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ADVANCE PROCESS CONTROL

Prof. Sachin Patwardhan

Department of Chemical Engineering,
IIT Bombay

Lecture No - 25

LQG and MPC

Sub-Topics

Model Predictive Control (MPC)

So in the last lecture we will look at linear quadratic optimal control and I also talked about a realistic combination before I move on to the last topic of this starting lectures for this model where it control I want to go or this realistic LQC formulation again this is our systematically then what I did in the last lecture, so that you understand the foundation.

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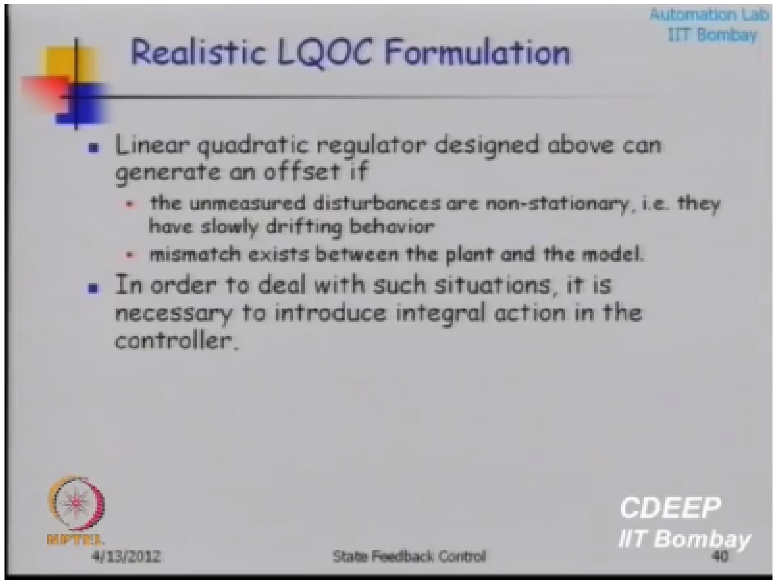

Why we modified the control law okay and its feedback control in the last lecture I founded difficult to explain so I went back and we derived and this derivations such are given in for the manual from the innovation bias approach are known actually we need book, so that there at we vary some of those ideas where there in different form that this difficult formulation that is given here in so w have two problems one problem was w had to deal with disturbances we have to deal with disturbances right now.

At the stage let me now qualify linear measured or unmeasured that means the disturbances okay we looked at the system you need to started doing linear quadratic optimal control that means remains you we just look at this model right, we just look at a model in which there are no disturbance there are no measurement error only problem that you wanted to looked at want to bring the systems on non zero initial state to final state is equal to 0 0 origin system and the system are different.

And then why we have done this I mean once you have done the whole derivation and com to the realistic formulation we realize that we have done this because we order to come up with the formula for coming g∞ controller gain okay, once you get the controller gain you can modify to do other things for example disturbance equation the quadratic all those things you can do for the modification, so now we derived the controller for this simplistic looking model okay now what I want to show.

That even when you have disturbances okay when you have disturbances or if you want to take it some arbitrary set points okay you can transform the model to a model which looks very, very similar to this you can done that then you know actually what I want to do rejecting disturbances and moving to any arbitrary set point that is okay.

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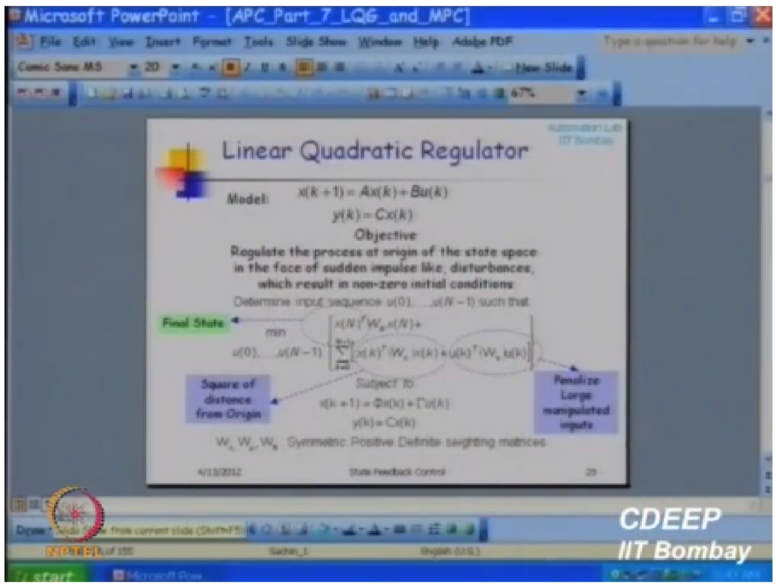

So I will move directly to and then well let me just show this on side again and then we wanted to move we order to design with the LQ controller which moves to the origin the way we did this work we took a finite where have in the formulation here and then inlet and get the infinity which is the derivation that it will finite variation formulation let m go to infinity and then you found your solutions okay so we go to our more realistic formulation okay so.

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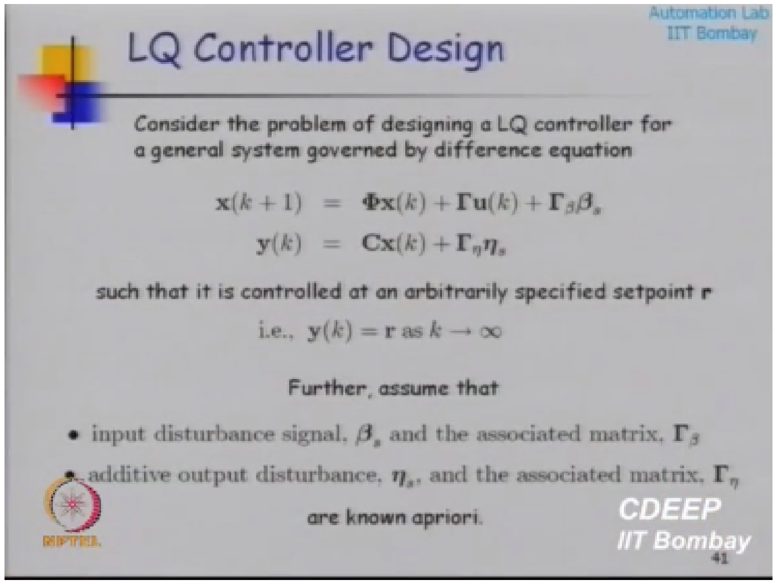

Let me start here problem of designing your linear quadratic optimal control okay for us system which is governed by this difference equations okay and I linked two terms here one term is a constant disturbance in the space and a correspondence supplement okay why I have use some different limitations here then eventually it become here okay I could have used γd you were using γd here in some cases you know the disturbance then you can use γ β = γd in some case we do not have disturbances.

Then what you do so always assumed here so right now I am keeping it leaving it for we and I am not saying like this is the measured disturbance or unmeasured disturbance or whatever I am using it okay similarly I am assuming that there is some adequate disturbance in the output there are two disturbances in the system one is the input disturbance and other one is output disturbance right now I am assuming them to be constant not time varying that change like okay I want to control the system I want to control this system.

At an arbitrary set point are okay right now if I assume the constant itself okay let us assume the constant set point is it clear now I am taking a more realistic formulation there are disturbance in the state which is constant where the disturbance in the measurement which is constant and I want to set the systems of set point are I say this thing to it I want to reach a fit point or your arbitrary set point where as looking at derivation variables like I say defeated if the copy to time set up.

Then 00 corresponds to the initiate solution you write and you might want to take it to some of the fit point which is 00 y = you know – 4cm and 3cm rather that to be so I should be able to specify any state point I should be able to reject disturbances that us my ultimate goal of any controller design and then I simplified the design for a very simplistic system where there are no disturbances and I will say directly control at the origin, so how do a tide is to together to actually what really I want to do okay.

So let us assume that somehow at this stage okay we know this we know this signal βx we know this Lβ we also go this signal βx and we know this Lβ where I should have see is like error.

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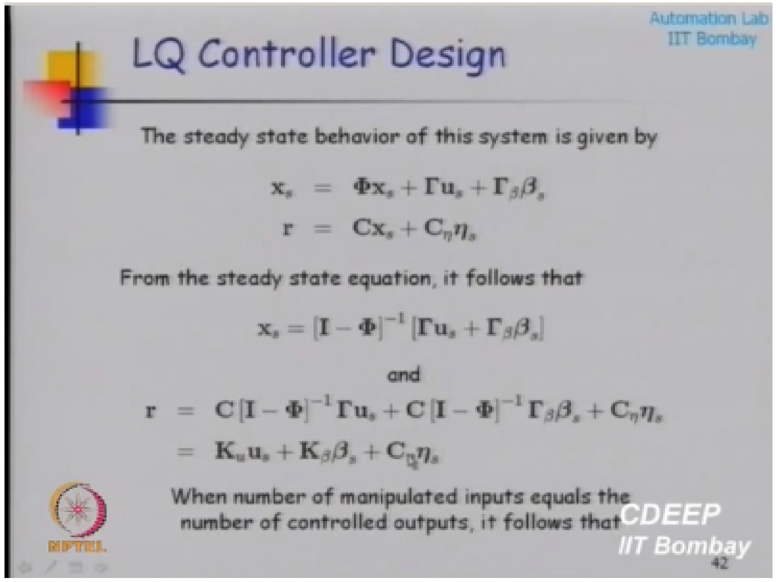

Slight error in the is the type of here this should be η so because it is C I have called this C subsequently and because this is γ and I call this γ is like that and here to wee just correct this series of oky so now what happens is what is the steady state of the particular equation this is the solution equations okay at the steady state okay x = ϕX into γU is γ β into β it is the steady state input disturbance okay this is the steady state output disturbance which is my steady state equation okay.

