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ADVANCE PROCESS CONTROL

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Lecture No. 03

Topic:
Development of Control Relevant Linear Perturbation Models (Part 1)

Sub-Topic:
Linearization of Mechanistic Models (Contd.)

So take a quick review of what we have done in the previous lecture and then we will take it forward to model development which are control relevant. If I put it in abstract form what we have discussed ordinary differential equations.

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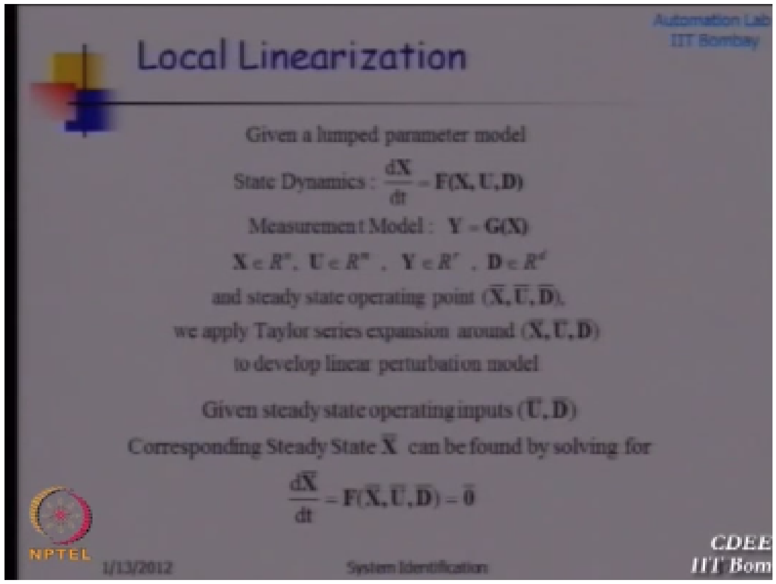

X are the states or X are dependent variables, U and D rep resent inputs independent inputs, U represent molecular variables, D represents the disturbances, Y here are measured outputs. Now I am using this notation here Y belong or N which means X is a vector which is N dimensional, U is a vector which is M dimensional, R is a vector, Y is a vector measurements are all dimensions, there are R measurements, there are N states, there are M inputs and there are D disturbances.

So we are looking at a system which is multivariable in general okay. then we are going to have a local perturbation model, this local perturbation model is developed in the neighborhood of some stead-state operating point. This is denoted here by $\overline{X}$, Ubar, Dbar where if you notice here I am using capital letters to denote all the variables, capital letters here indicate that these are absolute values okay.

We will move on to deviation variables, I will start using small letters for deviation variables. So these are absolute vales, and then I want to do multivariable LSDs expansion in the neighborhood of the operating point okay. The first question is bar these are dependent variables, independent inputs are only disturbances and manipulate variables okay. If I specify levels of Ubar and Dbar using the non-linear differential equation, I can solve for the steady-state.
I can solve for f($\overline{X}$, Ubar, Dbar)=0, I do this using Newton–Raphson method or Newtons method or some iterative methods for solving non-linear algebraic equations, steady-state is nothing but when you convert this differential equation and give it at stead-state you have a set of non-linear algebraic equations which have to be solved simultaneously. So that can be yes, done using standard techniques for non-linear equation solving.

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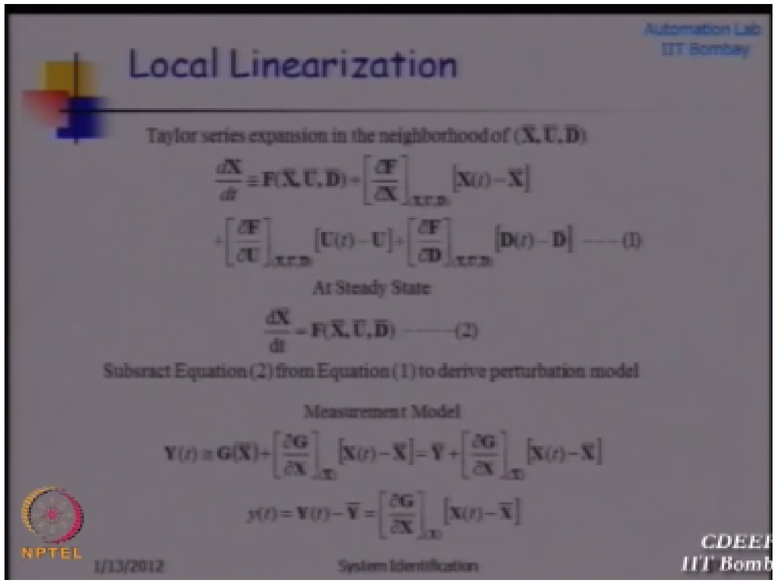

And then for given Dbar and Ubar you can find out $\bar{X}$ this is using a well known method such as Newton's method. I think found $\bar{X}$, Ubar, Dbar we want to use multivariable Taylor series expansion. So what is this multivariable Taylor series expansion. This is my multivariable Taylor series expansion, I am going to expand the right hand side in the neighborhood of $\bar{X}$, Ubar and Dbar okay.

Now here F is a function vector, F is a vector of functions. We know already about what it is, partial derivative of F with respect to X is going to be a matrix, it is going to be a matrix, what is the dimension of this matrix, F is a function vector there are N functions, N differential equations, and X has the dimension N. it will ne nxn, this will be nxn, mind you, when you write this equation you have to be very, very careful about the order in which variables appear.
I cannot write XT-$\bar{X}$, I cannot change this order of this matrix and this vector, this is a vector, this is a matrix okay. So matrix times vector, it has to be that way. You cannot even by mistake to change the order. What is the meaning of this $\bar{X}$, Ubar, Dbar written here, it means partial derivatives are evaluated at $\bar{X}$, Ubar, Dbar. Once you fix this $\bar{X}$, Ubar, Dbar I am also evaluate this partial derivatives at a particular point, this is a constant matrix okay.

Same is the case with $\partial F/\partial U$, $\partial F/\partial U$ is derivative of function vector with respect to inputs, this is a matrix which is nxm and you have a matrix which is nxd which is $\partial F/\partial D$. So there are two, there are three terms appearing here, two because of inputs, one because of the state itself. Just look at this equation, it tells you that this is not a algebraic equation, this is a differential equation.

A differential equation has a memory of past okay. If you try to analyze the terms in this equation, it has three terms on the right hand side. It says that the current derivative, current rate of change is function of what has happened in the past, where does the past information come from? X, X is the past information okay. Whatever is happened from time 0 to current time T is getting captured through XT or XT-$\bar{X}$ in this case perturbation.

What is happening new, what is the new input that is coming in, that is U and D okay. So it just says that rate of change of X or perturbations or small perturbations in X is given by this particular equation. Equation 2 I have just written this stead-state equation, I am going to subtract. Basically, you had steady-state we know that F($\bar{X}$ Ubar Dbar)=0 okay. So actually you are subtracting 0 from both sides.

But I am subtracting equation 1 from equation 2 okay, and I will get the perturbation model, I will get a perturbation which is X-$\bar{X}$ okay, when I subtract this equation from this equation F will vanish from both sides okay, on the left hand side I will get X-$\bar{X}$ okay, and right hand side will be again X-$\bar{X}$, U-Ubar and D-Dbar, this term will vanish when I subtract okay. in the same way I am going to linearize the map or the measurement model.

Measurement model is Y or measured variables which are some functions of states okay. I have taken the most general case where G is some non-linear function okay. And I am developing a perturbation models, so small yt is a perturbation from the steady-state Ybar which is given by ∂G/∂X, ∂G/∂X will be a vector which is rxn there are N states. I have just given these equations, these prices here where you are convenience.

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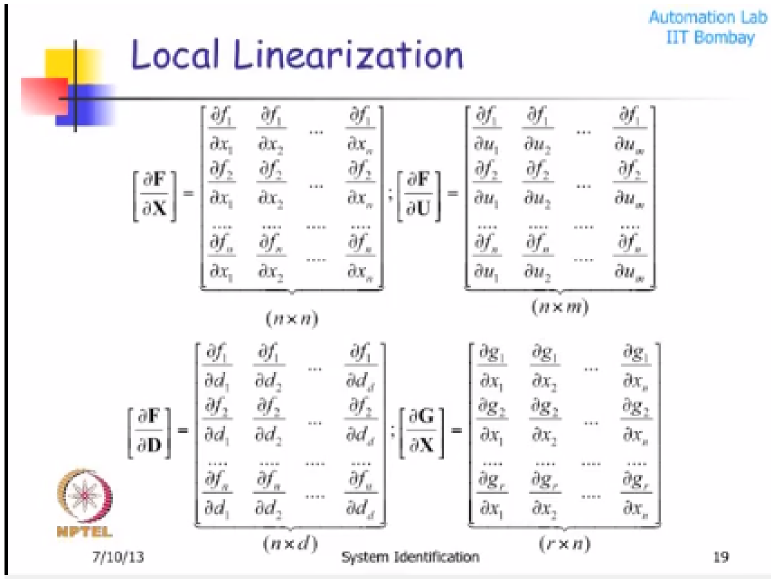

So these are nxn, mxn, nxd and rxn matrices. These are evaluated at the steady-state operating point. So once you evaluate them at a particular operating point you have fixed matrices okay.

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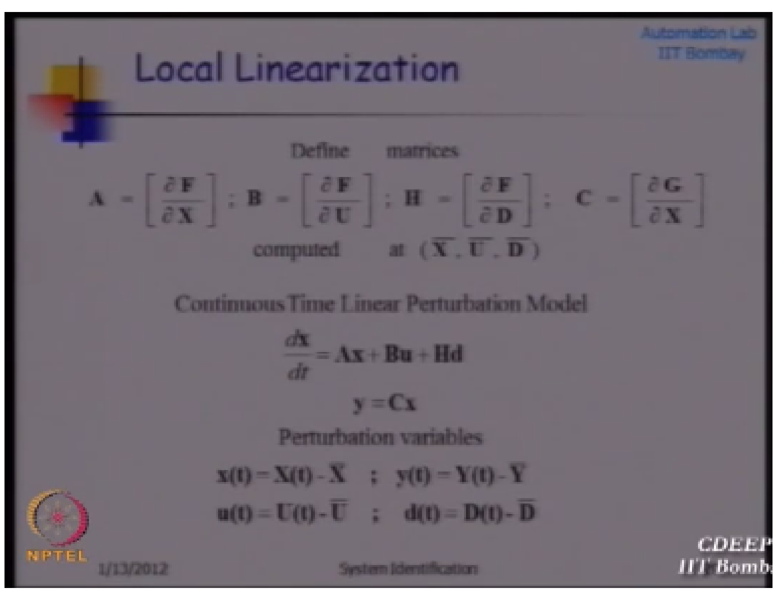

So I am going to, it is inconvenient to work with those ∂X, ∂F/∂X and all that, I am going to switch to a simplified notation ABC, you open any text book and control, they actually start with this model, they do not tell you how you got this model, they just start with this model X.=Ax+Bu+Hd and so on so forth. So but actually these are perturbation variables, there are small perturbations and they were the operating point.

And the connection with the models that come from physics that you know from your engineering background is actually through Taylor series expansion, linear approximation, local approximation and so on okay. So as far as controller design is concerned, I am going to use this simplified local linear model why, linear differentials are very well understood, even if they are multivariable.

I can solve them, I can manipulate them, I can play with them, you can solve them analytically okay. And that is going to be the strength when you develop linear control theory. Well what is my aim ultimately, what is the aim of control? Get to this point, somebody else can tell? Safety okay, I will put it in little more abstract term, I want to shape to dynamics okay, the way I want. Sometimes I want to maintain at certain point, sometimes I want to move from point A to point B right.

Where is that trouble here in this equation, one trouble is disturbance okay. What is it that you have in your hand is you. So I want to shape, I want to choose you in such a way that output Y has a desired transient behavior okay. So I want to manipulate and if I want to manipulate I should be able to turnaround this equation the way I want, that is possible very, I mean that becomes very easy when you get a differential equations.

It is not that you cannot do with non-linear differential equations, well for us to go there would take at least two more courses to finish the point where we started looking at non-linear differential equation manipulations, yeah.

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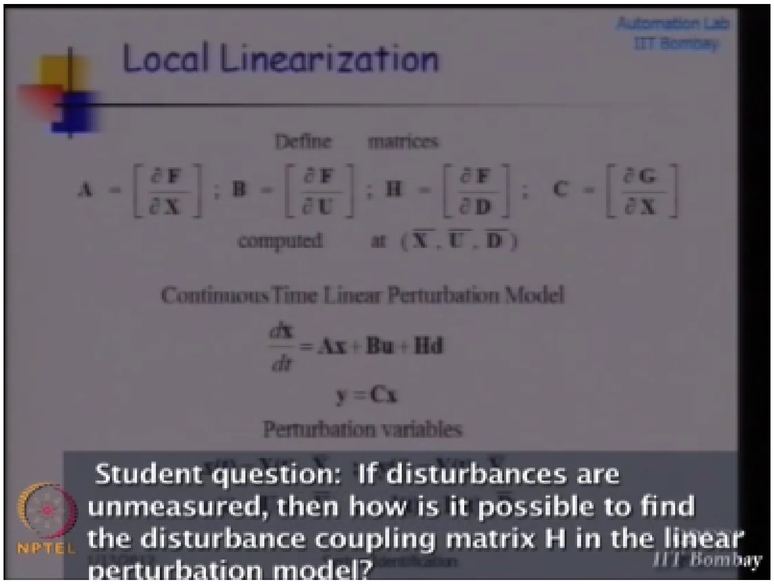

Yes we cannot many times capture H so I will be actually dealing with it much more elaborately in next few lectures so right now what I am saying is that see in this particular case assumption is that you have a good model coming physics available to you in that case I can find out H matters okay well when you do not have model what you do we will answer that question later.

