

NPTEL

**NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED L**

**EARNING IIT BOMBAY
CDEEP IIT BOMBAY**

**ADVANCE
PROCESS CONTROL**

Prof. Sachin Patwardhan

**Department of Chemical Engineering
IIT Bombay**

**Lecture No – 26
LQG and MPC**

**Sub-topics
Model Predictive Control (contd.)**

We will be looking at model predictive control and let me go about some of the ideas, moderate again, well we have modified population from linear optimal control.

(Refer Slide Time: 00:31)

MPC: Basic Idea IIT Bombay

- **Finite Horizon formulation:** Optimization problem is formulated over a finite window of time starting from current instant
- At each sampling instant, a constrained optimization problem is formulated over the window and solved

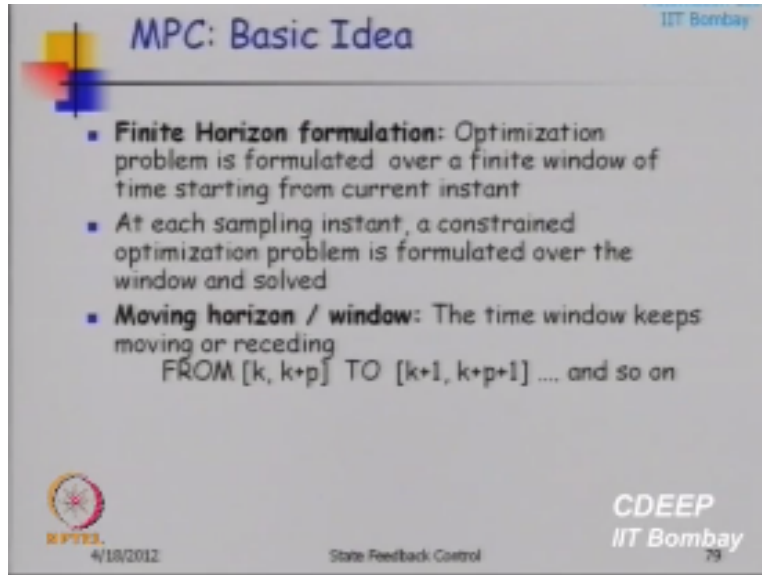
NPTEL 4/18/2012 State Feedback Control

CDEEP IIT Bombay 79

Linear optimal control is technically a formulation over a finite horizon whereas this is a formulation which is, you go a finite horizon and it keeps changing as a function of time. At each

sampling instant we are going to solve a constrained optimization problem over a window and this window keeps moving, this window keeps sliding in time, so that is important.

(Refer Slide Time: 00:55)



The slide is titled "MPC: Basic Idea" and is from IIT Bombay. It contains three bullet points explaining the concept of Model Predictive Control (MPC). The first bullet point is "Finite Horizon formulation: Optimization problem is formulated over a finite window of time starting from current instant". The second bullet point is "At each sampling instant, a constrained optimization problem is formulated over the window and solved". The third bullet point is "Moving horizon / window: The time window keeps moving or receding FROM $[k, k+p]$ TO $[k+1, k+p+1]$... and so on". The slide also features logos for IIT Bombay, CDEEP, and a date of 4/18/2012.

MPC: Basic Idea IIT Bombay

- **Finite Horizon formulation:** Optimization problem is formulated over a finite window of time starting from current instant
- At each sampling instant, a constrained optimization problem is formulated over the window and solved
- **Moving horizon / window:** The time window keeps moving or receding FROM $[k, k+p]$ TO $[k+1, k+p+1]$... and so on

CDEEP
IIT Bombay
4/18/2012 State Feedback Control 79

So you solve first the problem over window k to $k+p$, where k is the current instant, implement the optimal in to move. Move forward in time; reformulate the problem over $k+1$ to $k+p+1$. So the window size p remains constant and so on. So this window keeps sliding in time and then I was giving you analogies that this is what, actually we keep doing when you control the system, when you drive a car, when you drive a cycle or a motor cycle or when you drive your work you actually keep planning only over horizon and then this window keeps sliding. Of course our brain is much more complex computer than what you can implement.


So the window in time and space that we can have can be time varying and sometimes you might plan over hundred meters and ten minutes in future, sometimes you might plan only for three minutes in future, you know brain is amazing. You cannot teach a computer to do what a brain can do, but at least we can approximate what.

(Refer Slide Time: 02:13)

Automation Lab
IIT Bombay

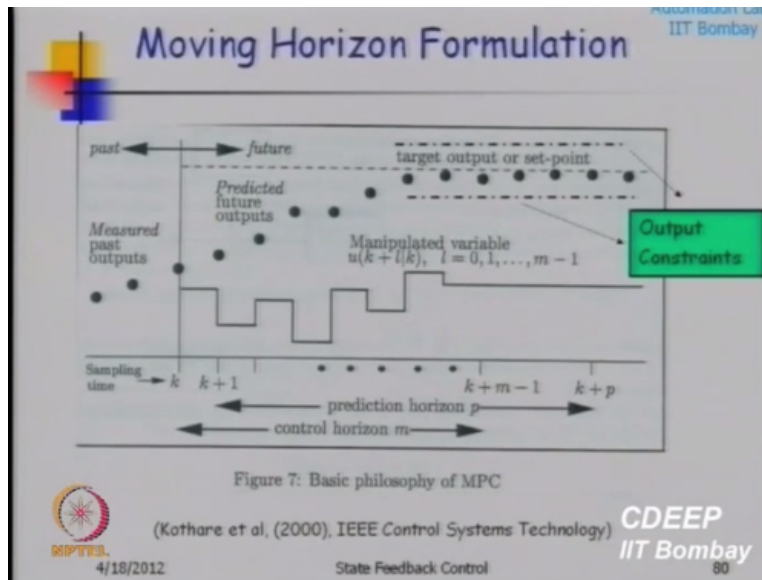
MPC: Basic Idea

- **Finite Horizon formulation:** Optimization problem is formulated over a finite window of time starting from current instant
- At each sampling instant, a constrained optimization problem is formulated over the window and solved
- **Moving horizon / window:** The time window keeps moving or receding
FROM $[k, k+p]$ TO $[k+1, k+p+1]$ and so on
- **Pro-active constraint management:** Given a good dynamic model, we can perform on-line forecasting and foresee any possible constraint violations over the time window

 4/18/2012 State Feedback Control **CDEEP**
IIT Bombay 79

So what the constant management we actually do this when you are driving, you actually do the constant management, you make most that will make sure that the car will not go outside the constrained boundaries. After some time, okay same thing you have to do now, we want to have a model which is running on board on the computer.

(Refer Slide Time: 02:34)



Is parallel to the plant online in the real time. You are going to do forecasting over the future and you going to check whether the constraints are obeyed or violated by the prediction model, by the predicted outputs, okay. So I have just modified a little bit this concept of what is called as control horizon and prediction horizon. Earlier I had talked about input blocking which is a little advanced concept. I have moved that to like appendix in the revised and I am introducing a simpler idea for control horizon here.

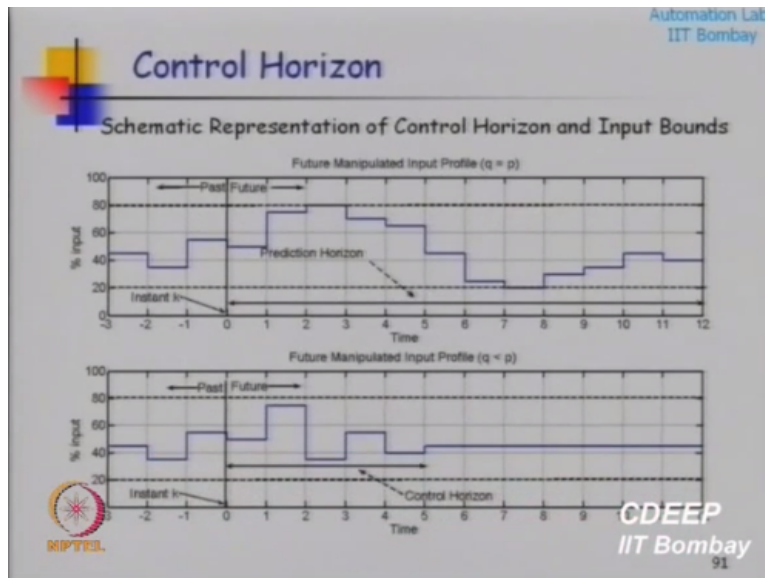
This is what originally it appeared and this is what is very commonly used, the blocking concept which I talked about is a little advanced and you can understand it maybe later. Let us take a step backward and understand the simpler concept which is control horizon. So what I am going to do now is that forget about, I am going to a special kind of blocking you can say. See I am going to plan over the horizon from k to $k+p$. Typically how much, how much long you plan, your plan this p is chosen based on the settling time on the system. What is settling time?

If you give a step change in the input the time it will take to settle. So roughly of course with different inputs the settling time will be different so you take the maximum of the settling time for different according to you. So how much you want to forecast in the future up to you know why settling time because you know the predicted effect of current input will be felt after the settling time. So that much ahead in the future you want to predict. So typically this could mean for, let us say this is a furnace, it could mean prediction over five hours or four hours, okay.

If it is just, if it just a vehicle it could mean just prediction over five seconds. It depends upon the system and so typically if you quantify in terms of sampling intervals this would be typically to 100 to 200 samples in future. Okay, so it will be about 100 to 200 samples in future. That is what you want to predict over. Okay, now other side this is one picture which tries to capture everything. So these are the future input that I am planning. Okay, now since I am going to implement only one move here, this one $u(k)$, okay, and discard the optimization results when I move to the next window.

Okay, what we do is restrict the degree of freedom into the future. In principle we can change or we can treat future inputs up to these points as the manipulated inputs which are available for manipulation. We can do that okay, but what we do is that gives rise to a large optimization problem. Okay, we want to restrict the CN variables so what we do is we typically allow input to change over first Q sample where Q could be 4, 5, 6, a small number and then we assume that after this point the input is held constant at the last value which is here.

(Refer Slide Time: 06:14)



Okay, so the blocking and I have tried to explain this using a different figure now. Yeah, so these are the two scenarios where you allow all the future inputs at the decision variables to the optimization problem. The other thing is you say that only first five you are allowed to change and after that you keep it constant, okay. So this first file, this window is called as the control horizon from here to here, okay. The prediction horizon is the window over which you are predicting the future behavior, control horizon is that horizon over which you are allowing money per input to change after which your rate is constant, okay.

This control horizon business is come mainly because of from the computation view point, you are trying to reduce the degree of freedom for optimization process. If you have to solve a large optimization problem and anyway if you are going to use only one move out of that. okay. Suppose I optimize next hundred input moves and through 99 I just use 1 then first of all I am formulating a huge problem, okay of which I only trust 1 move okay, then the idea is to reduce the dimensions but when we you reduce the dimension of course your maniple for that ability reduces.

Okay, if you give less degrees of freedom okay the way you can shape the future is get restricted fine, but you know there is a tradeoff between fast computation online because you have to do this if you are doing, implementing the predictive controller for a vehicle you may have to do this for a calculations, optimization calculations in fraction of a second. So in that case you know

that smaller dimension optimization problem convert this faster so these can be done, okay. So let us move back again and let us quickly go over the basic elements of MPC.

(Refer Slide Time: 08:08)

Automation Lab
IIT Bombay

Components of MPC

- **Internal model and state estimator**
 - Discrete Linear State Space Model developed from mechanistic approach or time series modeling (FIR or Finite Step Response models were used initially)
 - State Estimator: Open loop observer / Kalman Predictor / Kalman Filter / Luenberger Observer / Innovation form of state observer developed from ARX / ARMAX / BJ model
- **Prediction of Future Plant Behavior**
 - Key issue: Handling unmeasured drifting disturbances and plant model mismatch
- **On-line constrained optimization strategy**
 - Quadratic programming
 - Linear programming

RPTES
4/18/2012

State Feedback Control

CDEEP
IIT Bombay
81

1 is you have an internal model, well I have developed the formulation using an observer, close observer but it is very tough, you can use all kinds of things. You can use open loop observer, you can use filter Kalman filter, you can use Luenberger observer, you can use whatever, okay. So I have developed 1 in a particular way. Okay so you can use the model which is coming from ARMAX or BJ, actually I have uploaded yesterday to show you how it can be done for LQG using both models, one using at linearised models other using identified models. How it should be different.

So that was image files and you cannot copy from that, but you can view what is that. If we are able to copy tell me I will change it. So then you need a prediction scheme over the future, how do you predict your future? There are two components of prediction scheme, one is you have to use the model in to the future to predict future profiles. Second is you have to realize that the model is never perfect. So you have to have some scheme for compensating future predictions for plan more investment. Okay, so there are two things, one model is my un-major disturbances always present; the model we developed in the beginning of your project is they were going to be valid.

I mean it is roughly okay, but not you know perfect model, the plan conditions keep changing and anyway you have a non linear real plan is always non linear you are linearised so there are all kinds of possibilities. So you have to have a scheme for compensations of plan model mismatch and then you have to have a scheme for solving it optimally online. So for that of course linear programming, code programming, there are very efficient tools available and commercial course are available and you can use those commercial course, some of them are even oblige domain and okay.

(Refer Slide Time: 10:18)

Automation Lab
IIT Bombay

State Estimation and Prediction

Consider state estimation and prediction using prediction form of observer

$$e(k) = y(k) - C\hat{x}(k|k-1)$$

$$\hat{x}(k+1|k) = \Phi\hat{x}(k|k-1) + \Gamma u(k) + L_p e(k)$$

Such a observer can be developed using any of the following approaches

- Kalman predictor
- Luenberger predictor
- State realization of ARX / ARMAX / BJ model

Prediction estimate of the current state and innovation

$$\hat{x}(k|k-1) = \Phi\hat{x}(k-1|k-2) + \Gamma u(k-1) + L_p e(k-1)$$

$$e(k) = y(k) - C\hat{x}(k|k-1)$$

CDEEP
IIT Bombay

SIFTES
4/18/2012

State Feedback Control

83

So the said that the observer we are going to develop using an observer here and this observer could be developed to any enemy it need not be Kalman filter, it need not be Kalman predictor it can be Luenberger predictor, it can be state realization of ARMAX / BJ model, can be anything. Then I am going to use this to do current state estimation $\hat{x}(k|k-1)$ is the current state and then I am going to estimate the current innovation. This current innovation will contain the information about model plan mismatch. Okay and major disturbances if everything is perfect okay then innovation is the right choice but if it is not perfect, if the model is different from the plan innovation is not a right choice and we use that signal to compensate for the future predictions.

(Refer Slide Time: 11:35)

Automation Lab
IIT Bombay

State Estimation and Prediction

Innovation Bias Approach:
Effect of model plant mismatch and /or unmeasured disturbance signal is extracted by filtering the innovation through a unity gain low pass filter

$$e_f(k) = \Phi_e e_f(k-1) + [I - \Phi_e]e(k)$$

$$\Phi_e = \text{diag} \left[\alpha_1 \quad \alpha_2 \quad \dots \quad \alpha_r \right]$$

$0 \leq \alpha_i < 1$ for $i = 1, 2, \dots, r$ are tuning parameters

Given a guess of the future manipulated inputs
 $\{u(k+j|k) : j = 0, 1, 2, \dots, p-1\}$

model predictions over future time window $[k+1; k+p]$ are generated using the discrete dynamic model as follows

CDEEP
IIT Bombay

RIPTEL
4/18/2012

State Feedback Control

84

So how we do that, you do using this innovation bias approach, you filter the innovation okay and then you try to find out what is the zero means of the signal. This is done using this simple exponential filter this is what I have included now is, one minute, I suggest the flight for editing and filter, I do not know how it is not there, some problem in the version.

(Refer Slide Time: 11:59)

Automation Lab
IIT Bombay

Unity Gain Filter

Consider a unity gain 1st order transfer function used for filtering a signal $y(t)$ to generated $y_r(t)$

$$y_r(s) = \frac{1}{\tau s + 1} y(s)$$

In discrete time (with sampling interval T), this is equivalent to

$$y_r(z) = \frac{(1-\alpha)}{z-\alpha} y(z) \quad \text{where } \alpha = \exp(-T/\tau)$$

Note: $0 < \alpha < 1$ for any $\tau > 0$

Or in time domain

$$y_r(k+1) = \alpha y_r(k) + (1-\alpha)y(k)$$

The filter is often implemented by removing the unit delay, i.e.

$$y_r(k) = \alpha y_r(k-1) + (1-\alpha)y(k)$$

CDEEP
IIT Bombay

RIPTEL

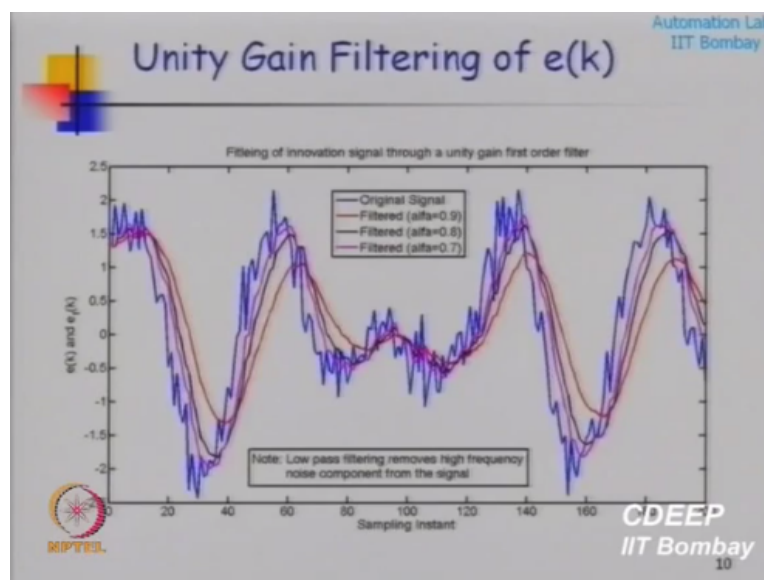
9

Okay, so what is the genetic aim filter I have explained this in this one line, unity gain filter is simply while in continuous domain of filtered signal Y the y is filtered through a first order filter simple $ts+1$ or if you are more comfortable with A . it will be $S+A$ and divided by 1 upon will be $1+A$ I think. So this is the unity gain filter, the gain of this filter is 1 . So the task of this filter is only to knock off certain frequencies. What are those frequencies that will be decided by how you choose this t . okay what is it map it to this particular differential equation or this one. First are the transfer function actually map so this is difference equation.