Now I can from the steady state first equation okay from the first equation it follows that you know I just said this ϕs in the left hand side I get r – 5x r – 5x in this is it clear from the first equation I and then I can going to me β is going to be this matrix is going to be all the matrices here I am going to okay r is given to the set point is given to me I want to set the point so appraisal give me a set point okay two set just we set the point apart u is the control engineering set point I will go reach into that particular point okay.

Okay you have given the set point so what is not non here is input us okay that should be maintained to achieve this set point in the phase of these disturbances okay that can be computed now, so I am just what I am doing to here is this particular complex matrix C into r - ϕ γ I – ϕ inverse into γ I am going to call it as Ku this is the rotation simplified rotation same thing is for this particular matrix I am going to call it as Kβ okay and then with Cη remain here okay

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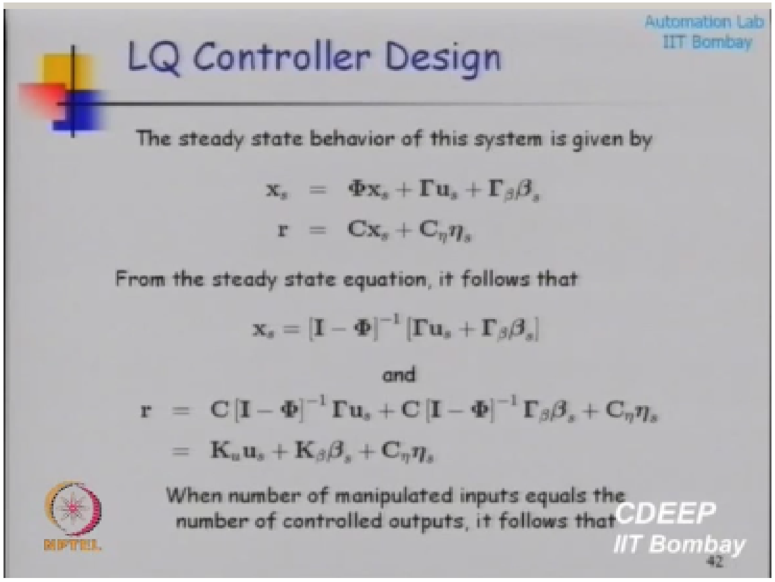

So when number of inputs equal to number of outputs that is number of controlled outputs is equal to number of manipulate variables this Ku matrix okay with gain matrix to this is a steady state gain matrix if you realize this is the steady state gain matrix this steady state gin matrix in this square and of the system is controller the particular system will be gain matrix to the invertible we should be able to go to the system to whatever finite steady state that you know okay this Ku will be invertible matrix in which case you can write Us to be in this substitute that we will get X_s.

Okay if you happen to somehow know the disturbances and if you happen to know the set point then you can find out and steady state that would be reached to achieve those set points and to reject the disturbance okay in slight over disturbances if I maintain this Us I should reach this x as maintaining this Xs implies I have reached suppose I wanted to be write I wanted to be at y = r at state okay.

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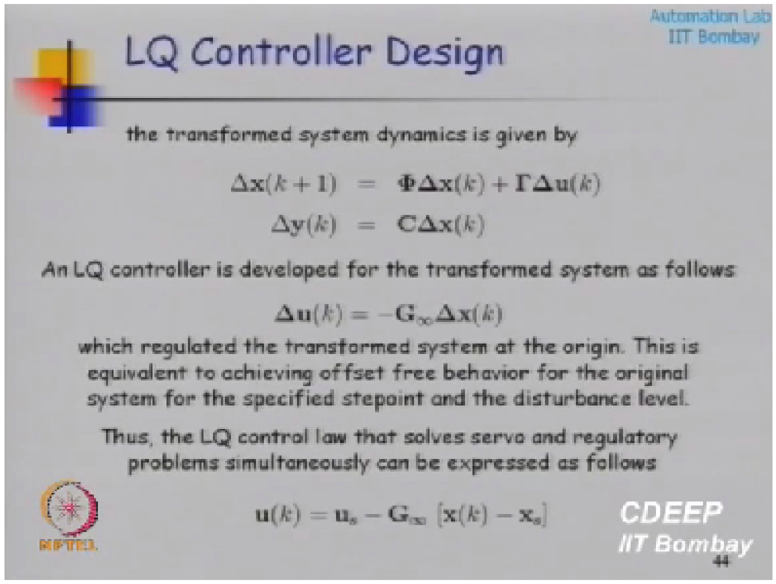

So if I am maintain this Us then I going to reach Xs if I reach Xs I would reach r that is what I want it I wanted to reach the fit point r if I just do this and reach the fit r and this write me on this okay and of course is Kue not inverted in if there is number of inputs are goes in number of outputs then you can already used and that is not a problem okay now what I am going to do is I am going to subtract this module I am going to subtract this model from the original model that I consider that is this model here.

There of course replace this by C, C here so this dynamic model and this steady models I am going to subtract okay I am going to subtract these two models once I subtract these two models I get this model the disturbance constant disturbance can disappear okay constant is it clear tricks that I am doing now I am subtracted the dynamic model and the steady models from the dynamic model, okay so what I got is let us called this delta model okay just called the delta model because this delta model.

So this delta model is xk + y – x is now this model this model looks like the model for which they have to developed LQ controller okay this model looks like so I can define its Δxk I can define Δuk I can define Δyk if you just translation of the model which you realized you are just translating model okay we are changing the origin that is all okay, now what is the meaning of controlling loops model at the origin if you control this model of the origin then that means xk = xs with xk = xs and uk = us okay that means you have achieved the set point of the desired set point in spite of the disturbances.

In the output and in that state okay so my controller design is done for this particular model and then you got this controller okay in times of Δ you got the controller which was the controller which is LQ controller okay your chosen W_x matrix and Wg matrix all that is you done and then you get this controller and then so origin of this particular sustain correspond to the desired set point okay when I am implement this control law I implemented like this okay and implement the control where I implemented like this okay we just clear so basically what I am going to th next.

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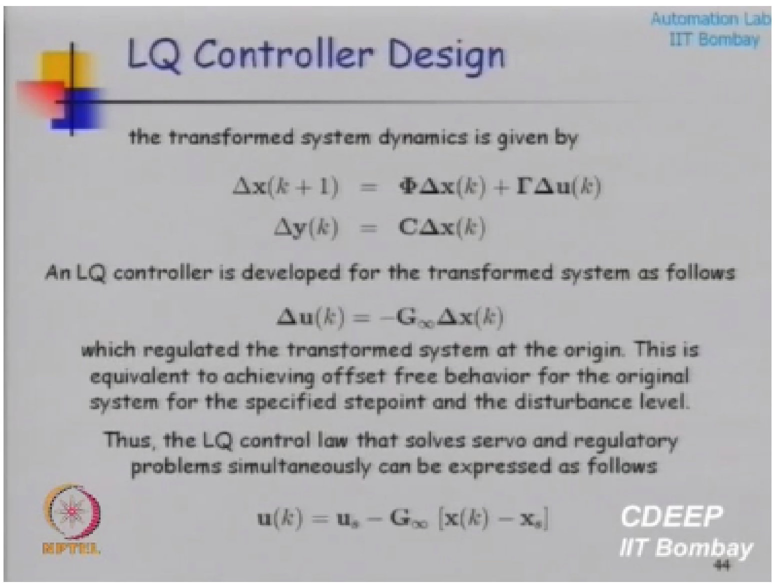

Is I am going to change my controller like this now there is a transition from the previous slide to this slide see here I said this is xs and us these are not time varying okay there are not time varying because I have made two simplistic assumption time simplistic assumption well disturbance levels are constant, but disturbances can be drifting okay disturbances can be drifting second I had made an assumption but the set point given is constant but you may have a set point as it 3 these are example if you are controlling.

A near clock and you want to take off okay then you want to go from height nearer to height what are there to and we have given some set point projector where you should go like this so pilot will give a set point arbitrary to the auto project and then outer part will through the aircraft from the given level to the higher level okay, when the situation comes from a set point it is constant in the sets of in aircraft cruising at a constant height.

Okay this set point is constant then you only have to worry about a disturbances like wind gases many of you are travelled in aircraft went to suddenly you will get from wind so stand and then aircraft is not checking then you need a controller which will and this is typically done by the automatic controller I am going to auto pilot will be un moved computer doing LQ controller predictive controller about this, so the pilot does not do this within their lead to the controller that capitalization at so the disturbances could be times varying the set time could be time varying so I am trying to know modify this control to.