Right now we are considering a scenario where you have good first principle model you have good mechanistic model and this is possible for some systems see it is not that I means some systems in robotics okay may the equations that come from mechanics can be written quite precisely okay so it is not that I mean this course the way we are going to go about is not related to any particular domain though it say advanced process control you know it is a advanced control of course.

So in some cases you have these models in some cases you can write those you can find out H matrix starting from a mechanistic model okay.

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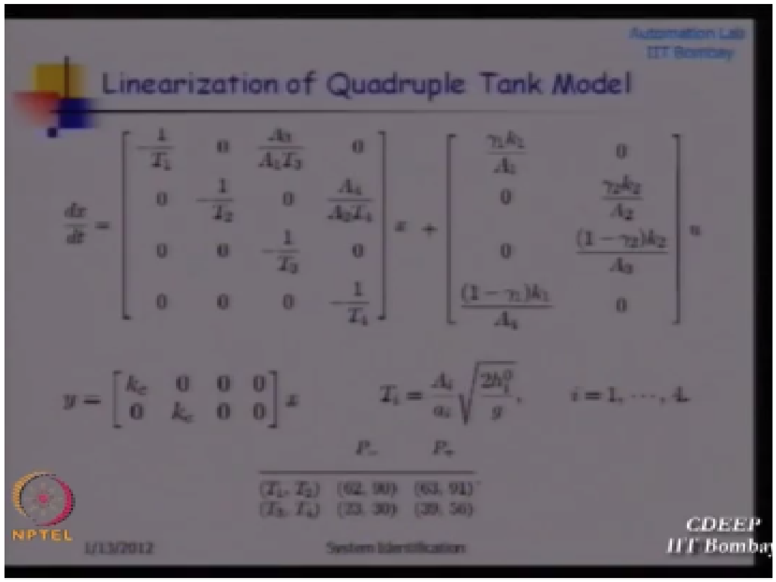

Well we have seen this matrix for quadruple tank set up.

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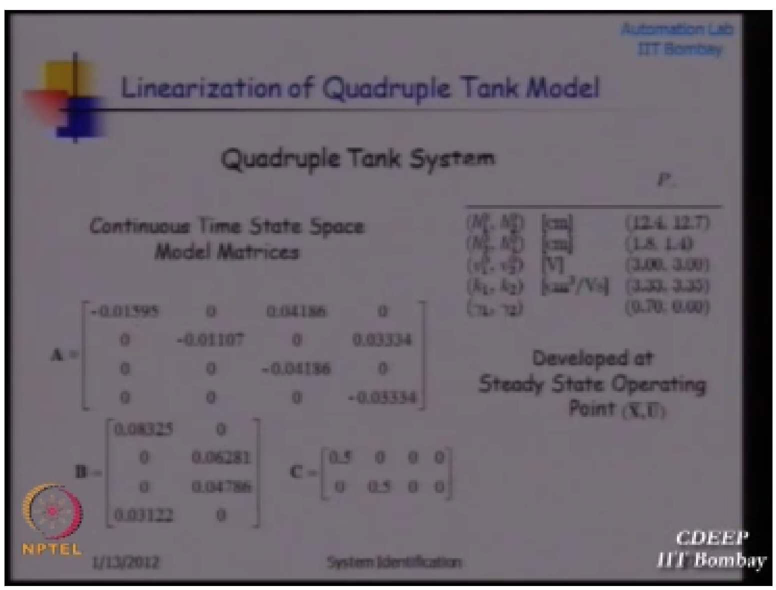

And then actually this is the steady operating conditions which have put them on the corner right corner the steady state y bar and u bar is nothing but 4 levels and 2 inputs which are given their okay and if I actually linearism, substitute values I get these 3 matrixes A, B, C there is no H there is no disturbance considered right now so there are 3 matrices that we get.

And then we said well we can actually convert this into a transfer function matrix I just wanted to map this to a transfer function matrix the model that we have is actually a this model that we have got here is called a state space model okay model linear differential equation it tells you something about what is happening inside through the states okay but all the states may not be measured in this particular case out of 4 levels.

Only 2 levels are measured okay so we know here this model is more powerful than input output model tells you something about things that are not measured.

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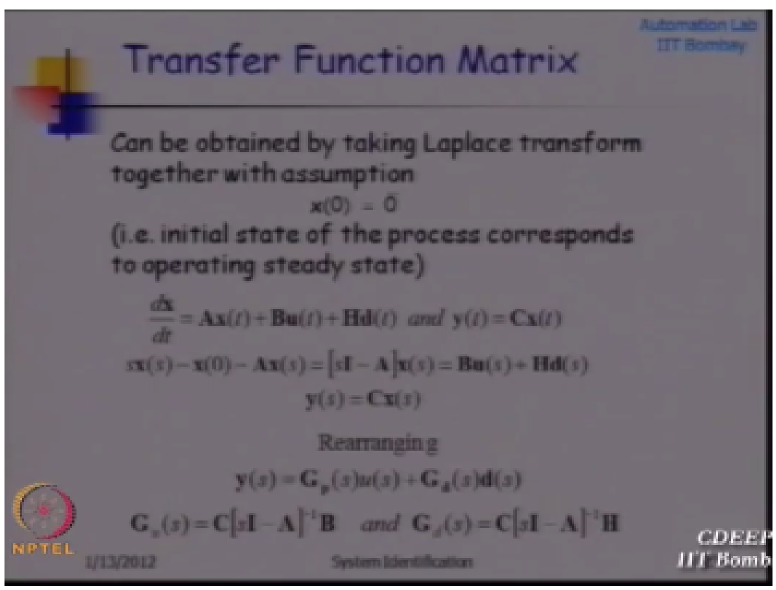

Now the next things is you know is through Laplace transforms if I assume that initiative 0 and if I take Laplace transforms we did this last time we get this transfer function tricks why which relates to you and to D.

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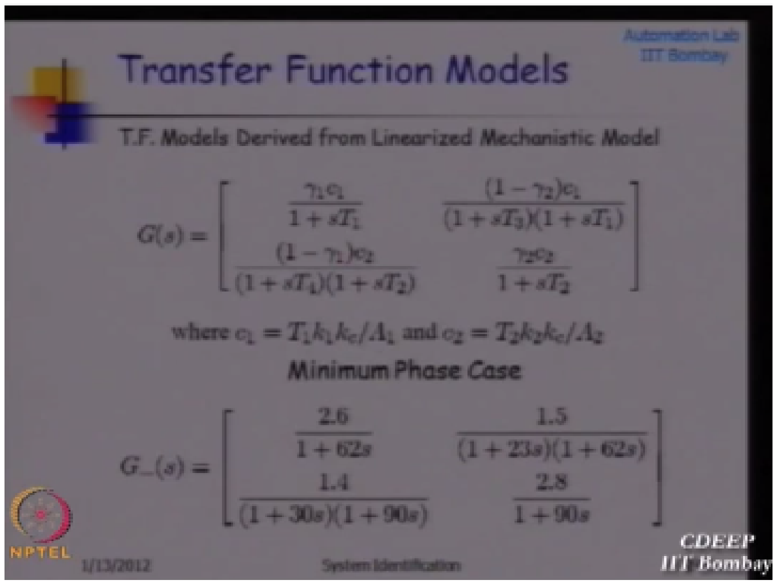

And for this particular case I calculated a transfer function matrix this turns out be this 2 x 2 matrix 2 level measured H1 H2 to level lower levels and 2 voltage inputs, you can see the transfer function matrix tells you that level 1 is affected by both you know V1 and V2 level 2 is affected by a both URL V1 and V2 okay matrix is full in reality when you go to a complex system you will have many measurements you will have many inputs and typically the matrix will not be you know diagonal.

It will be full many things affects many things it is very difficult to find a system in which only 1 input affects only 1 output every rare okay.

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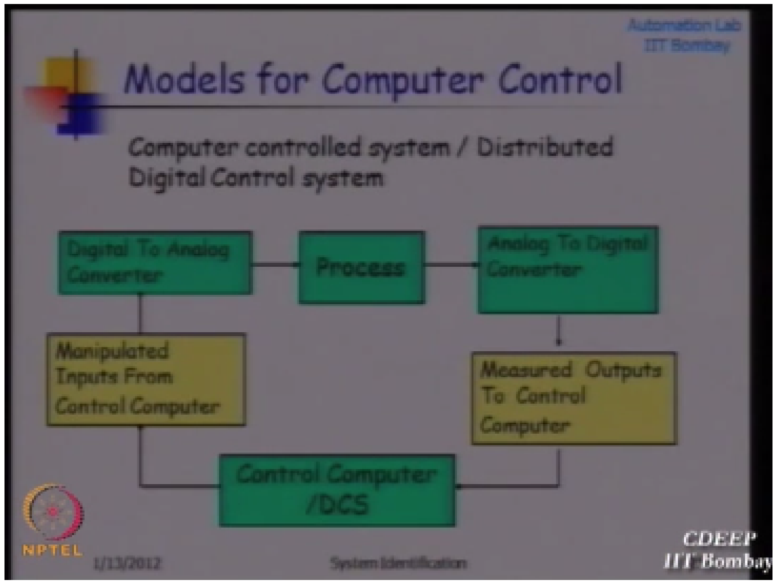

And we started looking at this computer oriented models and we said.

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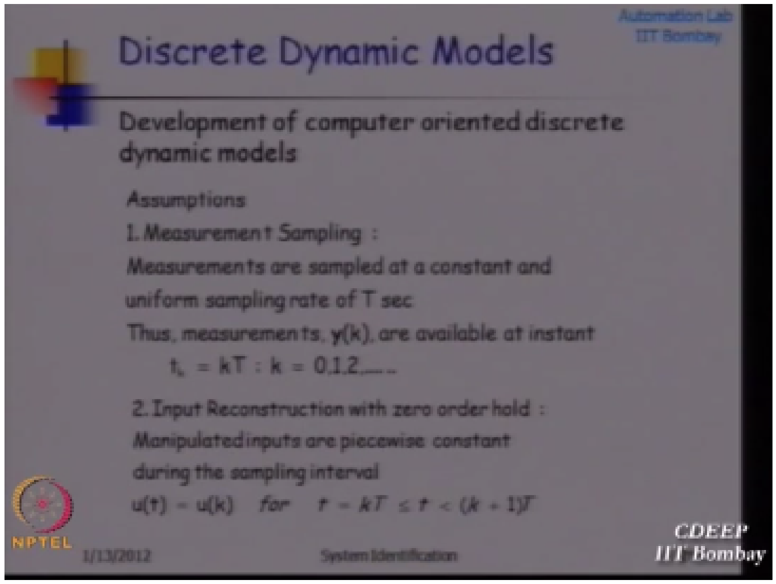

We have to worry about 2 things 1 is measurements are sampled okay you have you know regular interval at which the measurements are coming to a computer and I am going to deal denote them by yK that is my notation actually yK means measurement variable y available at time kT is k is the sampling instant I start from say 0, 1, 2, 3, 4 gab between 2 samples could be 1 second it would be 5 seconds could be 10 mille seconds.

Depends up on the system which are considering and then inputs I said are going to P wise constant that is because I am sending my from my computer, my computer can only generate sequence of numbers my computer cannot generate a continuous signal.

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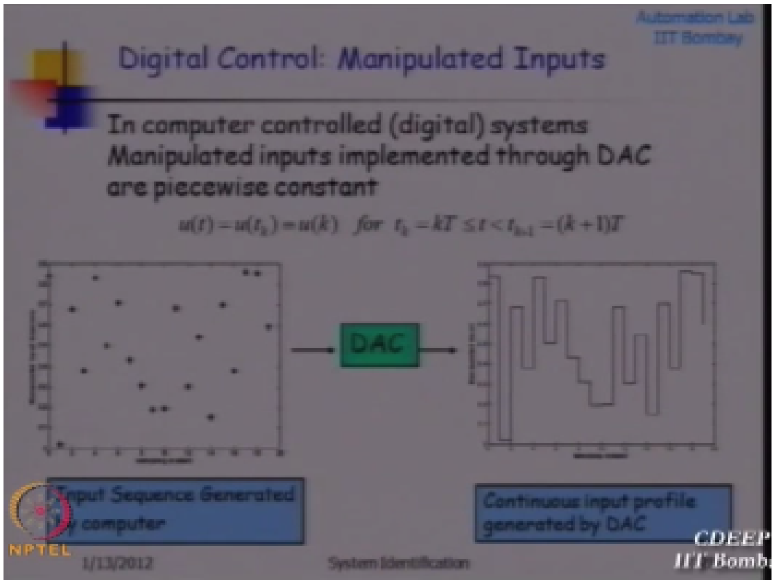

I need a device called as D to A converter digital to envelope convert and this digital to envelope convert will reconstruct a continuous time signal a wall control wall or a s stepper motor or whatever is actual element will receive a continuous signal it will not receive a you know pulses or will not receive impulses finite impulses it will receive a continuous time signal.