This is that domain transfer function which is nothing, but this filter okay first order filter and the map between this α and the time constant you might be more comfortable when it comes to filtering it might be more comfortable thinking in terms of a time constant or a frequency other than you know discrete time α . Okay so this here to work with this t here and remember this mapping at $\alpha = \exp(-T/t)$, where T is sampling time.

So this is how you filter a signal. Okay so this filtering of this signal this is the filtered value so new filtered value is α times old filtered value plus one minus α times the new input which is coming Y . So Y is filtered okay and depending upon how you choose α the signal gets filtered to this first order filter and then.

(Refer Slide Time: 13:40)



I will just showing here this first order how it looks. So for different values of α how does a filter signal look this blue signal is the original signal without filtering and the other three signals

are filtered signals for different values of alpha. So this alpha is a kind of tooling parameters we have to choose the tooling parameter between 0.8 and 0.99.

Okay and when you do your control LPUG or MPC whatever this particular parameter will have to change to get a good behavior. You can, I have given starting guess is 0.8 anywhere between 0.8 to 0.99. You have to try different values. It may happen that for higher values the system will start stabilizing, for lower values it may not. Okay so after we find out this business then comes you know suppose we are given a future side of manipulate inputs u_{k+1} , u_{k+2} .

(Refer Slide Time: 14:56)

Automation Lab
IIT Bombay

State Estimation and Prediction

Innovation Bias Approach:
Effect of model plant mismatch and /or unmeasured disturbance signal is extracted by filtering the innovation through a unity gain low pass filter

$$e_f(k) = \Phi_e e_f(k-1) + [I - \Phi_e]e(k)$$
$$\Phi_e = \text{diag} \left[\alpha_1 \quad \alpha_2 \quad \dots \quad \alpha_r \right]$$

$0 \leq \alpha_i < 1$ for $i = 1, 2, \dots, r$ are tuning parameters

Given a guess of the future manipulated inputs

$$\{u(k+j|k) : j = 0, 1, 2, \dots, p-1\}$$

model predictions over future time window $[k+1: k+p]$ are generated using the discrete dynamic model as follows

CDEEP
IIT Bombay

4/18/2012 State Feedback Control 84

Okay at the moment I have not put those constraints of control and all that. I am just checking that we are giving all the future input for manipulation. How will the prediction look like?

(Refer Slide Time: 15:10)

Automation Lab
IIT Bombay

Future Trajectory Prediction

Future instant (k+1)

$$\hat{\mathbf{z}}(k+1) = \Phi \hat{\mathbf{z}}(k) + \Gamma \mathbf{u}(k|k) + \mathbf{L} \mathbf{e}_f(k)$$

$$\hat{\mathbf{y}}(k+1|k) = \mathbf{C} \hat{\mathbf{x}}(k+1|k) + \mathbf{e}_f(k)$$

$$\hat{\mathbf{z}}(k) = \hat{\mathbf{x}}(k|k-1)$$

↓

Future instant (k+2)

$$\hat{\mathbf{z}}(k+2) = \Phi \hat{\mathbf{z}}(k+1) + \Gamma \mathbf{u}(k+1|k) + \mathbf{L} \mathbf{e}_f(k)$$

$$= \Phi^2 \hat{\mathbf{z}}(k) + \Phi \Gamma \mathbf{u}(k|k) + \Gamma \mathbf{u}(k+1|k) + (\Phi + \mathbf{I}) \mathbf{L} \mathbf{e}_f(k)$$

$$\hat{\mathbf{y}}(k+2|k) = \mathbf{C} \hat{\mathbf{x}}(k+2) + \mathbf{e}_f(k)$$

CDEEP
IIT Bombay

NIPTEL
4/18/2012

State Feedback Control

85

Okay so the prediction will look like this first that this $\mathbf{z}(k+1)$ is the first prediction I am going to correct this using the innovation filtered innovation and I am going to correct this \mathbf{y} predicted also using the filtered innovation. So this correction here this brings in somehow the effect of unmajor disturbances plan more a mismatch to the future predictions. Okay all that I have done is that recursively use this model.

Okay I have recursively; main thing is that the first point on the prediction is connected with the observer in the past. This is the connection this particular straight main is the connection between the prediction in to the future and observer which is working in over the past. Okay so this is what, where we connect. So initial point for the prediction is same as the last point of the observer that is what I am saying here.

(Refer Slide Time: 16:07)

Automation Lab
IIT Bombay

State Estimation and Prediction

Consider state estimation and prediction using prediction form of observer

$$e(k) = y(k) - C\hat{x}(k|k-1)$$

$$\hat{x}(k+1|k) = \Phi\hat{x}(k|k-1) + \Gamma u(k) + L_p e(k)$$


Such an observer can be developed using any of the following approaches

- Kalman predictor
- Luenberger predictor
- State realization of ARX / ARMAX / BJ model

Prediction estimate of the current state and innovation

$$\hat{x}(k|k-1) = \Phi\hat{x}(k-1|k-2) + \Gamma u(k-1) + L_p e(k-1)$$

$$e(k) = y(k) - C\hat{x}(k|k-1)$$



4/18/2012

CDEEP
IIT Bombay

83

State Feedback Control

You just see here I have this observer here. My observer at time k I predicted estimate of x , so I have to start my predictions from the current point. Where am I right now in terms of state I am at $x(k/k-1)$. Okay so once I do this match in between the past and the present okay then I am going to just reversal the use basic question over the future. Okay and carry out the predictions. So our raw implementations of predictive control will just involve a fore loop. Okay a raw implementation with involve the fore loop in which first this will be the first at you know $j=1$ this will be done then this value will be used here.

See you do not have to do all these expansion you just first in a fore loop you compute $z(k+1)$ use z at $k+1$ you will get z at $k+2$. Using z at $k+2$ you will get z at $k+3$ and so on. Just put it in the fore loop you will get the predictions. Okay for every guess how you able to solve this problem we have to release it once in the last lecture, an optimization problem. So you have to compute for a guess of the inputs you have to compute the future predictions. That you do in the fore loop. Then you compute the objective function for the optimization problem and due to the optimizer. Optimizer you will do the rest.

So here what you can do is that is a n optimization program and we can use the optimization program called `f` and `com`. Okay, so constrained optimization program in which you can specify the bounds you can and you have to give a you have to write a function in which the objective function is constructed using model predictions. Once you do that okay you can implement your model predictive control scheme.

(Refer Slide Time: 18:25)

Automation Lab
IIT Bombay

Future Trajectory Prediction

$$\begin{aligned}\hat{\mathbf{z}}(k+3) &= \Phi \hat{\mathbf{z}}(k+2) + \Gamma \mathbf{u}(k+2|k) + \mathbf{L}e_f(k) \\ &= \Phi^3 \hat{\mathbf{z}}(k) + \Phi^2 \Gamma \mathbf{u}(k|k) + \Phi \Gamma \mathbf{u}(k+1|k) \\ &\quad + \Gamma \mathbf{u}(k+2|k) + (\Phi^2 + \Phi + \mathbf{I}) \mathbf{L}e_f(k) \\ \hat{\mathbf{y}}(k+3|k) &= \mathbf{C} \hat{\mathbf{z}}(k+3) + \mathbf{e}_f(k)\end{aligned}$$

↓
↓
↓

Future instant (k+p)

$$\begin{aligned}\hat{\mathbf{z}}(k+p) &= \Phi \hat{\mathbf{z}}(k+p-1) + \Gamma \mathbf{u}(k+p-1|k) + \mathbf{L}e_f(k) \\ &= \Phi^p \hat{\mathbf{z}}(k) + \Phi^{p-1} \Gamma \mathbf{u}(k|k) + \Phi^{p-2} \Gamma \mathbf{u}(k+1|k) \\ &\quad + \Gamma \mathbf{u}(k+p-1|k) + (\Phi^{p-1} + \Phi^{p-2} + \dots + \mathbf{I}) \mathbf{L}e_f(k) \\ \hat{\mathbf{y}}(k+p|k) &= \mathbf{C} \hat{\mathbf{z}}(k+p) + \mathbf{e}_f(k)\end{aligned}$$

CDEEP
IIT Bombay
86

SIFTES
4/18/2012
State Feedback Control

So now I have explained this in terms now as well as program is in this concern I need only this first statement and the statement and the statement and the statement. I have explained this like this in terms of all the future inputs and also all the future and zhk that is because I want to do some interpretation. I said that prediction p step high prediction the future is function of two things. One is happen in the past that information is bought through or three things actually. What happened in the past that is initial stage of the model.

Okay then all the future inputs that you are going to input moves that you are going to make. Okay and this will bring in the information about the past disturbances and past model plan mismatch. So this is the compensation from model plan mismatch. Okay so this is how you do the predictions okay and this is the interpretation of those predictions.
(Refer Slide Time: 19:28)

Automation Lab
IIT Bombay


Future Trajectory Prediction

Interpretation of p step output prediction equation

$$\hat{y}(k+p|k) = [C\Phi^p] \hat{x}(k|k-1) + (C\Phi^{p-1}\Gamma) u(k|k) + \dots + (C\Gamma) u(k+p-1|k) + [\Phi^{p-1}L + \Phi^{p-2}L + \dots + L + I] e_f(k)$$

Future output prediction	=	Effect of the past state on future outputs	+	Effect of future inputs on future outputs	+	Effect of Plant Model Mismatch and Unmeasured Disturbances on future outputs
--------------------------	---	--	---	---	---	--

'p' is called as Prediction Horizon



4/18/2012

State Feedback Control

CDEEP
IIT Bombay

87

And here of course p is called as the prediction horizon. Now the next question is that so that is the guess know. See the way optimization works is that you guess. So you give initial guess from that it will compute the objective function compute the gradient and then there are methods to generate new guess from the old guess.

Okay so the way this is going to work is each time you have to do a prediction. Okay so actually what you do is you write a function in which given a guess you generate the objective function that is your job. Okay and what will the optimizer do, from the old guess to generate a new guess evaluate the objective function that is the all the optimizer.

So optimizer will keep generating new guesses till certain criteria, that is a necessary condition for mathematics to satisfy and then it will terminate the optimization. So it may in the relative process okay for every guess optimization generate I have to construct the collections evaluate the objective function and give it back to the optimizer.

Okay evaluate the objective function, evaluate the constraints and then tell the optimizer situation at there you are for be given guess whether you are inside the constrain boundary that is business of the optimizer. Okay so you I think of an optimizer as a supporting available to you to which you just supply an objective function.

Okay so what is what is in the objective function even guess of the inputs to carry out predictions over the horizon, you think those predictions you find out difference between the future head

point and the future predictions find thus objective functions calculations and then give it back to it. Okay so actually technically what we have done is something like this.

(Refer Slide Time: 21:47)

Automation Lab
IIT Bombay

Future Trajectory Prediction

Note: The predictions generated using the innovation bias approach is equivalent to carrying out predictions using the observer augmented with an artificially introduced integrated white noise model, i.e. prediction Generated using the following dynamic system

$$\begin{aligned}\hat{\mathbf{z}}(k+j+1) &= \Phi\hat{\mathbf{z}}(k+j) + \Gamma\mathbf{u}(k+j|k) + \mathbf{L}\hat{\mathbf{e}}(k+j) \\ \hat{\mathbf{e}}(k+j+1) &= \hat{\mathbf{e}}(k+j) \\ \hat{\mathbf{y}}(k+j) &= \mathbf{C}\hat{\mathbf{z}}(k+j) + \hat{\mathbf{e}}(k+j)\end{aligned}$$

Initial Conditions: $\hat{\mathbf{z}}(k) = \hat{\mathbf{x}}(k|k-1)$ and $\hat{\mathbf{e}}(k) = \mathbf{e}_f(k)$
for $j = 0, 1, 2, \dots, p-1$

CDEEP
IIT Bombay

State Feedback Control

4/18/2012

We have done prediction using this kind of a model. What we have done is that we have assumed that very well unknown disturbance. This unknown disturbance we have primed at EFK. Okay and we have assumed that is unknown disturbance remains constant over the horizon; okay actually this is called as integrated white noise model.

Okay I have practical implementation of this integrated white noise model and then what I showed you in the previous slides. Okay, but philosophically it means that I am actually doing this. I am actually solving this plus this equation together where as the initial condition for this is nothing but the filtered innovation.

Okay so whatever we have done at here can be written in a consectally like this. So actually what I going to highlight here is that to remove offset or do account for do model plan mismatch we have to introduce them integrating element into the controller. With integrating elementary to the controller is introduced artificially and this artificial business comes through this augmented equation.

Okay so this artificial state epsilon has been introduced into the predictions and then business use to compensate from model plan mismatch. Philosophically this is what has happen practically it

means of course we have you know this s coming here, if you eliminate all that p it is actually it is this, but conceptually it means that you are adding together.

How many integrated elements you added you near added integrating elements equal; to number of outputs, because ES. This is the innovation. Okay so you actually augmented the system with extra states which is equal to number of innovations is equal to number of involvements, so that is how you get rid of the offset.

(Refer Slide Time: 23:49)

Automation Lab
IIT Bombay

Constraints on Inputs


To reduce the degree dimension of the on-line optimization problem, degrees of freedom available for shaping the future trajectory are often restricted to first q moves

$$\{\mathbf{u}(k|k), \mathbf{u}(k+1|k), \dots, \mathbf{u}(k+q-1|k)\}$$

by imposing input constraints of the form

$$\mathbf{u}(k+q|k) = \mathbf{u}(k+q+1|k) = \dots = \mathbf{u}(k+p-1|k) = \mathbf{u}(k+q-1|k)$$

q is called the Control Horizon


 4/18/2012

State Feedback Control

CDEEP
 IIT Bombay
 89

Now constraints on the inputs and the simplified constraints. Okay, not input blocking. Input blocking I have moved to the end of the slides. So this simplified constraint, you are going to allow on the first Q moves to be freely changed after that I am going to make a constraint that after q , $q+1$ up to $q+p-1$ is equal to this. Okay so first q this q first moves with probably mean on the 5 moves or 6 moves or 67 moves.

That is because it is not be repeatedly solve the problem. Okay that's why we make the simplification if you have lot of computing power and if you have you know the computing time is not a constraint. Then you not have to put this constraint. This is moiré from a practical view point. Right so this in the control MPC terminology this is called as a control horizon. Typically industrial implementation this should be five or six certain and a prediction horizon will be 100, 150. So you allow next ten moves to be chosen freely for the optimizer and of which implement only one.

(Refer Slide Time: 25:19)

Automation Lab
IIT Bombay

Future Trajectory Prediction

Note: The predictions generated using the innovation bias approach is equivalent to carrying out predictions using the observer augmented with an artificially introduced integrated white noise model, i.e. prediction Generated using the following dynamic system

$$\hat{\mathbf{z}}(k+j+1) = \Phi\hat{\mathbf{z}}(k+j) + \Gamma\mathbf{u}(k+j|k) + \mathbf{L}\hat{\mathbf{e}}(k+j)$$

$$\hat{\mathbf{e}}(k+j+1) = \hat{\mathbf{e}}(k+j)$$

$$\hat{\mathbf{y}}(k+j) = \mathbf{C}\hat{\mathbf{z}}(k+j) + \hat{\mathbf{e}}(k+j)$$

Initial Conditions: $\hat{\mathbf{z}}(k) = \hat{\mathbf{x}}(k|k-1)$ and $\hat{\mathbf{e}}(k) = \mathbf{e}_f(k)$
for $j = 0, 1, 2, \dots, p-1$

Introduction of integrated white noise in predictions helps in achieving offset free closed loop behavior.

CDEEP
IIT Bombay

NIPTE
4/18/2012

State Feedback Control

88

Okay and here this one so this were you know see $\hat{\mathbf{x}}$ is the observer, so here deliberately it kept two notations, one for prediction and for observer. Observer is giving the past. So predictions for a given guess you will predict. That predictions may not happen, okay so that is why what I have done so will move here.