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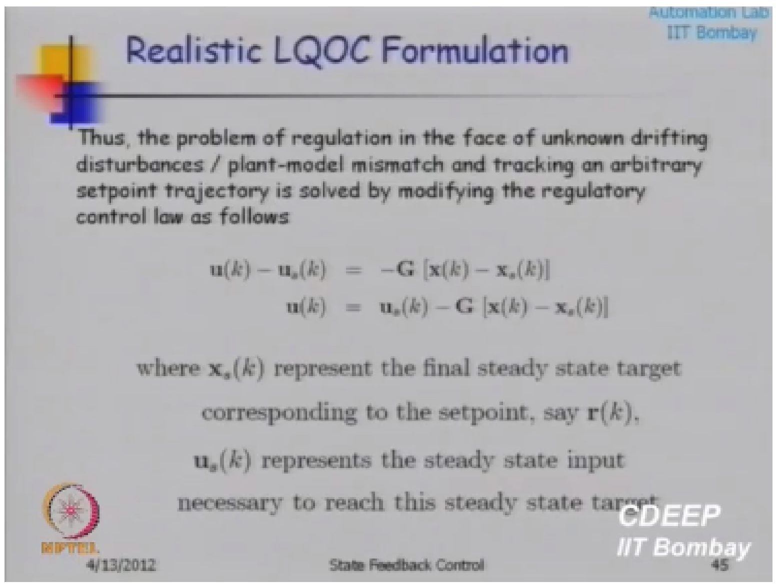

Accumulate both the possibilities so I am showing that this is xs and uy so that translation that I do also need to be time varying so the disturbance of the time varying and the set point of time varying in the translation of the linear system model that I need to do we also trying it, that is the logic which is in flow to you know deal with any kind of disturbances deal with set point track, okay so I am now talking about rk, rk is a set point trajectory it is changing at E time instant okay so here xs and us are the steady state that are estimated both in the current set point at time k.

And the current level of disturbance disturbances can changing in future I will adopt to that okay so this is the basic idea, so what is done in which innovation bias approach.

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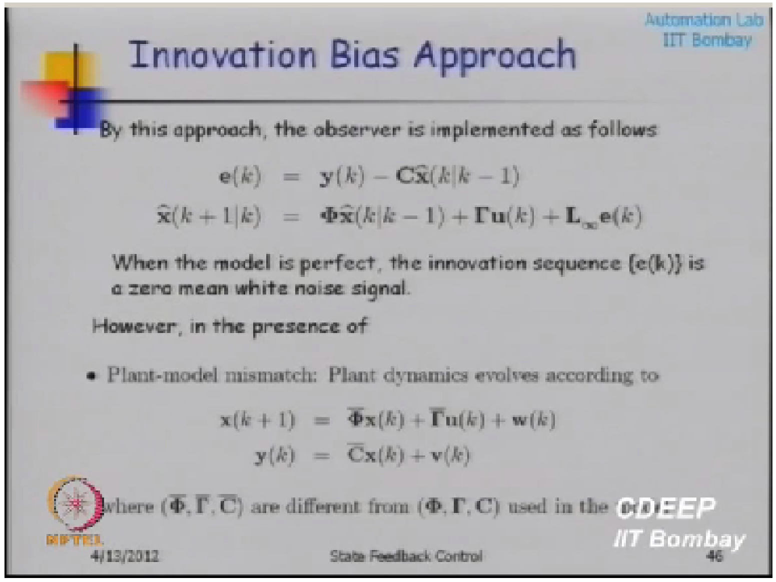

Is that I have this observer which is developed from this observer with I have here I have taken selections points the reason for taking selection form of course I said this simplicity in derivation but also a prediction form of this time can be arrived two times is modeling these are developed box in this model so you have to develop box model developed a equivalent state with realization will get this directly you will get case directly okay I use it directly that is what you get we go back and look at your notes.

And you will see that I made this connection there actually time selection model is nothing but it Carmen filter okay so have this observer this observer could be divide from 4 principles then you know riccati equation and this it could be derived directly from data box doing this model or model k realization we get this model okay does not matter, now what I said is that I do not have disturbance model right now, let us assume that there are all unmeasured disturbances okay if there are really unmeasured disturbances where really white now.

Then this we know that ek will be avoid okay but I showed you that if ek is not going to do white noise then the disturbances in the input sorry our stepwise or you know drifting disturbances then I am going to ek okay or if there is a model plan miss match ek is going to be right noise okay.

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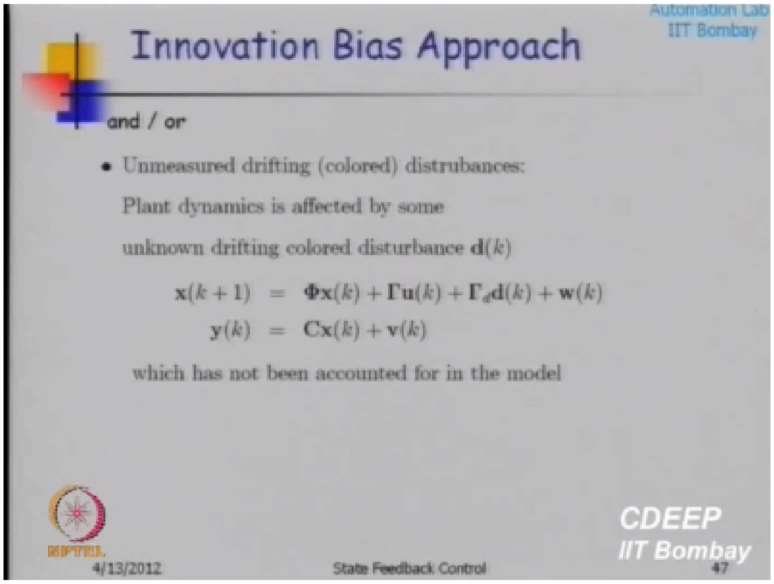

So what we do in such case is fallen I talk about this filtering of the disturbances so what I am going to know what I am going to now just okay let me go go back go back to our what I am going to now I am going to take this ek, ek to be same as β which I used earlier remember I use that β and γ β okay I am going to take this L∞ and ek to be represented you a disturbance in the states okay and L∞ to the corresponding coupling matrix okay see here what I needed relevant here.

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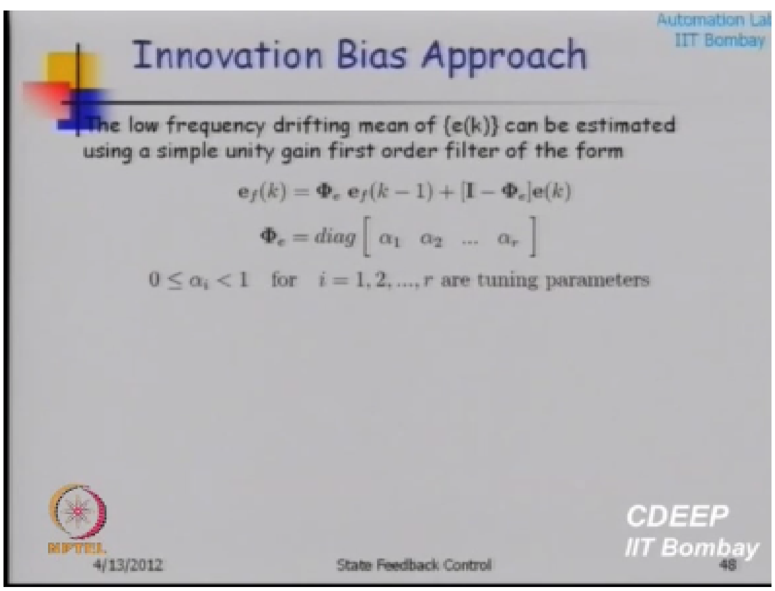

I needed this term γ β and β okay what I am going to here is I am going to pretend but this L∞ is nothing but γ β and ek is greater than that you have a unmeasured disturbance in fact drifting okay, so of course this unmeasured disturbances to components one a slow components other than in your fast components I want to knock out the all fast component I m going to look at the low drift low drift cigarette which I am going to do using this filtering levels I am going to filter this innovation.

Error and then filtered value I am going to representative of the unknown systems and then what then I am going to.

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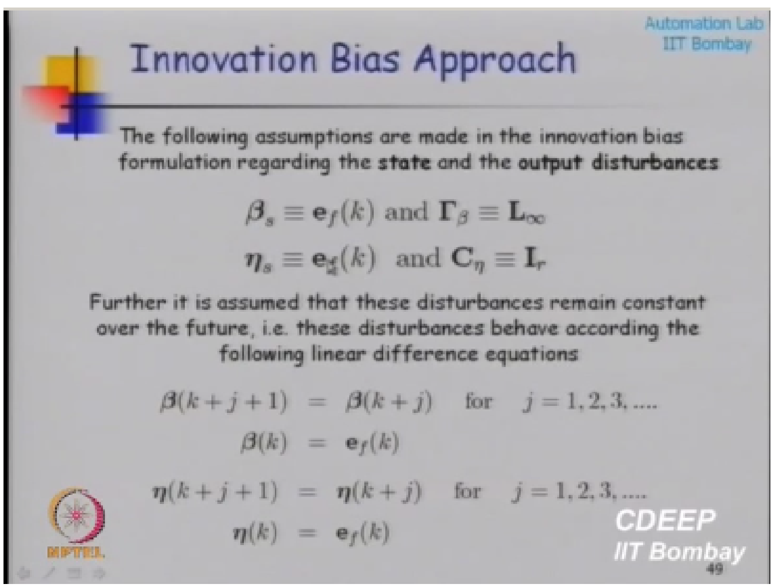

And then I am going to just do this mapping I just do this mapping okay I say that my βs is drifting that efk okay β γ or is nothing but L∞ okay then I will say that βs is nothing but efk and seen it has just use to identity matrix okay movement I do this mapping when I can use all that I have done earlier but I have to make one more assumption here okay what is the assumption I have to make the assumption that the disturbances do not change in future we know things about it inflated depressing here.