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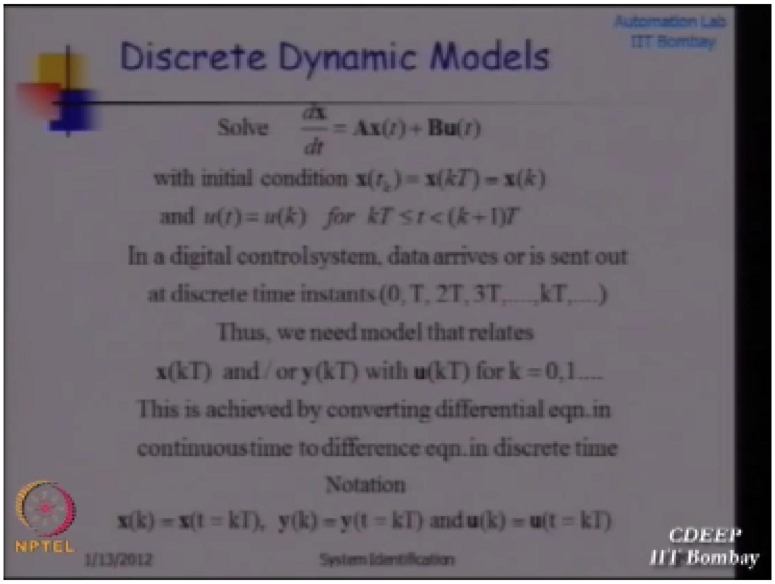

So now I need to adjust my model to the relativity that I have this differential equation and I want to develop a computer oriented model okay a computer oriented model which in which we convert a differential equation into a difference equation what kind of difference equation I want to hope in time okay I want to go from time instance 1 to 2, 2 to 3, 3 to 4 okay every time in my computer I am going to get one sample at you know let us 5 seconds interval I want a model that relates what happen now to the next instant what happen to the instant to instant after that okay so convert a differential equation into a difference equation.

What are the constrains or what are the at let us snow let us not worry about time 0 let us look at a situation where we are standing at some snapping instant k okay at current time okay and I want to I have though I know the state value at current time which is xk I am going to denote it has xk here.

This is a short term notion for x at time Tk where Tk is nothing but k times T, T here is the sampling interval okay T is sampling interval so sampling interval will depend upon how to choose a sampling interval is somewhat complex bossiness under is theorem quotient sampling theorem which light you can probably go back and read in the references that I a have given I do not want to spend time on theorem here but what is important I will tell you should sample fast in enough so that you do not miss out the major features of you signal.

Okay what is this you know from system it is different okay in a furriness which is very slow it might be sufficient to sample every 1 minute you do not miss too much if you take furious temperature measurement every 1 minute.

Because fermions are very slow dynamics okay automobile you need samples every you know 509 mille seconds or 100 millisecond okay we where controlling of fuel cell we needed samples at 10 millisecond so it depends up on what is so this t could be you know what ever depending upon the system and as a engineer you have to take a call what should be the sampling interval okay.

So digital control system the signals arrive okay a measurement arrives at instant time kT and input it send out to the system at time kT so let us call it instant k sampling instant k so I want to convert it differential equation into difference equation I am going to use a notion here xk is nothing but the state of the system at instant k okay it is a short hand notion. Yk is a output measurement at instant k uk is the input sent out to the plant at instant k.

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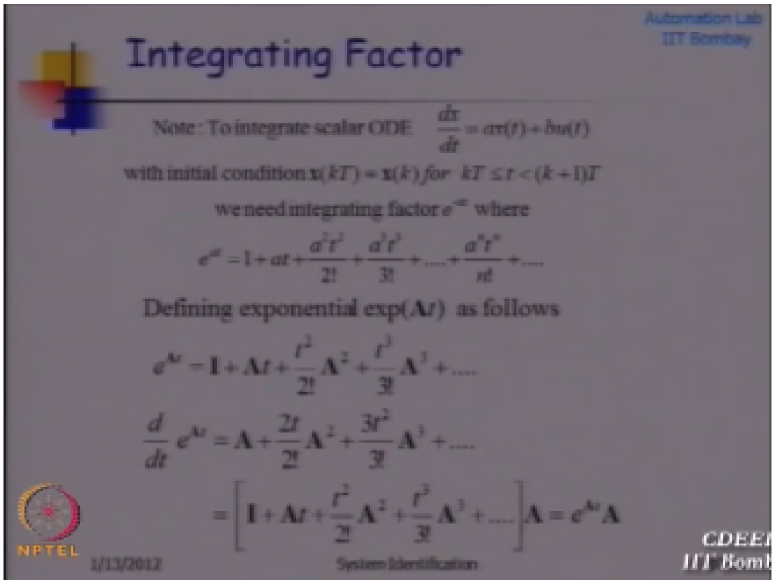

Okay now how do I convert will I want you to remember something that you have done in your 1^{st} year of engineer remember integrating factors how will you integrate so you take a xt1 on the left landside then you have a integrating factor e^{-1at} right then you multiply both sides by integrating factor and a new integrate both the sides I am going to do this same thing expect I have to do it for a vector differential equation.

Okay so I need something I need something that is equivalent to an integrating factor but now I do not have a scalar in this equation I have just put here a scalar equation a is scalar b is scalar okay this is a very simple system it is very easy to integrate this differential equations all of you know how to do it I need to define something equivalent to an integrating factor I need something equivalent to this what is nice thing about eat or et or if you differentiate what will you get back it is a very nice signal continuous time signal if we differentiate et will get back et okay I want to introduce what is called as matrix exponential now the analog you can see how it is defined see the factor here 1 here is replaced by I identity matrix okay then you have a times t, t is a scalar of course while it is convention to wrote it like this.

Probably when you do scalar and matrix multiplication you should not write scalar after the matrix we should write e^{tA} but books sometimes or it inter changing okay now look at this new we introduced matrix exponential well, let me tell you something e^{At} is we defined as limit of this infinite sequence on the right hand side just like e you know scalar At is defined as this is definition of e^{At}, now I need to find out what is a derivative of this particular you know matrix exponential is a stake derivative on the right hand side.

Okay we will see what have you done here I can take out A and derivative of e^{At} turns to b $e^{Aat - A}$ first a fall do loss get eAt is matrix, even though we write as eAt and e is a scalar eAt is a matrix the definition is i + Ap A^2 B^2 and so on, so this is a matrix, so when I differentiate this I get eAt post multiplied by A matrix you cannot writ pre multiplied expose multiplied by however we going to use this.

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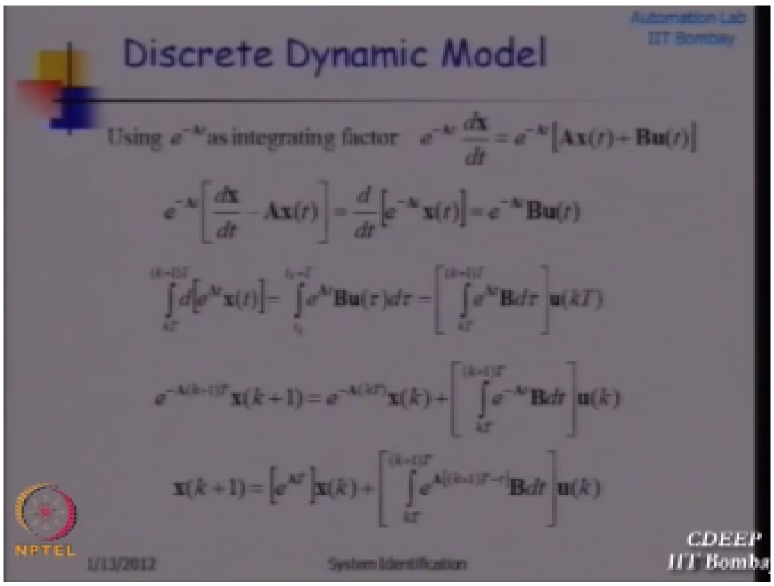

Okay now I am going to go to same way that what we do for the scalar case I am going to multiply both the sides by the integrating factor okay is everyone with me on this I am P multiplied by in this case you carry actually in this case you can in my derivation is convenient to write A in this particular case we can write A or either way because what I want to stress is when you probably I over a state when you deal with this and say have to be very, very careful you know order, order is to be maestro okay.

Now I am rearranging this equations I have just taken e^{At} on the left hand side okay, so the second step is everyone with me on this then and now I am going to integrate from where to where and standing at sampling instant A I am just worried about going from K to k + 1 okay.

I just want to tell that relates what is happening now with the next sampling in strict okay that is all I am worried about, so I am going to integrate for time KT to K+1t where t is of course the sampling interval okay, now here the right hand side integral cab be simplified because what we know is within the sampling interval my inputs are constant piece wise constant okay, within this sampling interval my inputs are piece wise constant, so I am going to take this U,K,T outside and then I only have to integrate e^{A} times B dT okay.

This integral I have to compute okay this everyone with me on this B of look at what is here this is B of this when I integrate this I will get you know e^{At} xt I apply two limits I have skip 1 in between step is anyone has doubt with this I have just apply two limits I have taken the integral apply two limits and moved one part to the right side okay yeah, yeah it is 8 out yeah that is a type of yeah please correct it this is $e^{A\tau}$ not 8t in the second term no it is important it squared important what you say is squared important.

Okay now or either Aτ or I have to change to dT either of the 2 okay I think changing to dT with the help instead of changing everything to τ okay, so is this step here okay I have just integrated the left hand side apply that two limits okay and rearranged right, now what I am going to do is I am going to multiply both the sides by e – K + 1t okay when I do that see what remains here it is – A, k, T this is + AK + 1t, so the difference will be only e^{At} that what is remains okay here my integral changes to this.

Is it okay I am just going slow because we should you know understand your steps this is clear yeah I am going to multiply +A both sides now A^{K+1t} K + 1 is a fixed value, so I can take it inside the integral without making any it is – this should oh there is 1 more up to here yeah we should have yeah in this equation I should have e^{-80}.

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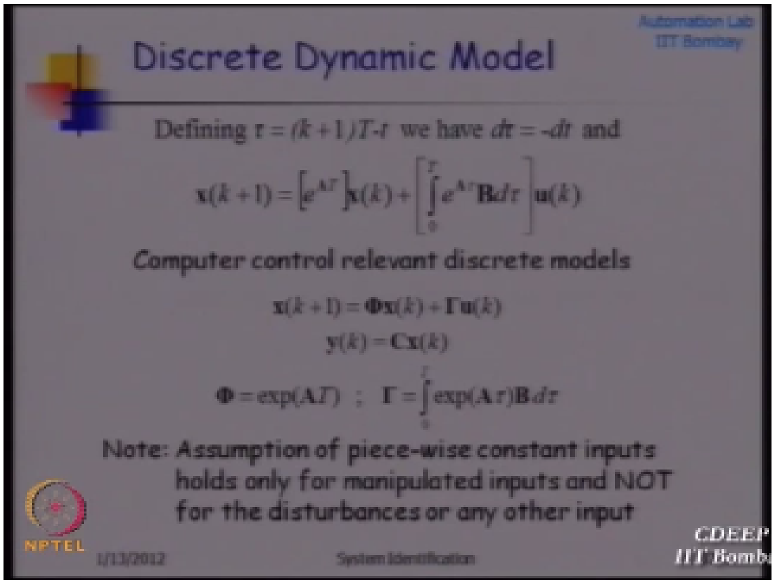

I show and that is better is that okay now yeah okay so for we have reached this point this now consistent yeah, so I need to integrate I need to find out e^{At} I need to integrate this right side equation well now how do you do that I am going to do some change of variable I am changing I am defining τ as K + 1T – t so ß will become – dt and end with a little bit of algebra you can show that a integral can be change to 0 to t $e^{A\tau}$ B dtτ okay so with this particular modification you can change this to okay.

So nice thing about these two matrices is that here and here K has disappeared okay this is only function of B A and sampling interval okay it is only function of B A and sampling interval so now I need to worry about I am going to define two more matrices ϕ and γ okay and I am going to use this ϕ and γ throughout to denote this linear difference equation model this is my linear difference equation model which is compute oriented computer relevant okay.

Digital control relevant whatever you want to call it output map does not change there is no integration output map is a measurement map is a static map why, t=c(xt) so yk at instant k is c(xk) so that does not change I do not want to do anything for changing the measurement model I only have to change the differential equation into difference equation, okay.

