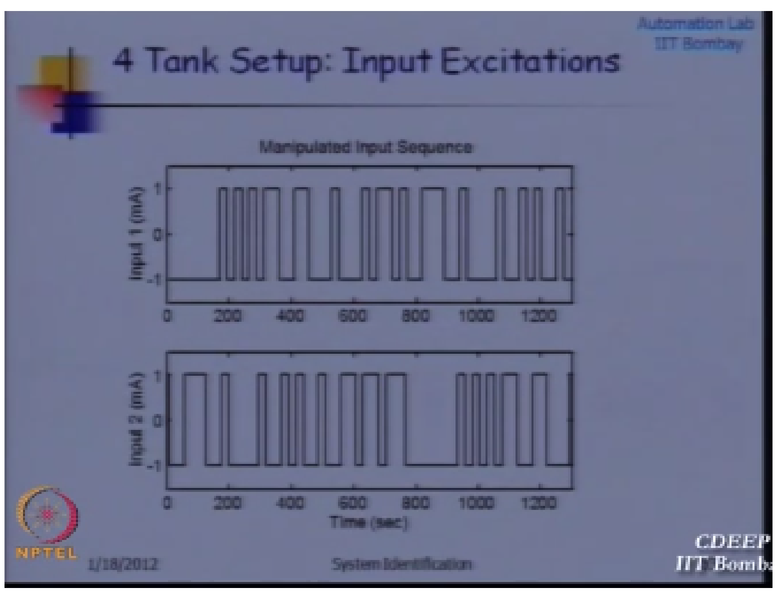

Now I actually converted the experimental my okay so what I read this I took initially the system to a study state and in the rejected part of excitations 0 here is not really 0, 0s 12 millions as this plotted the parts of the inputs, so I have given I have both the system to a steady state okay by giving 12 million which to both the control valves the level has complete study stat okay and then I inject perturbations I would just plotted here the perturbations you see the perturbations in by 1 and valve 2 how did the system respond okay.

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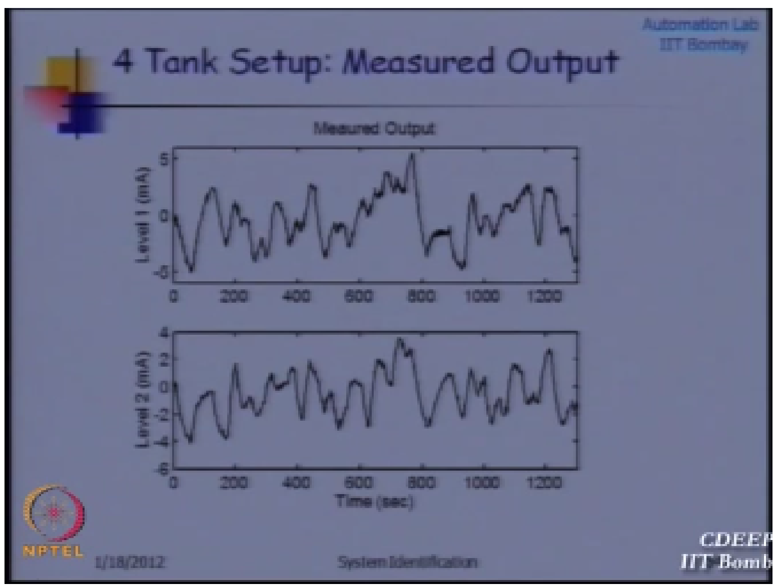

These are the measured outputs we have to measure the outputs again perturbations from the study state level okay, all of these data which means for this particular system I have plate this system I have you know manipulated the valves according to certain.

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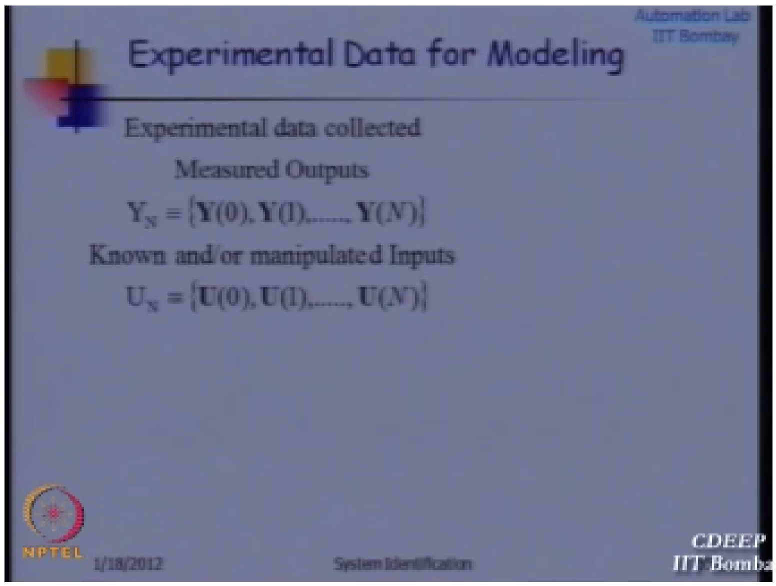

According to certain scheme this particular signal which I have given here is called as sudo random binary signal and randomly changing between two values +1 a milli amp to -1 milli amp, okay my system the control wall it input between 4 to 20 milli amps okay, if you give 20 milli amps the valve is full open if you give 4 milli amps the valve is full closed okay, I am first taking the opening the valves to 12 milli amps, so 12 milli amps is my base light okay.

At 12 milli amps I wait till the system levels come to steady state the system is perfectly at the steady state at 12 milli amps okay. Now was start my perturbation studies okay, and then around third milli amps so this when you say here plus or minus 1 it actually means I am perturbation the input to the valve from 12 to 13 and 11 for each of the valves, okay this is the perturbation are injected to the system and how does the levels react.

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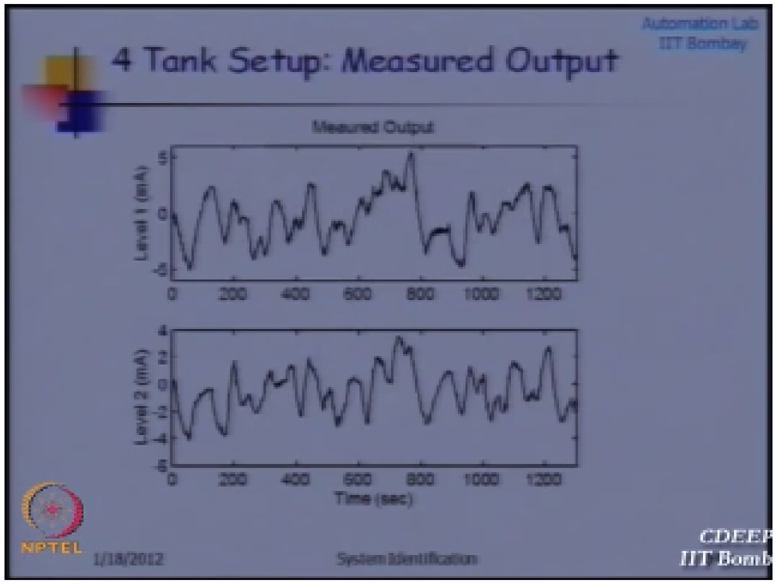

Levels have reacted like this, this is the perturbation in the level around the steady state okay.

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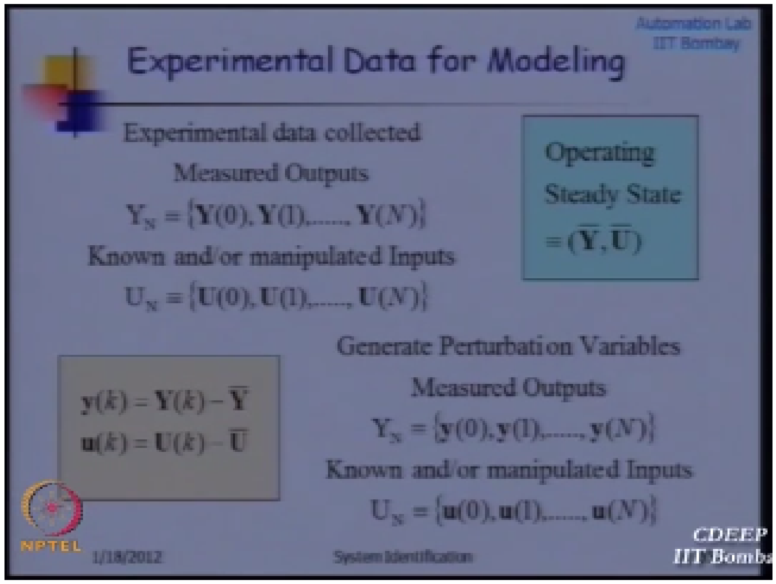

So what I have done actually that we put it mathematically what I have done is our collected data for y0, y1, yn, y capital is the absolute value U is the absolute value of the input, absolute value of the input is between + or – 11 volts, + or – 11 amperes milli amps and the level is measured again in terms of milli amps okay, the level is measured between milli amps. Now I am going to develop the input output model okay, now what is input here, the input here is the current input that goes to the control valve very, very important input right now is not really the flow okay.

So whatever is going out of my computer is the input whatever is coming to the computer is the measurement so measurement includes level dynamics sensor dynamics everything okay, the measured inputs that control you know the manipulate input here is the value which is send out of the computer which means process includes valve dynamics posses dynamics, sensor dynamics everything, okay.

It is not just okay, then well I should know what is my operating steady state I can take a measurement initially what is the steady state level and what is the steady state input I told you in this cases 12 milli amps input is the steady state input so $\bar{U}$ is 12 milli amp/ milli amp and $\bar{Y}$ how to find out experimentally whatever I get, I subtract that so I define this negation variables and finally I generate perturbation inputs what I plotted here is this perturbation inputs okay.

I have collected data which is the data collected is not in terms of perturbation data collected that is what variables. For the purpose of modeling I am going to converted into perturbation variables why, because they are going to develop up a model using Taylor's roxy perturbation, Taylor's roxy perturbation talks about perturbations in the input to perturbation in the output okay, yeah.

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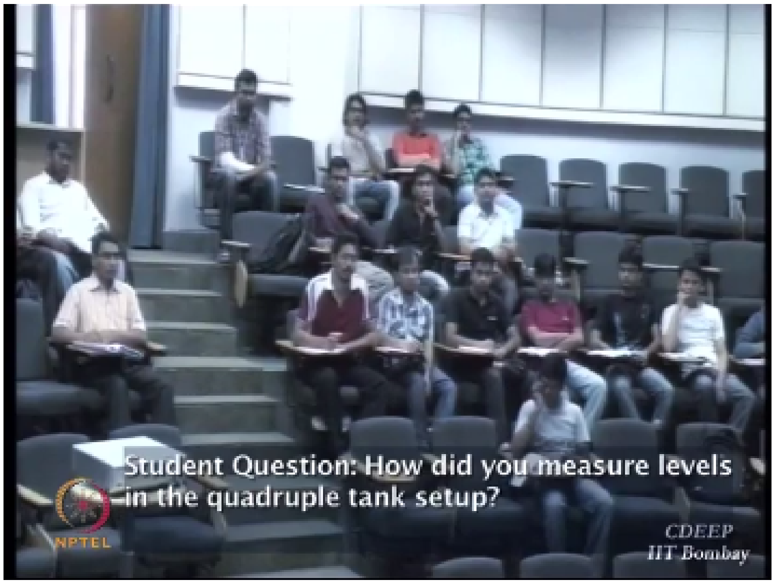

In this case we have for the level measurement we have differential equation transmitters okay, which convert the 0 to 100 cm water column into 4 to 20 milli amps so 100 cm water column is or 60 cm water column I think 60 cm water column is 20 milli amps 0 cm water columns 4 milli amps, so that is our and then that transmitter is connected to my analog to digital convertor okay, which is giving me measurements at every five second interval okay. So every five seconds I get one measurement from.