Then I make an assumptions that disturbance at whatever is a current filtered value of the innovations that is not going to change in the future that removes constant it is like a step it is like a step that is all I am going to assume right now okay, just we simply the assumption to add or this I am so looking at it adding as in integrator into the questions but that in formally realized later that I am actually making an assumption but this β is actually remaining constant over the variance or from near to ∞ okay.

I am adding the simplifying assumptions here so same things which to about output disturbance and I am saying that there is a constant of disturbance your constant input disturbance and both of them can be approximated using with filter value okay even the making the simplify assumptions okay then.

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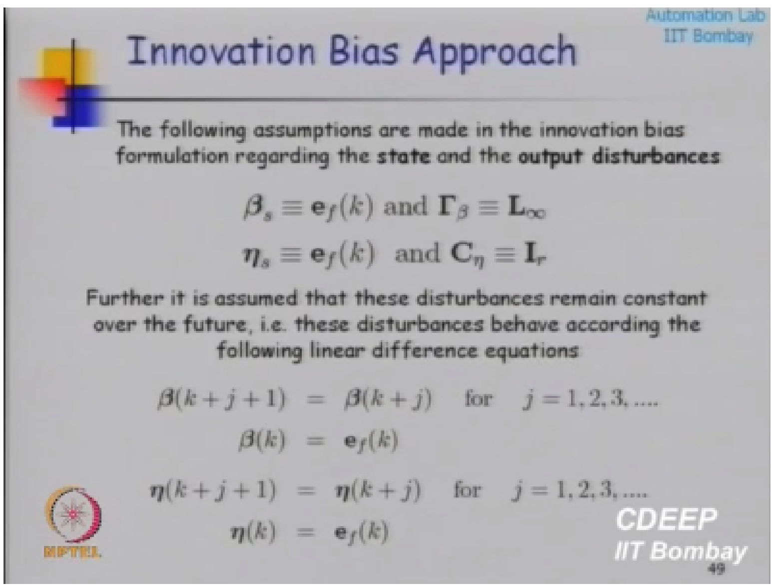

My control can be you know you just substitute here for γ β and C β and then β and β is you substitute those values will take the control okay this particular control law can deal with drifting disturbances it can deal with changing set points is that okay, so that the trick to arrive at this goes to shift the origin okay in fact shift the origin continuously on the fly okay we will design okay and then try to use the law that was derived to control at the origin to deal with the sustain which is actually changing.

Which are changing disturbances or to track the set point okay so the trick all these things are possible up to because you are already with linear systems you can do all kinds of manipulation okay liner algebra as I just to all do all kinds of nice things and that is why I ma going to you know design the controller only for a simplified model and then complicated the model and use the same controller all these things are possible become linear okay the real world of course is not linear non linear.

But when the approximations of linearity hold you can use this manipulations and implement this control okay, yeah which is it yeah, so actually I have taken one shortcut her because I have assume that by I am just trying to give you analogy and then this one more depressing here because you are using observer and a plan together and then that the real derivation would be more complex from this if I do showed the analogy that we might I just actually that there is model which I had $\beta k = L\infty$ and if you going to assume this and then I get the same controller.

That is well okay so to do what that you show through it gives the offset to everywhere I should consider the plant which is differentiate this model observer which is exact and then so that how offsetting is removed, but which will I doing that I am taking the kind of that was parallel okay so hope that you will understand it and then you actual derivation from these are both together in between or complex and we got more algebra or it can be derived and that is not a I am using better form and 1.

Liner to do one of the variations I am going to write two equations every time for a observer so I can just do with only one single equations right okay, so let us now move to predictive control which is the till that I have been working to teach work is that I want okay so as I said in my last lecture arbitrary control is.

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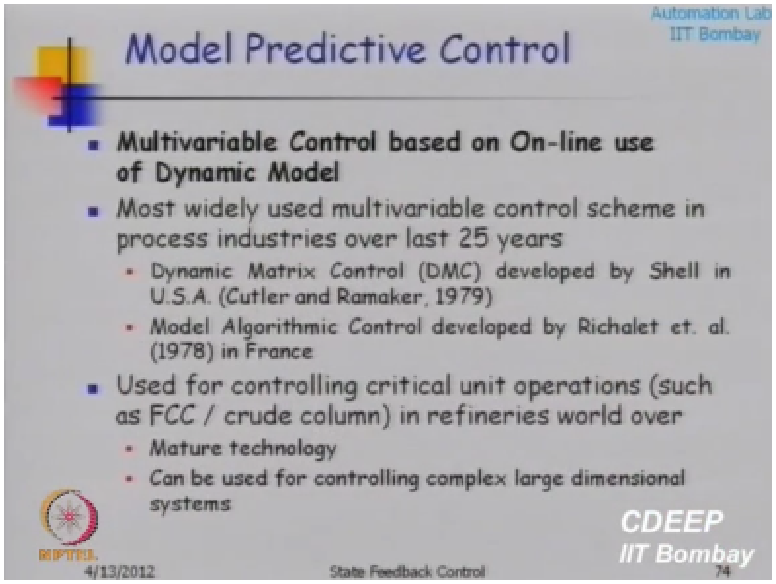

It is a multi variable controller on-line multi variable controller that we use the dynamic model okay it one of the most widely controllers in the industry today started with the chemical industry now we just correct to everywhere that we used in all kinds of uses well these two are the first to implemented in the theory one work done in shell in 1976 that already where implemented on the real problem but there are the paper in automatic the ideal group from France and I think it was closely helped technology.

Because it could do online control of a complex learn you know the computer and probably well of the close large scale compute controls system applications why it was done in the chemical industry for well because a chemical process are very slow okay and you have time to compute okay so my Furness is time controlling or disclosing column the times also never dissolution got the column to be 45 minutes and the settling time can be you know 5 hours in a so the sampling times where control can we have where just time units 5 minutes 10 minutes.

And then in the computers are those times you could only solve problems which are if it is area time to compute, so all the computations could be probably done in 30 seconds in main computations to 30 seconds its 30 large and if I want to compute which I want to controller reward using these methods okay, then I need very fast computation and that is now possible and so that is why the technology with started with chemical industry as now move to distance which I verified there are control and all.

But it started in the chemical industries because chemicals which is are very there are time to compute why do you need time to compute time lines, so the departure here from the philosophy of control that we have been used to we had 1 fixed controller PID controller for ample so that three parameters PID okay one differential equation or difference equation we saw online okay which is very simple you can w do not require the you know bit computing time for this if you do it very, very quickly okay.

And two controller going to define a g∞ matrix once you finite g∞ matrix what I will certain need for inline implementation matrix multiplication you need to an observer and then you need matrix multiplication from ut to multiply matrix by vector matrix whatever if you have those things you can you have some few characteristics to be done at each control time okay do you how do you commutate the predictive to estimate x^ k given k – 1 that involves L∞ we compute L∞ g∞ we have to gain matrices.

You just need you know three equations what is the observer equations first to find out the innovation then find the observer accusations and then implement the control law okay, so that easy if we just 2 3 equations to be solved online it very here we are going to depart from that okay we are going to have controller in which calculations will be done at each time instant okay.

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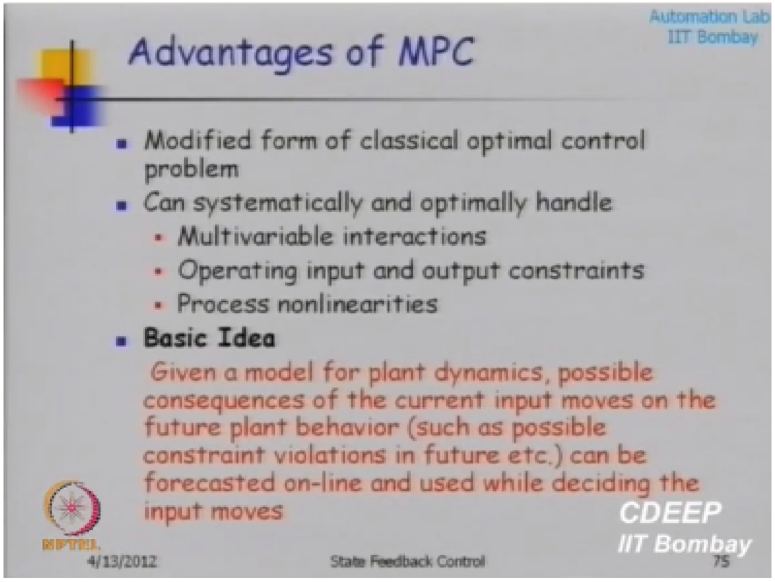

And then I have used in very large dimensional problems so we will come to that little later so what is this controller by this controller is same as optimal controller except from modification okay it is not going to b different from the quadratic optimal control in which this you know the concern way of quadratic optimal control then it is not practical filtered for real problems and quadratic optimal control well what is the distance little quadratic optimal control the first point is order to diffuse multi variable interacting multiply input multiple output systems quadratic linear quadratic optimal controls and deal with it not a problem we are using a multiple input multiple output model.