(Refer Slide Time: 25:47)

Automation Lab
IIT Bombay

Moving Horizon Formulation

Figure 7: Basic philosophy of MPC

(Kothare et al, (2000), IEEE Control Systems Technology)

CDEEP
IIT Bombay

NIPTE
4/18/2012

State Feedback Control

89

See, where is a governing equation for the plant dynamic equation. Okay what is that we are going to use the observer for asset model for prediction? Okay I want to use the observer now observer in the past and observer in the future. Okay I want to keep them separate so that your understanding is clear. Okay so I am going to use n two different notations. Okay so when I am saying x here when I am saying x so this is my observer so I am using observer to do that current state estimation. This is at k and standing at the k is instant. Okay I want to estimate the state at the current point and then using this state as a initial point I want to predict into future.

Okay so what I am going to do now is this clear up to this point. This point is clear. So now this so this innovation, that is okay. That is fine. Okay now look the equation, so now I am going to use the same model for future prediction. See u is the future input. I have not actually made it yet. I am just contemplating depend inputs. Okay suppose I will do implement UK given k. what would be the prediction? The difference equation is given me. I know five, I know gamma, I know L, I know C. Okay one mistake I have made is that it should be, here it will be z. So this z here. I will change that. I will correct that.

It should be z (k+1) should be here. Okay and then what I am doing is initial state of this. See you need z (k) here to go to z (k+1). Okay where are you going to get that from the observer in the past? That is the connection between the prediction and the past. Okay so that is the error here this should be z (k+1) here. So here I have corrected that.

(Refer Slide Time: 27:58)

Automation Lab
IIT Bombay

Future Trajectory Prediction

Future instant (k+1)

$$\hat{\mathbf{z}}(k+1) = \Phi\hat{\mathbf{z}}(k) + \Gamma\mathbf{u}(k|k) + \mathbf{L}e_f(k)$$

$$\hat{\mathbf{y}}(k+1|k) = \mathbf{C}\hat{\mathbf{x}}(k+1|k) + \mathbf{e}_f(k)$$

$$\hat{\mathbf{z}}(k) = \hat{\mathbf{x}}(k|k-1)$$


↓

Future instant (k+2)

$$\hat{\mathbf{z}}(k+2) = \Phi\hat{\mathbf{z}}(k+1) + \Gamma\mathbf{u}(k+1|k) + \mathbf{L}e_f(k)$$

$$= \Phi^2\hat{\mathbf{z}}(k) + \Phi\Gamma\mathbf{u}(k|k) + \Gamma\mathbf{u}(k+1|k) + (\Phi + \mathbf{I})\mathbf{L}e_f(k)$$

$$\hat{\mathbf{y}}(k+2|k) = \mathbf{C}\hat{\mathbf{z}}(k+2) + \mathbf{e}_f(k)$$



NIPYER

4/18/2012

State Feedback Control

CDEEP
IIT Bombay

65

So here there is a type 1. Okay I will correct that. So you get it now so once I get this prediction I can use this prediction into the next prediction. Okay and then I can go you know jumping in to r hoping in time in future. Okay from $k+1$ to $k+2$, $k+2$ to $k+3$, $k+3$ to $k+4$ and I am going to do this up to $k+p$ in the future. So actually what I have done if you look carefully when I have, see this one I am saying this future error is equal to e_f . Okay I move to the next point. Again what is the future error I do not know? Okay so is the best guess for the future error? Current error. So that is what I have done.

So see all these other signals I am using future you see here, but what about the future disturbance? Can I ever predict future disturbance? I can never predict future disturbance. Okay so I am saying that the best estimate of the future disturbance is the current value of the disturbance. Okay and then I am going to just use it see even three state, I am using this e_f and then if I go p step I am still using e_f .

(Refer Slide Time: 29:28)

Automation Lab
IIT Bombay

Future Trajectory Prediction

$$\begin{aligned}\hat{\mathbf{z}}(k+3) &= \Phi\hat{\mathbf{z}}(k+2) + \Gamma\mathbf{u}(k+2|k) + \mathbf{L}e_f(k) \\ &= \Phi^3\hat{\mathbf{z}}(k) + \Phi^2\Gamma\mathbf{u}(k|k) + \Phi\Gamma\mathbf{u}(k+1|k) \\ &\quad + \Gamma\mathbf{u}(k+2|k) + (\Phi^2 + \Phi + \mathbf{I})\mathbf{L}e_f(k) \\ \hat{\mathbf{y}}(k+3|k) &= \mathbf{C}\hat{\mathbf{z}}(k+3) + e_f(k)\end{aligned}$$

↓

Future instant $(k+p)$

$$\begin{aligned}\hat{\mathbf{z}}(k+p) &= \Phi\hat{\mathbf{z}}(k+p-1) + \Gamma\mathbf{u}(k+p-1|k) + \mathbf{L}e_f(k) \\ &= \Phi^p\hat{\mathbf{z}}(k) + \Phi^{p-1}\Gamma\mathbf{u}(k|k) + \Phi^{p-2}\Gamma\mathbf{u}(k+1|k) \\ &\quad + \Gamma\mathbf{u}(k+p-1|k) + (\Phi^{p-1} + \Phi^{p-2} + \dots + \mathbf{I})\mathbf{L}e_f(k) \\ \hat{\mathbf{y}}(k+p|k) &= \mathbf{C}\hat{\mathbf{z}}(k+p) + e_f(k)\end{aligned}$$

CDEEP
IIT Bombay

4/18/2012 State Feedback Control

See all the other things are future, but the disturbance estimate is current. Okay so philosophically what does it mean? It means that you are making a model which is of this form.

(Refer Slide Time: 29:47)

Automation Lab
IIT Bombay

Future Trajectory Prediction

Note: The predictions generated using the innovation bias approach is equivalent to carrying out predictions using the observer augmented with an artificially introduced integrated white noise model, i.e. prediction Generated using the following dynamic system

$$\begin{aligned}\hat{\mathbf{z}}(k+j+1) &= \Phi\hat{\mathbf{z}}(k+j) + \Gamma\mathbf{u}(k+j|k) + \mathbf{L}\hat{\mathbf{e}}(k+j) \\ \hat{\mathbf{e}}(k+j+1) &= \hat{\mathbf{e}}(k+j) \\ \hat{\mathbf{y}}(k+j) &= \mathbf{C}\hat{\mathbf{z}}(k+j) + \hat{\mathbf{e}}(k+j)\end{aligned}$$

Initial Conditions: $\hat{\mathbf{z}}(k) = \hat{\mathbf{x}}(k|k-1)$ and $\hat{\mathbf{e}}(k) = \mathbf{e}_f(k)$
for $j = 0, 1, 2, \dots, p-1$

SIPTES
4/18/2012

State Feedback Control

CDEEP
IIT Bombay
88

This model predictions okay in which so you are saying that the future is going to remain constant is equal to the current distance. Okay so conceptually doing that is equivalent to this model. Okay these two are one and the same. From the theory view point it is important to write this, actual implementation is what though the equations you are not going to you can actually substitute this here you can substitute this here and get rid of this equation. Okay, but that is all fine, but philosophically you have to do this. Okay philosophically you augmented the system with extra integrating states. Okay that is all.

(Refer Slide Time: 30:37)

Automation Lab
IIT Bombay

Constraints on Inputs

To reduce the degree dimension of the on-line optimization problem, degrees of freedom available for shaping the future trajectory are often restricted to first q moves

$$\{\mathbf{u}(k|k), \mathbf{u}(k+1|k), \dots, \mathbf{u}(k+q-1|k)\}$$

by imposing **input constraints** of the form

$$\mathbf{u}(k+q|k) = \mathbf{u}(k+q+1|k) = \dots = \mathbf{u}(k+p-1|k)$$

$$= \mathbf{u}(k+q-1|k)$$

q is called the Control Horizon

In a practical implementation
control horizon (q) \ll prediction horizon (p)

CDEEP
IIT Bombay

4/18/2012 State Feedback Control 89

So then I have this future manipulated variables which I have some degrees of freedom and then I of course have constraints. I can see unlike all other control schemes. Why I manipulate the control schemes have become so popular. Because all the other control schemes you just do with one gain matrix. Okay it is difficult to or gain matrix for one transfer function for control the transfer function or whatever it might be. You cannot systematically hide the constraints.

(Refer Slide Time: 31:06)

Automation Lab
IIT Bombay

Constraints on Inputs

Bounds on the manipulated inputs

$$\mathbf{u}^L \leq \mathbf{u}(k+j|k) \leq \mathbf{u}^H$$

Bounds on rate of change of manipulated inputs

$$\Delta \mathbf{u}^L \leq \Delta \mathbf{u}(k+j|k) \leq \Delta \mathbf{u}^H$$


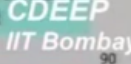
$$j = 0, 1, 2, \dots, q-1$$

$$\Delta \mathbf{u}(k+j|k) = \mathbf{u}(k+j|k) - \mathbf{u}(k+j-1|k)$$

$$j = 1, \dots, q-1$$

$$\Delta \mathbf{u}(k|k) = \mathbf{u}(k|k) - \mathbf{u}(k-1)$$

Since predictions are carried out online at each control instant, it is possible to choose future inputs moves such that the above constraints are respected

4/18/2012 State Feedback Control 90

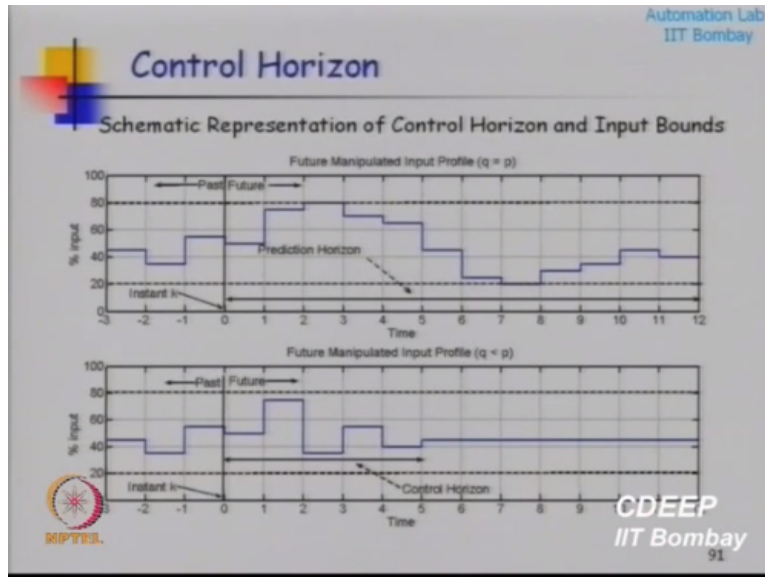
Here you are solving optimization problems so tell the optimizer that do not choose a move which is outside the bounds. It is so simple. Okay you are actually doing time domain selections so you can say that do not choose a move that will let the prediction output to go beyond. Okay so it is expressed you know your control problem as you think about it you can transfer it into an optimization problem. It is very easy. You can give bounds on the inputs and real systems are always bound from the inputs and all the analytical control theory cannot deal with it systematically.

Okay so what you do there you know when you are actually suppose you have to implement or LPG controller with bounds. All that you do is that you know you put a leave statement. If LPG asked you to implement the move which is higher than what is fusible then do not implement it, it is equal to okay, so that is called you know, it is called reset wind of mechanism, but do there of half majors you know you are putting if then as statements.

That is not maths. Right where as here when you put here a optimization problem you are actually using formal mathematical techniques. Okay so you can constrain delta u you can see many times you cannot change.

You cannot open the valve from say a 50% opening to 100% opening in 1 second. Okay are you cannot change a step promoter from you know at beyond a certain rate. So there are always physical limits and that controller should know that there are physical limits. When you just put a gain times sometimes if the error increases the delta u increases that they not happen here. We actual can constrain the inputs.

(Refer Slide Time: 32:52)



Okay so I will just try to pictorize this control horizon input constraints and all that.

(Refer Slide Time: 32:58)

Automation Lab
IIT Bombay


Future Setpoint Trajectory

In addition to predicting the future output trajectory, at each instant, a filtered future setpoint trajectory is generated using a reference system of the form

$$\begin{aligned} \mathbf{x}_r(k+j+1|k) &= \Phi_r \mathbf{x}_r(k+j|k) + [\mathbf{I} - \Phi_r] [\mathbf{r}(k) - \mathbf{y}(k)] \\ \mathbf{y}_r(k+j+1|k) &= \mathbf{y}(k) + \mathbf{x}_r(k+j+1|k) \\ \text{for } j &= 0, 1, \dots, p-1 \end{aligned}$$

with initial condition $\mathbf{x}_r(k|k) = \bar{\mathbf{0}}$

Here, $\mathbf{r}(k) \in R^r$ represents the setpoint vector.



4/18/2012

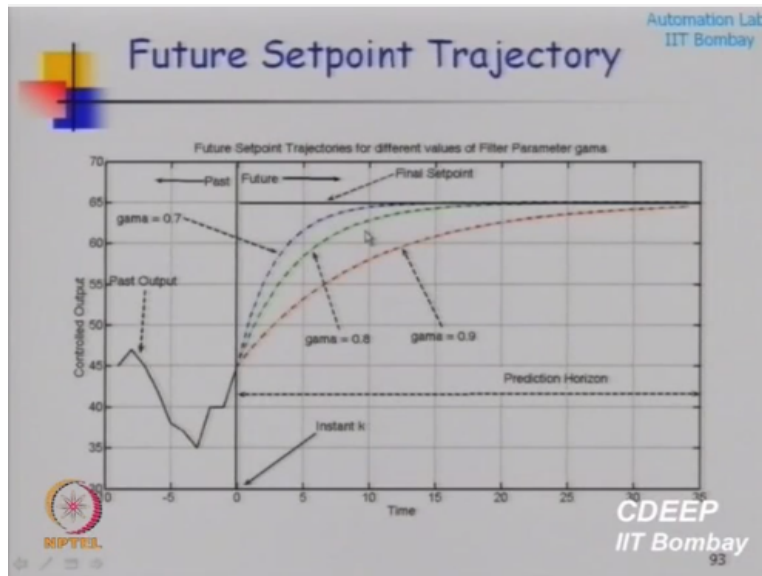
State Feedback Control

CDEEP
IIT Bombay

92

You know I talked about last time about future set point trajectory. Okay how do you want to go from the current point to the final point? Okay like you have cruising an aircraft and then you want to go very slowly it is the new set point, new height or whether you want to shoot very quickly and go up it depends upon application you can actually decide of future trajectory which starts from the current point and takes you to the final set point. So this is this can be done using a first order filter and then different values of first order filter will be view.

(Refer Slide Time: 33:34)



If you do not put any filter it means the set point trajectory is the step. If you put a filter it means you are taking it gradually. Okay if you do not put any filter if you say that the set point at the next instant should be equal to the set point. Okay if there is no filtering it is like a state function. If there is a filtering then you are taking it gradually. Okay so it depends using parameter this is not this.

(Refer Slide Time: 34:06)

Automation Lab
IIT Bombay

Steady State Target Computation

$$\min_{\mathbf{u}_s(k)} [\mathbf{r}(k) - \bar{\mathbf{y}}(k)]^T \mathbf{W}_E [\mathbf{r}(k) - \bar{\mathbf{y}}(k)]$$

subject to

$$[\mathbf{I} - \Phi] \bar{\mathbf{x}}_s(k) = \Gamma \mathbf{u}_s(k) + \mathbf{L} \mathbf{e}_f(k)$$


$$\bar{\mathbf{y}}(k) = \mathbf{C} \bar{\mathbf{x}}_s(k) + \mathbf{e}_f(k)$$

$$\mathbf{u}^L \leq \mathbf{u}_s(k) \leq \mathbf{u}^H$$

Case: Number of manipulated inputs equals the number of controlled outputs and unconstrained solution exists

$$\bar{\mathbf{u}}_s(k) = [\mathbf{C}(\Phi - \mathbf{I})^{-1} \Gamma]^{-1} [\mathbf{r}(k) - (\mathbf{C}(\mathbf{I} - \Phi)^{-1} \mathbf{L} + \mathbf{I}) \mathbf{e}_f(k)]$$

$$\bar{\mathbf{x}}_s(k) = [\mathbf{I} - \Phi]^{-1} [\Gamma \mathbf{u}_s(k) + \mathbf{L} \mathbf{e}_f(k)]$$



4/18/2012

State Feedback Control

CDEEP
IIT Bombay

94

Then of course I talked about steady state target business in LQ medial quadrant optimal control, same thing you have to do here if you want to do a, except that I have put this an optimization problem, because it is quite likely that you have set point may not be reachable within the bounds. Okay so then you have to come to the target which is not equal to the set point, but as close as possible to the set point.

Okay operator might give a set point which is not reachable, okay within that input bounds so that you have to modify that target problem here little bit. When you implement mpc you do not have to do this forget business. Forget about of course if the un constrained solution exist that is the same as what you get from LQ part. That you gone different. That is the same.