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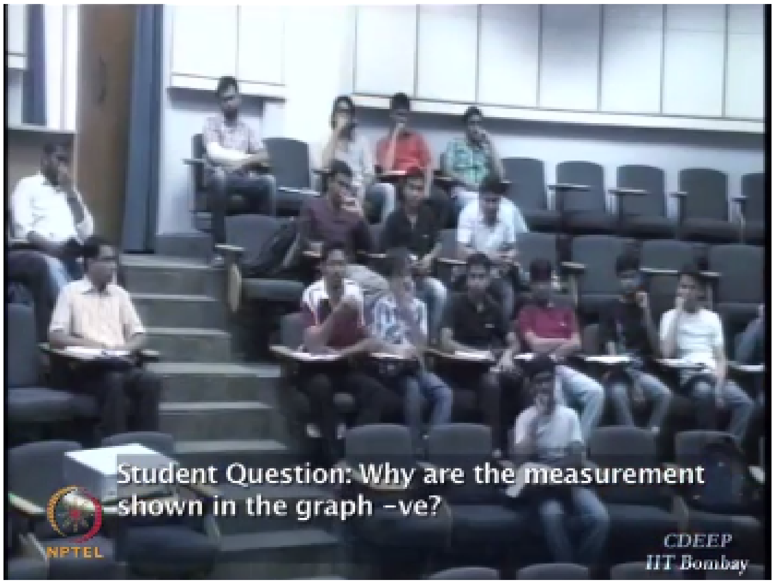

Because it is portability, it is deviation so see here my original output is not negative values but if I take a steady state okay.

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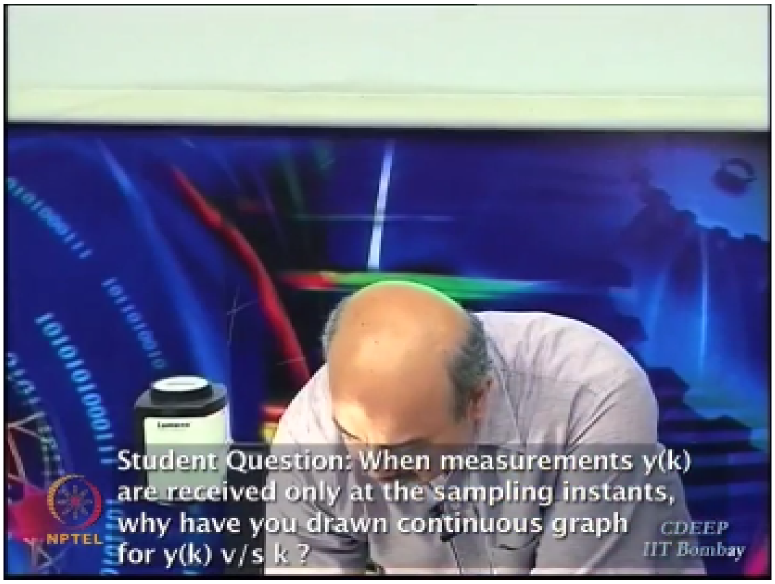

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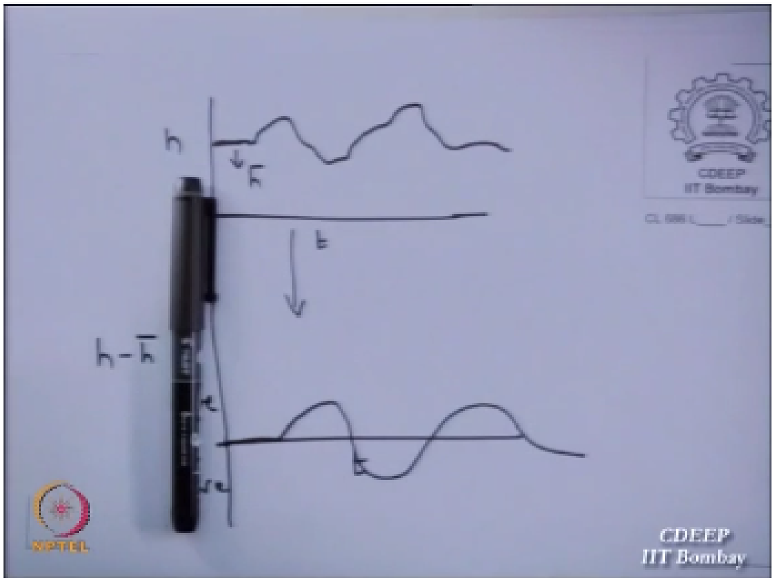

See where original output is h verses time is something like this so let us say my initial unit is at steady state then it starts changing, okay so this is the steady state. Now if I plot this in terms of you know h so let us call this point as $\bar{h}$, so if I plot h-$\bar{h}$ verses time then it will look like this whatever this is negative this is positive okay, but original signal is not like that original signal is you know let us say it is varying between some 10 milli amps to 14 milli amps or something like that, okay is varying between 10 to 14 milli amps.

I removed the mean value or the steady state value then that gives me signal which is positive and negative okay, is everyone clear on this. you get perturbation signals which can be positive, negative original signal may or may not be positive or negative it depends up on the system okay. If you have induction motor you may have currents which are negative or positive but if you have temperature, temperature is not good to be in, I mean temperature can be negative if you take depended on the scale but something is like concentration is never going to be negative, okay.

But perturbation concentration can be negative because there is perturbation around some mean value okay, so that time be negative so okay, yeah. No, I should now be drawing output as a continuous signal I should be drawing it to the discrete points okay, it is only that I will connect the discrete points okay, so idea of their drawing this would be just points. There are 250 measurements there should be 250 points okay, it is just for the sake of you know, so look at here this is my initial steady state, okay, yeah.

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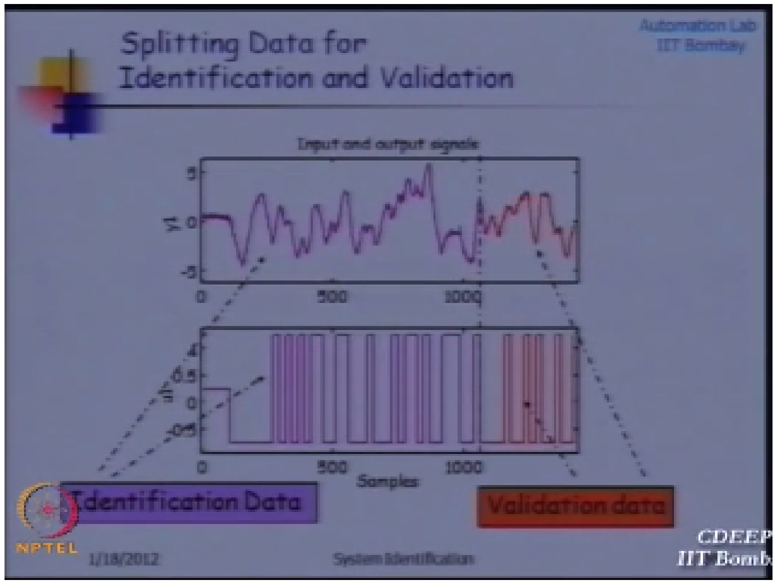

So this is my initial steady state because is at 0 I have subtracted this steady state value and then this is the perturbation around the steady state these are the perturbation inputs okay.

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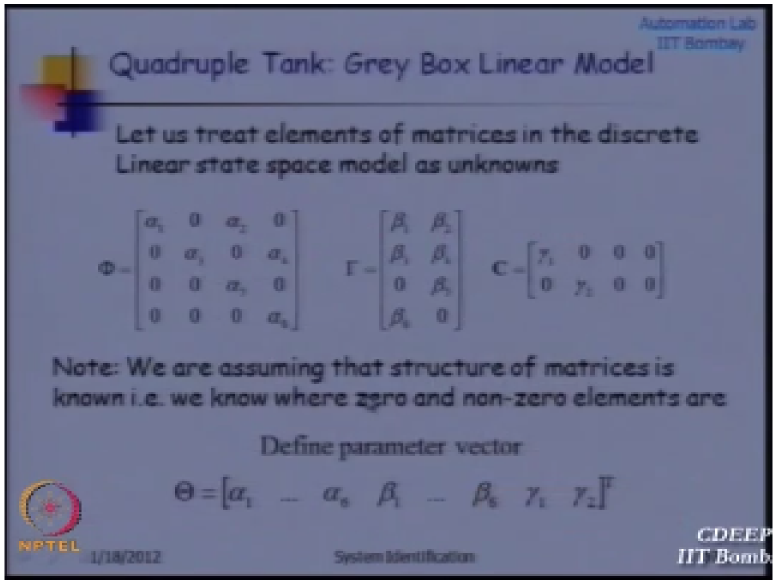

Well, now what I can say is let us go back to this, let us go to the model that we have developed earlier and in connect, we develop this model from eagerization and finally yeah.

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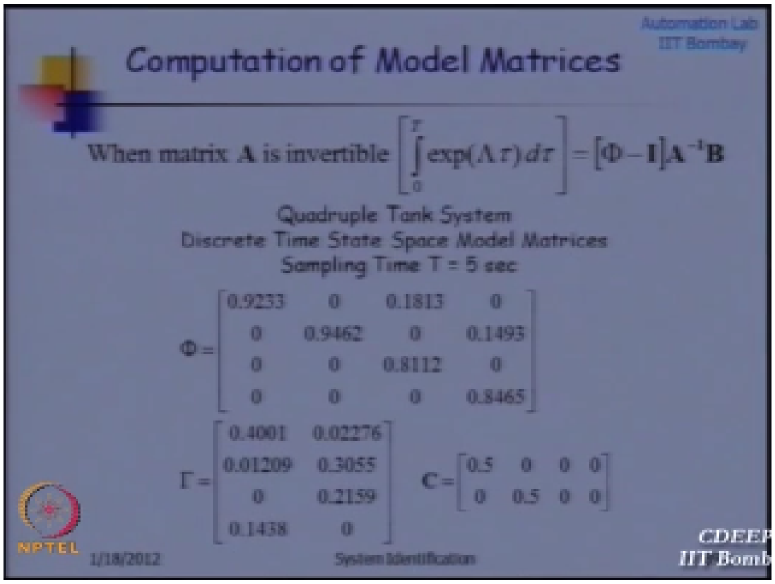

Finally I got this matrix okay, I want to do look at this matrix closely okay, there are certain elements which are 0 here why they are 0, because they decoupled there is no relationship that is why those are 0, okay. There are only few elements in fact if you look at ϕ matrix there are only six elements which are non-zero right, the six elements how many it is a matrix which is 4x4 only six elements no not six, yeah only six elements are no zero okay.