So this ϕγ matrices are defined as exponential e^{AT}, where T is the sampling interval and γ is this complex integral that we have instead here and we will now talk about how to compute these integrals. Well just finder that piece wise constant inputs it only holds for manipulated variables the disturbances are not piece wise constant what is going out of my computer is piece wise constant okay, so developing a model for disturbances which are not really piece wise constant is a tricky business when it comes to computer oriented modeling where we are going to look at it this tricky business. But let me just free and let me just say that it is not an easy task, okay.

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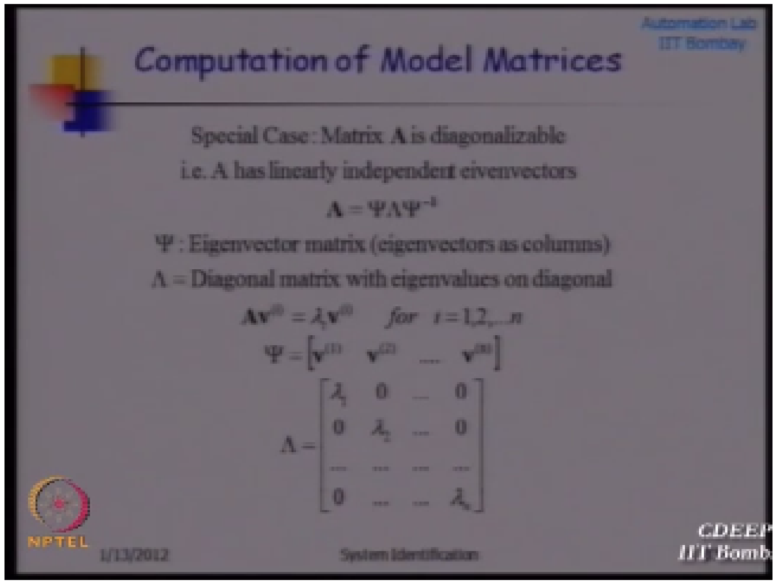

How do I compute lot of algebra, I assume that all of you are familiar with this grand equation you can I am going to give a special case first then I will talk about how to do it for general case through another method why I am doing at this way is because many times it is easy to get some insides. Let us assume that matrix A that we did after linearization of non linear differential equation let us assume that it is ideal value is, it is Eigen vectors are such that they are linearly independent.

If they are linearly independent you can diagonals matrix, matrix diagonalization is a very, very powerful tool it is used all over in applied mathematics in engineering. What I am going to do is I am going to A as matrix ψ Λ ψ inverse, Λ is a diagonal matrix it has all the Eigen values appearing on the diagonal okay, ψ is a matrix in which you keep Eigen vectors next to each other okay, so this ψ is a matrix in which this Eigen vector is a column vector okay, v1 is the column is a Eigen vector for Eigen value $\lambda 1$ v2 is for $\lambda 2$ there are n Eigen vectors I am just keeping them next to each other this equation talks about diagonalization of matrix A, okay is this clear it is okay. I am going to use this equation to compute matrix exponential it makes it very, very easy.

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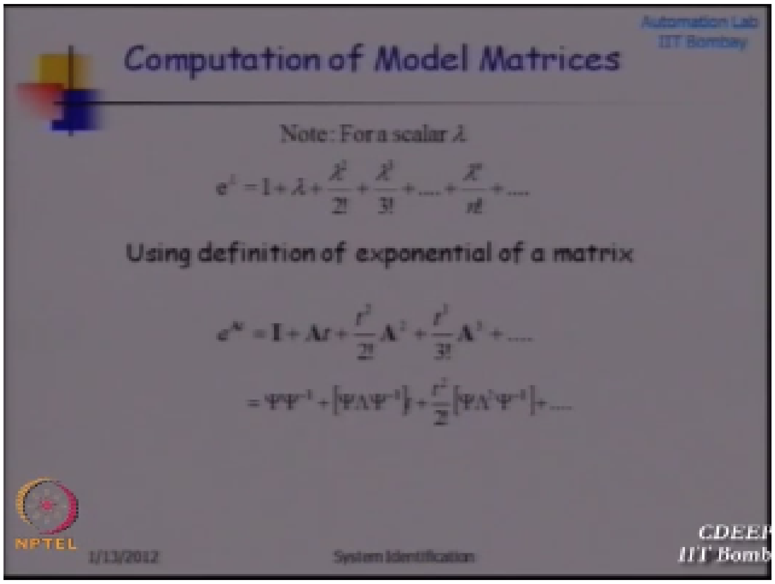

Okay, just a reminder what is scalar case okay, this is my e^{AT} I want to compute this I going to just rewrite this, these everyone with me on this what is A^2.

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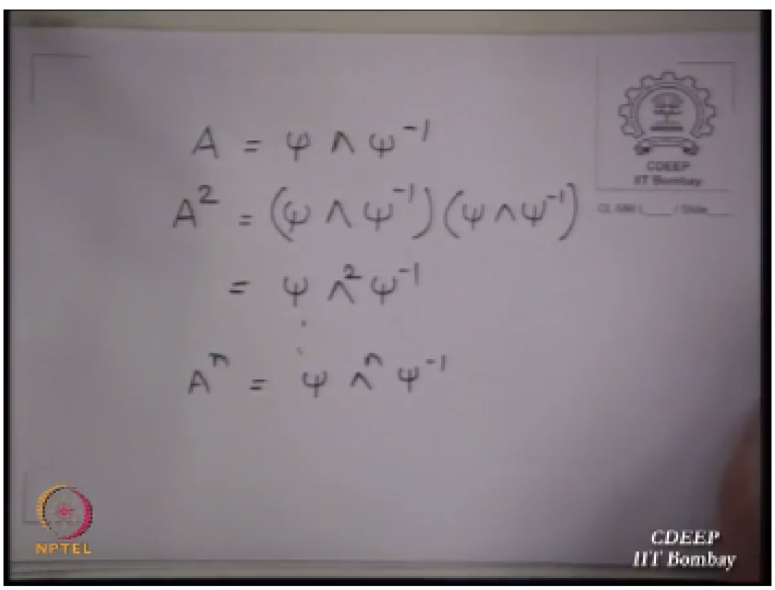

See we have this, so A^2 is $(\psi\Lambda\psi^{-1})(\psi\Lambda\psi^{-1})$ which is $\psi\Lambda^2\psi^{-1}$ okay, so likewise I can write A^n as $\psi\Lambda^n\psi^{-1}$ okay, this is a trick which I am going to use.

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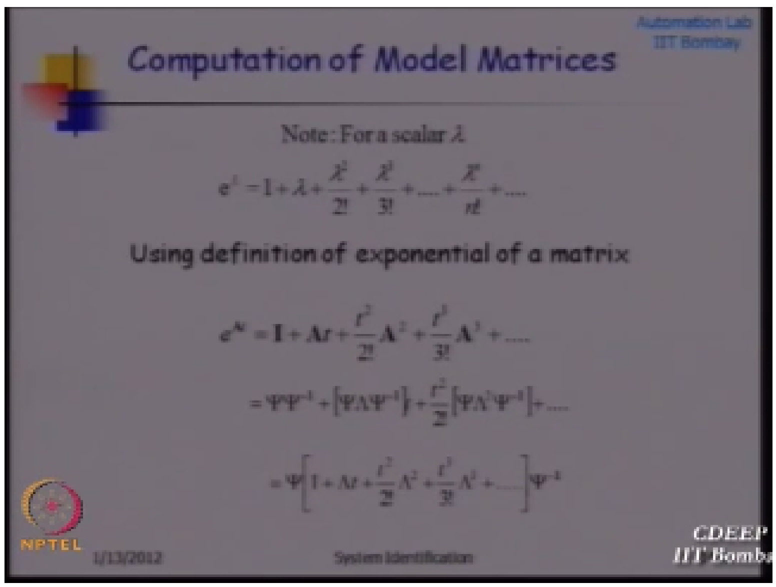

So this is the trick which is going to use to convert I can just pull out ψ and ψ^{-1} I get this i+ this in term you know the matrix inside the brackets is very nice why it is nice, because λ is a diagonal matrix okay, λ is a diagonal matrix.

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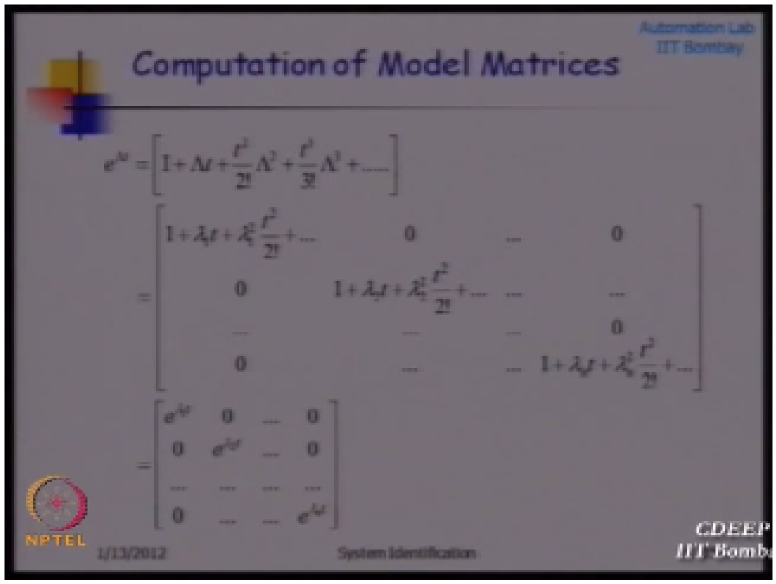

So what is $e^{\Lambda T}$, $e^{\Lambda T}$ is nothing but $i+\Lambda T+$ now look at this matrix if you actually compute this matrix okay, look at these diagonal terms, diagonal terms will be this is the diagonal matrix, this is the diagonal matrix, this is the diagonal matrix okay, I am just combining that into one big matrix this one big matrix is $1+\Lambda+1T+\Lambda 1^2$ but this is nothing but $e^{\Lambda 1T}$ right, and so on so for each one of them tabs into $e^{\Lambda T}$ so this is very nice.

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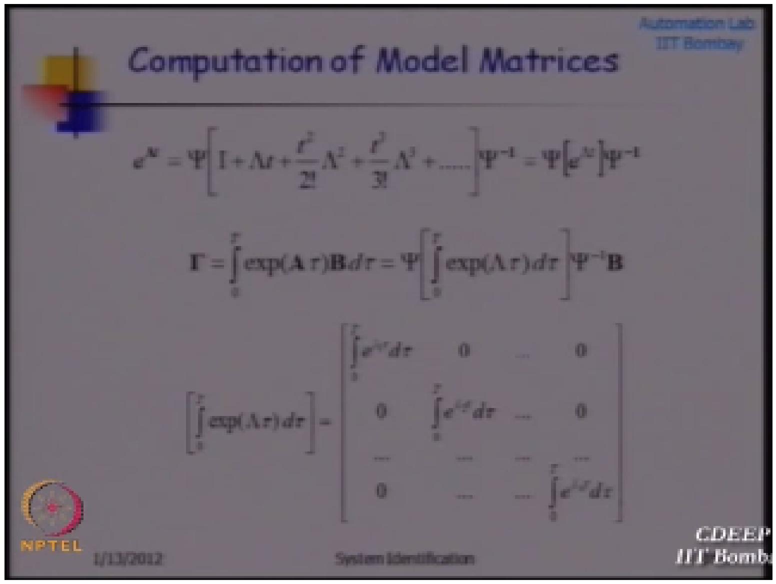

So which means you know computing matrix exponential is not that difficult if you know Eigen values and Eigen vectors you just compute Eigen vectors you compute $e^{\Lambda T}$ you are there. Well computing γ once you know how to compute this computing γ also becomes very easy okay, is everyone clear about this is this fine any difficulties no, okay. so computing this matrix is inside matrix here the one inside this.

Now once you have Eigen vectors decided, the sixe the constant matrix then computing these integrals is not at all difficult it is very, very easy you just have to integrate each term separately.

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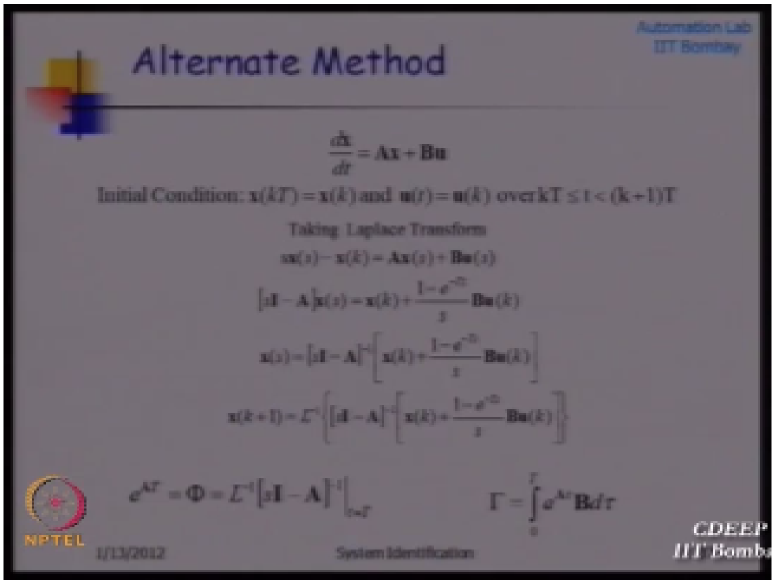

I am talking about an alternate method of doing the same thing how do I get this ψγ well I have this differential equation and then I am going to now go through Laplace transforms, okay. I show how to do it in time domain let us also look at Laplace transform if you comfortable with that okay, take Laplace transforms on both sides okay. The difference is when you look Laplace transform of a differential equation you will get x0 okay, what is x0 in this case no it is not at time 0, it is at time k I am solving differential equation from time k to time k+1.