(Refer Slide Time: 35:06)

Automation Lab
IIT Bombay

Constrained MPC formulation

Given the prediction model, input constraints and desired set point trajectory, the MPC problem at sampling instant k is formulated as follows

$$\min_{\mathbf{U}_f(k)} \mathbf{E}_s(k+p|k)^T \mathbf{W}_\infty \mathbf{E}_s(k+p|k) + \sum_{j=1}^{p-1} \mathbf{E}(k+j|k)^T \mathbf{W}_E \mathbf{E}(k+j|k) + \sum_{j=0}^{q-1} \Delta \mathbf{u}(k+m_j|k)^T \mathbf{W}_{\Delta U} \mathbf{u}(k+m_j|k)$$

$$\mathbf{U}_f(k) = \left[\mathbf{u}(k|k)^T \quad \mathbf{u}(k+m_1|k)^T \quad \dots \quad \mathbf{u}(k+m_{q-1}|k)^T \right]^T$$

$$\mathbf{E}(k+j|k) = \mathbf{y}_r(k+j|k) - \hat{\mathbf{y}}(k+j|k)$$

$$\mathbf{E}_s(k+p|k) = \hat{\mathbf{x}}(k+p|k) - \bar{\mathbf{x}}_s(k)$$

CDEEP
IIT Bombay
95

State Feedback Control

SIPVES
4/18/2012

So what is this constrained MPC formulation constrained MPC consist of an objective function at 3 terms. One term this first term is distance of the predicted output from the set point trajectory. Okay what I am going to say is that minimize the distance between this is some of the acquire of distance. This is error transverse w error. What is this error, this error is set point trajectory minus the predicted output.

Okay so I want to minimize the difference between the future set point trajectory which have given and the future predictions that is done over p . You put one more term on the final terminal point okay, you put one more term on the terminal point. This terminal point term is this is between the target states.

The target state business now and all that leaving the implement LQOC will be clear to you target state business. So you take it the system as close as possible to the target state so these two will beg sure that you know you are cruising or you are doctoring the future behavior as close as possible to the desired trajectory. You want to do it such that no excessive moves are made in the future. So you do not do it by you know making large input news.

So that is handling to this input waiting. Okay you put a waiting on rate of change of input. Okay typically into a controller. So data u is different between two subsequent I think this modified norm, because no that is no input block and while change this. Okay what are the constraints of this model equation, of course the constraints set to the model equations.

Okay every time you give a guess the optimizer have to compute predictions using basic questions. Okay that is and bounds on the outputs. You can bound the future outputs which are something completely different from what other control can do. You can bound the future predicted outputs and then of course input bounds input rate bounds everything you can listen the optimization problem.

I have given you one way of formulating optimization problem as quadratic norm optimization problem. Somebody might say why here this is two norms square right. Why two norm square? Why not one norm? Why not infinite norm? Why you can use infinite norm? You can use one norm, you can use all kinds of, in fact people also use out MPC operative function as a profit maximization.

So you can have very open problem were you know the optimization problem is a big decide the future so that the profit is maximized. Okay conventionally of course you do this optimization business. There are some more important things which come in MPC, which are not there in the other. I want to highlight this to this.

(Refer Slide Time: 38:25)

Automation Lab
IIT Bombay

Constrained MPC formulation

Subject to following constraints

(a) Model Prediction Equations

$$\hat{\mathbf{z}}(k+j+1) = \Phi\hat{\mathbf{z}}(k+j) + \Gamma\mathbf{u}(k+j|k) + \mathbf{L}\hat{\mathbf{e}}(k+j)$$


$$\hat{\mathbf{e}}(k+j+1) = \hat{\mathbf{e}}(k+j)$$

$$\hat{\mathbf{y}}(k+j) = \mathbf{C}\hat{\mathbf{z}}(k+j) + \hat{\mathbf{e}}(k+j)$$

Initial Conditions: $\hat{\mathbf{z}}(k) = \hat{\mathbf{x}}(k|k-1)$ and $\hat{\mathbf{e}}(k) = \mathbf{e}_f(k)$

(b) Bounds on future inputs and predicted outputs

$$\mathbf{y}^L \leq \hat{\mathbf{y}}_c(k+j|k) \leq \mathbf{y}^H \quad j = 1, 2, \dots, p$$



4/18/2012

State Feedback Control

CDEEP
IIT Bombay
96

Okay sometimes you do not want to control an outside precise at a set point. You do not mind if it fluctuates with in a bound. Okay so for example you know some concentration or some you know some temperature of in some reactor is very, very important for the product purity. What the level inside the reactor it need not be exactly at you know 7.2 meters.

It can be with in 7.5 and 7 meters. Okay so what I can tell the controller here is that do not control level at the set point as longer it is within the bounds its fine with me. So this is called zone control variables and this is something different about MPC. You need not give a set point or the particular output you can just say that maintain it with in a bound.

Okay which is move practical in many situation, so you can actually allowing giving freedom to the controller not to take certain outputs exactly to certain values, but allow them float with in a bound. So all these things are possible with this. Okay so the controller specification is so transparent and so straight forward from what you think. See if you ask a operator or even a control engineer as you go there and said translate your controller requirements in to frequential domain design criteria, but so you know popular techniques in control of frequential domain, very difficult to translate that.

That we will take care. Okay what is the bound on the input you know what is the bound on the input what is the rate at which you can change you know what rate you can change. Okay what is the physical bound on the predictor of, but you know what it is. So all these things can be very transparently you know convert it in to a controller specification.

Okay it can be flexible if a frequency domain controller is to be redesigned you need an expert who understand frequential domain who can who will go back do the calculations, here is an optimization problem which is solved at every instant. Right suppose I formed I had one side of constraints today I can change them tomorrow. So this particular formulation predictive control formulation is very very flexible. Nothing more flexible all you can think of. Okay so how do you compute this term here w infinity know you control if you do it using solving Lyapunav equation then you know you can guarantee some properties.

(Refer Slide Time: 41:11)

Automation Lab
IIT Bombay

Constrained MPC formulation

W_E is a symmetric positive definite error weighting matrix and W_U is symmetric positive semidefinite input weighting matrix. These matrices are treated as tuning parameters, which are used to shape the closed loop input and output behavior.

The terminal state weighting matrix W_∞ can be found by solving discrete Lyapunov equation. When poles of Φ are inside the unit circle, W_∞ can be found by solving discrete Lyapunov equation

$$W_\infty = C^T W_E C + \Phi^T W_\infty \Phi$$

When some poles of Φ are outside unit circle, the procedure for computing the terminal weighting matrix is given in Muske and Rawlings (1993)

CDEEP
IIT Bombay

NPTEL 4/18/2012 State Feedback Control 97

Controller requirements into a frequents element defined criteria, popular techniques in control, my control of frequents element very difficult to translate that. That is on the, what is bound on the input? You know what is the bound on the input, what is the rate that you can change, what is the rate you can choose. What is the physical bound on the predicted output? You know what it is.

So always things it can be very transparently converted into a controller specification. It can be flexible. If the frequency domain controller to redefine you needs an expert who understand peoples element. You will go back do the calculations; here is an optimization problem which solved at every incident.

Suppose I formed I had one developed constants to that I can change them tomorrow. So this particular formulation predicted control formulation very, very effective. Nothing more flexible we can think of. So how do you compute this down here w infinitive you can show that if you do it using Lyapunov equation then you can guarantee from properties.

So I am not getting into that political aspects, but just believe right now that how do compute, you can solve this Lyapunov equation and get daily activity. Well which has been implemented over the years and found to be stable and working and all that of course know academician all that well that improving properties and then where have been done a work on theory to establish theoretical foundations of evitable.

When it works 1000 of cases now we are trying to satisfy all the theory of Lyapunov works here and I am not given those all the details applicable to show stability, remember to upload some more theatre this year reference. Well then, what you do of course implement this in moving around the foundation, you only implement the first move optimal move and he told the rest and then you move on repopulate the problem again resolve it and so do the implementation.

(Refer Slide Time: 42:40)

Automation Lab
IIT Bombay

Moving Horizon Formulation

- Optimization problem transformed to Quadratic Programming (QP) problem for improving computing efficiency on-line and solved using efficient QP solvers available commercially.
- MPC formulation can control Non-square multi-variable systems i.e. systems with number of controlled outputs not equal to the number of manipulated inputs.
- In many practical situations, not all outputs have to be controlled at fixed setpoints but need to be maintained in some "zone". Such zones can be easily defined using constraints on predicted outputs.

IPTRE
4/18/2012

State Feedback Control

CDEEP
IIT Bombay
99

Now are there efficient ways of solving this, well raw optimization problem which I discussed is, can take lot of time to solve. There is something called quadratic programming, if you have done a course on horizon optimization for Lyapunov programming. Quadratic programming can solve, the same you have do reconvert this problem into the quadratic programming that is algebra I just showed you the procedure. I am not going to go detail into that equation that is just lot of algebra. You have patiently certain keep doing this equation rearrange all the equations into certain forms and then you can solve it very efficiently.

So the nice thing about predictive control is that, it can be used for a systematic non-square. The number of inputs, number of output may not be equal. There can be more outputs than the input, there can be more output than the input, that can be input than the output, that can be equal, that can whatever. Does not matter. Same ideas, same optimization formulation work for any kind of input, output mapping. So when I said when the number of outputs are more than the number of inputs you have do a variable, you cannot maintain all output to the point. They were some disk can be predicted to theoretical. What is this quadratic program in business?

(Refer Slide Time: 44:07)

Automation Lab
IIT Bombay

Quadratic Programming (QP)

A constrained optimization problem is called as Quadratic Programming (QP) formulation if it Has following standard form

$$\text{Min } \frac{1}{2} \mathbf{U}^T \mathbf{H} \mathbf{U} + \mathbf{F}^T \mathbf{U}$$

Subject to

$$\mathbf{A} \mathbf{U} \leq \mathbf{b}$$

NPTEL

CDEEP
IIT Bombay
100

I will just briefly mention it and then go into too much this detail. Details have given in the notes. A mathematical constrained optimization problem is called as a quadratic program when it is of this form. So the optimization problem that we talked about just now manipulative control

problem can be transformed this one. The u here or the future it use H is a complex matrix of ϕ γ c and all that and then this F is another vector and if you will again the future input, so if you have this quadratic equation as objective function.

Subject to this bounds AU equal to B then this is called the quadratic programming problem and these quadratic program problem can be solved very, very easily in short time in mathematical terms. They can be solved in fraction of a second. So if for example if you do this exercise that you use mat lab's, this is a constrained optimization problem. I think it is a programming.

If you use the mat lab traditional constrained optimizer and if you use quadratic programming of mat lab program called quad prompt with the quadratic programming, then quad practice 10 times faster than the normal optimization and of course for how many information when you want to solve one optimization problem each sample an instant you can do very fast computing.

So this transformation of the original problem is the quadratic problem is desirable. Of course when you do your assignment do not do this, you do it as simple write a prediction equation. Do it in the raw way when you do it your assignment, but actually real time implementation you would be use in quadratic program. So you are going to transform the original problem into this kind of a problem and there are very efficient QP course available commercially or even in public domain and you can use them to solve your problem very complex problems, very large scale problem.

(Refer Slide Time: 46:30)

Automation Lab
IIT Bombay

Quadratic Programming (QP)

A constrained optimization problem is called as Quadratic Programming (QP) formulation if it Has following standard form

$$\text{Min}_U \frac{1}{2} U^T H U + F^T U$$

Subject to

$$AU \leq b$$

A large dimensional QP formulation can be solved very quickly using an efficient search method

Through a series of algebraic manipulations, the Constrained MPC formulation can be transformed to a Quadratic Programming (QP) Problem.

SIPTEL CDEEP IIT Bombay

100

So this controllers I will show you they might have 10,000 different variables and you can solve them in few seconds if you use this efficient quotes. So how do you do this?

(Refer Slide Time: 46:46)

Automation Lab
IIT Bombay

QP Formulation

To understand how the MPC optimization problem can be transformed to a quadratic programming problem, consider MPC formulation without terminal state weighting

Defining the future input vector $\mathbf{U}_f(k)$ and
the predicted output vector $\hat{\mathbf{Y}}_f(k)$

$$\mathbf{U}_f(k) = \left[\mathbf{u}(k|k)^T \quad \mathbf{u}(k+m_1|k)^T \quad \dots \quad \mathbf{u}(k+m_{q-1}|k)^T \right]^T$$

$$\hat{\mathbf{Y}}_f(k) = \left[\hat{\mathbf{y}}(k+1|k)^T \quad \hat{\mathbf{y}}(k+2|k)^T \quad \dots \quad \hat{\mathbf{y}}(k+p|k)^T \right]^T$$

(Note: QP formulation can be carried out with terminal state weighting also. It has been neglected here to keep the expressions relatively simple)

ODEEP
IIT Bombay
101

What I am going to do here is develop, just show you very quickly how to transform the problem into quadratic problem. I am going to neglect right now that terminal target set business, but that can be included I am just removed it because keep the algebra simple. So do not think that it is cannot be done with that.

So what do you do is you define a vector of all the future inputs start one below each other. For this U, U future F is all the future inputs that one below each other, this is why future F is all the future predictions start one below each other. And then all those prediction equation I am going to start below each other and write one joint equation.

(Refer Slide Time: 47:33)

QP Formulation

$$\hat{Y}_f(k) = S_X \hat{x}(k|k-1) + S_U U_f(k) + S_e e_f(k)$$

where

$$S_x = \begin{bmatrix} C\Phi \\ C\Phi^2 \\ \dots \\ C\Phi^p \end{bmatrix}; \quad S_e = \begin{bmatrix} CL + I_r \\ C\Phi L + CL + I_r \\ \dots \\ C\Phi^{p-1}L + \dots + CL + I_r \end{bmatrix}$$

Matrix relating the effect of past states to future predictions

Matrix relating the effect of past unmeasured disturbances and model plant mismatch on the future predictions

CDEEP
IIT Bombay
102

Why future is some matrix into the initial stage less some matrix into all the future inputs, less some matrix into year. So what are these matrix as SU and SE, these of course if you sit and write all those equations one below each other and then take u common. If you do the algebra you can find those matrixes. So the matrix is turnout to be some huge matrix.

(Refer Slide Time: 48:03)

Automation Lab
IIT Bombay

QP Formulation

$$S_u = \begin{bmatrix} C\Gamma_u & [0] & [0] & \dots & [0] \\ C\Phi\Gamma_u & C\Gamma_u & [0] & \dots & [0] \\ \dots & \dots & \dots & \dots & [0] \\ C\Phi^{q-1}\Gamma_u & C\Phi^{q-2}\Gamma_u & \dots & \dots & C\Gamma_u \\ C\Phi^q\Gamma_u & C\Phi^{q-1}\Gamma_u & \dots & \dots & C(\Phi + I)\Gamma_u \\ \dots & \dots & \dots & \dots & \dots \\ C\Phi^{p-1}\Gamma_u & C\Phi^{p-2}\Gamma_u & \dots & \dots & C(\Phi^{p-q} + \dots + I)\Gamma_u \end{bmatrix}$$

Matrix relating the effect of future manipulated inputs
On future predictions
Consists of impulse response coefficients of the model
Referred to as **Dynamic Matrix** in MPC literature

CDEEP
IIT Bombay
103

So this matrix S_u will have dimension equal to number outputs into number of prediction arising. Suppose you have number of outputs are 5 and prediction arising is 100 this matrix will be 500 clause whatever, number of inputs trying to control arising. So suppose number inputs are 5 and control arising is 10, so it will be 500 cast 50 matrix, huge matrix and then doing this metrication mat lab is not difficult, 500 cast matrix this stage at the mat lab.

So this matrix is often called as dynamic matrix and if you observe carefully and if you know what are impossible of coefficient from your previous understanding of system theory and actually this matrix consist of all the system impose called coefficient system and it was initially called dynamic matrix of the system.

(Refer Slide Time: 49:07)

Automation Lab
IIT Bombay

QP Formulation

$$\hat{Y}_f(k) = S_x \hat{x}(k|k-1) + S_U U_f(k) + S_e e_f(k)$$

where

$$S_x = \begin{bmatrix} C\Phi \\ C\Phi^2 \\ \dots \\ C\Phi^p \end{bmatrix} ; \quad S_e = \begin{bmatrix} CL + I_r \\ C\Phi L + CL + I_r \\ \dots \\ C\Phi^{p-1}L + \dots + CL + I_r \end{bmatrix}$$

↓

Matrix relating the effect of past states to future predictions

↓

Matrix relating the effect of past unmeasured disturbances and model plant mismatch on the future predictions

CDEEP
IIT Bombay
102

So basically you are written in the system in terms of three things. All the future predictions starting to one is equal to one matrix into all future inputs and these two matrix are bring in the effect of past state and past disturbances. You have written by the one all the future predictions into one joint equation, one joint matrix equation.

(Refer Slide Time: 49:28)

Automation Lab
IIT Bombay

Unconstrained QP Formulation

Defining the future reference trajectory

$$\mathbf{R}(k) = \left[\mathbf{y}_r(k+1|k)^T \quad \mathbf{y}_r(k+2|k)^T \quad \dots \quad \mathbf{y}_r(k+p|k)^T \right]^T$$

the prediction error vector $\mathbf{E}(k)$ at instant k can be computed as

$$\mathbf{E}(k) = \mathbf{R}(k) - \hat{\mathbf{Y}}(k)$$


Using these notations, unconstrained version of MPC problem can be stated as follows

$$\min_{\mathbf{U}_f(k)} \mathbf{E}(k)^T \mathcal{W}_E \mathbf{E}(k) + \Delta \mathbf{U}_f(k)^T \mathcal{W}_U \Delta \mathbf{U}_f(k)$$

$$\mathcal{W}_E = \text{block diag} \left[\mathbf{W}_E \quad \mathbf{W}_E \quad \dots \quad \mathbf{W}_E \right]$$

$$\mathcal{W}_{\Delta U} = \text{block diag} \left[\mathbf{W}_{\Delta U} \quad \mathbf{W}_{\Delta U} \quad \dots \quad \mathbf{W}_{\Delta U} \right]$$

CDEEP
IIT Bombay
104



Then you can write your MPC problem in terms of this joint vectors. So huge matrix and then you are doing some algebra.