Out of 16 six elements are no zero okay, look at γ matrix γ matrix has two elements which are o whatever sampling interval we choose for this particular system structurally these matrices I am going to look same then I am going to look different, is it clear. Structurally these matrices are going to look exactly like this whatever we will do same is that case for C matrix I am measuring only two levels okay, in that only two elements I am going to be non zero remaining are going to be 0, okay.

So what if I treat these sic elements here as unknowns to be estimated from data okay, can I do that so what I am going to do now.

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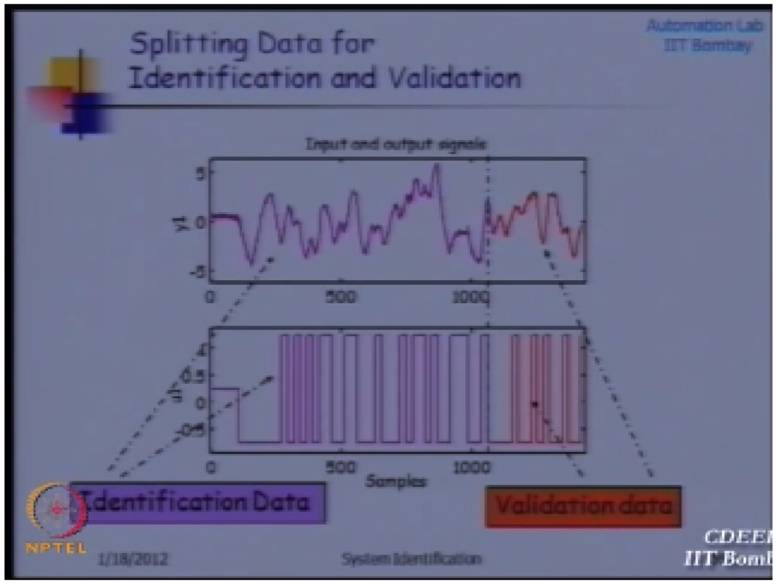

I have this data with me okay, I would just plotted here Y1, U1 there will be data for Y2, U2 okay, I have this data for two inputs two outputs and I am just showing here one portion of it okay, so I have data for Y2U2.

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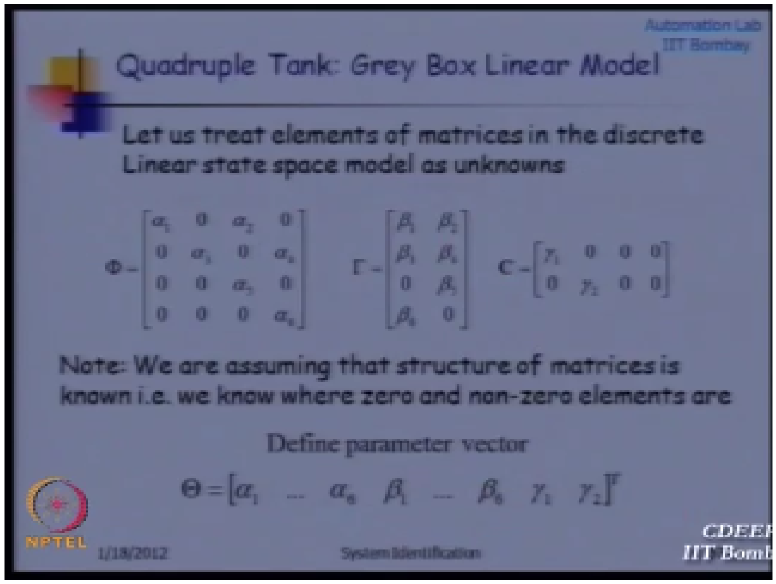

Okay, now I am going to say that if I discretized a model coming from physics okay, it has a certain structure and that structure is given by this you know matrix these three matrices. Now what do I mean by structure is given by three matrices I have put 0 there I know there going to be 0 okay, and an events which call me or go of determine through linear addition of a model I am going treat them as a unknowns okay.

So I am treating α1, α2, α3, α4, α5 and α6 the six unknowns in five matrix the six unknowns in γ matrix the two unknowns in C matrix okay, so now question is can I estimate these 6+6, 12+2 14 parameters only from data without worrying about starting from physics coming with a non linear differential equation model nothing. But starting from that model if I had a clue of what that model is I can come up with this structure, okay.

If I want what is the model from physics is okay, I can come up with this structure I am going to call this model as a grey box linear model, why grey box because some information from first principle has been used, but I do not have parameters of the first principle model okay, but I have used the first principle information to arrive at a structure of this linear perturbation model, okay. The structure has been formed from physics but the parameters values are unknown and I am going to estimate the model parameter values from data so how do I do that, okay.

Let me define a vector called θ this θ has 14 elements α1 to α6, β1 to β6 and γ1 and γ2 all 14 elements put into this θ vector I am going to use least square estimate. Do you use you know least square estimation if it cure fitting okay, instead of curve fitting I am going to do a differential equation fitting or a difference equation fitting, okay there is only the difference between what you know and what are going to do now, okay.

I am going to fit a difference equation linear difference equation into this data of Y1 Y2 and U1 U2, okay treating these 14 parameters as unknowns. How do you do this if you are given some data and if you have look fit a line okay, what do you do. Do not tell me I use excel and I just click and I get a print that is not the answer what we do? Linear re square estimation remembers what is the linear re square estimation, what you minimize? That are between measurement, predicted estimated output and actual measured output okay so I minimize difference between I minimize the difference between estimated output and the measured output, estimated by what? Estimated by the model okay.

What is the raw way if doing this? You can guess the parameters okay you can guess parameters then for every guess you can generate model estimated model outputs you have measured model outputs you compare the two okay if you think that the fit is not god you revise your estimate and go on doing this till you know the estimated output goes close to the measured output okay so this is what I am going to do next okay I am going to do it through optimization, optimization how to solve the optimization problem I am not going to get in to that in this I am just going to formulate you can solve it using met lap tool box.

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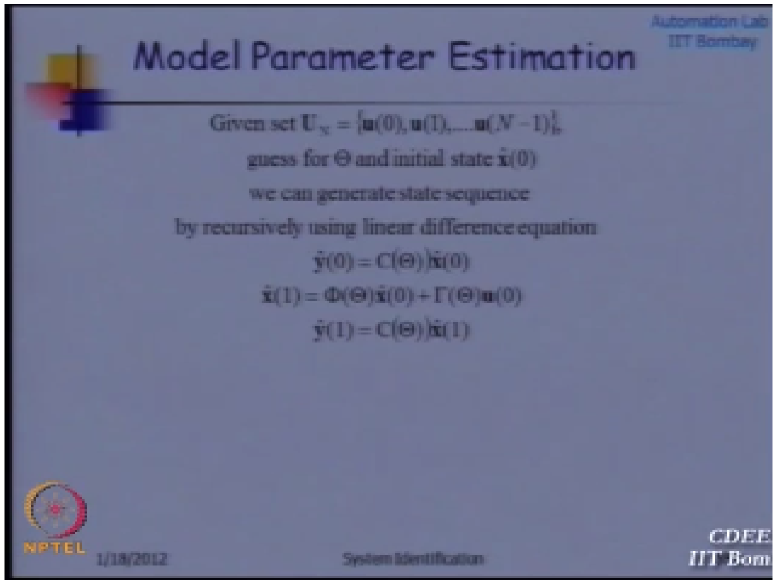

So let us see how this can be done, now what is known to you the inputs are known to you input sequence is known is you know you are introduce the inputs right you have change the wall position so wall position as a function of time is known to you also know that it is p2 constant okay. So you the wall positions okay, what is not known to me, what is not model?

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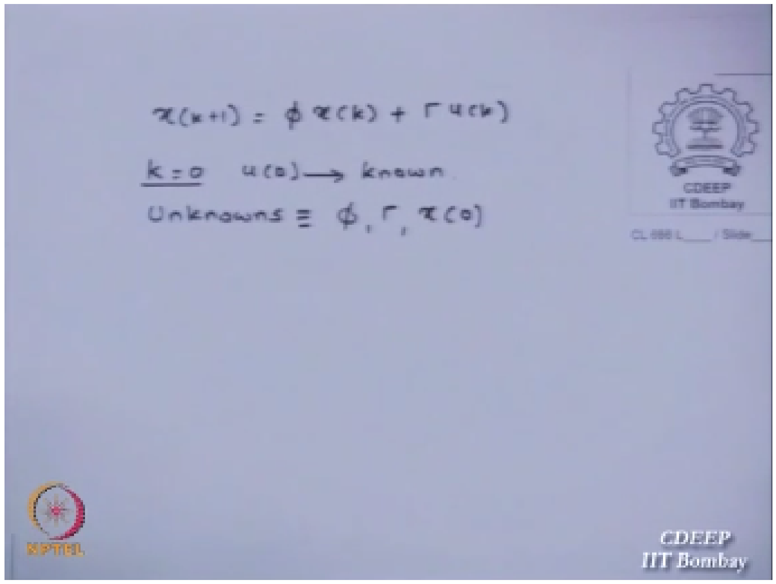

So now model is okay this is my model, let us say I am write time 0 okay so at k = 0 what is known to me I know u0 okay, I do not know okay so this is known what are the unknowns? One thing which I do not know is right now is φ matrix I do not know what is φ matrix so I do guess what I have to guess α1 to α 6 I have to give x for α1 to α 6 I have to guess elements of γ, I do not know what is what are the elements of γ right.

I also do not know what is x 0 I do not know x0 what if I guess all three of them okay what if I guess all three of them let us see what we can do.

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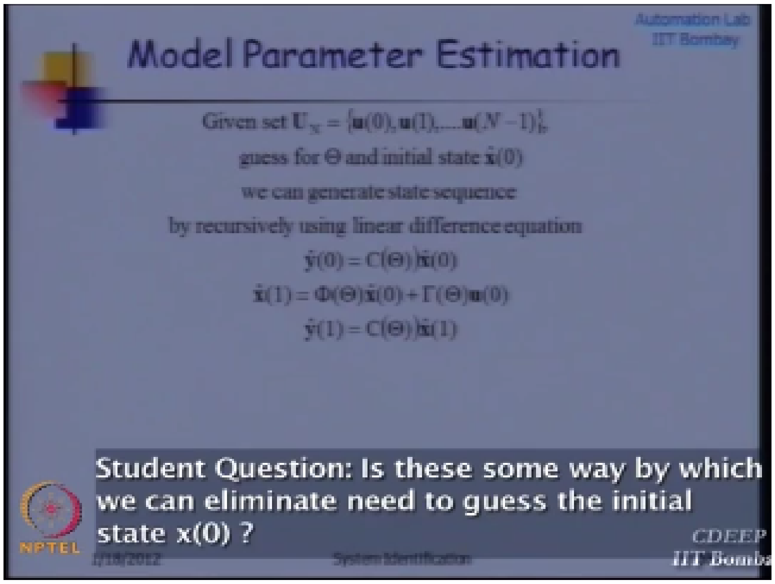

So initial state one way of eliminating is making in a simplistic assumptions that x0 is 0 you are starting from steady state and x0 is 0 that is typically done.