The controller which we design in LQ could be handling multiple inputs multiple outputs okay so that is not an issues that can think in the problem what about constraint what are the constraints let us take a simple example of devising a controller okay to navigate a vehicle within IIT okay what are the constraints this translates a real problem into control constraint what are the constraint speed relevant good it has to be on the left side of the role left half of the role you know the trajectory cannot be arbitrary okay is only certain path it has to follow it we my have an ideal part of the set point.

But it may not have a set point loop it has a boundary units it has to be there manipulation what are the manipulations can you have infinite manipulations can you increase the speed at any rate as you want no, you cannot will be some limit on the fuel injections okay so their maximum level on the project injection they will d 0 limit you cannot go below you cannot have negative full industrial and that is what I am talking we cannot have full injection higher than cretin values is a maximum fuel industries.

At rate of fuel injection okay so then rate at which you to in the ramp the 12 injection which is the limited at you may you drive what is r1 are called you know that you it is not that it just because it press the calculator at whatever speed you want the real injection is going to be at that you know it is going to be whatever maximum rate it can deliver that is only rate is deliver okay and there are real constraint and though system there are other kinds of constraints for example when I ma driving the car.

There is a constraint that I should run the motor or I should run the engines such a way that carbon monoxide initially should be at small for or should be fine okay if you shake the reward the engine so fuel to air ratios has to be very carefully maintained otherwise you know will have un bound hydro carbons we need that atmosphere so there are atmospheric or environment due to the relations which will tell you how to use it from the engine then same thing is to be about some of the systems that chemical plant you know what you release into the atmosphere either it is actually you cannot relapse any amount of you know atmosphere or to the water okay.

So where the regulations on coming from environmental considerations there are real problems we are calling a Furness okay there is a maximum related which you can inject the fuel you cannot inject the flow rate is.

But when you write a LQ controller it just says that controller output is gain times you must write there is no more saying in that LQ controller where actually uk is bounded uk is not any other any value it constraints then what you do if you want to implement LQ controller with bound then what you do is u in a software liquid artificial limits and we say that if controller calculate input which is higher than this value then do not send that value to the will acuter you know send a value which is equal to or reject limit.

So there are I have to that I you going to do to maintain the system within the bounds is it possible then there are input bound and tell about output bound what about the fact that I am trying my LQ controller I make a move and my vehicle go outside the boundary after sometime not know after 15 minutes if I make a wrong move in and my vehicle is going to go outside the row okay what about this my LQ controller cannot really project after sometime the violation of constraint on the output.

Not just the input and this is going to happen after sometime not now okay so always other examples I want to run up the interfering with formal okay and there is a you know that there are over shoots many lines that desirable choose okay we want to go from 800 to 850 okay we start injecting more fuel the temperature over flows to 900 okay you do not want to go to 900 it might be touching the limits of you know from testing limit you do not want go to we beyond say 870 but when you start doing this.

You are not look when it is going to go to 870 it may happen in future okay so basic idea in modulate the control is that if you have a model online you use to do it forecasting okay you can use and for se what going to happen in future and then manipulate your control action now such that the future violations of the constraints are eliminate okay I am going to do this online all the time every happening constants I am going to use this model few predictions over the future okay so how much you will take how do you drive a car then I am driving the car okay starting from my hostel or starting from my home to look into our I do not have a plan for the entire trajectory I have a plan for this okay I implement that and then I have a plan from extend okay now implement that in the plant some next 10m.

May be very somebody crossing in between and I have to change the plan so I cannot have on plan I can have one optimal plan so that entire trajectory I need to have a plan which is changing in the function of time looking at the disturbances looking in a trajectory you know many in between there is some work going on and the road is that I need to change my policy for you know accelerate it arbitrary okay I need to that depending upon the situation current situation so what is the basic idea.

The basic is very, very simple actually meditative control is much more easy to understand that LQ and you know this is something that we keeping all the time okay then we keep working we have a dynamic model for the plan there are whether you do it doing in the unknowing the difference may be a occurrence reflection we realize that which will not able to know the time in different context whether it is driving a car and then they are giving you of course are when you dealing with your friends.

Using what we can do so given in the dynamic models of the plan what we do is we actually try to fore fee what are the possible consequent of actions that I take now and then based on the fore cast I decide what is the next move that I should okay so what is happening here.

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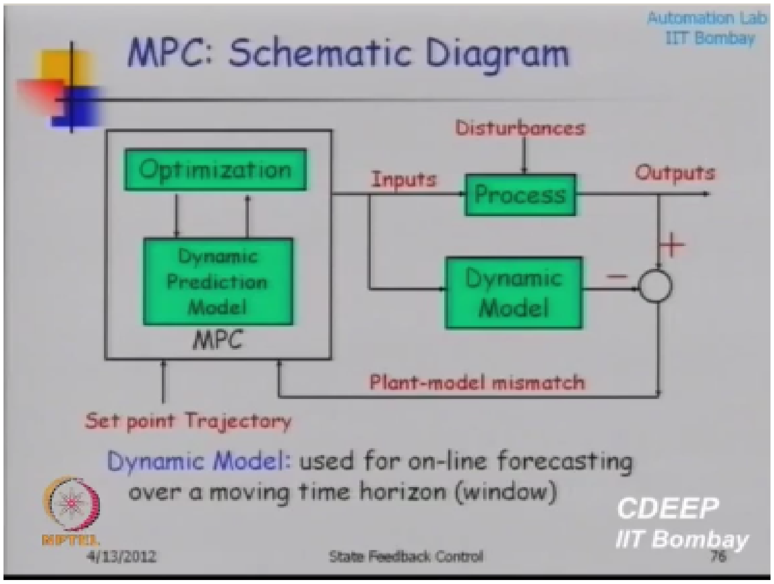

In model equations control in model predictive control there is always a dynamic model which is running in parallel to the plant this model is used for forecasting okay actually you solve or you formulate and online optimal is in problem at each cancelling is filled you formulate the optimization problem.

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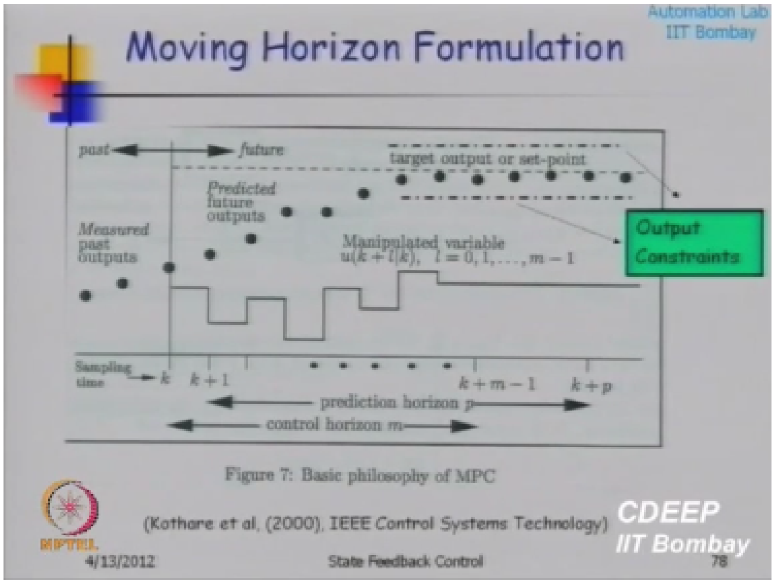

Okay over finite will know that means pictures this some interview so what I am going to do is I have I m finding here at in filled k okay K+1 k+2 is all feature okay K – 1 K – 2 K – 3 or got and standing right now or in stand k okay now this is inside my computer well I am setting in the computer I have this different equation models okay I can play with difference equations model I can be forecasting what if I give this inputs equation that will happen after sometime okay so I can for every forested.

Inputs in the future I am putting here at final of g uk + L given k what does it mean it means that input that I am contemplating I have not implement it okay I am going to implement this contemplated input only I am not modeled inside the computers okay if I use this contemplated input on the model okay model will tell me what will happen in the future okay model will tell you what is happen and then that forecaste is used to meant the okay, so if I can fore cast I can see that it is a cross input actually the predicted behavior.

Will cross the target suppose which is not constraint suppose this is the temperature limit okay then I want to move from this point current point tom this set point in such a way that I never cross this boundary okay we cannot fixed controlled law it is difficult to see what is going to happen each other here I can explicit see what going to happen then I am gaping to simulations in future okay see when you are driving a car you have constraint that you should not treat an object right so just imagine what you do when you drive the car and apply break and you actually have a dynamic model.