(Refer Slide Time: 49:40)

Automation Lab
IIT Bombay

Unconstrained QP Formulation


where

$$\Delta \mathbf{U}_f(k) = \begin{bmatrix} \mathbf{u}(k|k) - \mathbf{u}(k-1) \\ \mathbf{u}(k+1|k) - \mathbf{u}(k|k) \\ \dots \\ \mathbf{u}(k+q-1|k) - \mathbf{u}(k+q-2|k) \end{bmatrix}$$

$$= \Lambda \mathbf{U}_f(k) - \Lambda_0 \mathbf{u}(k-1)$$

$$\Lambda = \begin{bmatrix} I & [0] & [0] & [0] \\ -I & I & [0] & [0] \\ \dots & \dots & \dots & \dots \\ [0] & \dots & -I & I \end{bmatrix} \quad ; \quad \Lambda_0 = \begin{bmatrix} I \\ [0] \\ \dots \\ [0] \end{bmatrix}$$

CDEEP
IIT Bombay
105



You convert this into mapping between all those matrixes and so this is just a lot of patience and you can convert all those equations into this complicate version, nothing great about it. So basically you have transform the problem into this quadratic problem. If there were no constraints this quadratic problem can be solved automatically. If they were no bounds on the inputs there were no constrain then how do you call this what is the definition of this?

If you transform the solution okay if you is it is f vector as exact k/k-1, e, e f(k) and all that. Okay you can rearrange the solution unconstrained solution you can rearrange in to this half. Though in suitable algebra you can rearrange in to this form.

(Refer Slide Time: 51:25)

Automation Lab
IIT Bombay

Unconstrained QP Formulation


Since only the first input move is implemented on the process

$$\mathbf{u}_{opt}(k|k) = \Lambda_0^T \mathbf{U}_f(k) = \Lambda_0^T \mathbf{H}^{-1} F(k)$$

With some algebraic manipulations, the above control law can be rearranged as follows

$$\mathbf{u}_{opt}(k|k) = \mathbf{G}_u \mathbf{u}(k-1) - \mathbf{G}_x \hat{\mathbf{x}}(k|k-1) + \mathbf{G}_e \hat{\mathbf{e}}_f(k|k) + \mathbf{G}_r \mathbf{R}(k)$$

CDEEP
IIT Bombay
107



Okay I am showing you this is because I want to point out that MPC actually the state feedback control. Let us see here. \mathbf{U}_k is \mathbf{g}_x times negative state feedback control. The same it is a follow for state feedback control the design. Okay unconstrained MPC will tell not to be state feedback controller. Constrained MPC is a state feedback controller, but not in the close form. Unconstrained you can show it in the close form.

Okay if you are doing unconstrained solution okay then you do not have to do in a solve optimization problem every time you just compute this matrices \mathbf{G}_u , \mathbf{G}_x , \mathbf{G}_e and then this multiply you will get the solution. Okay so there is nothing so you get what I am saying. So this unconstrained MPC is actually is a follow for state feedback controller. Okay and then you can of course with unconstrained MPC you will not real to be use in typically in reality. This only to give you inside that actually MPC the follow here the state feedback controller and reality.

(Refer Slide Time: 52:14)

Automation Lab
IIT Bombay

Unconstrained QP Formulation

Since only the first input move is implemented on the process


$$\mathbf{u}_{opt}(k|k) = \Lambda_0^T \mathbf{U}_f(k) = \Lambda_0^T \mathbf{H}^{-1} \mathbf{F}(k)$$

With some algebraic manipulations, the above control law can be rearranged as follows

$$\mathbf{u}_{opt}(k|k) = \mathbf{G}_u \mathbf{u}(k-1) - \mathbf{G}_x \hat{\mathbf{x}}(k|k-1) + \mathbf{G}_e \hat{\mathbf{e}}_f(k|k) + \mathbf{G}_r \mathbf{R}(k)$$

From the above expression, it is easy to see that unconstrained MPC is a form of state feedback control law

Advantage of unconstrained formulation: closed form control law can be obtained and, as a consequence, on-line computation time is small



CDEEP
IIT Bombay

107

When you use MPC you will use with the constraints actually sure have input bounds if nothing else. The bounds are always, yeah all the difference is the final horizon. This is the final horizon problem whereas LQG is a infinite or other problem and the what see here in the LQG here you are able to give constraints on delta u. okay so this is then you know very nice handle on module plan mismatch. So it is not too different if you were if you ask me if you have to do implement and you know a scheme which is unconstrained.

I do not see too much advantage over LPG. I would then implement LQG why go for. Okay again LQG does it give close form solutions I do not know. This can be LQG unconstrained MPC why LQG is a form of LQG with idea moving horizon. See LQG remain the idea for move horizon. Okay so basically the idea is that this original problem can be recast as to be problem and then you can actually solve it as very, very efficiently.

(Refer Slide Time: 54:07)

Automation Lab
IIT Bombay

Alternate Formulations

- To achieve offset free control, it is possible to develop MPC formulation based on the **augmented state space model** (see Muske and Rawlings, 1993; Yu et al., 1994) as described in the case of LQG (on slides 46 to 50).
- Early formulations of MPC, such as Dynamic Matrix Control (DMC), were based on 'open loop observer' and were meant for open loop stable systems. These formulations can be derived by setting the observer gain to zero in the innovation bias formulation.
- MPC formulation in this presentation has been developed using Kalman predictor only for the sake of convenience. It is straightforward to develop similar formulations based on the Kalman filter with innovation bias approach or state augmentation approach.

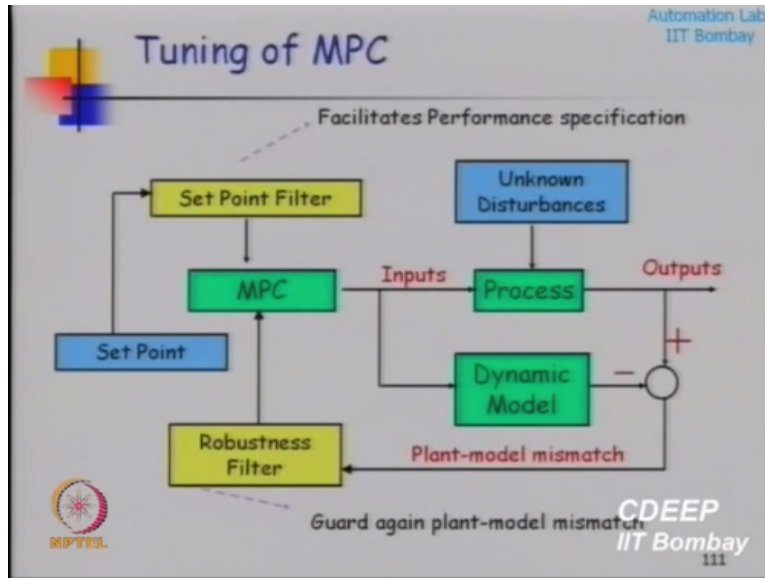
MPTCL

CDEEP
IIT Bombay
110

Okay so yeah one can do of course I have developed one way of doing MPC using this innovation by his you can do his state augmentation and then formulate MPC were I mean MPC has been a very rich area almost 30 years of research and this area around. There are so many ways of doing it. I have just showed you one possible way which I like which it is more of personal test. I have done it using close up observer, but originally it was not done using close up observer. Originally it was all done using open loop observers. So originally all these methods could be used only for open loop stable systems.

Now of course you do not have to do all that. Then of course you can, I have done it using Kalman predictor but I just wanted to note that. That is not a restriction it can be done it using any form.

(Refer Slide Time: 55:05)



So there are different tuning parameters, you can choose the set point filter trajectory, you can choose the Robustness filter this innovation by his filter there you can tune. Then you can tune the control horizon prediction level. You can actually set the delta u moves so all these are tuning parameters and then this tuning parameters are very transparent meaning in terms of performance they are very, very transparent meaning.

(Refer Slide Time: 55:37)

Automation Lab
IIT Bombay

Tuning of MPC

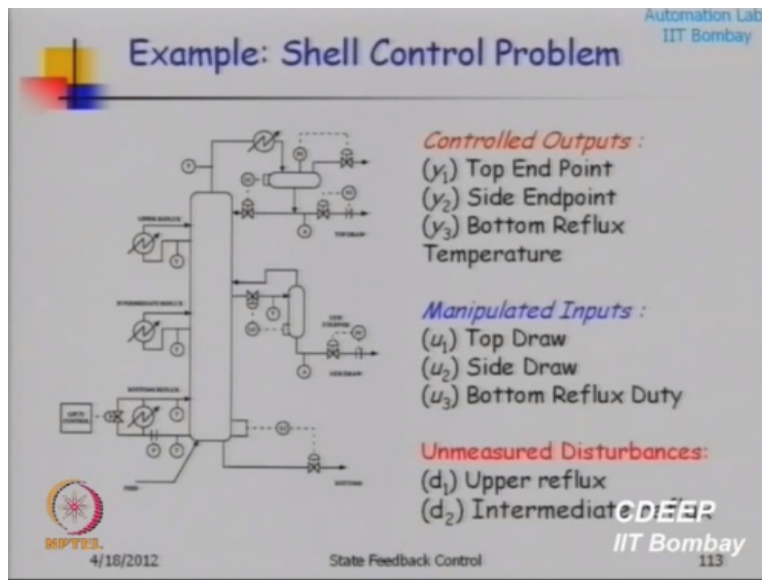
- **Prediction Horizon:** Typically chosen close to open loop settling time (60 to 100 samples)
- **Control Horizon:** Typically chosen small (5 to 10) to avoid model inversion problems
- **Input rate constraints**
- **Zone / Range Control:** Not necessary to specify set points on each output. Instead, high and low limits can be defined within which output should be maintained

MPTCL

CDEEP
IIT Bombay
112

So typically prediction horizon you choose between 60 and 100. Control horizon is between 5 and 10. These are from industrial implementations, I am just giving some numbers which are you can give zones instead of giving set points. So all kinds of.

(Refer Slide Time: 55:54)



Just to show you one example, this is a problem from which is quoted by the Shell refinery, so I want to control three set points. There is a, there is some heavy oil which is being separated into lighter products and heavier products. Okay this is one of the typical operations in the refinery, so the feed is coming here at the bottom and the steam is being injected here at the bottom, so I can manipulate the flow that the top draw that is the flow rate here. I can manipulate the flow rate out here. This is the feed which is coming in.

And then I can manipulate the heat input here. Okay this is called u_3 is bottom reflux duty is nothing but the heat input here to this feed exchanger and I can manipulate this product rate, I can manipulate the side product rate. Okay there are two disturbances, some part of the liquid is taken it is sub cooled and put it back. Same thing is done here, now this cooling fluid for this is coming from somewhere else. So these two are actually disturbances. If you do not understand the Physics do not worry, basically as a control engineer I have three end appointments during at I want to control the purity of the product here.

Okay and I want to control the temperature at this point okay, just look at the blood box three things, two product compositions, one temperature. I have measurement as a level for the two product compositions and the temperature. I can manipulate three inputs, the top draw I can manipulate this flow rate here again manipulate this flow rate again manipulate the heat input. Three inputs, three outputs, two disturbances. Okay with particular system has large time dealers to very typical system to control it has very heavy interactions. Okay so it is a problem by the

people work in control and they have given this model and then you can convert it in to state plus model in to specifics and so they also specified.

(Refer Slide Time: 58:20)

Automation Lab
IIT Bombay

SCP: MPC Tuning Parameters

Operating Constraints

Input Limits $-0.5 \leq u_i \leq 0.5$ (for $i = 1,2,3$)
 $-0.5 \leq d_i \leq 0.5$ (for $i = 1,2$)

Rate Limits $-0.05 \leq \Delta u_i \leq 0.05$ (for $i = 1,2,3$)

Output Constraints $-0.5 \leq y_1 \leq 0.5$
 $-0.5 \leq y_3$

$W_e = \text{diag}[1 \ 1 \ 0]$
 $W_u = 1.5 \text{diag}[1 \ 0.1 \ 1]$

Sampling interval
(T) = 2 min

Prediction Horizon : 40 Control Horizon : 5 CDEEP
 IIT Bombay 115

4/18/2012 State Feedback Control

The input rate constraints input constraints everything they have given either description on the problem. Okay so they have set that the input should not cross limits of (+) or (-) 0.5 disturbances will not cross (+) or (-) 0.5 at any control move should not be in more than 0.05. Okay so designing an LQG control that will make sure that this happens, will not possible. Okay will have trouble, so or even a bid controller or you cannot design you can enforce either you know I hope major that if it gives you higher then do not use it and discard, but there are constraints on the output.

Okay it just says that the y_3 will not be controlled at the set point y_3 can be above (-0.5). They have given everything into these variables, so we do not know at physical variables and then they have said that y_1 should be between (+) or (-) 0.5 will the constraint. So you have to operate the cont system under these constraints. Okay so I have developed the MPC controller for the system using 40 sampling constraints. The model is I have done the same thing that you have done in your course I took this as a plant then I injected the inputs.

(Refer Slide Time: 59:45)

Automation Lab
IIT Bombay

Shell Control Problem (SCP)

$$y(s) = G_u(s)u(s) + G_d(s)d(s)$$

$$G_u(s) = \begin{bmatrix} \frac{4.05e^{-27s}}{50s+1} & \frac{1.77e^{-28s}}{60s+1} & \frac{5.88e^{-27s}}{50s+1} \\ \frac{5.39e^{-18s}}{50s+1} & \frac{5.72e^{-14s}}{60s+1} & \frac{6.9e^{-15s}}{40s+1} \\ \frac{4.38e^{-20s}}{33s+1} & \frac{4.42e^{-22s}}{44s+1} & \frac{7.2e^{-19s}}{19s+1} \end{bmatrix}$$

$$G_d(s) = \begin{bmatrix} \frac{1.2e^{-27s}}{45s+1} & \frac{1.44e^{-27s}}{40s+1} \\ \frac{1.52e^{-15s}}{25s+1} & \frac{1.83e^{-15s}}{20s+1} \\ 1.14 & 1.26 \\ 27s+1 & 32s+1 \end{bmatrix}$$

Characteristics

- Large time delays
- High degree of multivariable interaction

CDEEP
IIT Bombay

State Feedback Control

114

4/18/2012

Okay I use system in to the central box develop a straight model. So I am reading this plant as a blood box. Okay I am deleting data using it to develop a prediction model.

(Refer Slide Time: 1:00:00)

Automation Lab
IIT Bombay

SCP: PID Tuning Parameters

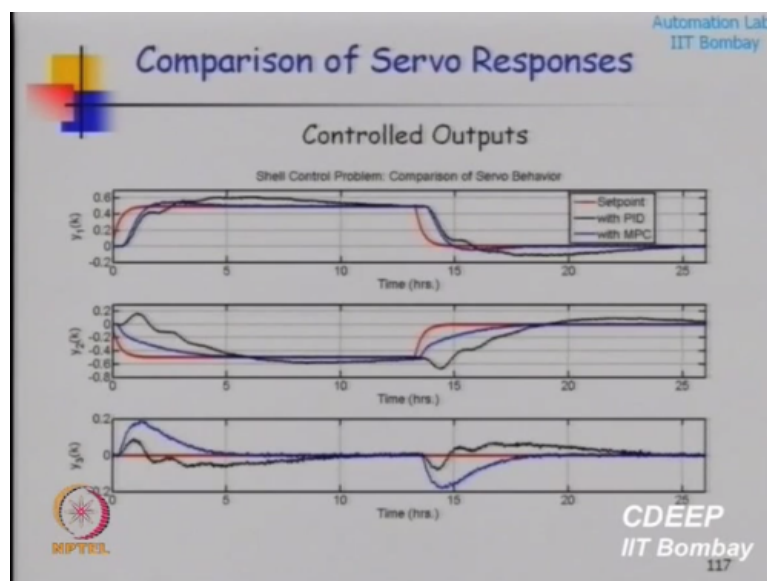
- Multi-loop PID control: Three independent PID controllers with no coordination among them
- PID Pairing and Tuning
 - (y1) Top End Point - (u1) Top Draw
Kc = 0.3 , Ti = 13 min, Td = 0
 - (y2) Side Endpoint - (u2) Side Draw
Kc = 0.23 , Ti = 30 min, Td = 0
 - (y3) Bottom Reflux - (u3) Bottom Reflux Duty
Kc = 0.28 , Ti = 9 min, Td = 0

CDEEP
IIT Bombay

116

That prediction model I gave use for controlling, I am going to compare my performance with 3 PID controllers. Okay and what I have to show you is that three well tuned PID controllers cannot manage the system so well as the model predictive controller cap. Model predictive controller is a multi variable controller with more constraints. It is a very, very advanced controller. There is exactly three PID controllers which are like three drivers in the car who do not know about each other. So I have three controllers which you can see here.