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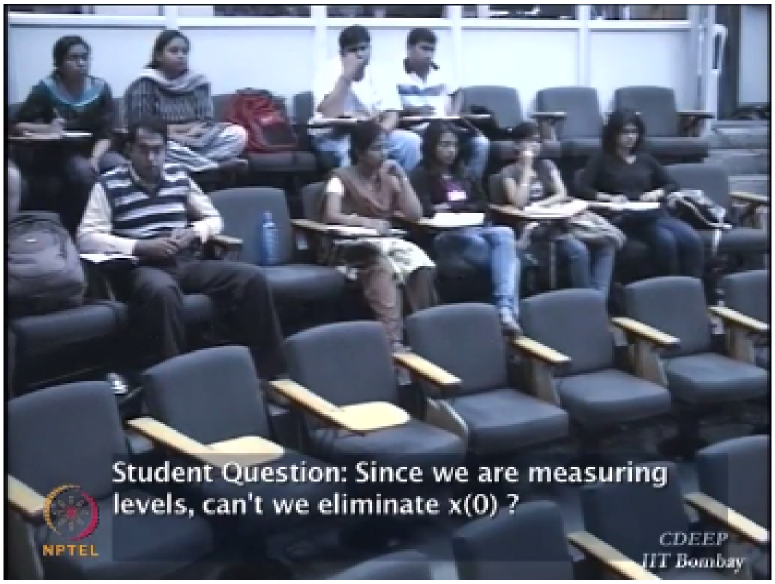

No you cannot illuminant, you can make simplifying assumptions as x0 is 0 we cannot really illuminant x0, but you are not measuring x you are only measuring y so I am only measuring two levels below I am not measuring all the four levels see because y is there because see level transmitter cost okay, a level transmitter a d to b transmitter could cost something of the order of a good quality to be transmitter but cost something like 70 to 80000 okay.

So I do not have money to put all four level transmitters okay so I cannot measure x0 I can measure only two of them I can know initial level foraminate two not for all four so x0 has to be some of guessed okay some part of it might be measured you can actually put those values other parts we have to guess or make a simplifying assumption initially you are starting from 0 steady state okay and x0 is 0 they have a two ways okay.

So coming back to this equation if you look here I have given a guess for this θ vector and what is now y 0 y 0 is see I am now make the notation little bit complex because I want o just emphasize on this slide that c is guessed I guess θ then I get c I guess θ then I get φ I guess θ then I get γ, okay that is why I have made the notation little complex here, so starting from this guess I know u0 I have guess this I have guess this I have guess this I can compute x1 right this is a difference equation.

And then y1 becomes y 1 becomes c θ, cθ will have based x1 bar on a computer here okay so I get y1 okay. So I got x1 now using x1 and u1 I can estimate x2 okay and then arching in time now okay I know input sequence I know u0 u 1u 2 u3 u4 okay just see I have put this little march above here y ^ this is x^ y^ x2^ y ^ because it is an estimate, you do not know whether it is correct or not these are all estimate, estimate based on your guess θ because your guess θ you have estimated x okay yeah.

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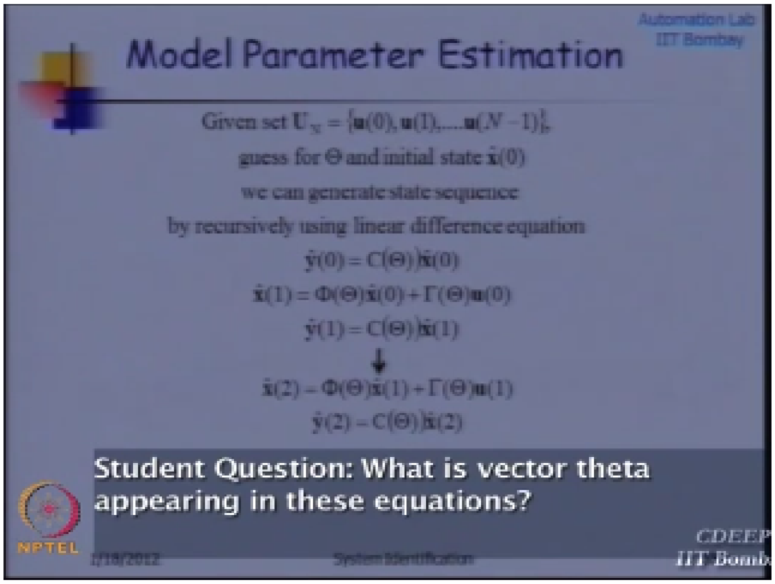

So θ is this parameters let us go back θ is this parameter side okay θ is this α 1 to α 6 β 1 to β 6 γ1 to γ1 γ2 so once I guess this look at this slide once I get this φ γ and c get defined okay.

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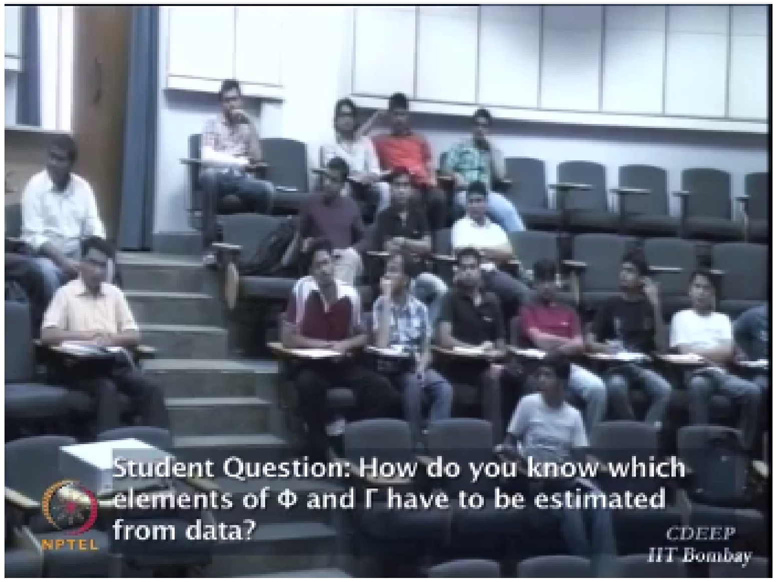

That is because it is coming from physics so I am assuming that I have a model from physics but I do not know it is parameters that I know enough to come up with this structure okay that is why I am calling it gray box okay, afterwards I am going to relax this also I am going to say when I have this data and I do not know anything about the model structure then what that is the next question.

So I am going in a gradual manner you know I am starting from physics I have full model how do I get control the level model okay second is well I have a model but I do not know it is a parameters but I do not know something about the structure then how do I explode it okay, next is well I do not know anything okay then what I do? So three stages okay.

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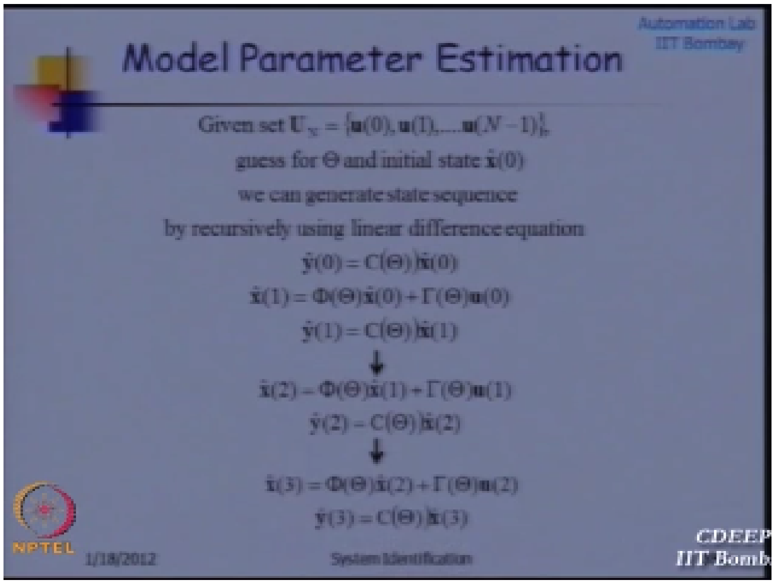

So any one with me on this I can go on now watching in time for every guess of parameters and every guess and every time I guess parameters I also get x0 okay I can generate complete state sequence right, if I can generate state sequence I can also estimate y okay and then well the conventional v2 estimation what I want to minimize I want to minimize difference between y measured and y estimated I defined in error epsilon which is difference between y measured and y estimated okay.

And now I have to minimize some of this square of errors, troubles epsilon is a vector what do I do? Take the norm take two norm again take a vetted norm that was I can defined a vatic matrix w I can take a vetted norm, vetted norm is many times you know you will have this y consisting of variable of different magnitudes one could be pressure the other could be temperature third could be you know some concentration the physical values will be completely different.

If you just take some of the square of errors you know pressure values will be Newton per meter square 10^5 and you know concentration might been fraction and you will have problem numerical problems okay so we can use the waiting matrix to balance elements of this and then what I am going to do is to solve this parameter estimation problem as an optimization problem it is an optimization problem okay,

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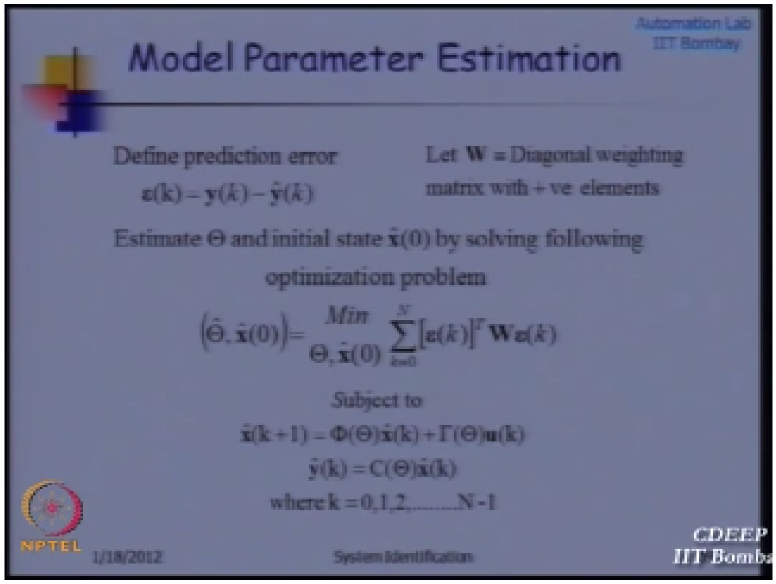

In which I am going to minimize this objective function this is a objective function we get some of the square of errors because this is you know because this is a vector of taking vetted transpose w x vector so this will give me a scalar value okay some of this from time 0 to time n I have data 250 points okay I want to minimize this some of the square of errors with respect to θ with respect to x0, one simplification I can do is assume x0 is 0 then it with respect to only θ okay.