For the car behavior in your mind you dynamically construct the models for the object which is moving somebody comes from the way see there are times then somebody is crossing the road okay you do not break there are times going somebody is crossing their do not break why because if you have a model for the car you know when you will reach certain point okay and then you develop a model mentally and though they are part of the computer only so we can you know we can predict when somebody is going to cross the road and whether are car to be there at that point before or after and accordingly.

We can plan our actual and this is the decision only call that particular window of time you do not you know one is the objective gone you change your policy again you that actually it is gain okay so you have more over plan which is dynamic okay so the ideas is that standing at this point I only forecast what are the finite window in future okay I do forecast.

Over let say next 20 minutes or next 15 minutes okay using this forecast I am going to find out somehow this future optimal state of future inputs such that I reach the desired straight point without violating the bound without violating the bound from the inputs okay I am going to do this without violating the bound so the inputs I am going to do without varying it bound from the states what are we going to do then you I am going to implant only on move I am going to implement only one move into the plant.

Inject on move into the plan okay I move to this loop point K + 1 okay I reformulate my problem of deciding controller in the form K + 1 to K + p + 1 so I have moving window in time okay I have formulate my control problems over a moving indoor in time and each time I solve it.

I will just make I do not truck my see if you make this decision that you know you have some object coming and then you are going to slow down for next. Let us say you take a decision every 10 seconds and you keep it for 10 seconds okay, you have a policy for you know 10 seconds for next two minutes you are going to slow down okay, and then you will accurate after sometime this is your optimal policy right now looking at the object okay.

But then after 10 seconds or 20 seconds what happens here the fellow you is crossing see there a car is coming okay, it is actual run and cross the road okay, then you suddenly have you revise the policy you do not say that well I decided to go for next two minutes I am going to go slow you did not do that right, you after 10 seconds after 20 seconds you revise your policy you say that okay, now looking such that okay, let me now accelerate again so let me change my policy.

So you do not trust your moves future moves for a long time you just consume for a short period and then you change them right. Look at the way we deal with you know let us say exams okay, so you know k+1, k is today, k+1 is tomorrow k+2 is day after tomorrow and what are the disturbances quizzes and you know yeah, real life problem and then you know depending up on what happens in a particular quiz today you decide your plan.

What is the method input the preparation that you do for a exam to actually certain goal, what is the set point of course 10 out of 10, so and then what you do is you change is you change a policy of preparing for a particular course depending upon there are multiple inputs, one is what is being thought in the class okay. Second is, what is the quizzes and your performance. So there are so many in puts that keep coming to the system.

And then you would not say that I have decided to work only one hour for a day for make to feel this, you do not do that right? You probably implement your plan only for today, tomorrow something else happens your particular quiz you forget and then you look at, no, no, no I am going to change my plan, and you do it for two hours a day or three hours a day whatever okay. So we have a moving plan and at any time with your semester you will have a plan for a next seven days let us say okay.

And then that keeps changing depending upon the situation, depending upon what is happening in the classes, what is happening in the exams or in the results. And so you keep adapting and you do not have a plan over a longer horizon, you know plan over shorter horizon, so this is called prediction horizon in predictive control okay. You have a prediction horizon, you keep predicting your moves to predicting behavior over the next seven days, and then you keep adapting.

You only implement one move at a time, you do not trust your optimal moves for a long time okay. So my idea is going to be this, my optimization problem is formulated over a finite window of time. And this window is going to move okay, as I move in time okay. So another analogy for this would be, if you are walking in a dark okay, and when you have a torch with you, with the torch you can see only 50 meters okay.

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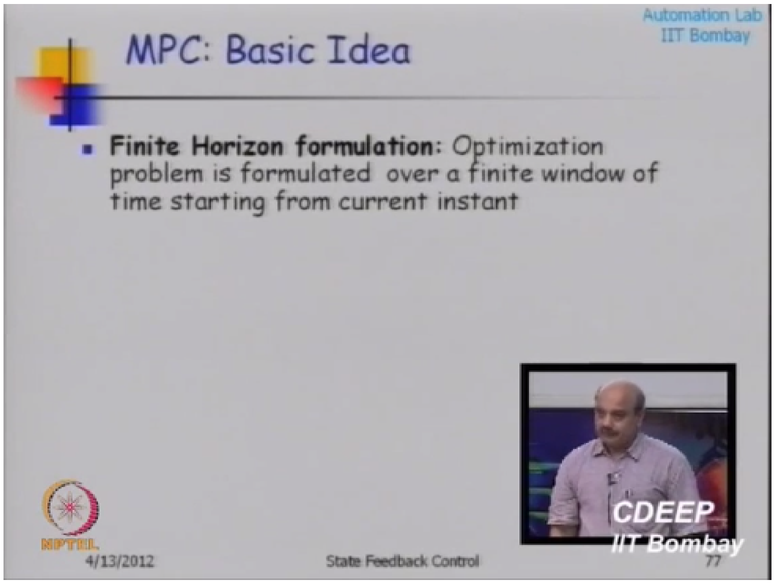

You take one more step, you see next 50 meters okay. So you are just every time going ahead into future and you move one step ahead and you see 50 meters next yeah.

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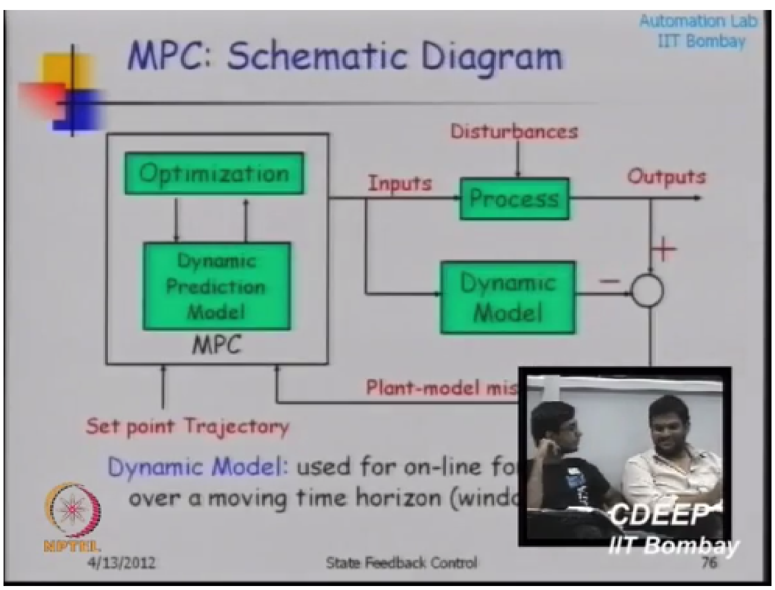

Table of P here, what I am saying here.

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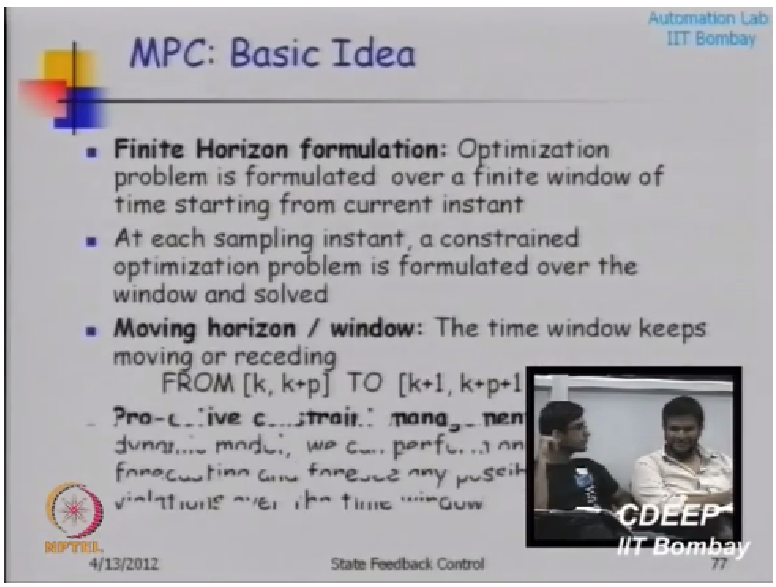

K+P.