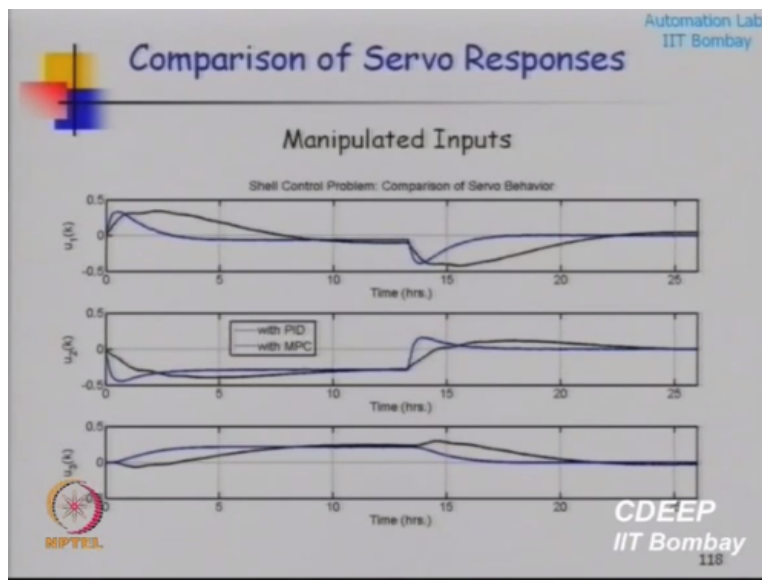
(Refer Slide Time: 1:00:34)



That black line is the PID controller, this here, in this black line. I have given a set point in here. The MPC controller very quickly set. This blue line is the MPC controller. Black line is the PID controller. PID controller that is either side, you have almost 12 to 15 hours, MPC cycle period is 5 hours. You cannot see it here. This is the set points I have changed. Here also this blue MPC with cycle period in 5 hours, PID that either side.

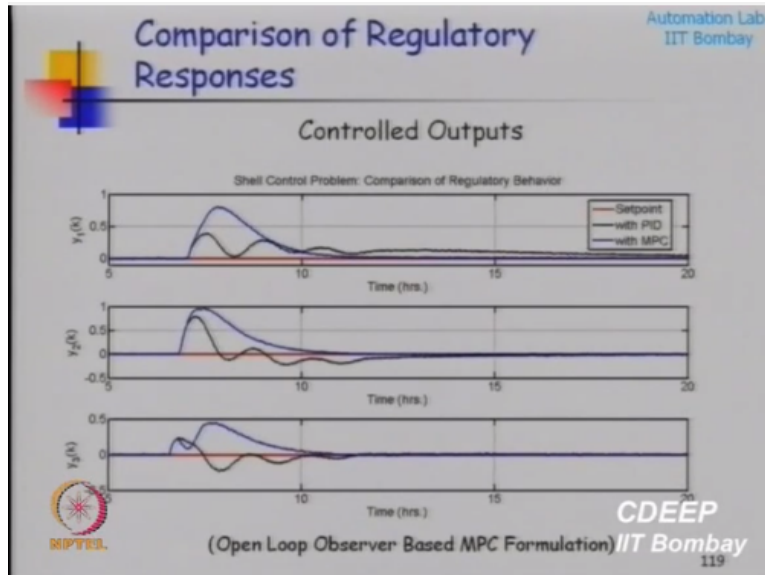
So same thing is whenever you converting, MPC is typically, it takes you to from one set point to the other set point very quickly because the multi variable controller, it avoids any editing for especially for avoiding multi level interaction. The model itself is a multi level model, it may have interaction. They are standing on the inputs to well connecting. The input profiles are continuously different for MPC and PID.

(Refer Slide Time: 1:01:48)



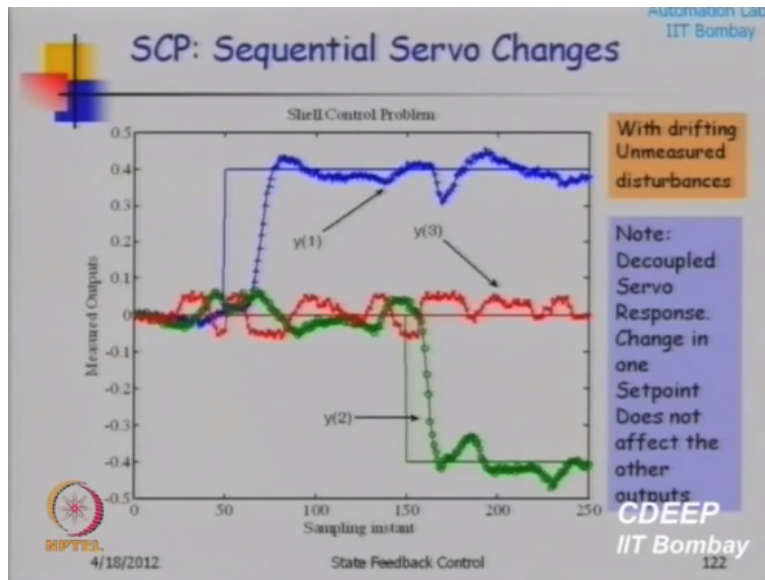
So what about the regulatory cut phase, idealic cut change in the input differences and then they also given you a part of the description. They also given what disturbances include and all that. So we get this include disturbance reduction in using MPC.

(Refer Slide Time: 1:02:08)



Here is just the disturbance you have given and they send we were clear that it could be.

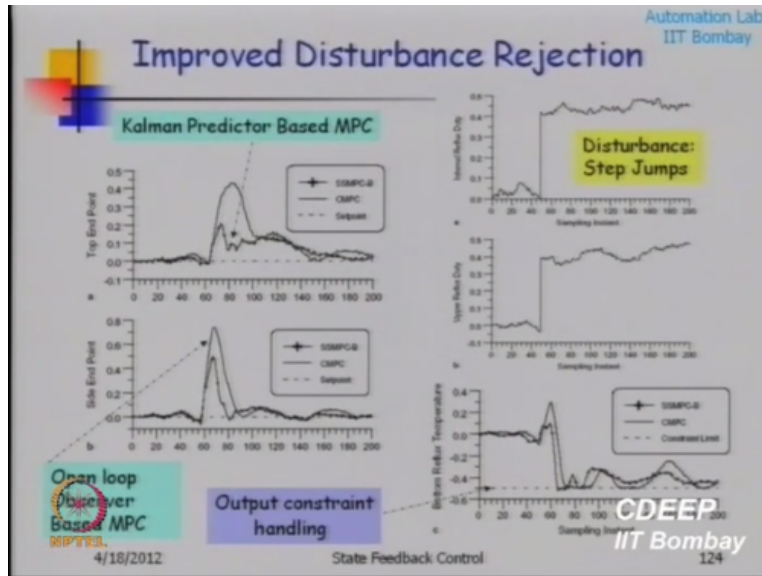
(Refer Slide Time: 1:02:13)



When I change one set point, the other two set points, the other two outputs. I have now introduced different disturbances. Earlier I had cut off WTI/2. This will surely clean behavior. Now I am doing simulations here with undelayed disturbances noise, everything is here. What I want to show here is that when I change one set point other two variables nothing happens.

When I change this set point this variable and this variable are around this control. It is same as not good here. So it give the kind of a decoupled response. I think the other group does not exist. It is well known that there is one controller which is looking at all three things simultaneously. This MPC is much more, and then of course you can do better reactions.

(Refer Slide Time: 1:03:12)



What I have I told you is just basics, there is a lot more to learn.

(Refer Slide Time: 1:03:32)

Automation Lab
IIT Bombay

Linear MPC: Nominal Stability


- **Approach 1:** Formulate infinite prediction horizon and finite control horizon problem

$$\sum_{i=1}^{\infty} \|e_r(k+i|k)\|_{W_0}^2 + \sum_{i=1}^m \|\Delta u(k+i|k)\|_{W_0}^2$$

Stability is proved by using Lipunov's approach
- **Approach 2:** Impose terminal Constraints on predicted state $x(k+p) = 0$ or introduce large weighting on terminal state
- **Approach 3:** Impose contraction constraints

$$\|x(k+p|k)\|_R \leq \alpha \|x(k|k)\|_R$$

R : +ve definite, $\alpha \in [0,1)$



4/18/2012

State Feedback Control

CDEEP

IIT Bombay

125

Well as control engineers you should bother about nominal stability and then impose the constraint that, state constraint that $x=0$ and then we can establish stability one end have to be favored. I will really reference it towards the end if you are interested in pursuing that. Then otherwise there is an approach which is called as introduce contraction constraints into my MPC combination.

What are the things shown is that, you can construct Lipunov's function directly with the MPC objective function. So you can actually proof stability agent, the MPC object you need to observe. So I am not going to go into that. So you can.

(Refer Slide Time: 1:04:21)

Automation Lab
IIT Bombay

Nominal Stability

Infinite Horizon Formulation (Muske and Rawlings, 1993)

Let $(\Xi_s x)$ and $(\Xi_u x)$ be projections of x on to stable
unstable eigen - spaces of Φ matrix, respectively

$$\psi = \left\{ \begin{array}{l} \sum_{i=1}^{m-1} x(k+i|k)^T (W_u) x(k+i|k) + \sum_{i=0}^{m-1} [\Delta u(k+i|k)^T (W_u) \Delta u(k+i|k)] \\ + x(k+m|k)^T \Xi_s^T \bar{W}_s \Xi_s x(k+m|k) \end{array} \right\}$$

Subject to


$\Xi_u x(k+m|k) = \bar{0}$

Input and state constraints

\bar{W} : Positive definite solution to Liapunov Equation

$$(\Xi_s^+)^T \Phi^T (\Xi_s)^T \bar{W}_s (\Xi_s) \Phi (\Xi_s^+) - \bar{W}_s + (\Xi_s^+)^T W_u (\Xi_s^+)^T = [0]$$

$$\Xi_s^+ = \Xi_s^T (\Xi_s \Xi_s^T)^{-1}$$



4/18/2012

State Feedback Control

CDEEP
IIT Bombay

126

This paper by Muske and Rawlings in 1993 one in IEEE conventions of automated control. And other one is AICTE journal both of them are same frequency I have given reference in the end. They will they this particular paper sorted out most of the theoretical issues so they, they are considered to be very and put it into the states space framework LPG framework.

So details you can see that, basic idea is that you know you put this constraint that $x_{k+t}=0$ so it has been the 0 state sometimes if you put that constraint and you can prove liapunov stability. If a critical solution exists for the system and liapunov state control.

(Refer Slide Time: 01:05:07)

Automation Lab
IIT Bombay


Linear MPC: Robustness

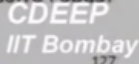
- **Approach 1:** Uncertainty modeled as interval uncertainty / ellipsoidal bounds on FIR coefficient
 - Optimization problem formulated as min-max problem (minimize worst case value of objective functions taken over the set of uncertain plants)
- **Approach 2:** Multi-model description of uncertainty


$$x(k+1) = \Phi x(k) + \Gamma u(k)$$

$$y(k) = Cx(k)$$

$$[\Phi, \Gamma] = \left\{ \sum_{i=1}^s \lambda_i [\Phi_i, \Gamma_i] \mid \sum_{i=1}^s \lambda_i = 1; \lambda_i \geq 0 \right\}$$


 ▪ Use Linear Matrix Inequalities framework to solve robust control problem




 4/18/2012
 State Feedback Control
127

And then how do bend in robustness into a system and so on, so there are already many, many commercial products I am looking that Dr. Jogish Naveen talk about one such product. So I am trying to organize this lecture after the exams are over and those of you are here should attend that lecture.

(Refer Slide Time: 01:05:29)

Automation Lab
IIT Bombay

Commercial Products

Company	Product name	Description
Adersa	HIECON	Hierarchical constraint control
	PFC	Predictive functional control
	GLIDE	Identification package
Aspen Tech	DMC-plus	Dynamic matrix control package
	DMC-plus model	Identification package
Honeywell Hi-Spec	RMPCT	Robust model predictive control technology
Shell Global Solutions	SMOC-II*	Shell multivariable optimizing control
Invensys	Connoisseur	Control and identification package

(Ref.: Qin and Badgwell) **CDEEP**
IIT Bombay

128

SIPTER.
4/18/2012
State Feedback Control

So this a home of survey paper machine in back bell and apologies paper in model if you are interested. So and these companies there are many such companies which actually implement these controllers in India. They want people who are trained, who know about predictive control system identification for information and there is work for people who know this. So these controllers are very much there not just global neediest, cControllers are here now been implemented occur so.

(Refer Slide Time: 01:06:09)

Automation Lab
IIT Bombay

Linear MPC Applications (2003)

Area	Aspen Technology	Honeywell HiSpec	Adersa ²	Invenys	SGS ²	Total
Refining	1200	400	200	25		1985
Petrochemicals	450	80	—	20		550
Chemicals	100	20	3	21		144
Pulp and paper	18	50	—	—		68
Air & Gas	—	10	—	—		10
Utility	—	10	—	4		14
Mining/Metallurgy	8	6	7	16		37
Food Processing	—	—	41	10		51
Polymer	17	—	—	—		17
Pharmas	—	—	42	3		45
Aero-space/Defense	—	—	13	—		13
Automotive	—	—	7	—		7
Unclassified	40	40	1045	26	450	1601
Total	1833	606	1438	125	450	4542
First App.	DMC:1985 IDCOM-M:1987 OPC:1987	PCT:1984 RMPCT:1991	IDCOM:1973 HECON:1986	1984	1985	
Linear App.	603 × 203	225 × 85	—	31 × 12	—	

(Ref.: Qin and Badgwell, 2003)

CDEEP
IIT Bombay
129

SIPTES
4/18/2012

State Feedback Control

So it in the survey of functional diagram paper where these are implemented, you can see that you have been implemented in refining petro chemicals, ball point paper, chlorine gas, utility, miming metallurgy, food processing aerospace and defense, applications at 2003 well as I am sure now there are exponentially grown. If it is this is changing with the engine speed of macro process and computing possible. But 10 years back you had 80 old computers or you can almost say that and what is the available now on your mobile probably you could be alone with desktop 10 to 12 years back. So things are moved very, very fast.

So you can because it is used in automatic applications or robotics. as I said, the latest thing I have heard about was using it on Google. Google is hiring people with okay, you want Google jobs you could go with Model Predictive Control method London they could like to hire you, you can do prediction models for you know hoe see you have to allocate resources okay, of computers through service to the customer demands.

And they are stochastic disturbances and you can develop a Time Schedule Module, do Predictions and forecasting and when you research allocation okay, so they have so many problems. Ultimately Predictive Control is not only for a success plant, it is for predicting where you can develop a model do forecasting for future horizon and then do allocation. So and then you implement your move only for let us say next 10 minutes to take call for relocating after 10 minutes.

So Predictions horizon could be you know some 2 hours or 3 hours into the future. You can develop a model on the fly data is coming you can adopt the model within time series approaches develop the model on the fly do predictions. What is the largest application of MPC? How many output control and how many input manipulated okay that is there in the Canada in one of favor in Canada largest one is 600 outputs.

(Refer Slide Time: 01:08:31)

Automation Lab
IIT Bombay

Linear MPC Applications (2003)

Area	Appt. Technology	Honeywell Hi-Spec	Aders ^a	Invents	SGS ^b	Total
Refining	1200	400	280	25	—	1905
Petrochemicals	450	80	—	20	—	550
Chemicals	100	20	3	21	—	144
Pulp and paper	18	50	—	—	—	68
Air & Gas	—	10	—	—	—	10
Utility	—	10	—	4	—	14
Mining/Metallurgy	8	6	7	16	—	37
Food Processing	—	—	41	10	—	51
Polymer	17	—	—	—	—	17
Pharmaceutical	—	—	42	3	—	45
Aerospace/Defense	—	—	13	—	—	13
Automotive	—	—	7	—	—	7
Unclassified	40	40	1045	26	450	1601
Total	1833	686	1438	125	450	4542
First App.	DMC:1985	PCT:1984	IDCOM:1973	—	—	—
	IDCOM-M:1987	RMPCT:1991	HECON:1984	1984	1985	—
	OPC:1987	—	—	—	—	—
Linear App.	602 × 283	225 × 95	—	31 × 12	—	—

(Ref.: Qin and Badgwell, 2003)

CDEEP
IIT Bombay

State Feedback Control

129

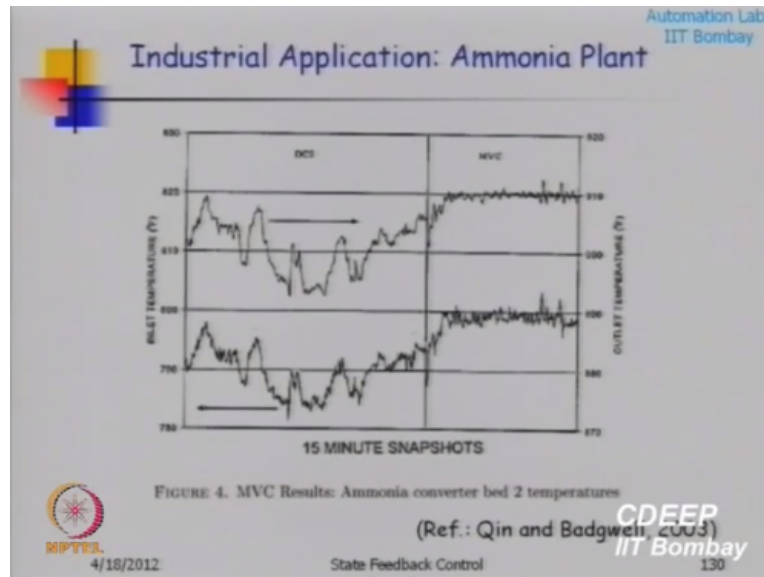
MPTER
4/18/2012

And 283 inputs, 603 outputs and 283 inputs. The huge controller well my cousin happen to work on the controller. He is detected from IBM and he works in a control company in Canada. So he was telling me that the model for this particular plant is like a book. If you want to look at the transfer function you have to scan a book, because it is a huge model. It is a transable model or whatever a model which is DMC they have solving developed a transfer functional kind of a model which is 600x283 inputs matrix and to look at the state response itself is a trouble and then to fix which part of the model is bad his work was to fix which part of the model is bad which is to forget off.