This is the optimization problem that I need to solve you can solve this using mat lap tool box you can solve this using any one of the known methods it is enough to understand the formulation you know there are tools to solve this problem, so yeah.

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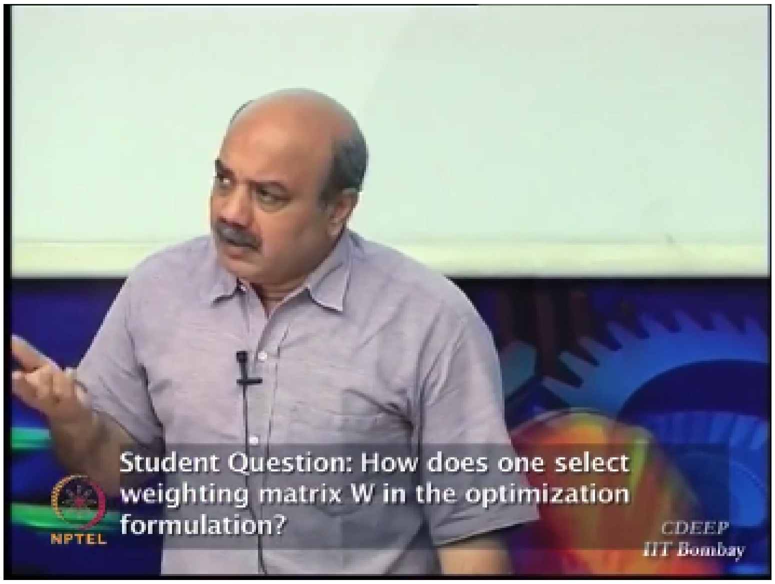

W matrix is typically decided to balance between different magnitudes and brim them on a same scale okay, so you know you would put it one simple way of doing it is find out the range or variability and then take elements of w matrix inverse of the range see for example if my pressure is changing between you know plus or minus 10 4 you know + or – 1 x 104 then you divide and bring it to + or – 1 variation okay.

So you try to scale it is a scaling matrix okay see because you may have a temperature in your model in you y estimated you may have a temperature value which between you know + or – 100 and pressure is varying between 10 4 to 10 you know -4 deviation pressure okay if you take just temperature square + pressure square you know the temperature square will get buried it will not be seen, so you scan it in such a way that temperature variation become + or – 1 pressure variation become + or - 1 so you chose w matrix such that you sort of balance numerically the values okay.

Is this optimization formulation clear? I am going to find out these parameters by numerical optimization okay.

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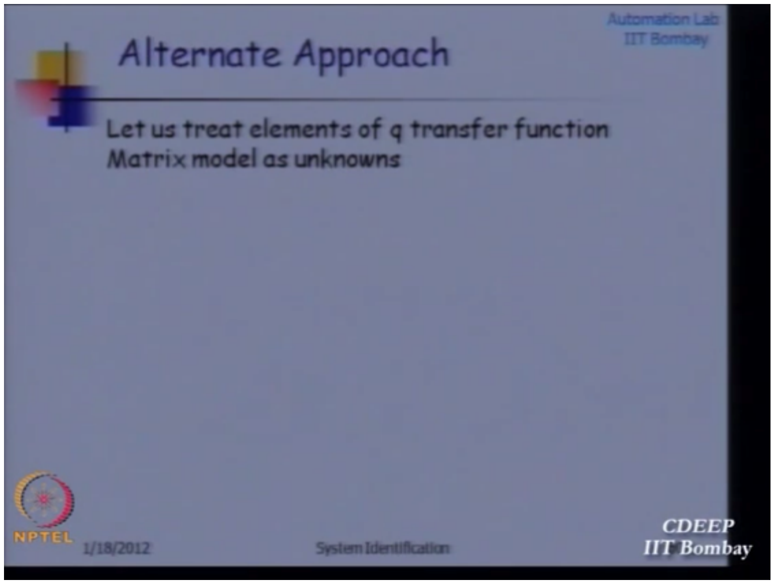

Well like a like he said how do you know the structure of the matrix okay I do not know such a matrix so I am relaxing even that constraint now I am going little okay now let us look at I do not know the structure of the matrix with let us assume that I know the structure of the transfer function matrix okay.

This can be known because you know first order you know what the first order is in second order transfer function I can know whether between y domain u1 there is the transfer function is first order or second order see just go back to.

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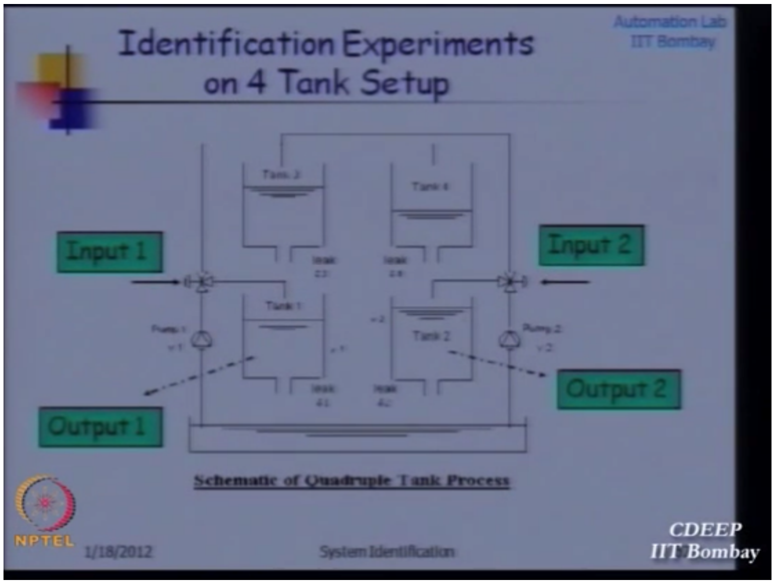

Let us look at this program okay now between input 1 and type 1 what do you expect the transfer function order to be first order okay same thing between in an output input 2 and output 2 the first order okay what do you expect between output 2 and type 1 it will be second order because the input is going through this type and then going to this dynamics okay.

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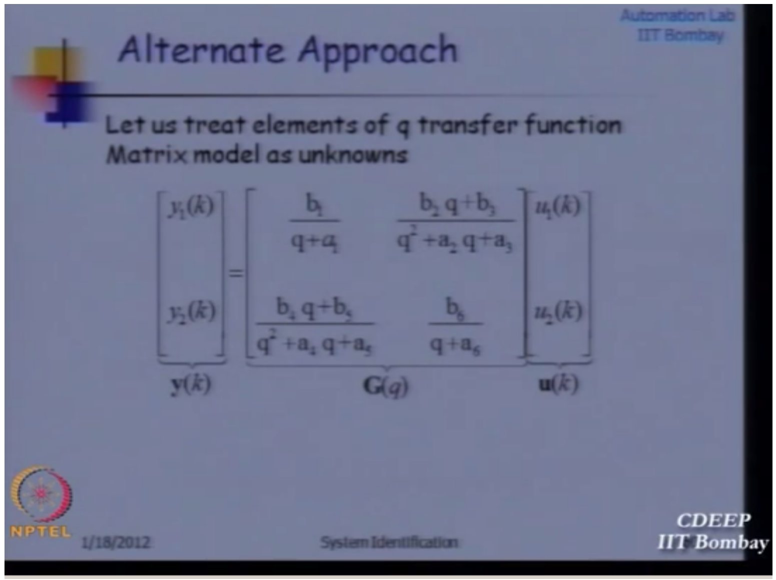

So now I have come up with these model abstract model with consume parameters is the quadrapation model okay I have come up with this abstract model with unknown parameters just that argue in like this you know this is this is between this and this only an type okay so model should be first order so the title elements of first order you see here y1 u1 expose order y2 u2 is first order okay.

But y1 u2 is second order because it is going to 2 times is simple is derived in this okay dynamics are sensor and neglecting dynamics are control valve so that simplification okay whether 1 should neglected or not difference upon because your judgment of the engineer whether you know this dynamics is negligible that depends upon this, this system okay.

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This is this was a distillation column system what the model order you will select yeah that is the good question with answer that will be later but in our distillation model well you can count the number of trace and say it is hundred out of model but it is impossible develop 100 order model okay then we do reduce certain model we approximate the second order of the model with the dead time and so on okay.

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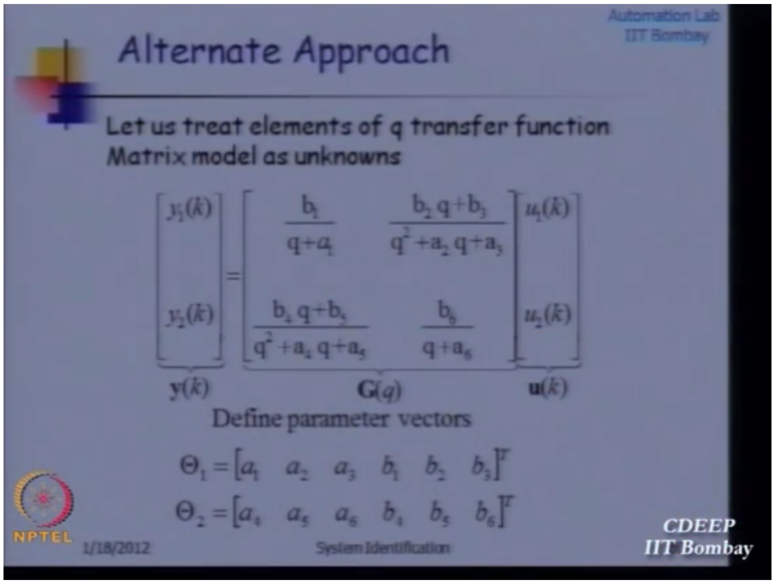

So now what are we doing now is I am going to identify these parameters of this model this transfer function model okay I am going to look at one output type okay so problems of system identification I m going to develop Miso models, Miso multiple input single output MISO you probably have words so SISO what is SISO? Input and output right.

I am going to do Miso model between y1 and u1 u2 and y2 v1 and v2 I will reform two parameter vectors here θ1 is a1,a2 a3, b1,b2,b3 so a1,a2,a3 and then b1,b2,b3 so further first row estimation I am going to do

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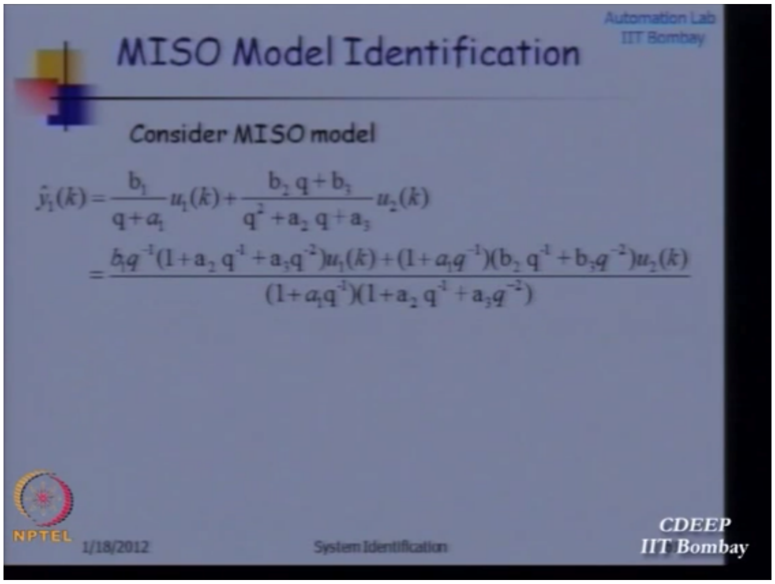

Okay I m considering this MISO model I am considering this MISO model okay now I have to convert this into time domain this is our I can use the algebra of transform functions if you see here then next step is to convert this into operator q^{-1} and then I have taken common denominator you see the common denominator okay and then after doing this I can multiply cross multiply and convert it into difference equation.