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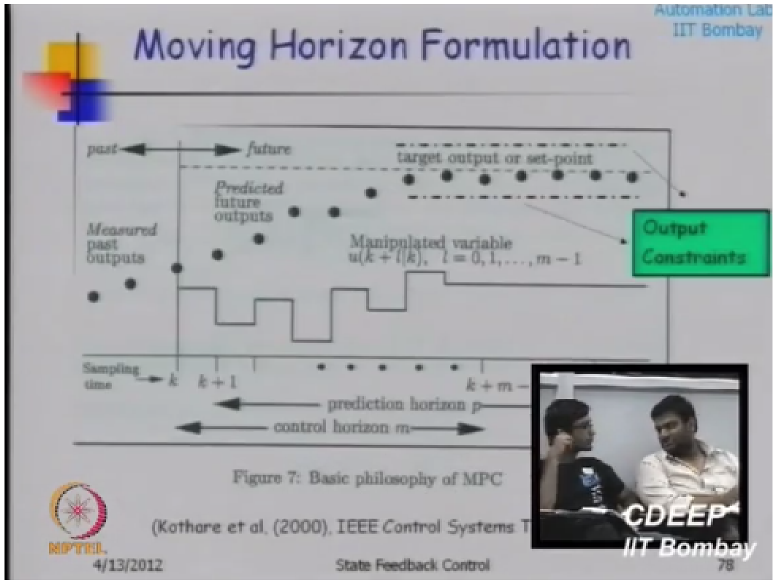

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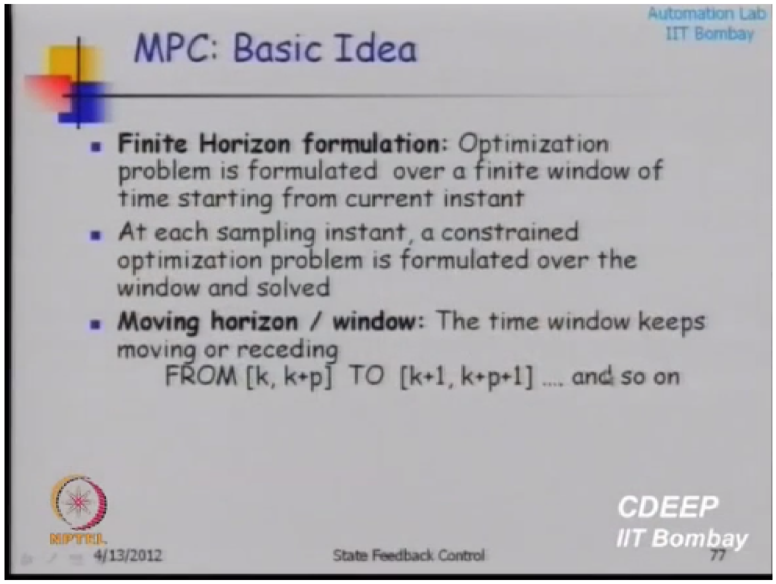

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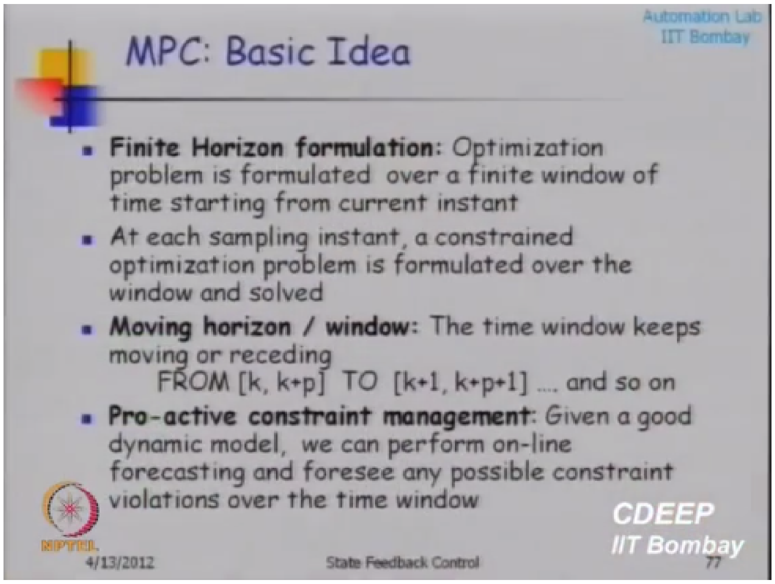

So at each optimization, at each point I am going to form an optimization problem and solve it online okay. I know the solve is for them online, optimal solution of this particular problem is not going to be a close form control law. It is solution of an iterative optimization scheme okay. So what becomes very, very important here is to have very efficient online optimization procedure, what might if they are available you can solve very large optimization problems in 1000s of variables in fractions of a second.

Now with the available computers, with so much advances in the optimization, you can do that and that is why all these things are feasible now. And then you move on in time as a moving window okay. So what is important here is that you are doing proactive constraint management, every time you make sure that the inputs are within the constraint. The projected output, the forecasted outputs are within the constraint okay.

So that you keep ensuring every time and this is where it actually replaces a program logic controller where you handle constraints who adopt these. Here the model, the control problem knows that there are constraints and it tries to handle it in a very systematic manner.

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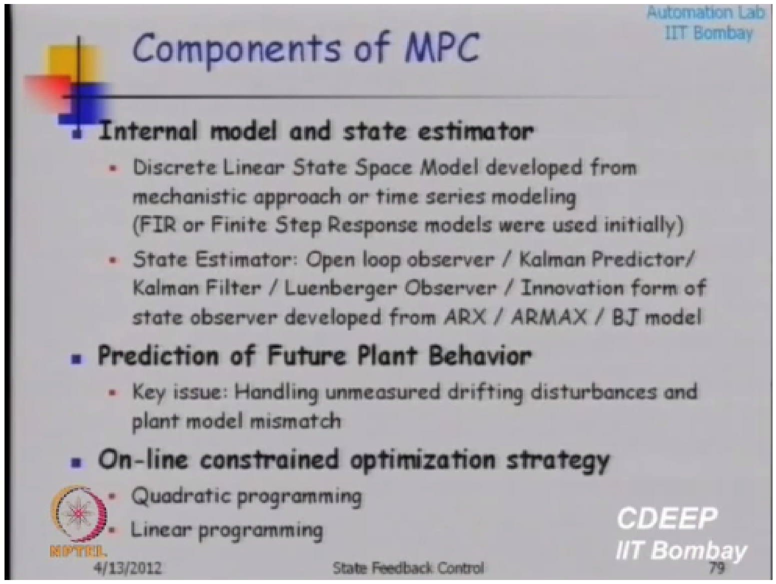

Okay, so what are the basic components of a modifier to the control? Of course, one is the state estimator okay, or the internal model. Well, I am going to present them using in state format, I am going to present using observer and all that, that is not how historically developed okay, historically developed through different ways, people first started using what are called as finite in the response model, and finite step response models.

And then later on the connections to the state space model were shown and then now there are packages available in the industries which are based on the state space models. So all these merging of all different ideas which I have done in the course has not happened exactly at the time the whole thing got developed. It was developed in the different way and then, you know people merge different ideas and they never model form now.

So earlier it was when, those who actually developed these controllers in the shell did not try to use ARMAX, well it terminates in lower ARMAX, but it did not try to use the ARMAX, there are certain difficulties when you go to noise modeling. So they use some simplistic noise models and that is not used observer the way we are using now. So it was in early 70s, not that those theory was not there, but it is that the components we use it was not there for large scale times.

And then state estimators, the next thing is that how do you handle predictions? How do you predict over the future okay. And the third thing is online optimization, how do you solve the problem, online in fraction of a second using an optimization scheme okay. So there are three components. So observer, modeling this I have covered painfully to main lectures okay. And in my opinion, in any predictive control scheme the key thing is the model okay.

You start from a scratch and develop a model which gives you good prediction okay, you can appreciate when you are driving a car, the key thing is the mental modeling are for the dynamics. Dynamics of the car and dynamics of the object which is moving in front of you. it could be another car, it could be a person who is crossing whatever okay, that model that we are able to develop is so critical in driving the car, that if that model is not good, there is accident okay.

So the modeling is the key to predictive control. Then you know, you are going to use quadratic programming which is the efficient way of solving a problem, or linear programming LP, so all those course can be used, those can solve a large scale problem.

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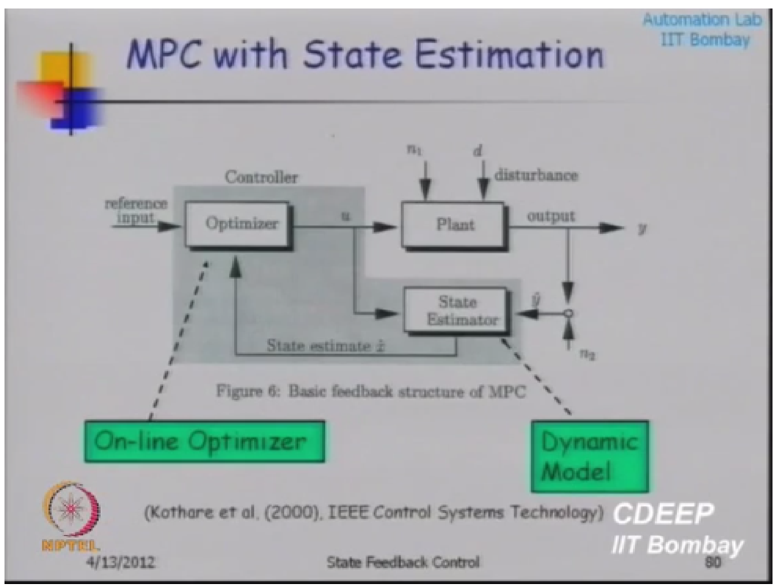

So we are going to pose this control problem finally as we are bringing a quadratic optimal control or LQ, quadratic programming problem or linear programming problem LP okay. And so, I just want to emphasize, now the controller will consist of a state estimator which is running in parallel, and an optimizer, online optimizer. Online optimizer will solve the control problem at every sampling instant.