Then honey well application largest application they had implemented in 2003 was 225 inputs and 95 inputs and controllers with 30, 40 outputs and 20 inputs in common and now actually invents are using and NPC yourself of the self module micro pace case model where you can do 5 input and 5 output control. So basically you should know how to develop a model. The key

thing is if you develop a model, prediction model and you can get going multilevel controller. So this is 2000 period of time nine years I am sure.

(Refer Slide Time: 01:10:05)



We just they are not receiver of very closely guarded technology, not too much thing is available on the open literature because these controllers would cost in crores. These are not cast these are not cheap control. Implementing them maintaining them unique specialist who have done advance process control course and you have to have you know the cost of this controller is very high that has been one of the reticulum of this thing that you need a specialist, see a periodic controller now is need a specialist implement. But that is on the capability, so still that is a limitation which say limitation that you need but then we are in business because you need specialist to implement this and you can see here this figure speaks for itself this is before advance process control which means MPC on the implemented and after enter flowed node.


So these two control outputs are all over the place before it was implemented and is the industrial data. Okay and when the implemented MPC controller this control output is just hugging the set point then worry about what part of it reduce look this visually this figure communicates what it can do in a real industrial plant.

(Refer Slide Time: 01:11:23)

Automation Lab
IIT Bombay

Need for Nonlinear Control

- Linear prediction model based MPC: limits applicability to small regions around operating point
- Real systems are nonlinear: use of linear controllers can generate sub-optimal performance
- Nonlinear MPC
 - Need to achieve tight control of highly nonlinear systems
 - Control of time varying (batch / semi-batch) systems
 - Grade transition problems in polymer processing


 4/18/2012

 CDEEP
IIT Bombay
131

State Feedback Control

Okay when there are many, many things you need as we have develop using linear models okay linear models have limitations and then you can actually do everything using non linear models. So one can actually this linear models can be developed when there is planned which is operating at a point. Okay when there is something which is continuously in transition like an air craft? You cannot have one linear model will describe the dynamics. So you have to have modifications and there are several, several modifications.


(Refer Slide Time: 01:12:00)

Automation Lab
IIT Bombay

Models for Nonlinear MPC (NMPC)

First Principles / Phenomenological / Mechanistic / Grey Box

- Based on physics of the problem
 - Energy and material balances
 - Thermodynamic models
 - Conservation laws: conservation of charge
- Valid over wide operating range
- Provide insight in the internal working of systems
- Development and validation process: difficult and time consuming, requires a domain expert for development


 4/18/2012

 CDEEP
IIT Bombay
132

State Feedback Control

So there is you can use I use those stake place models to explain you can use non linear first principle models to do MPC. Okay and not again new they are already part of it has do it. At least 7 or 8 products in the market which actually sell MPC based on first principle models. Okay developing non linear models is a research problem and even the recently here we write finding from develop nonlinear identified models non linear black box models for some plants.

(Refer Slide Time: 01:12:34)

Automation Lab
IIT Bombay

Models for Nonlinear MPC

- **Data Driven / Black Box Models**
Dynamic models developed directly from *input-output data*
- **Model Forms**
 - Nonlinear Difference Equations
 - Artificial Neural Networks
- **Limitations**
 - Valid over limited operating range
 - Provide no insight into internal working of systems
- **Development process: much less time consuming and comparatively easy**

RIPTES
4/18/2012

State Feedback Control

CDEEP
IIT Bombay
133

So you can use data doing models you can use neural networks you can use all kinds of support ventral machines.

(Refer Slide Time: 01:12:45)

Automation Lab
IIT Bombay

Discrete Time Series Models

Dynamic Models: Given observed data


Set of past Inputs: $U^{(k)} = [u(1) \ u(2) \ \dots \ u(k)]$
Measured Outputs: $Y^{(k)} = [y(1) \ y(2) \ \dots \ y(k)]$

we are looking for relationship


$$y(k) = \Omega(U^{(k-1)}, Y^{(k-1)}, \theta) + e(k)$$

such that **noise** (residuals) $e(k)$ are as small as possible

$\theta \in \mathbb{R}^d$ represents parameter vector identified from data

 IPTER
4/18/2012

State Feedback Control

 CDEEP
IIT Bombay
134

So there is a lot of research on how to develop control the relevant MPC relevant models for I am just skipping this not going to too much in to deep.

(Refer Slide Time: 01:12:52)

Automation Lab
IIT Bombay

Black Box Models: Examples

Nonlinear Output Error (NOE)

$$x(k) = F[x(k-1), \dots, x(k-n), u(k-1), \dots, u(k-d)]$$

$$y(k) = x(k) + e(k)$$

Example : Dynamic model using Recurrent ANN
Volterra Series Models
Block Oriented Hammerstein and Wiener Models

Nonlinear ARX Model (NARX)

$$y(k) = F[y(k-1), \dots, y(k-n), u(k-1), \dots, u(k-d)] + e(k)$$

Examples : Dynamic model using Feed Forward ANN
Nonlinear State Observers with NARX structure

CDEEP
IIT Bombay

SPYEL
4/18/2012

State Feedback Control

135

So you can have generally a model which is current state is some non linear function of fast states and fast inputs. You can have a non linear ARX model you can have non linear BJ models there are all kinds of.

(Refer Slide Time: 01:13:07)

Automation Lab
IIT Bombay

Nonlinear MPC Formulation

Let $r(k+i|k)$, $i = 1, 2, \dots, P$ denote desired future setpoint trajectory
Then, defining future prediction error

$$e(k+i|k) = r(k+i|k) - \hat{y}(k+i|k)$$

MPC problem at the k 'th instant is formulated as a constrained nonlinear optimization problem


$$\min_{\{u(k|k), \dots, u(k+m_q-1|k)\}} \Psi\{e_r(k+1|k), \dots, e_r(k+p|k)\}$$

Subject to

- Model Prediction Equations
- Input Constraints
- Output Constraints

Variety of Control objectives can be specified.

CDEEP
IIT Bombay



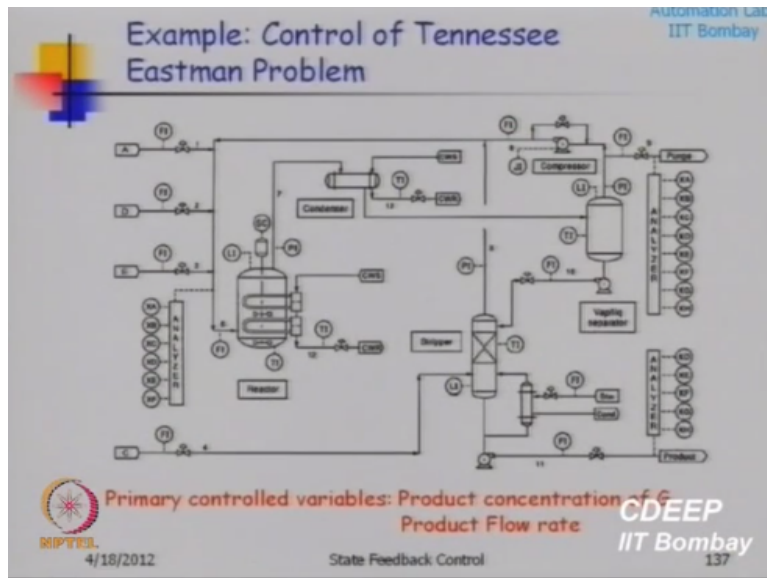
4/18/2012

State Feedback Control

138

And then of course MPC formulation is the same. You have optimization problem formulation subject to constraints on inputs, outputs stayed equations, modeling equations. It becomes a non linear optimization problem much more complex to solve and then how you can solve it in real time is a big problem. Okay so if you know your maths while you are in business you have must an example of this plant.

(Refer Slide Time: 1:13:37)



This plant again in a model equations are available if you right to them they will give you stimulate and then have included the k study of this controller which we have implemented one M tech student had implemented this controlling.

(Refer Slide Time: 1:13:44)

Automation Lab
IIT Bombay

TE Problem: Objective Function

Table 6. TE process: Weights on error and input rate in AMPC objective function

Controlled variable	Penalty on error	Manipulated Input	Penalty on input moves
		Reactor level setpoint	1
		Stripper level setpoint	1
		Separator level setpoint	1
		Reactor pressure setpoint	0.2
% G in product	5	F1	1
production rate, F11	5	F2	1
% B in purge	10	F4	1
% A in feed	2	F5	1
% E in feed	5	Reactor temperature	1
reactor pressure	5	Separator temperature	1

CDEEP
IIT Bombay

State Feedback Control

138

This thus 6 outputs and 10 inputs to be simultaneously control and the problem is for example top move from one what they call as one product grade to other product grade. Okay so there is some it is a product called G which you do not know what G is? Not know what it is?,

(Refer Slide Time: 1:14:08)

Automation Lab
IIT Bombay

TE Problem: Operating Constraints

Table 7. TE process: Input and input rate constraints in AMPC objective function

Manipulated variable	Input constraints	Input rate Constraints
Reactor level sp (%)	10 to 90	-5 to 2
Stripper level sp (%)	10 to 90	-0.2 to 0.2
Separator level sp (%)	10 to 90	-0.2 to 0.2
Reactor pressure sp (kPa)	2650 to 2850	-0.2 to 0.2
F1 (kmol/h)	0.1 to 46	-0.1 to 0.1
F2 (kmol/h)	1 to 181	-1 to 1
F4 (kmol/h)	1 to 681	-2 to 2
F8 (kmol/h)	550 to 1460	-2 to 2
Reactor temperature (°C)	115 to 128	-0.01 to 0.01
Separator temperature (°C)	75 to 88	-0.01 to 0.01

CDEEP
IIT Bombay

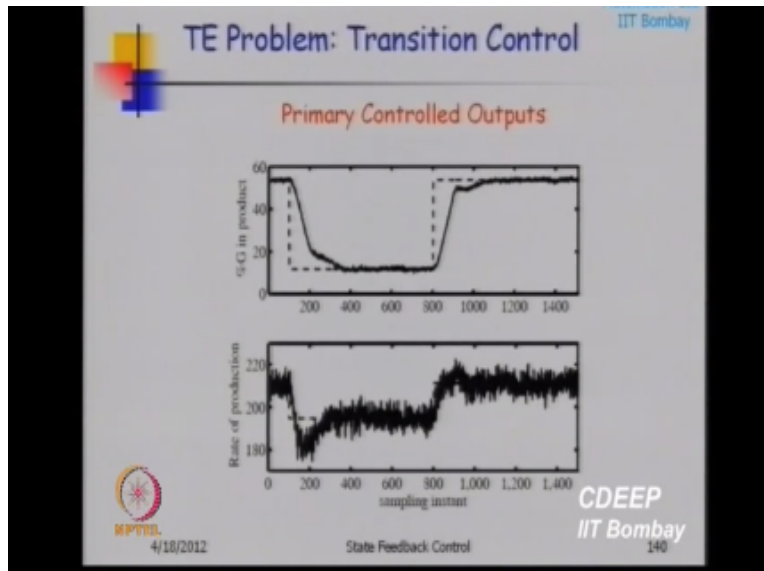
MPTEL
4/18/2012

State Feedback Control

139

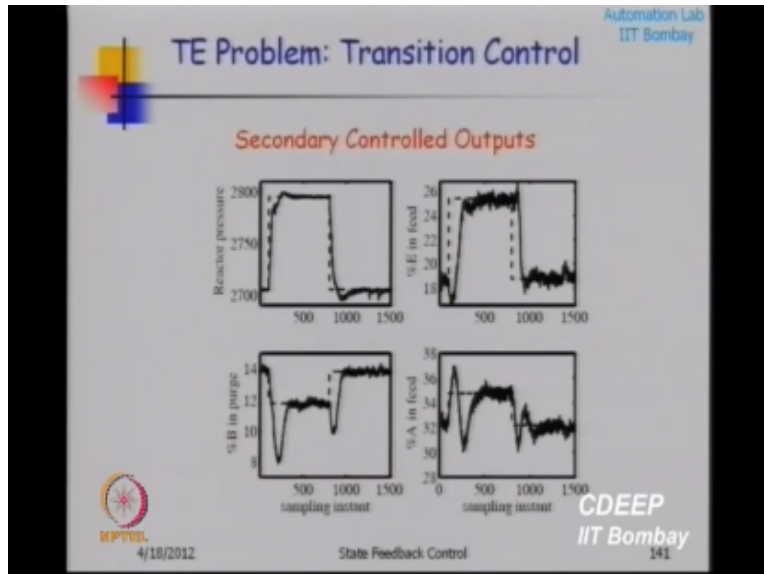
But and then they have given the constrains how you know output should be constrain? How input should be containing? What are the bounds? Everything is given. So this is a problem defined like a bethsprtrum you have a new way of solving MPC you implement on this and show that it works.

(Refer Slide Time: 1:14:24)



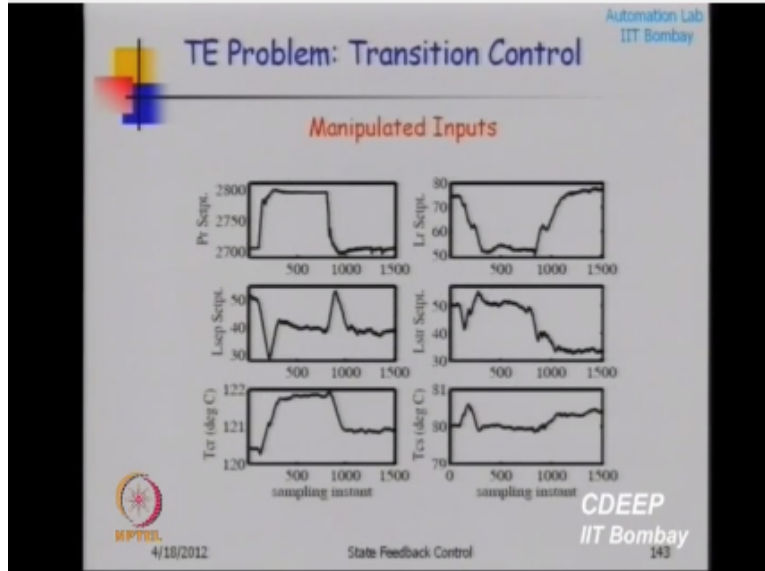
So we are taking it from 50% product split of G to 10% products to be a major transition of the system from one operating condition to the other operating condition and we are able to manage it to using our redeveloped this entire M Tech student developed. So you are going from certain product purity to certain other product purity certain product trade to certain another product trade and so on and.

(Refer Slide Time: 1:14:53)



This has done by simultaneously moving all the set points. In the new points all the ten input simultaneously. So subject to the all those constrain rates and all that. This is using some time varying non linear models we have developed.

(Refer Slide Time: 1:15:06)



And so.