Because ultimately you computation is for optimization I need a difference equation so I need to go back to difference equation okay so what is my equivalent equation very, very easy to write if you look at, look at the numerator and the denominator you can just I mean the equation looks because there is lot of algebra you multiply all the cross terms basically you know when you have q^{-1}.

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You know operating of uk use that u1k-1 and you will get u2k-1 nd so on if you understand that simple thing this is just this is for algebra getting the difference equations starting from yeah you have to assured that.

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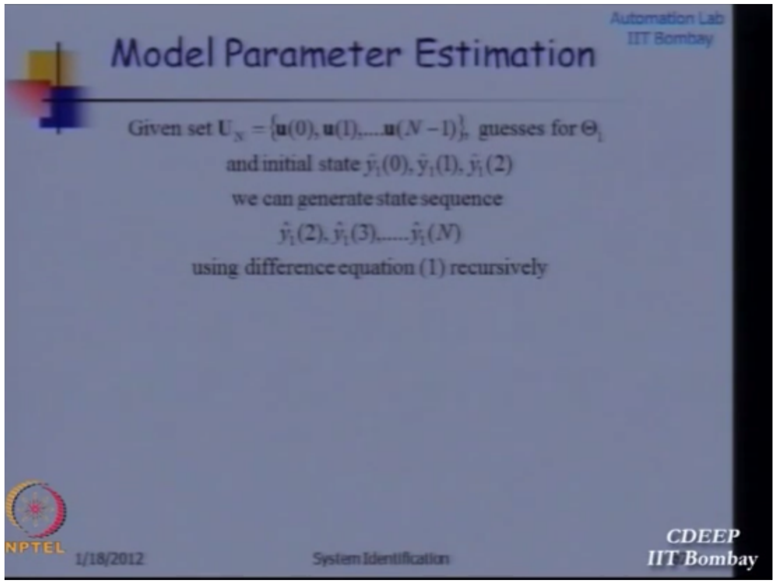

See this model will tell you that will be doing it on next page okay see because uf1 affix only u yf3 okay and the separate more technical will b your problem is that you can you do not have to assure equal to 0 you can liquidate in the list of unknown inputs or unknown variables and optimize so do that okay.

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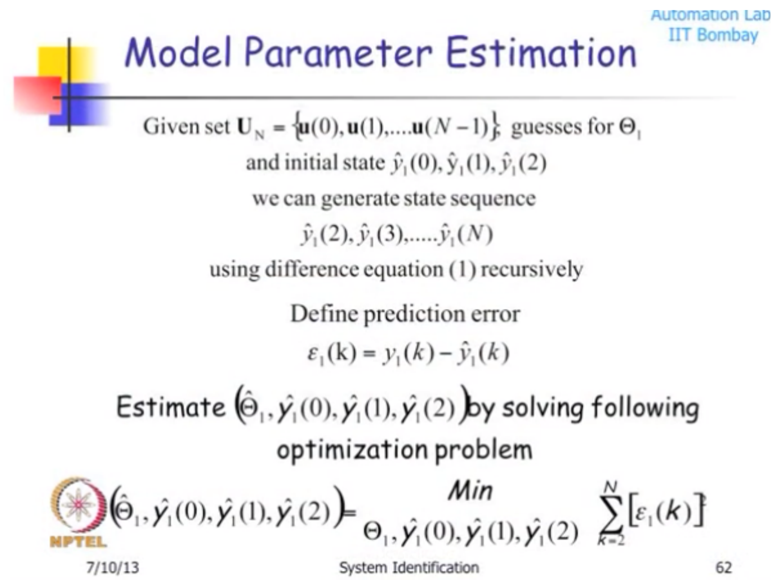

You can read this y0 y1 y2 to be unknown optimize guess an optimize okay so gain same thing if I have inputs sequence okay if I give you x path e path I can use a difference equation requires in times I can estimate y1^ what is y1 ^ level parameters as function of time okay estimated using the difference equation I find out the difference.

Now guessing the output xk were I do not need any matrix okay so unless you have problem is I am including $y_{0,}$ $y_{1,}$ y2 as you gets in that original case we have to base x_0 in this case I am guessing y_0 y_1 y_2 if you want to assure that it has go to 0 but you can do provided this system under consideration is open good stable.

I will talk about some little aspects later so I am not do it right now okay if it is open good statement it is fine to assure that this equal to 0 you will get some error initially but if you have large data that initial error will vanish okay on stable systems you cannot do this but identification of unstate system you have fairly complex is just an still you know into the domain of research not really okay.

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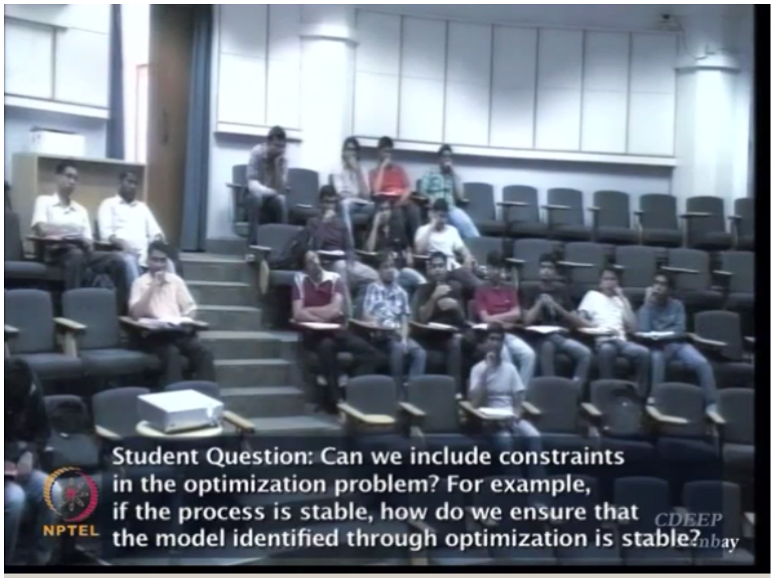

Is this clear is this optimization problem clear whether I want to do yeah, that is the good question so you have to introduce constraints into this problem that is not so easy okay that is a tough problem you probably have one way you can do it is to give constraints on θ okay but giving constraints of θ is also a difficulty okay.

So best till here is to start with a good guess okay now how do you start with good guess okay so well whether I can do this demonstrate some of these things has using a tool box you have the point you know extract just okay you have the point becomes at and you have to be profession till the hard you have to see the data.

You have to see how to do good guess is okay it will solve a problem it will keep it inside the meaningful range and so on but there are ways of initializing a module you know given a good guess of the module then particularly when the model is coming from physics see for example this kind of a model.

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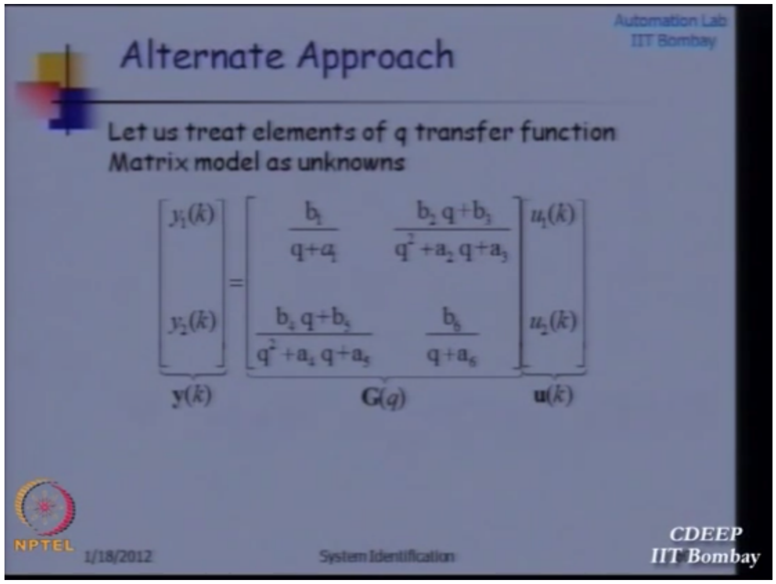

This kind of a model I can put constraint that the steady state gain should be negative or positive rather than no form physics if I increase the valve if it increase or decrease at the steady state okay so to find out the gain here for this you put Q=1 you will get the gain so I can put it explain that b1 of power 1+a2 okay the steady state gain.

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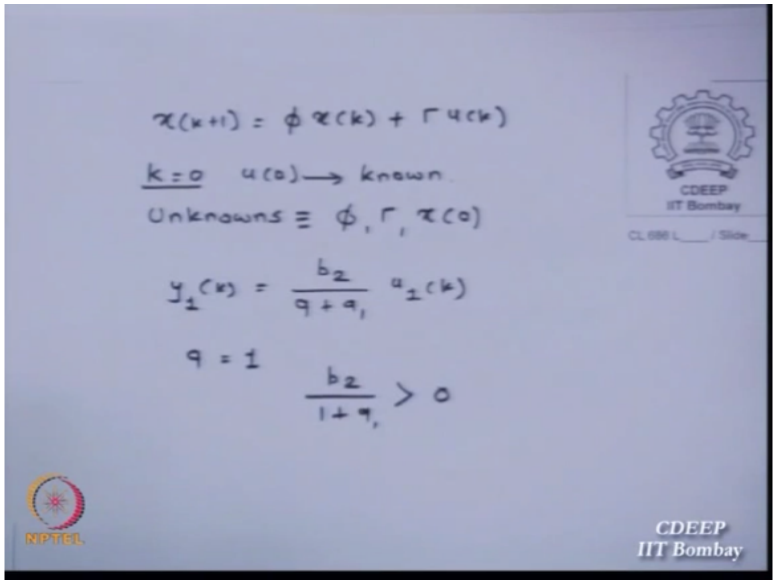

So if I for this model the steady state gain for this model is obtained by setting q=1 so the steady state gain is b2 upon 1+a1 okay now this argue from system if I change the valve 1 there is going to happen to level 1 so the increase or decrease it is going to be increase always going to increase so that gin steady state gain between the all 1one position and level 1 is of course going to b positive okay.

So I just put the constraint that this is always greater than 0 okay so likewise we can argue about each element and put meaningful constraints you can give constraint optimization problem which that is a good question and to get meaningful for the solutions you may have to do such okay, okay so this is, this is the problem for level 1 and input 1 input 2 these four model likewise I can develop same thing for level 2 input 1 input 2 to measure the models.