So here instead of x0 I am getting xk we assume that x0 over 0 but xk is not equal to 0 I cannot ignore it okay, I have to take into consideration. So this equation I have this xk appearing which cannot be ignored of course this brings in information about the passed into the system okay, how I am going to take a Laplace transform of U which is starting at time kt and going up to time k+1t okay, I am model as a pulse so pulse of duration t is entering the system, okay.

So it is like a forward step and then a delayed step in the negative direction so you are staring from 0 going to Uk coming back to Uk-1 so that is how it is modeled, okay. So to go back to time domain I should take Laplace inverse and if I take this Laplace inverse I will get ϕ, if I go to time domain from this equation I will get ϕ^{AT} is nothing but Laplace inverse of $sI\text{-}A^{-1}$ okay $sI\text{-}A^{-1}$ yeah, that is because of pulse.

I am looking at input as a pulse square pulse, okay and my γ would be e^{T} integral 0 to T you have two integrals coming here into picture and then combined will give you this particular integral I will just show it for one particular example yeah, yeah.

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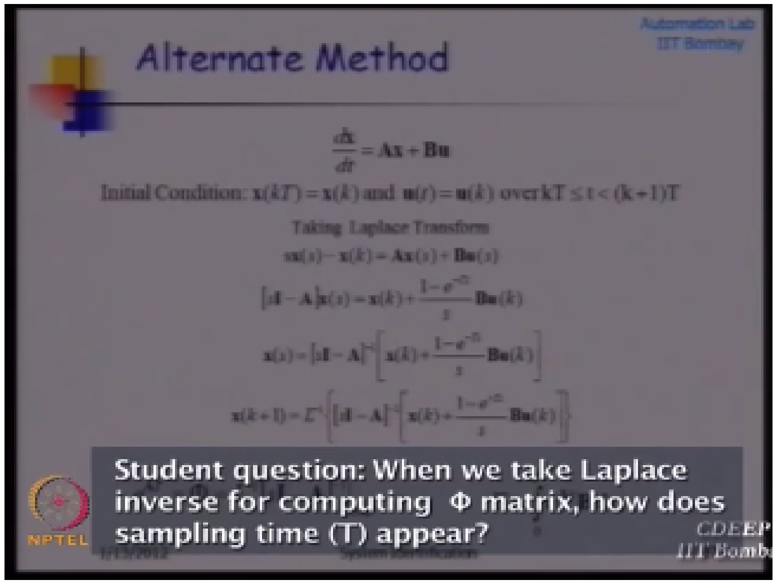

Except T you will get x at time you will get x(t) and then you find out x at time k+1t okay, I have skipped in between two, three steps because you cannot go to that detail you know you have to fill in the blanks okay, you will actually when you take lap lose inverse you will get x of t then you put t = k + 1 t and then you will get the next equation okay, is that clear? Yeah.

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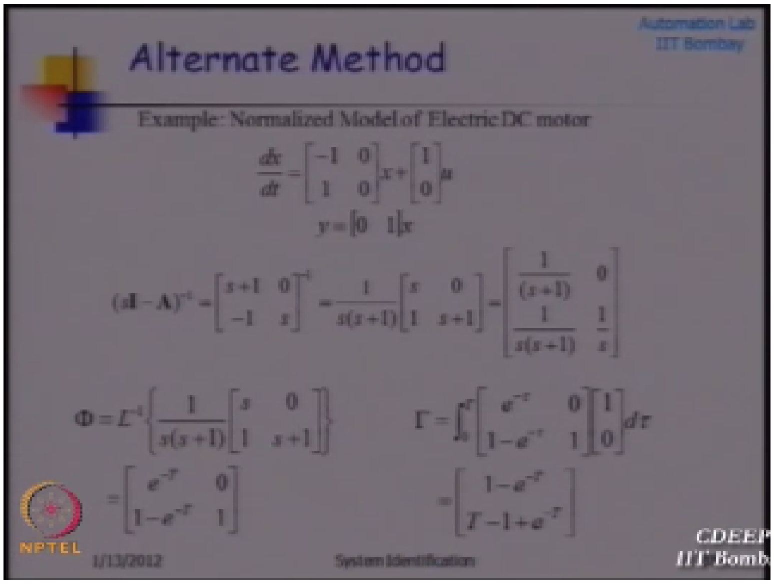

Okay let us take simplified model this is a simple problem from Astoven Vitim Mark one of the classical books in bio central, so let us not worry about what is x n what is by this is dc motor model you know just look at the algebra that we want to learn now okay, I want compute five so I should compute si – a inverse now these kind of matrices probably you have an some of you at least may not adult with earlier I am going to do algebra on these matrices as if elements are numbers.

So how do you find out matrix inverse? You take adjoin you take determinant I have done the same just look here this matrix si – a inverse okay, in fact you will notice lot of similarity with Eigen value equation and si – a what is Eigen value or how do you find Eigen value λ I – a okay, so this keep that in mind they are not going to be that they are not there same things. Okay so if I actually do this algebra of si – a inverse for this particular matrix I will get this matrix here on the right hand side one upon and then I will take a lap lose inverse.

Now how do you take lap lose inverse element by element to take lap lose inverse that is very easy if I do that at lap lose inverse of that I will get this equation I chose one particular sampling interval t let us say one second point was dc motor maybe pointy 001 second then I will get these elements you know right now I am not computed for a specific t but you will get disable this is how compute this is how I, what about γ?

So γ what I do is I write it a general expression for five okay e a τ and then I integrate, if I integrate I will get this right hand side here is this clear? Is that example clear? This is just go worried okay. This is how I can compute I can convert now when you are going to do it actually the example possible system use this one of these whichever you are conformable with Eigen.

But actually when we the main thing in this course is going to be you know using met lab programs and met lab you do not have to do all these equations writing this is a mat lab common call c to d continuous to discrete you just give matrix a, b, c, matrices it will convert for you and will give you 5 γ c you do not have to do any of these complex computations when it comes to a real big problem, so yeah.

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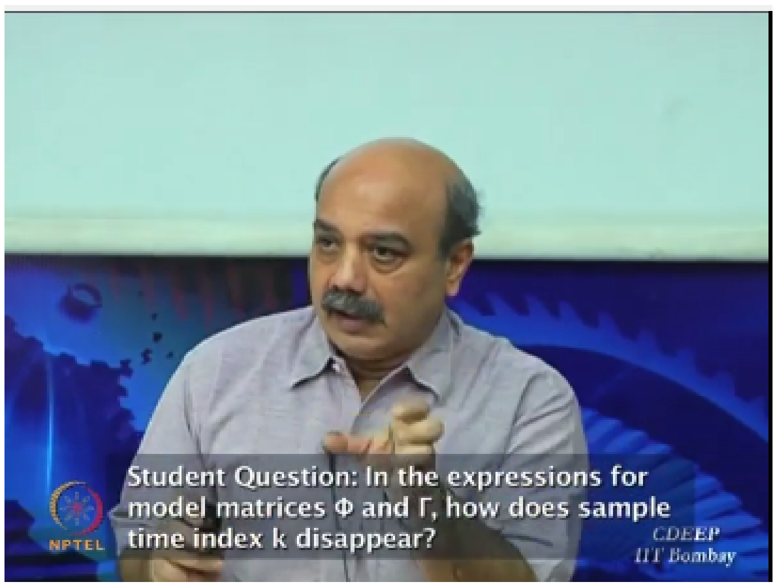

Not kt k is gone see first of all five and γ are not functions of k at all they are functions of only a b and sampling interval there see you will get only the difference you will get only the difference between the two time instruments you will never get k itself I mean variant systems very simple okay.

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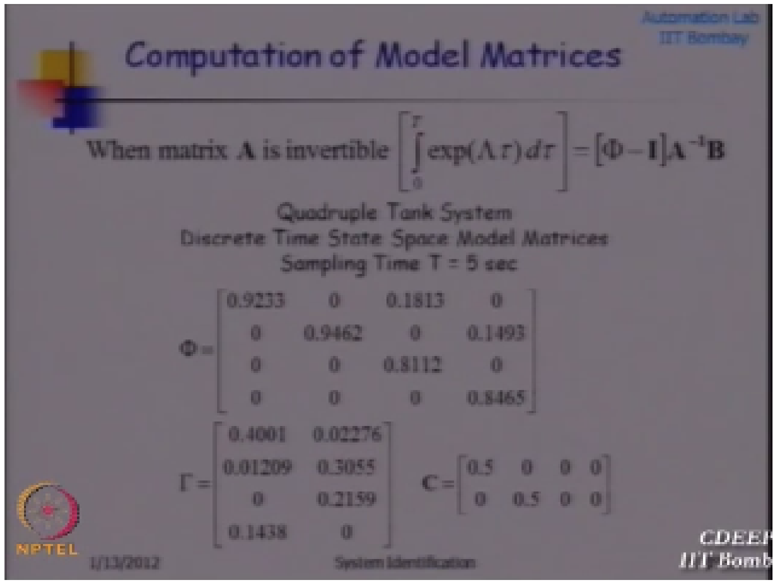

Well when matrix a is invertible you can derive this formula for γ okay, so if a has linearly independent Eigen vectors does not mean it is invertible it may have a0 Eigen value it may not be invertible but if it has linearly independent if it has all Eigen value which are non 0 then you have simplified formula for computing γ, okay quadruple tanks setup what I did of course is asked mat lap to do this I just give a b c matrices and mater lap just it be raise proceed to d I do not have to do bigger and bigger matrices you cannot start writing your own programs you just use mat lap to convert I get these numbers this is a okay.

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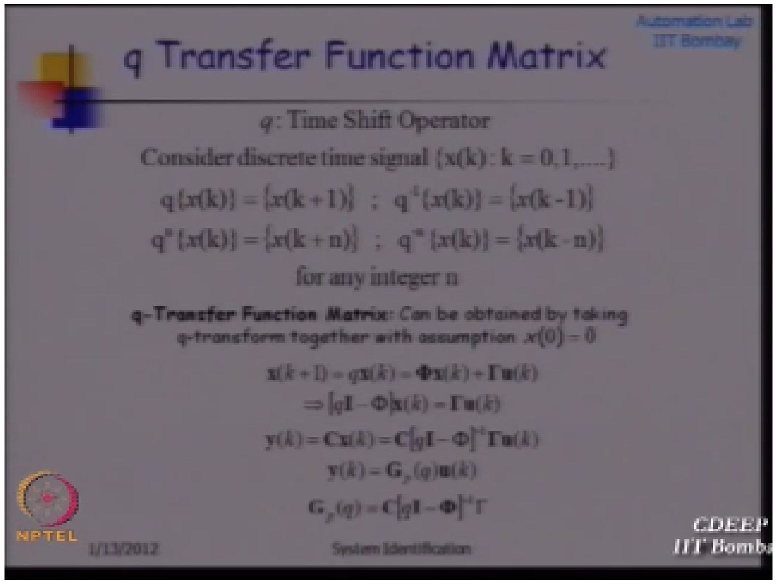

Now I want to go and introduce something which is going to help me throughout the course I have to use some kind of transforms to denote to do some of the algebra that I need to do later on and for that I am going to use something called a shift operator a time shift operator q is going to denote time shift operator okay. now what are we considering here we are considering a discrete time signal okay a discrete time signal x k is a collection of vectors I have put it in curly brackets okay.

So see when you write when you write sin t where t goes from some you know 0 to some value o to infinity what does it indicate? Does it indicate one value of the sin it indicates the entire function going for a time goes from 0 to infinity it is the entire time function from 0 to infinity, in this case I am considering discrete sin signals so I have to look at sequences, were sequences of vectors oaky k is the index I have sequences of vectors, I am going to define a shift operator q okay.

Q is the time shift operator when q is on x I will get one time in future you see this here when q operates on this signal xk I will get signal x k + 1all that is one okay so you get the shifted sequence, in general when I operate okay so I have shifted sequence make this correction appear small conceptually it is means a lot okay. So when q operates q is a shift operator time shift operator you know I want to look at signals which are shifted in time now I am going to develop something called as a q transfer function okay.

Why I need a transfer functions you know progressively it will become clear to you why I need transfer functions, so x k + 1 I am going to now skip writing curly bracers okay it is understood we are dealing with sequences every time I am going to write curly bracers.