(Refer Slide Time: 1:15:07)

Automation Lab
IIT Bombay

Model for Predictive Control

Model Type	Origin	Linear/Nonlinear	Stable/unstable
PDE, ODE	Physics	L, NL	S, U
State Space	Physics, Data	L, NL	S, U
Transfer Function	Physics, Data	LTI	S, U
ARX, ARMAX	Physics, Data	L	S, U
Convolution (Impulse, Step)	Data	LTI	Stable
Neural Network	Data	L, NL	S, U

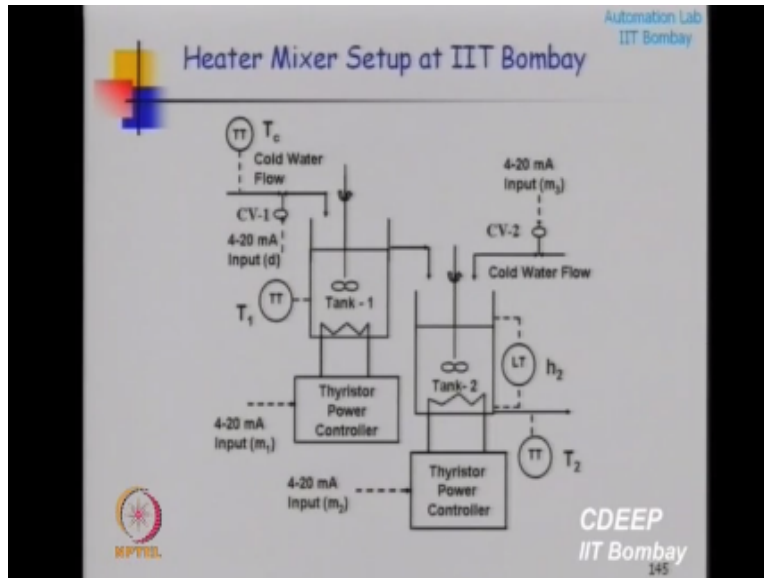
CDEEP
IIT Bombay
144

NPTEL
4/18/2012

State Feedback Control

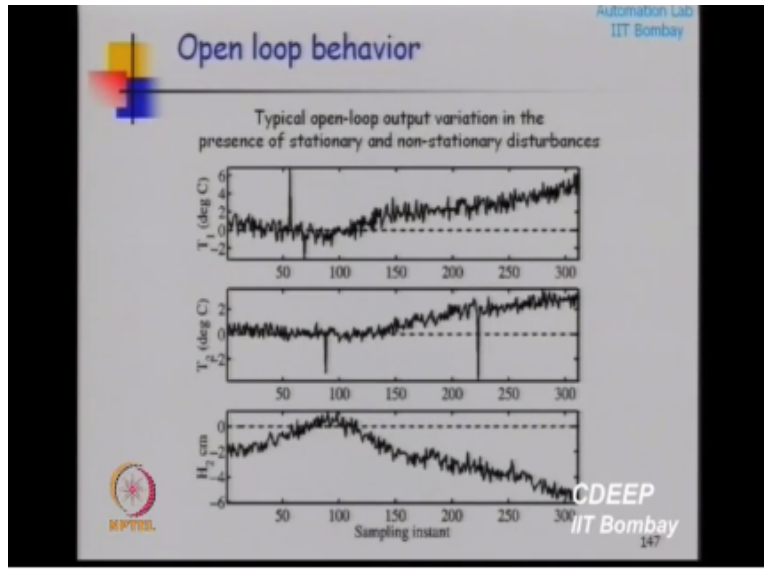
All you can use MPC using all kinds of models you can use data driven models you can use mixture of mechanistic models and it adverb models. You can use only mechanistic models depending upon what kind of you know what level of confidence you have in your model and so what I just described is just keep of the just is expired to work I just showed you visually.

(Refer Slide Time: 1:15:33)



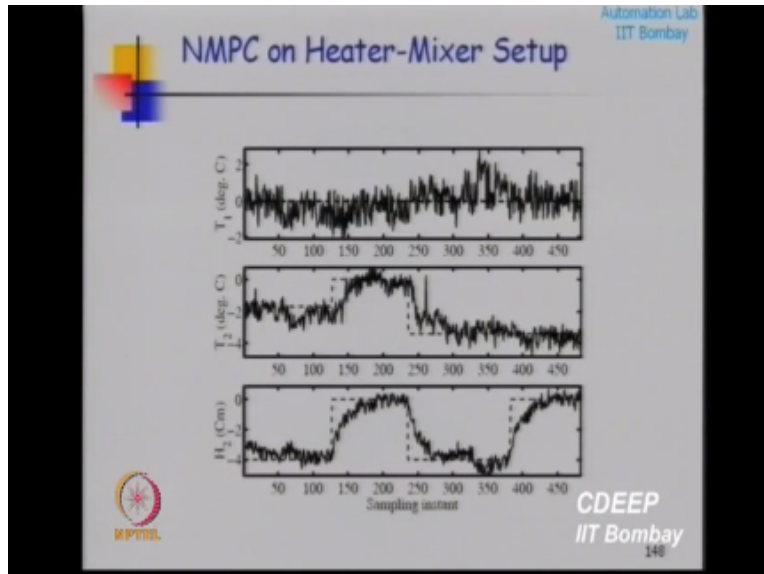
What different it can make. So this is an expanded state of which some of you are simulating right now in the assignment. I have two heat inputs has my manipulate inputs and one slow rate cold water flow rate and mixing hot water and cold water here. I want to control temperature from these two times and level here. So there are three control outputs, four manipulate inputs and I want to so there are two disturbances what happens this visually see that.

(Refer Slide Time: 1:16:07)



If I do not put a controller the temperature is obtained will open the behavior the level the temperature is changing here alone by 6 degrees here by 2 degrees in the second time and the level change in by 6 cm. this is a time of 15 cms. 6 cm they will drift with huge drift of this particular time.

(Refer Slide Time: 1:16:33)



Okay let put MPC I can just control it in within (+) or (-) 0.5. This is a non linear MPC implementation actually o the lab. So again part of were you are developing some non linear time series models to show that how they can be make to work on the real setup. So these are inputs and outputs.

(Refer Slide Time: 1:16:55)

Automation Lab
IIT Bombay

Nonlinear MPC: Vendors

Company	Product name	Description
Adersa	PFC	Predictive functional control
Aspen Tech	Aspen Target	Nonlinear MPC package
Continental Controls, Inc.	MVC	Multivariable control
DOT Products	NOVA-NLC	NOVA nonlinear controller
Pavilion Technologies	Process Perfecter	Nonlinear control

(Ref.: Qin and Badgwell, *CDEEP*)

IIT Bombay

State Feedback Control

151

MPTEL
4/18/2012

There are products non linear modified decontrol there are already product s in the markets last ten to fifteen years and those are the major companies in to who are already in to linear model base predictive control also in to non linear model effective categories. So these are again 2003survey and an low survey has come resign please so this is sure this much more work now many more commercials are let us to recheck to me.

(Refer Slide Time: 1:17:26)

IIT Bombay

NMPC: Applications (2003)

Area	Adress	Open Technology	Commercial Controls	DOT Products	Patent Technologies	Total
Air and Gas			18			18
Chemicals	2		15		5	22
Food Processing					9	9
Polymers		1		5	15	21
Poly & Paper					1	1
Refining					13	13
Utilities		5	2			7
Unclassified	1		1			2
Total	3	6	36	5	40	91

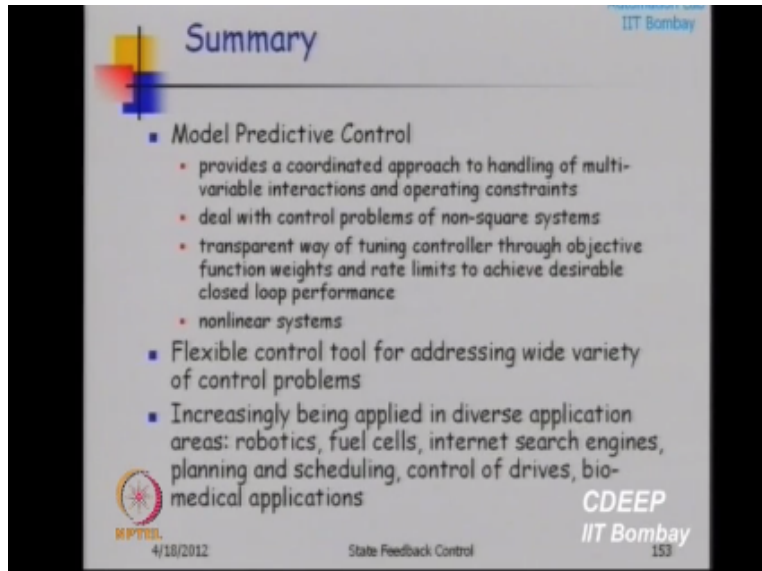
(Ref.: Qin and Badgwell) **CDEEP**
IIT Bombay

State Feedback Control 152

NPTEL
4/18/2012

That time these are the actual implementations of non linear MPC now it has go exponential.

(Refer Slide Time: 1:17:33)



Summary

- Model Predictive Control
 - provides a coordinated approach to handling of multi-variable interactions and operating constraints
 - deal with control problems of non-square systems
 - transparent way of tuning controller through objective function weights and rate limits to achieve desirable closed loop performance
 - nonlinear systems
- Flexible control tool for addressing wide variety of control problems
- Increasingly being applied in diverse application areas: robotics, fuel cells, internet search engines, planning and scheduling, control of drives, bio-medical applications

CDEEP
IIT Bombay
153

MPTBL
4/18/2012
State Feedback Control

So this is a very flexible control scheme. Okay one of the most potent and well one of the schemes which has which is the major commercial success. Okay so no other controller multi variable non linear or linear controller which has spread so much. So now of course in control all the control journals will always there always paper on only on model predictive control. The special workshops of number of books which are come out on this and the main thing is that if you know this you are in for a very good job because doing this knowing.

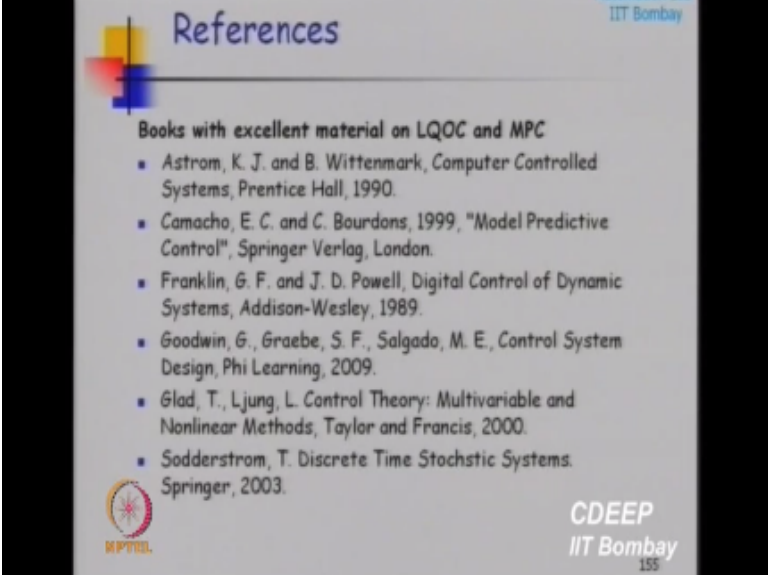
This technology is there are many current research directions you can work on how you model unmeasured disturbances you can work robustness you can work on fall diagnosis. People have use this MPC now for all kind of things. They are using it for scheduling, planning. Scheduling planning of you know to deal with the market conditions. Okay you want to plan the production over next to when horizon planning conventionally was they were control the engineers job with MPC.

You know you have moving horizon idea you have a prediction model you are in for a business for and you are decisions are what to produce were the manipulated inputs concept has to change under time slots you know how much you produce for which time were times all. There are huge applications on those who like embedded control work how to embedded MPC on a chip. How to embed a state estimated on a chip. Okay all these things are very fast in MPC.

So light maths how do you make and non linear optimization program which can very quickly solve some thing on mind. So each one of them actually has help me to define PhD problems. So

some body is worked on fast and MPCs somebody has work from disturbance modeling. So I am just listing here my PhD problems.

(Refer Slide Time: 1:19:55)



References IIT Bombay

Books with excellent material on LQOC and MPC

- Astrom, K. J. and B. Wittenmark, *Computer Controlled Systems*, Prentice Hall, 1990.
- Camacho, E. C. and C. Bourdons, 1999, "Model Predictive Control", Springer Verlag, London.
- Franklin, G. F. and J. D. Powell, *Digital Control of Dynamic Systems*, Addison-Wesley, 1989.
- Goodwin, G., Graebe, S. F., Salgado, M. E., *Control System Design*, Phi Learning, 2009.
- Glad, T., Ljung, L. *Control Theory: Multivariable and Nonlinear Methods*, Taylor and Francis, 2000.
- Soderstrom, T. *Discrete Time Stochastic Systems*. Springer, 2003.

NPTEL CDEEP IIT Bombay 155

And I have given lot of results here there are excellent books which give you exposure to MPC and also linear quadratic optimal control.


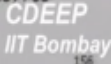
(Refer Slide Time: 1:20:07)

Automation Lab
IIT Bombay

References

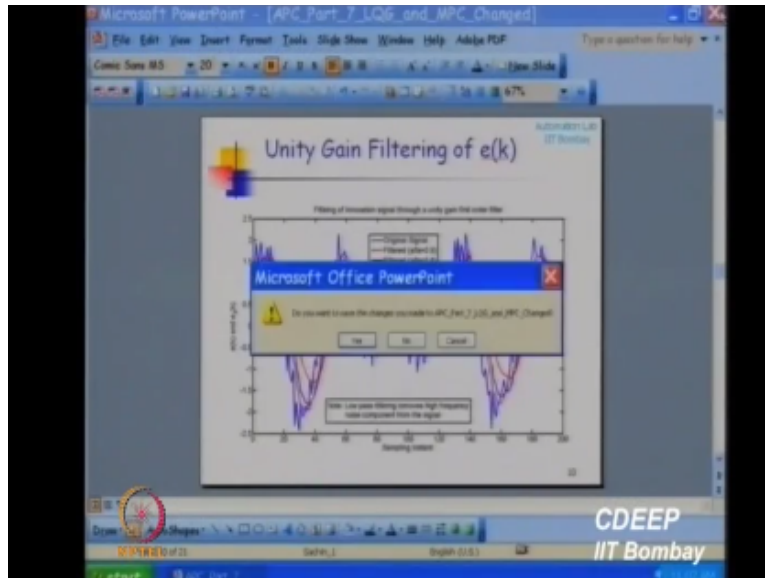
MPC and Related Important Review Articles

- Garcia, C. E., Prett, D. M., Morari, M. Model predictive control: Theory and practice - A survey. *Automatica*, 25 (1989), 335-348.
- Morari, M., Lee, J.H. Model Predictive Control: Past, Present and Future, *Comp. Chem. Engg.*, 23 (1999), 667-682.
- Henson, M.A. (1998). Nonlinear Model Predictive Control: Current status and future directions. *Computers and Chemical Engg.* 23, 187- 202.
- Lee, J.H. (1998). Modeling and Identification for Nonlinear Model Predictive control: Requirements present status and future needs. *International Symposium on Nonlinear Model Predictive control*, Ascona, Switzerland.
- Meadows, E.S., Rawlings, J. B. Nonlinear Process Control. (M.A. Henson and D.E. Seborg (eds.), New Jersey: Prentice Hall, Chapter 5, (1997).
- Qin, S.J., Badgwell, T.A. A survey of industrial model predictive control technology, *Control Engineering Practice* 11 (2003) 733-764.

 
156

These are some of the nice articles which have appeared and which can help you get going if you want to go to working into this direction.

(Refer Slide Time: 1:20:20)



So within that let me close these lectures.

(Refer Slide Time: 1:20:27)



Whole very frankly I have come this course four, five times, but this batch I really enjoy teaching you guys and you girls not just. So it was fun because you kept asking questions and help me to change my notes all the time, particularly thanks to Venkatesh and sourav who kept bug in me all the time. So it is good to have a class with this so responsive. So I had fun and I hope you also learned something.

So main thing is that do this assignments more than those exam problems which are not going to be more than two cross two matrices. Okay so I cannot do in the exam cannot give you bit, at the most I will give you some free but you will learn only when you actually dirt your hands in programming. Okay now some of those programs you are never developed. So that is why I have put my programs for your reference.

Okay so what you complex other thing you should did that for when you are developing this control scheme in simulation or in reality. You should never attend to do a gram integration divide and root. Okay take one component tested separately. Okay then take 1+2 tested separately. Test them together like that. So first develop observer the open loop no controller. Okay then develop a controller exchange with perfect state feedback.

No observed states. Then and tested with linear plant stipulation if it is working then you go to the observer based on the linear plants simulation. So one by one by one you should relax even now after working in this area for so many years if you have to start the new thing I start if there is no way you can test your program by a grand integration of something would take each

component tested separately when you know integrate. That what you would do if you have to do hardware project same thing you have to do here.

Okay you any software when you are developing a test each component separately and then small integrate them in to a bigger problem. Okay then you know where you are going wrong otherwise it is will become very difficult to just look at the notes and said I am right to do write this one program. It never works. Okay so now just to find up this is my prospective of advance process control. I think most of the control books are closes start by assuring that several of models already. I do not think that is correct.

I think it start from data okay and then come up with the control regard them. So from data to model to observer to controller. Okay so even though the development here all of it for MPC is based on it looks like it based on the linearized first principle model how to do it using identified model I have already uploaded my notes yesterday of how to implement LQG is that identified models. Okay same thing would go to for MPC using identified models. Okay may be I will add that one to ten for it.

So with that you know a complete view point from data to control. Okay and if you go somewhere and happen to implement this you should be in business from okay. So thanks for your nice interactions and hope we will meet again after the exam. I am going to organize this lecture by about real time implementations of MPC. It might be on his convenience could be on a Saturday. So it could be on fifth, fifth to the Saturday I think. So I will ask him and then organize I will be in the department have you all know to record it okay.

**NPTEL
NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED LERNING**

**NPTEL
Principal Investigator
IIT Bombay
Prof. R. K. Shevgaonkar
Prof. A. N. Chandorkar**

**Producer
Arun Kalwankar**

Project Manager
M. Sangeeta Shrivastava

Sr. Cameraman
Tarun Negi

Sr. online Cameraman
Sandeep Jadhav

Digital Video Editor
Tushar Deshpande

Technical Assisants
Vijay Kedare
Ravi paswan

Copyright
CBEEP IIT Bombay