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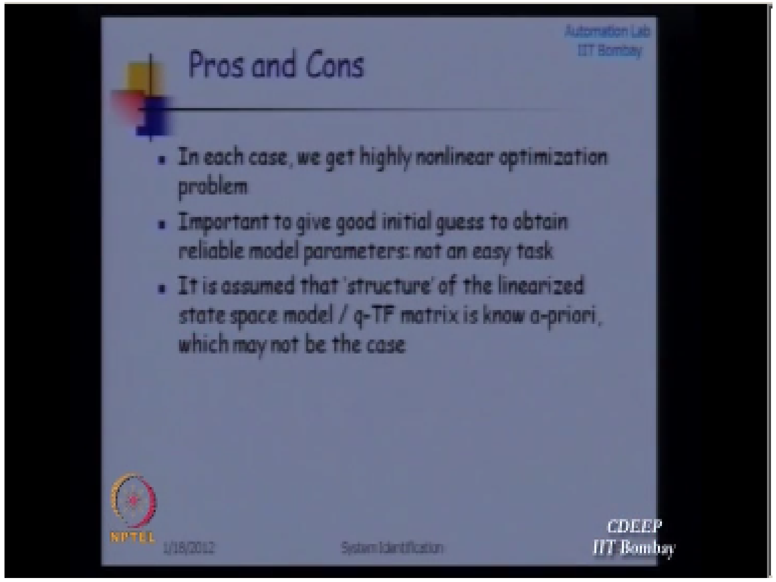

I can combine and went into grand model and use of control okay well what about pros and cons of doing this in each case you have to solve an optimization problem and you have to give a good initial guess so highly non linear of optimization problem not at all easy to solve you were looks stake forward here how easy to solve so you did not get good guess is very, very important how did you good guess trouble it is not so easy to give good guess next thing is right.

Now I have assure that structure of five model could be arrived at by some arguments okay you know these are zero and these are non zeros so I had a great model what if for a complex system you cannot do that it is quite slightly that for a complex system I cannot come up with this structure okay so, in that case I have no clue as to how this five matrix is going to look like okay could be anything, it could be full matrix could be in a fast matrix diagnose.

So there is one more thing I am just modeling here inputs that are no use okay any system will have inputs that are unknown okay for example inverse in this particular system okay you know there is a wall and either pump a pump gets voltage from the source and the source voltage is never perfectly contented to 20volts it is frustrating between 200 to 240 because of that the pump speed is vary because the pump speed is varying the flows frustrating.

So even if I keep the wall open in 50% actually flow is got constant flow keeps you know frustrating now this frustration have the impact on the level right it is not going to that the level is going to be constant because the flow is frustrating you know the wall position is constant the level is going to frustrate.

So my output measure as something else which is coming from some other input which is not measured I am not going to measure everything that happens right I am not going to measure the voltage in the it is not possible to measure everything so there are unknown disturbances and then I need to model them so what is the solution well so I am going to have relax even the assumption that I known the structure of the model nothing I want the flat form okay I only have data I only have this.

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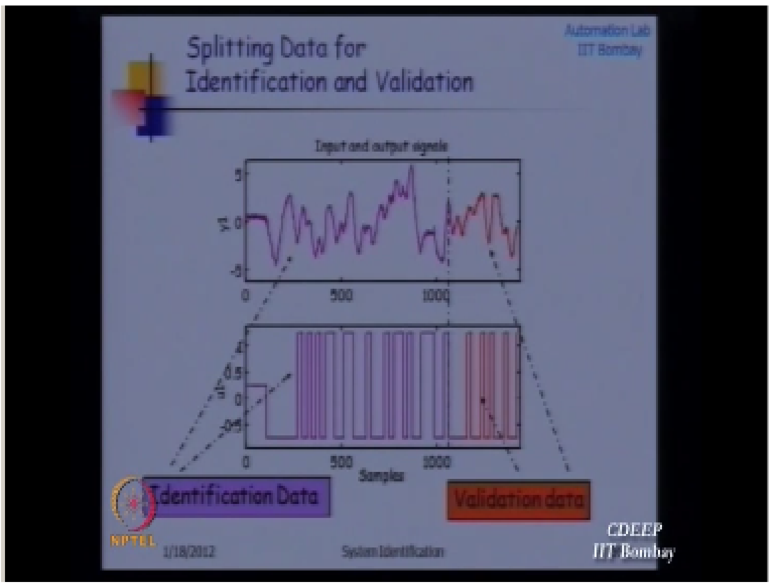

Okay I have this which I can play I can give inputs I can collect data now can I derive with the model okay that is what we are going to see in the next series of lectures yeah.

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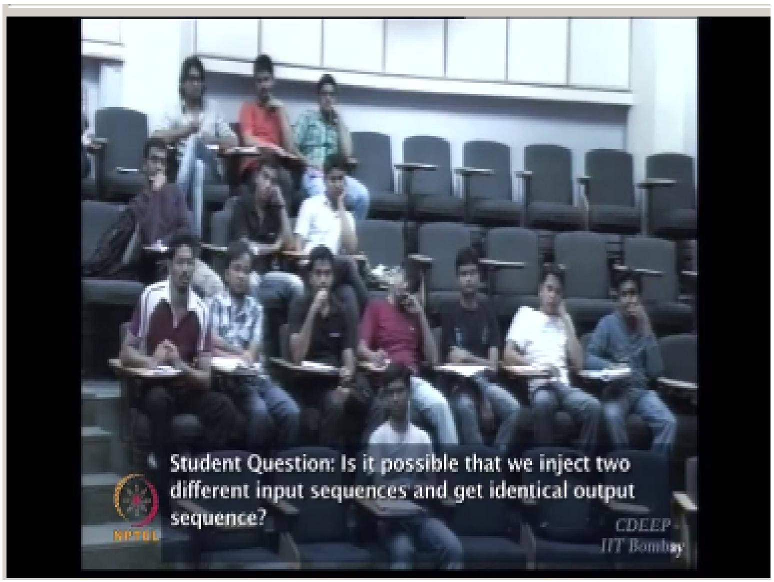

To different inputs sequence I have the identical output sequence mathematically yes no added two different equations not mathematically because it depends upon how many number of input how many number of outputs are there if you have in a differential equation because of the uniqueness of the solutions okay you will not get if you have input sequence different you will get output sequence different.

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We assume that the differential equation have a unique solution yeah even then a having non unique solutions with the same it is possible in non linear system it is possible in linear system I cannot think of if you have number of inputs equal to the number of outputs if greatest amount but in a non linear system it is possible to have same input sequence for example a non linear system can have multiple studies state for the same input we can multiple steady states for the same input okay very much possible in non linear systems.

And technically it is what you are saying same input multiple solutions so how does a system will behave will depend upon locally where it is okay in the state place or where it goes now those kind of things are beyond the scope but it is only possible in real non linear systems in this ideal world of real systems we will get a but even for the systems if the disturbances are different every time then you will not get the same output.

Even if you give the same input okay so in this system time set up if I give this same sequence every time I beyond on get dextrally saying output because even the voltage fluctuation were every time different so the level fluctuations that is arising out of voltage fluctuations are been time different okay if there are voltage fluctuations what so ever perfect system if you same input should get the same output okay yeah.

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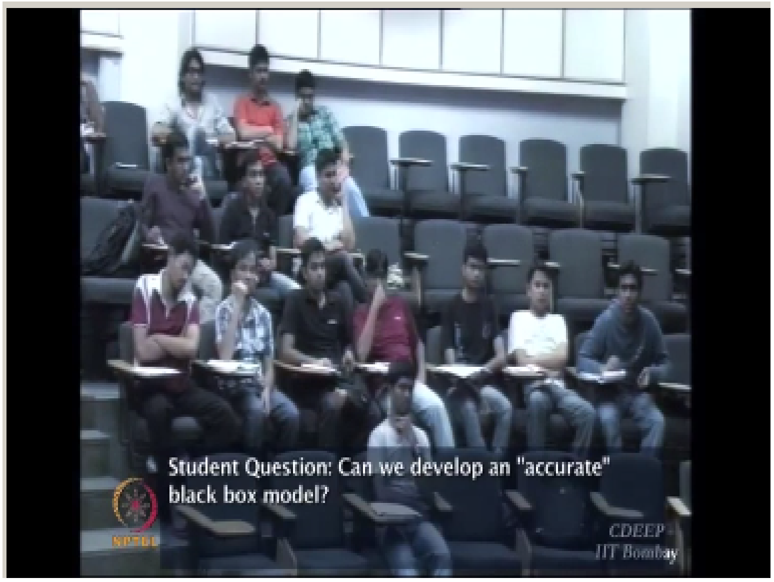

For about accurate black box model.

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Well it is it is very difficult to review an offline answer what you mean by accurate black box model? It is an oxide model but if you can develop a black box model and if you have a black box structure you might in depend box again you have done the optima ion problem you are saying that after the optimization you do not get big black box model then what you do? Use black box model which is yes but then you know if there is a mismatch between the plant and the model.

And if you use the grey box model that is your fault okay the diagnose algorithm will say that something is quality all the time because there is a mismatch between the model and the black so is the grey box model is very helpful but, it has to be very good otherwise we had problem diagnose using a grey box model we had lot of problem because if there is a mismatch for control there is no problem for diagnoses there is a problem control you can handle model plant management.

And do a good control as long as you know order of magnitude changes of the model and in the plant are same okay but so control mismatch does not matter so much in diagnoses if you are saying diagnose is the application I would go for a grey box model and improving it for diagnoses it is very difficultly used by black box model so the application depends what is that for control I can use black box model.

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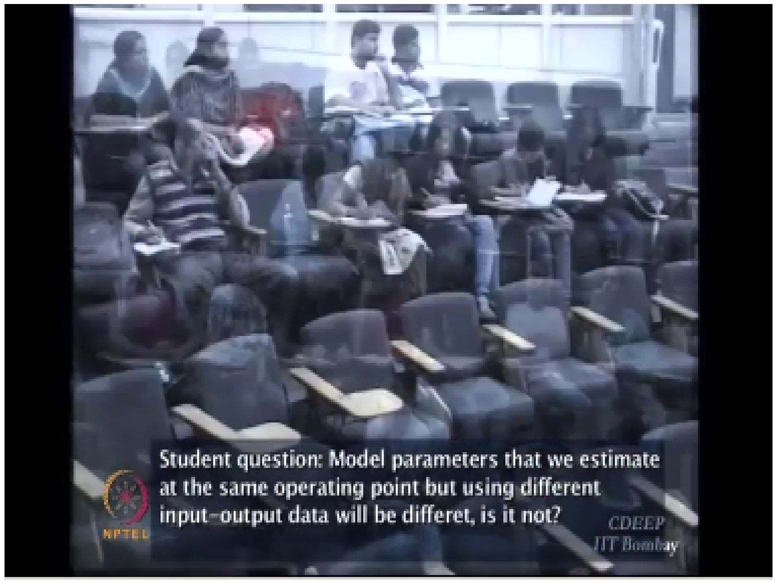

Yeah so thus the linear know if I change the linear action point I will get the difference model

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Yeah yes.