So I am going to take a q transform so x k + 1 is nothing but q times x k is this step clear? With this definition, x k + 1 is q times xk, q transform of xk is nothing but xk xq 0 okay you are not shifting forward you are not shifting backwards is the same sequence okay. So this equation I can rearrange as q i – 5 okay times xk okay = γ times uk okay. And then y = c of xk I would just eliminated x k, xk is q I – 5 inverse times uk I have just substituted that here okay.

I got this equation which is between only input and output okay y = g q x uk okay this is clear any doubts here? No, okay so this new animal is called as pulse transfer function, pulse transfer function matrix there going to deal with two different notations one is Q transform or Q operator shift operator and Z transforms and while we have to use both the reasons are different remember here I am still in time domain my signals are in time domain okay.

I have not gone to frequency domain when you do Laplace transform what happens you would start working in frequency domain right Laplace transform you start working with frequency domain here this Q operator is only time shift operator I am just representing into time okay suppose I do not want to work with X I do not want to internal variables state variables I just want to work with input and output okay.

I just want to look at measured levels to voltage which are going to the full time set up I am not, I am not all four levels so I have got read of you know the states now my module is only between measured outputs and okay now what does this Q how does it help me.

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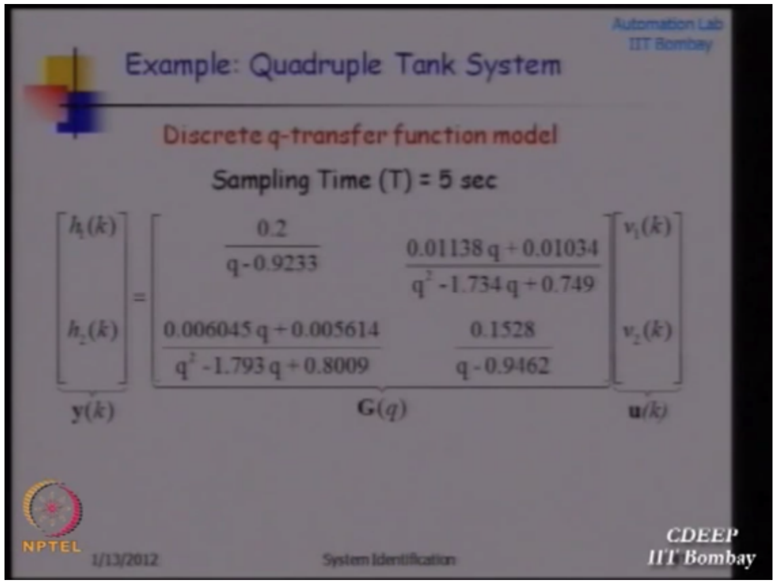

Well in one lecture I am trying to pack too many concepts but well with some practice you get what is if I do this, if I do this calculations see I have already have five matrix I already have γ matrix I have calculated that for quadruple time set up so I can just do this calculations again mat log will help in to do this calculations but for simple system two cross system you can do it by hand.

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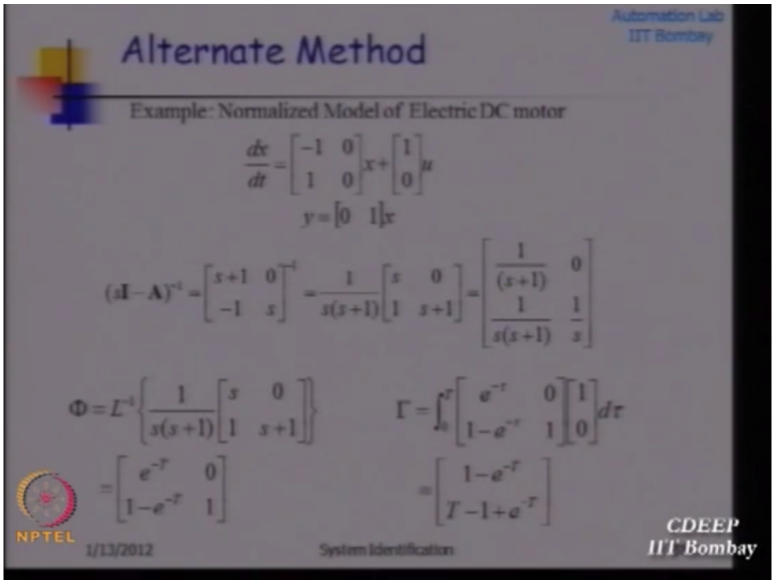

It is not so difficult okay do you want to try this for the model that we derive may be you should do that let us go back to the Dc motor model so you have this Ø and γ you are going to try converting with.

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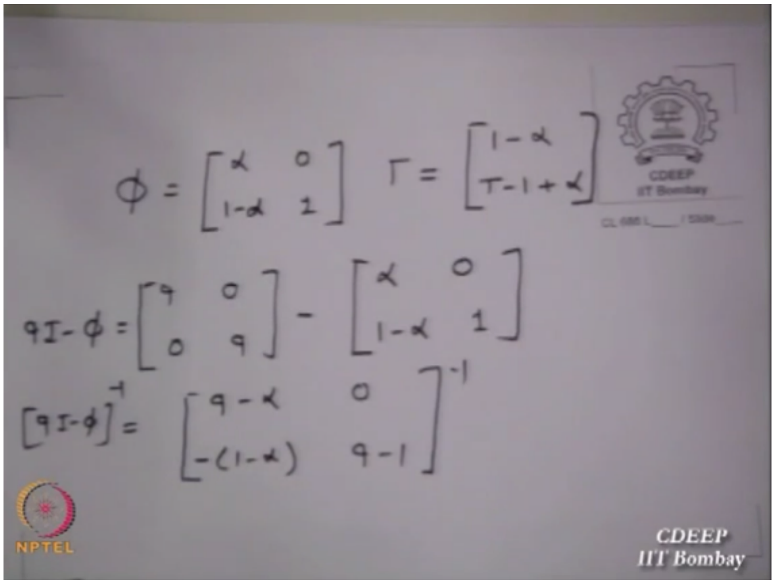

Let us put this you know Ø=you know this is α1-α 0, 1 and this γ matrix is of the form 1-αT-1 +α right okay so I just try to compute what will be this matrix this is qI- Ø so this will be q 0 0 q –α 0 1-α 1 this just try we do it know so this is q-α, 0 q-1, and -1-α and then you have to find out inverse of this okay.

So I want to find out qI- Ø inverse how will you find inverse of this yeah so find out co factor find out determinant well upon determinant times co factor transpose that will give you adjoint, adjoint transpose or adjoint so that will give you the you know and then you know you can, you can multiply this by multiply free multiply by C and γ okay.

What will be the transfer function matrix in this case well it be one cross one, two cross two what will it be in dc motor how many outputs are there in dc motor, in dc motor there is only one output there is only one input you will get the scalar transfer function Q transfer function okay you get the scalar Q transfer function what is the relevance why am I so much.

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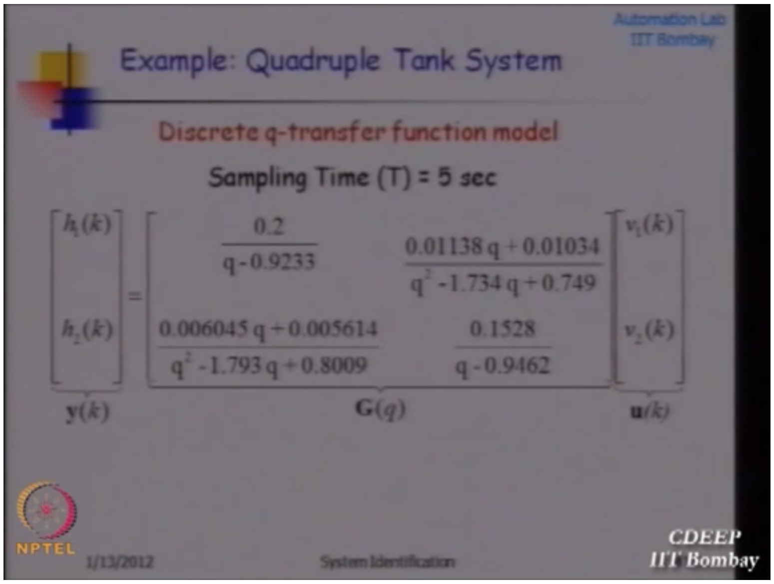

So I did this calculations for finding out the Q transfer function between h1 and h2 and V1 and V2 okay if I have a Laplace transfer function if I have a Laplace transfer function what does it indicate in time domain what does it indicate in time domain let us say.

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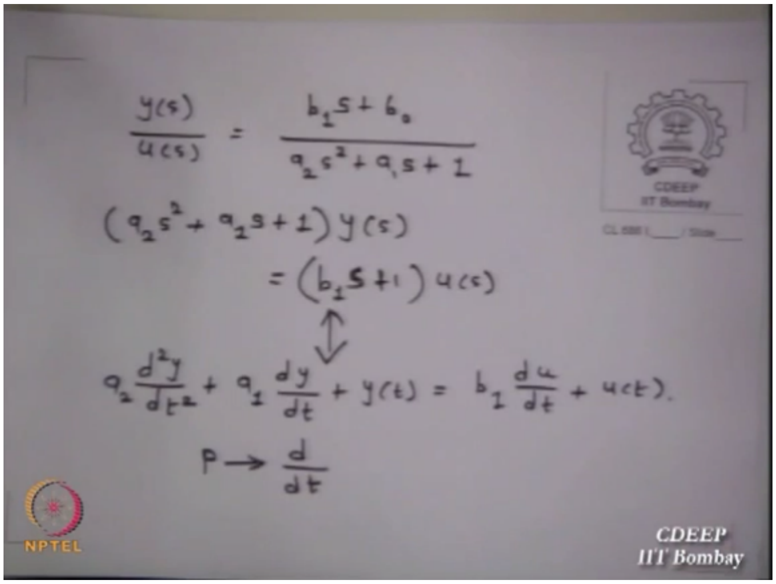

Let us say well I want to give an analogy because we learn much better much faster with analogy see if I have this Laplace transform function say $ys/us=b_1s+b_o/a_2\ s^2+a_1s+1$ what does it indicate what does it tell you what is that time domain equivalent I multiply both sides I will get $a_2s^2+a_1s+1*y(s)=b_1s+1*u(s)$ okay now this one is equivalent to differential equation okay.

So a_2s^2y will give me $d_2y\ y_dt^2$ so this is what was achieved when you go from time domain to Laplace domain my differential operator becomes an algebraic operator in terms of s okay in some sense well time domain you can use two notations sometimes they use P instead of S and Pe is defined as d/dt okay.

And you can write a P transfer function, P transfer function is b_1p+b_0 and denominator will be P2 a_2p^2 and so on okay when you write a P transfer function you will not write y and ys and us you will write yt and ut you are in time domain okay so the advantage that I have got when I went from you know time domain till Laplace domain algebraic expression okay I could do manipulation okay in control or design and all that so the same advantage I want to gain in discrete domain that is what I have put this Q operator yeah.

So now this gives me a relationship between inputs and outputs and I would convert this analog to time domain difference equation okay I want to convert the transfer function what happened in Laplace transform case I did the transform function between input and output okay I could convert the transfer function into differential equation they were actually one and same only they have a different representations.

A transfer function Laplace transforms function and an second order differential equation which I wrote okay let us just go back here this transfer function and this differential equation or not different okay is the same thing with two different terms you know it is just change of it is not that you are looking t two different entities.

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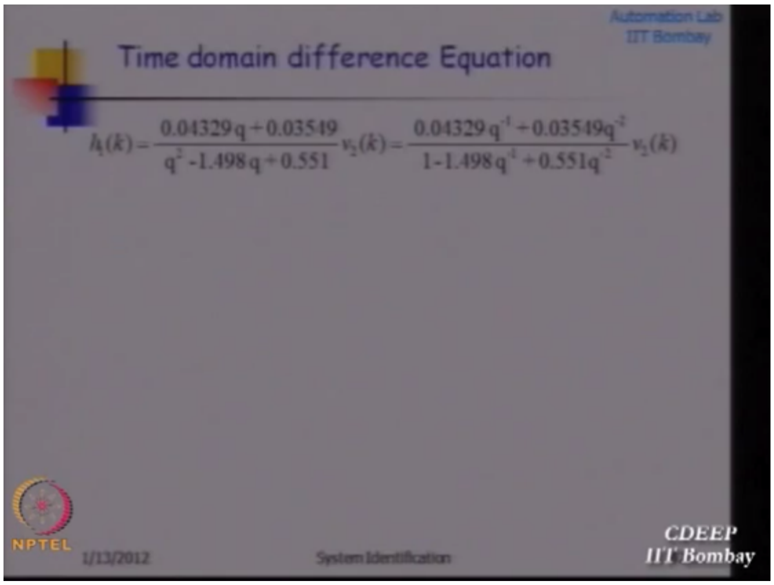

So it is only the convenience of algebra that is why we were to Laplace domain so coming back here I want to be use the same advantage here I want to go back to time domain difference equation okay I will just picked up I will just picked up one of them right now let us assume let us go back here.

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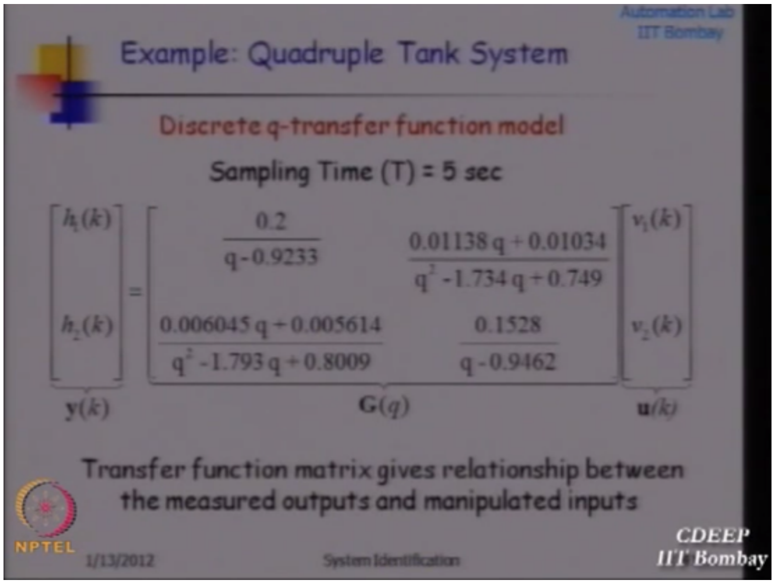

Let us go back here H1 is affected by what does this matrix tell you quadrapation in level 1 is influence by both quadrapation in voltage 1 and quadrapation in voltage 2 how, how does it effect through this you transfer function matrix q transfer function matrix. Let make a simplifying assumption that perturbation v1 is 0 okay, only v2 is changing and to find out how h1 is effected by v2 okay.

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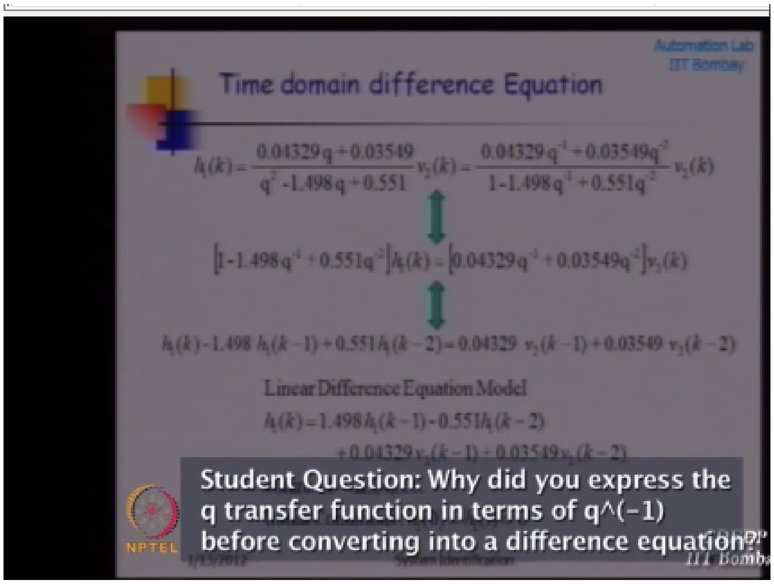

So I am just looking at the sub component of this model h1 effected by v2, v1 is 0 okay, now I have just divide here if you look here I converted this from equation in q to equation in q – 1 okay just algebra. What I do next? There is a small error here okay well what is q- 1, time shift / 1, what is q -2, time shift / 2, so I am going to concert this into a linear difference equation.

This linear difference equation here and this q transfer function are equivalent no difference, there is difference representation, the same advantage I get doing algebra of function I want to derive here using this q shift operator okay there is no other purpose but this give me relationship between only input and the output okay.

Basically I want to write current see I have done it 2^{-1} that is because I want to write k is the current instant okay, so my convention is going to be k is current, k -1 past, k-2 is two in past k+ is future okay. So when I am writing a different situation particularly for a behavior at a current time instant, see we will be talking sometime whether about future predictions but right now I want to write in terms of what is current in terms of what is happened in the past.

I could have written converted as h k +2 and al that but k is my present k + 1 is future, k + 2 in two future, k -1 is 1 in past, k – 2 is past so on. So I want to write different equation which talks about current = something in past, actually this different equation very nicely tells you about the dynamically systems okay. It tells you that current what is happening now okay is effect of what happened in past okay.

See level one 1 at time instant k current is a linear combination of 4 terms here, it has a memory of two levels in the past h k – 1, hk- 1, h1 – k -2 those two coefficients are waiting practices okay. Then it is also influenced by the new inputs what are the new inputs? V2 k but remember if my computer sense out the signal v2 k at instant k it\ will not have an instantaneous effect on y. there is always a delay of one, if I take an action it is effect will be seen one sample later okay.

Our module wants x k + 1 is function of u k okay, what do you do now with the effect of one sample later, so memory and this evident from this equation dynamical I get a linear different equation module here, is everyone with me on this is clear, why I went to q transfer because I could now just talk about relationship between an output and an input okay. Now should I do it in this class or should I not okay let us do in next class.

Frequency domain we are doing controller design we need plots I do not think you fondly remember but you have to use Nyquist to do control, my course is going to be on time domain, so you will understands all of these. Frequency domain is minimal but there is now ay you can escape frequency domain, in some cases you need frequency domain interpretation which are very nice and there is no other way of going about.

So we have to touch frequency domain, so we have to study this q transform is only at a time domain arrangement I would we talk about shifting signal and you deal this using time shift operators difference equation representation. Here I have to talk a Z transform okay, now z transform is defined as signal infinite signal, starting from time 0 to infinite okay.

We are dealing with signals which are time 0 to infinite right, see when I start my level control system, I will start getting level measurements, 1, 2, 3, 4, 5 , 6 and I f I do not stop I am going to time infinite right virtually. So I want to talk about signals which are 0 to infinite, infinite is some kind of idealization here in reality you will stop the plan after 1 day or few days are whatever. Now here z is a complex variable

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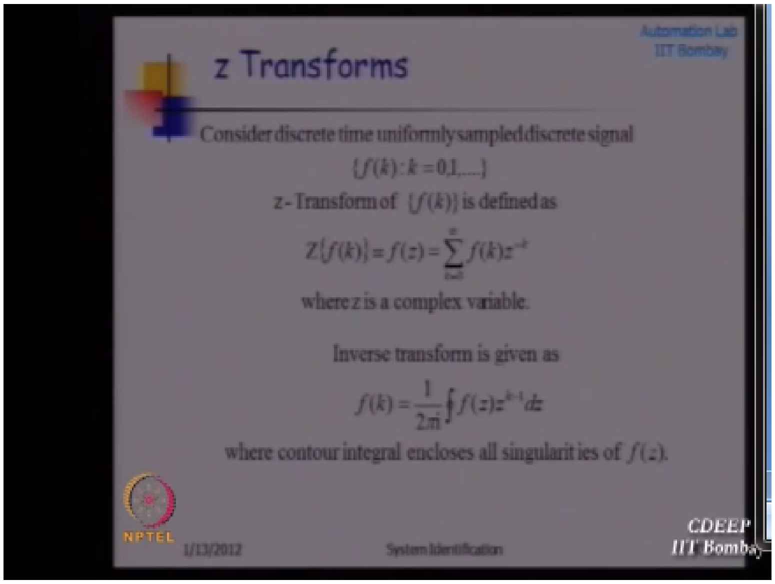

I want to understand z transform at a working level not from the complex view point or complex analysis view point take this with a pinch of salt we are going to compute those contour integrals in this course. Once you define transform you have to design reverse transform. Transform I will give you analogy for transforms.

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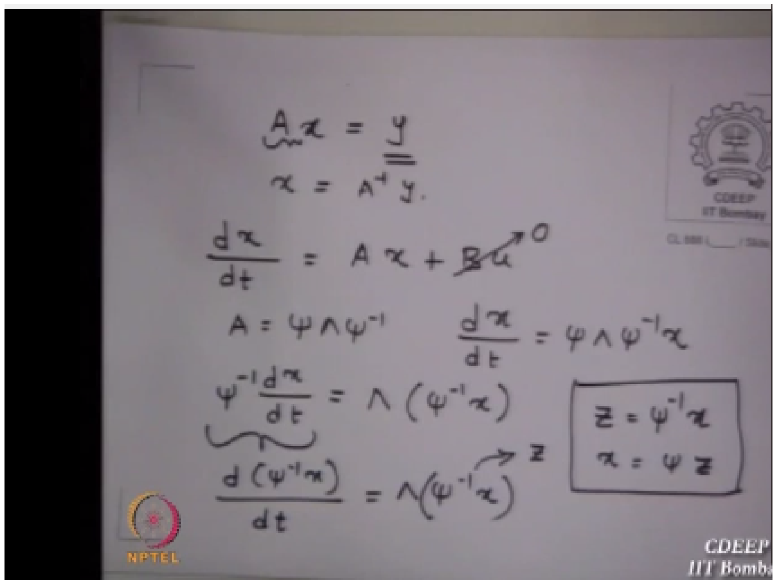

You have some signal say x okay and many times you want to work with some kind of a transform signals, so you multiply this by a matrix A which is inventible and you get a new vector y okay. so this is the transform I transform every x to vector y by pre multiplying by a inventible matrix A okay and then how do I recover, x you know A inverse y =transforms are useful they help you.

Sometimes simplify the equation okay I will give you an example, we are looking at this dx/dt = Ax we also have Bu right ignore Bu, just for a second actually we have this when we are looking at equation we have Bu let assume for time being is it 0 okay . Now my A is a full matrix n x n matrix. Let assume this is diagonal, I can write ψ ∇, so I am going to write this equation as d x/ dt = ψ ∇ x everyone with me on this.

A little bit of algebra will tell me that dx/ dt = γ ψx fine everyone with me on this, so this term is nothing but dx/ dt = γ ψx = λ, you might wonder why I am writing like this. I am going to define a new variable called z so this is my z, so what is this z? Z is the vector which is obtained by ψx how do you get x back, ψ z fine this is my transformation okay. Now with this new definition what happens is dx/dt=Ax okay with this ψ defined as will give dz/dt=it is not z transform it is just a vector which is λz what is λ? Is a diagonal matrix there are zeros here what is the advantage of looking at this differential equation through z over through x here this is nothing but dz1/dt=λizi where zi is the i^{th} element I get n differential equation which I decoupled which are not relate to each other.

If a is the full matrix which has you know all many, non zero elements then a solving the original problem is much moiré difficult then solving this problem and I can solve for z and I can go back to x using inverse transform right I can go back to x through inverse transform so transforms help transforms really help in solving problems what is lapse transform how it is defined? If I have signal FT

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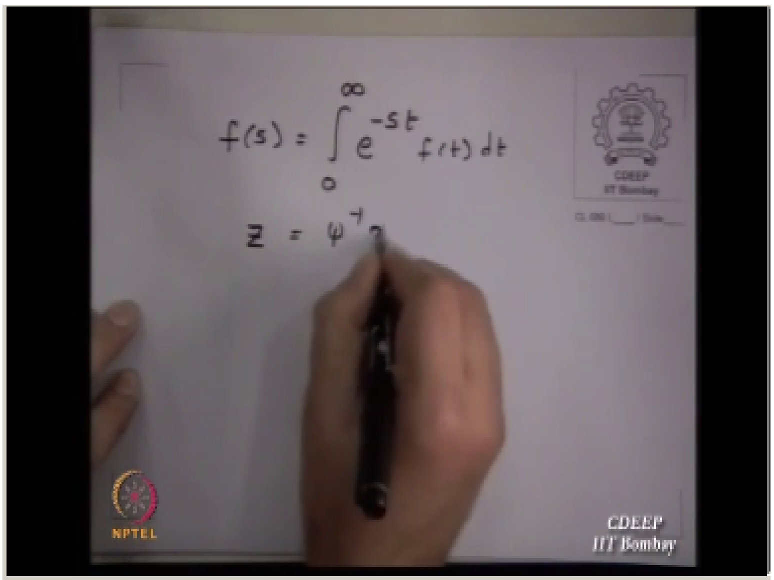

Ft $\int e^{-st}$ f(t)dt well conceptually this is not different from you know x is the vector when operated by a operator ψ^{-1} gives me vector z okay what is equivalent to matrix here ∫ is called ∫ carnal this ∫ carnal is equivalent to this multiplying by a matrix okay why do we get ∫ carnal here we are dealing with a functions not finite dimensional vectors if we are dealing with the function ft is the function from time 0 to time ∞ ft is the function so conceptually these two are similar equations they are not different they are one and the same conceptual okay so yeah.