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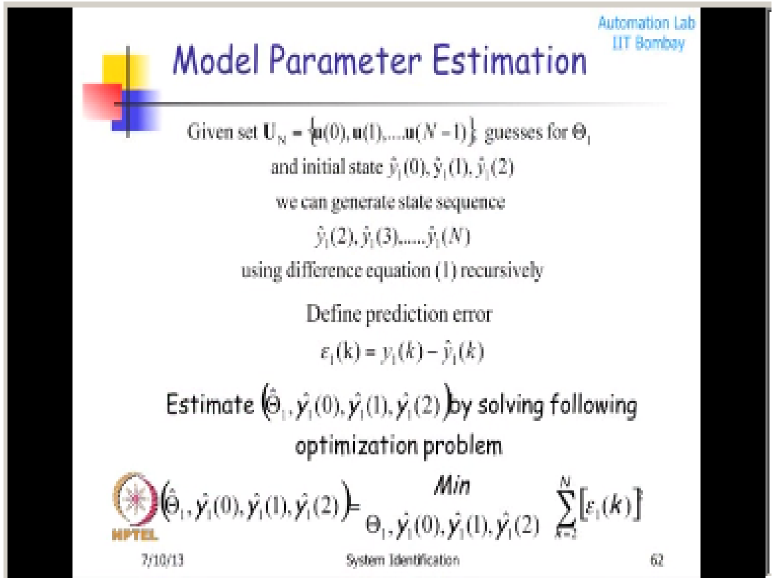

So every time you get a slightly different model.

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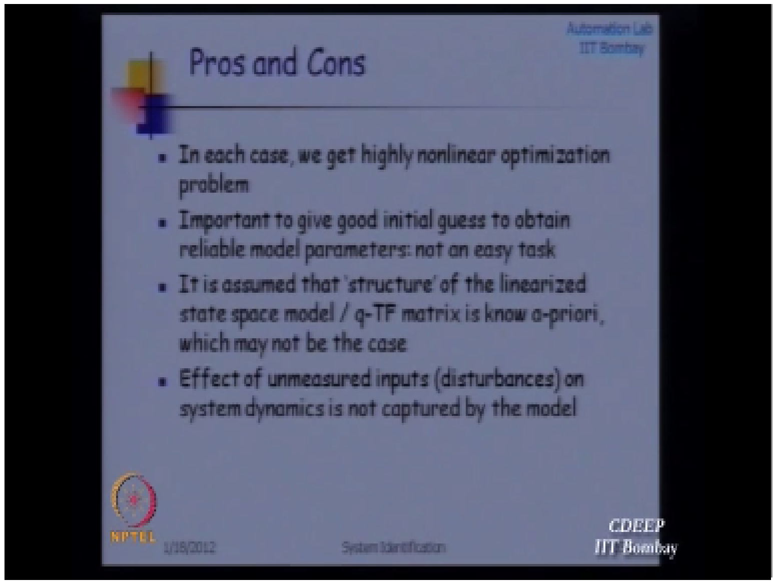

Hopefully when you are developing a black box model I am assuring that you know you are operating condition remain more or else same then only you can use it for design otherwise you cannot but then if you have a situation vary on nothing we just have this motor car drive and you did not have you know similar coming in metal lab then uses you can use simple models okay so I do not want to go into this today black box model and start with the in the next class I just want to show you this tutorial problems.

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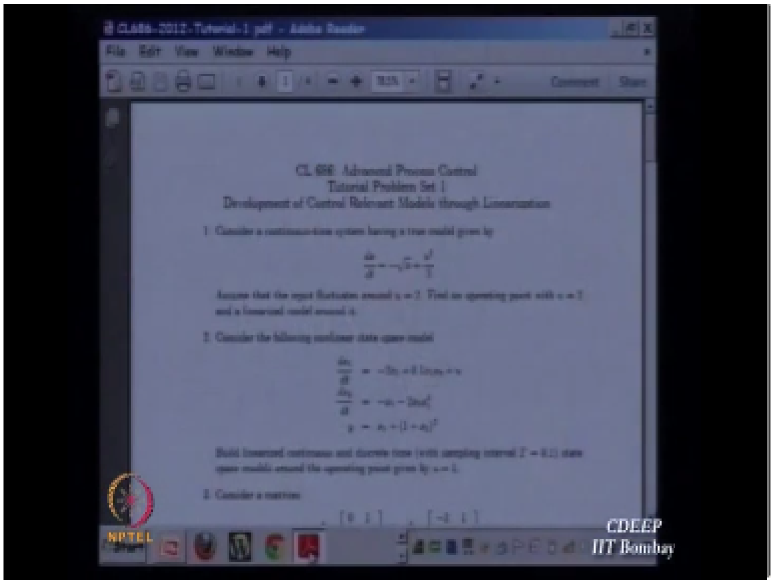

Which I have uploaded so I upload this tutorial problems there are about 10 problems here either about liberalizing the differential equations then converting them into difference equations finding out q transfer function matrix or z transfer function matrix first trans function matrix while out of same problems which are given you here 9 problems are here mathematical problems just talking about x1, x2, y1, y2 like in that text book last two problems are.

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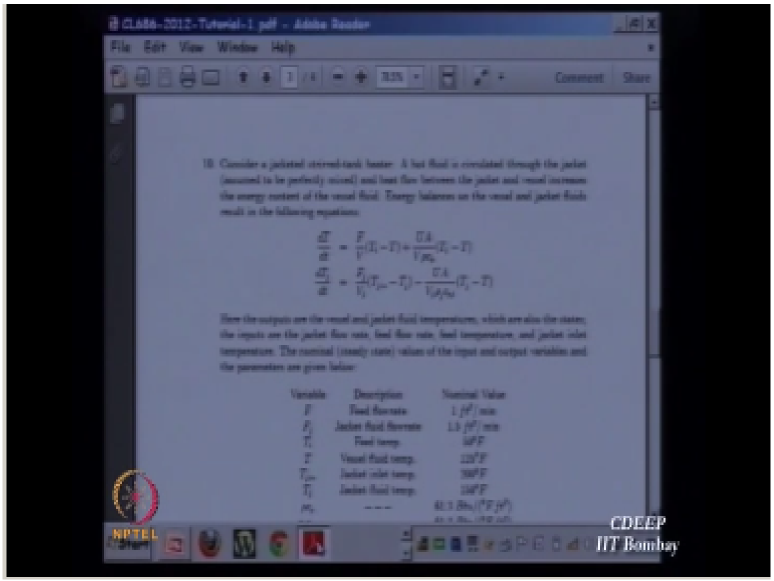

Real physical problems okay those are few words chemical engineers can appreciate both the problems the second problem actually it is not difficult to understand but you may face some difficult in appreciating what is the second model the first one here is just a system which is I will just draw this.

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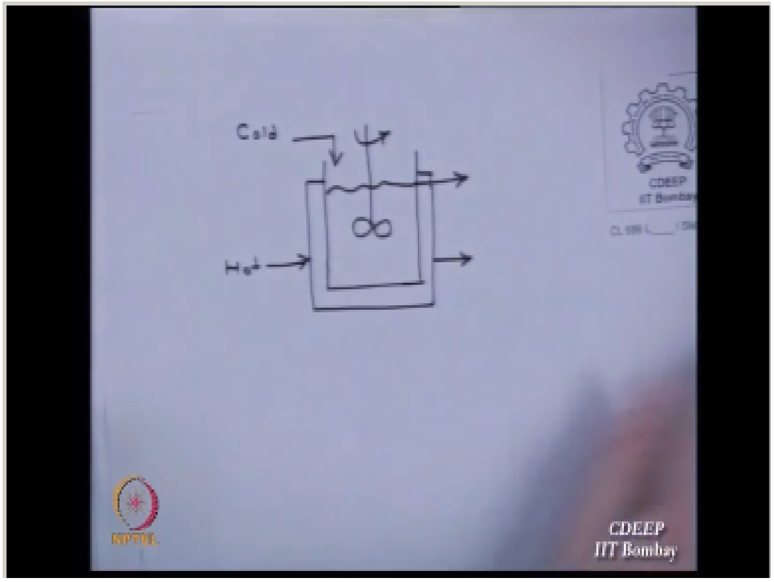

We have this symbol means this is well system okay and it moves out and then there is a jacket here the hitting flow could be stream for example okay some hot fluid in industry they use some hot fluid to let say heat the condense inside this time so the cold stream here and this is hot fluid okay so the cold fluid becomes hot becomes cold there is a exchange of heat between the two and the model for that is.

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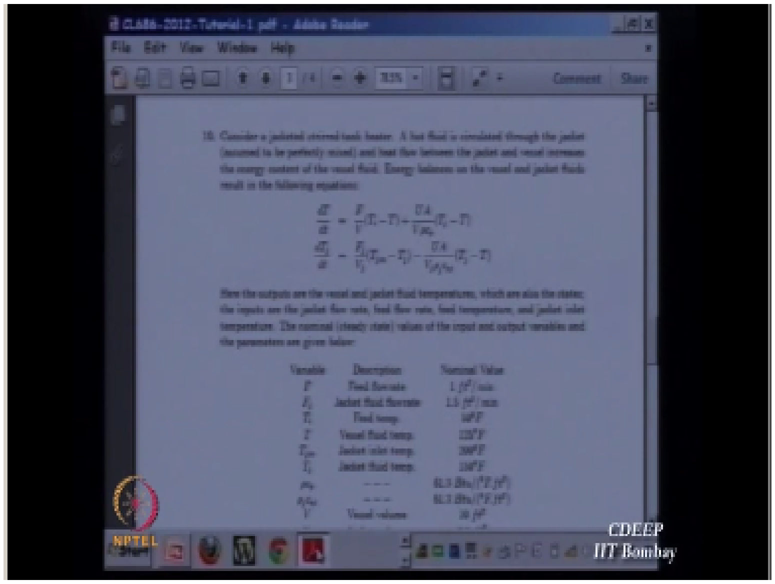

Is given by this two coupled equations so deviates this differential equations convert them into a model of this form x=a+bu okay and I given them potential variables you then you convert it into different equation by discrete and I get the model okay for this last two problems which I coming from physics you may need to use metal lab okay first 1 to 9 problems you can do by hand because in those problems I have given 1 to 3 you will get some nice Eigen values and Eigen vectors you can see here.

So 1,1 and you will get some 5 problems are mainly for exam and for doing some practice and practice and then the last two problems are real problems and I accept that all those who are non chemical engineers should appreciate the problem 10 chemical engineers should appreciate both the problems 10 and 11 okay next time I will try to put some more problems from electrical mechanical some other problems which I have I had some models from electrical circuit but I do not have values.

Because you have linear at some point you need actually some values of and so on or spring mass systems which are modeling some car and all those models are available but I am trying to get values for physical parameters so what we have been learned here okay we actually implement it on the project in metal lab okay so it is the problem from different domains and then what will you do if you do really here design a container you design a container for your system and so on everything that you do on in the class you do it in the type of okay.

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