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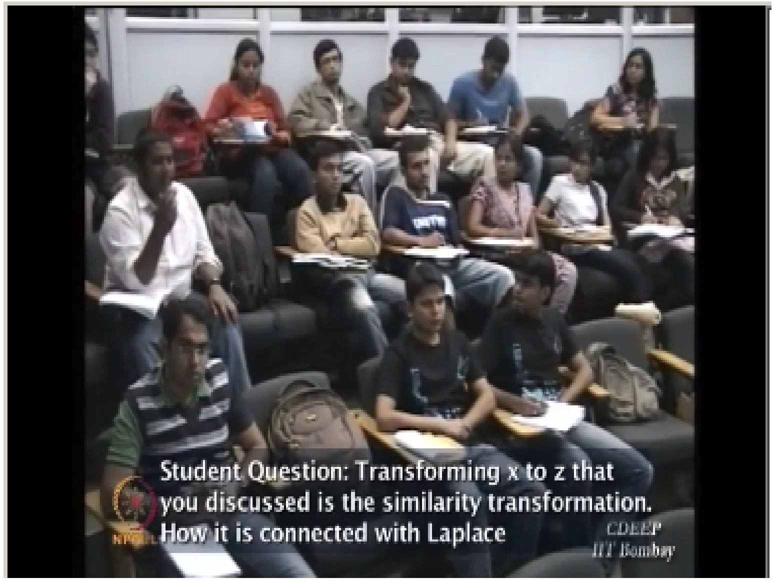

No, no I want you to understand the philosophy of transforming what I am saying is that why do we transform one is algebraically similarity transformation but both are inversely transformation yeah so just draw a line one is off course in a finite dimension other one is in infinite dimension so okay so just do another vertical this is to understand what is happening in transforms then, see what was the point here the point here was when I transformed it is easier to work with a transform signal.

Okay you too solve the differential equation at transform domain and then you could go back to the original domain you could do some algebra in transform domain go back to the original domain that was the idea so when I am going to define this transforms the idea is that I should be able to do something simplified simple algebras which is in the transform domain.

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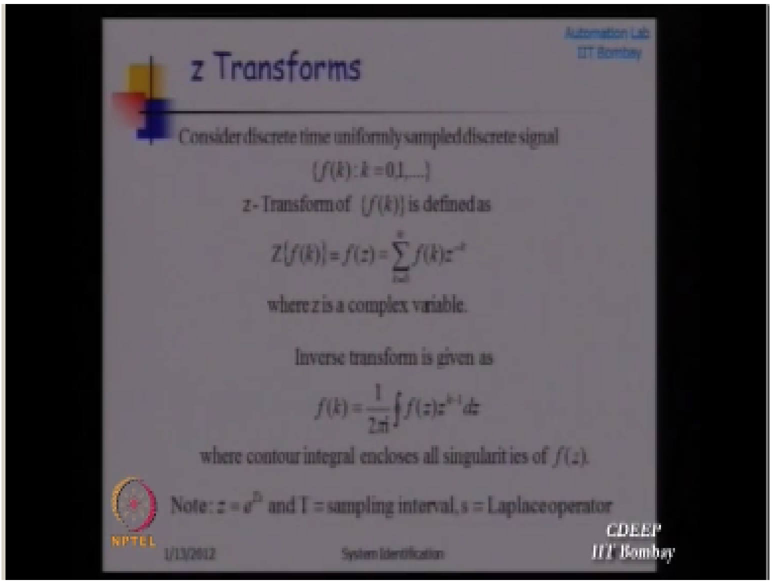

And then move back to the original domain so this is my z transform my z transform is defined as a you know fk where fk here is the time domain signal at time instant k and z^{-k} z here is a complex variable it is relevant to Laplace variable to this equation z=eds as I said I am not going to go deep into this you can trouble the effort to mark or Franklin and Paul well I have given these books in my I will be giving this reference as a part of my lecture notes.

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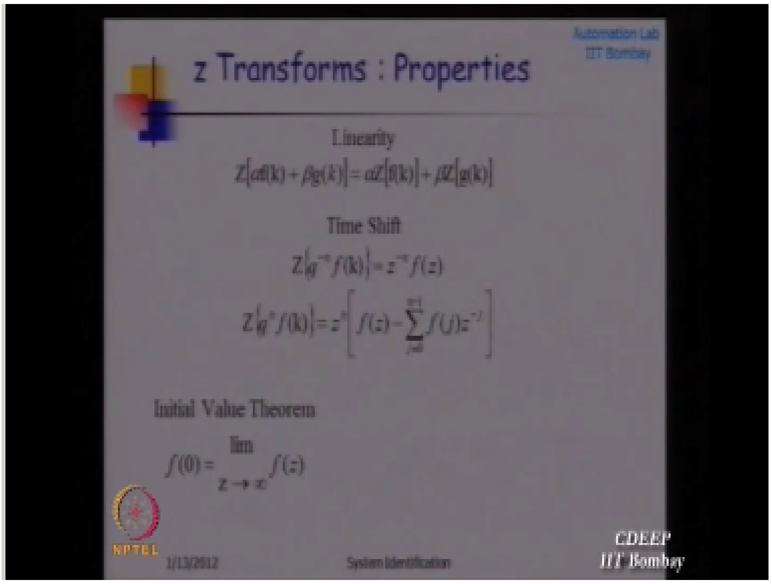

So, well like Laplace transform has some nice properties z transform also has some nice properties one is linearity so you add two signals and take its z transform it is equivalent to linear combination of individual z transform property that is going to be used very often by us is time shift okay z transform of a time shift signal is nothing but z- you will find for practical purposes q and z are similar okay as per as two differences equation is consent that turned out to be quite similar not identical but quite similar.

So you have z transform of signal which is shifted in the past and you have z transform of signal which is shifted in future okay so I have given here the expressions the derivations are not so difficulty and you can find them in a standard textbook well what do you have in a when you study Laplace transform you study value theorem initial value theorem? Same things are here we have final value are initial value theorem and so on what is going to be most important for us is this particular time shift you take z transform of time shifted signal you will get this expression you will get z^n and you will get z^{-n} when they shifted in the past okay

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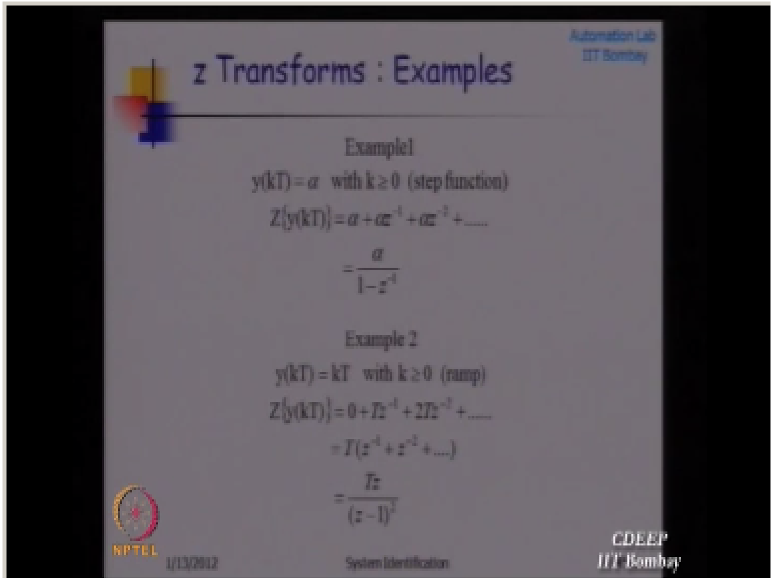

Just some examples of z transforms if you take this simple signal you know a step functions then I can take α out and then take summation of 1 that this is so this in any digital control book you will have z transform tables which is similar to what you have in Laplace transforms for sinusoidal and then you can go to book on digital control you will find listing of all these transform.

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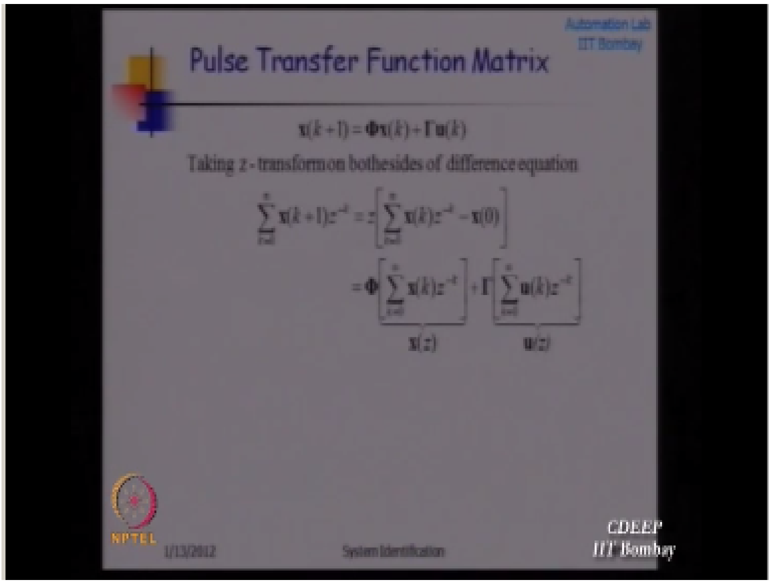

The way I am going to use where I will again revisit this my next class but I will just remit what I want to do I want to take an z transform of my difference equation okay and I wanted to get a z transform function matrix just like s transfer function matrix I want to get a z transform function matrix when you have this q thing why do you want z because xz is relevant to e^{ts} where s is complex number and then you can draw frequency responds using that interpretation that is not possible equal to time shift in time shift you are working in time domain.

Okay when you define these operators at some point we are going to use the frequency level interpretation and that is why we need this z here okay so my z transform is I am going to take a z transform of both the sides and this equation will finally you need something which looks very similar to the two transform nothing different okay final expressions are going to look not different from q transform just q is replaced by z philosophically they are completely different because you are working with time domain in other case you are working with frequency domain expressions turn out with similar so I can find out first transfer function for my system

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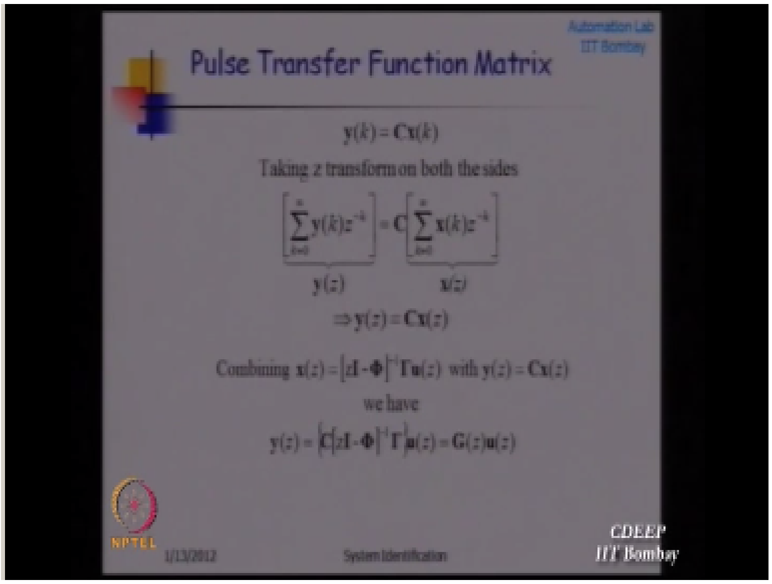

By converting my difference equation into first transfer function.

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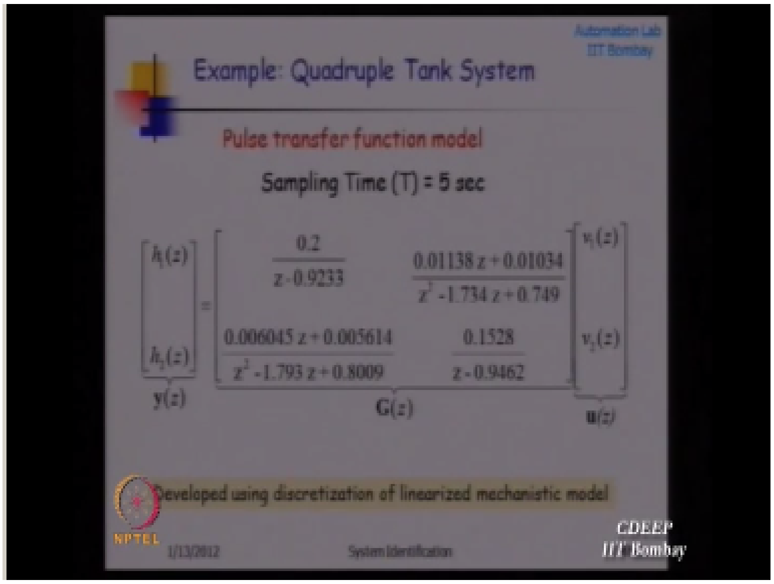

And I will visit this again in my next class in the beginning but I get pulse transfer function all that I have done is something except q is replaced by z here okay but notice I do not have any more here you know h1 .k and h2.k that is h1.z h2.z and v1 of z v2 of z which is my z domain pulse transfer function this is been obtained using linearization of the first principle model okay so I will stop here and then we will produce from here in the next class.

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