Uniformly it is a fraction of a second okay that is what we assume. The time required for solving the control problem, even though it is optimization problem, is significantly small compared to the sampling instant, sampling interval. Sampling interval, if it is one minute, I should be able to solve this control problem in one second, if sampling interval is one second I should be able to solve this problem in 0.0001 second, it is possible now, one millisecond is not a big deal.

You can have microprocessor which can solve an optimization problem iteratively online fraction of a second. Entire window, we will only use one and then we will move to the next optimization problem. So we will be solving optimization problem one after another, there is no one optimization problem. See what are the problem with LQ formulation, we made one giant problem.

We said from time 0 to time n and n goes to infinity. So which means in LQ formulation we try to solve the problem as one mega problem from time 0 to time infinity. It is like saying, you know design one controller that means move you from, that is move your car from your hostel to here okay, in solving one optimization problem off line. There we did not solve it online, we use lots of tricks, we cannot do this record the equation, and then we came up with the solution is the optimal solution, unconstrained optimal solution which lies there.

But now, you know I am going to say that well, I cannot have one controller which is like parents do moving system from initial point to some final point, that is possible okay. So now let us start doing the predictions, they are at the point K, is everyone with me this equation, we have this equation. Well, I intend to say this to L infinity to be consistent with the floatation.

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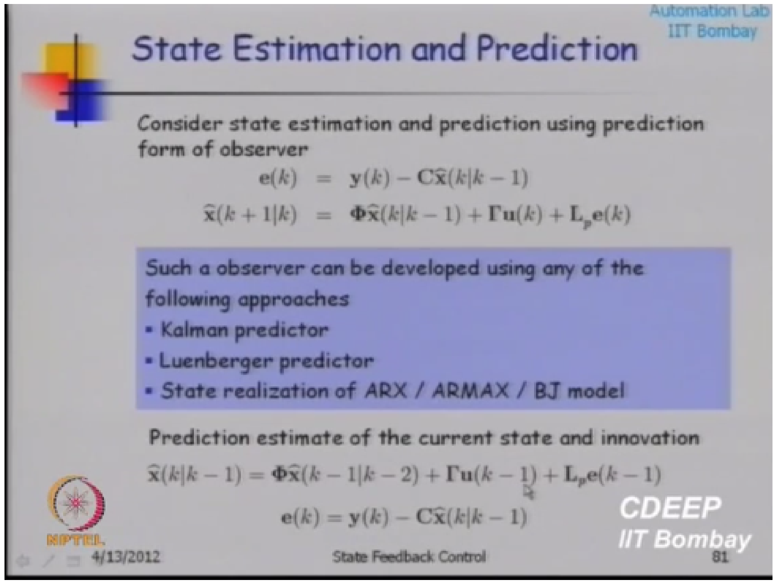

I have made some small errors, so those are correct. I will upload a corrected version and sorry, if you have to take the printout of sound P again. But notations have to be very, very consistent, if I make one, let us take the notations. But LP the humor infinity, the same as that infinity okay, the observer get. So this kind of a module could be obtained as the Kalman predictor, it could be Luenberger observer, I am not qualifying this observer to be of any particular type, it can be whatever is your choice, you can do the Luenberger observer.

It can be a realization of a ARMAX model, it can be Kalman predictor whatever, I do not care. I have this model, I have this state observer okay, I am going to use this state observer to do predictions okay. Now the current at time K my estimate of X is given by this equation, it should be yeah, ek is this, so e(k-1) will be y(k-1) and corresponding, this is the, when you are using e(k-1) here it will be the e(k-1) and X at k-1, k-2 okay.

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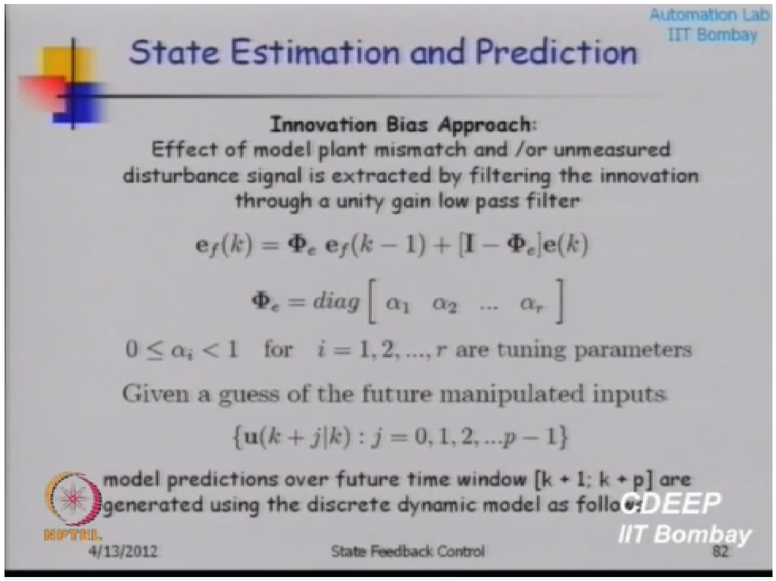

Okay, now we are at a time point XK, we have the state estimated of state at the current time point okay. I want to move in the future okay. So what I am going to do now, I am going to call this, there are again two ways of doing it, one way this innovation bias approach, the other way is the augmentation. And not only this kind of set of augmentation and you will say innovation bias approach okay.

So the same things which I did earlier, I am going to filter this innovations and take a filter signal. So this is what we have done earlier for the LQ controller. And now okay, now look at this problem of simulations into the future, or a forecasting used in the future okay. If you give me a guess of future inputs, I am going to call them as u(k+j) given K, this notation means this is a contemplated future inputs, not actually implemented on the real systems.

It will be implemented only on the model okay, on a model inside my computer okay. It is only for internal forecasting okay. So this is my, let us say you give me one set of such inputs, what should be important that is this inputs are within the bounds okay. Then how can I use the model to do forecasting. If I know the input okay, if you give me a future input, can I use my model to do forecasting okay.

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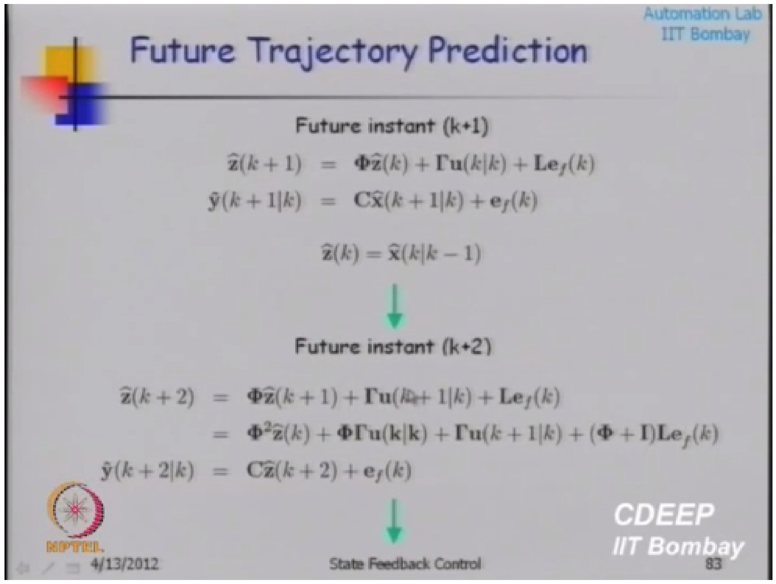

What I am going to do is with forecasted trajectory I have given here a different notation Z, just to differentiate the fact that this is forecasted, this is not really going to happen, may or may not happen, I am just forecasting okay. So I am using my old difference equation, we just go back here, I am just using this, I am going to use this difference equation okay. In slightly different way I am going to use this equation, this same equation.

So what is known to me here ϕ okay , γ okay, LP is known to me, the gain is known to me. And instead of E I am going to use the filtered value of E, I want to knock of 520 from this, I want to use a filtered value of K. So my prediction is going to look something like this z(k+1) okay, when I implement this forecasted input, it has not been really implemented, it is only in the model okay.

The forecast will look like this okay, the forecast will look like this, this is LP that location of L I will make it consistent okay yeah. Engineers model can also yes, correct. But then that similarly will get captured in E, E will become biased okay, and then that information will come out as if it is a disturbance into the signal E okay. So this E signal is actually trying to compensate for plant model mismatch, for unknown disturbances.

So it also compensate for ϕ and γ not being same as the plan okay. So that is a very good question. So these two compensations are essentially added to compensate for model plan mismatch okay, that the true plan will not have ϕ, it could be ϕ' or something, it could be γ', it could be C’ okay. So that is implicitly compensated through this signal okay. And it helps, it is not that it does not work, it works with this works okay.

See what I am doing here is, I am tying up this prediction with the observer, what are the observer given me, x(k-1). If you remember here, I got XK given, (k-1) I have got from the observer, I am tying this up with my prediction as the initial point, initial state. So what I am saying is that the initial state of my prediction is same as the inertial state of the observer. Observer brings the in formation from the past two current, current to future is through this prediction okay.

And this is the meeting point okay. Now if I implement the first move, then the predictor output will be this. What would be at (k+2) one more instant in future, I am going to use this z(k+1) here, you see here okay. And then I am going to implement one more input u(k+1) given K okay. And I am going to assume that this disturbance remains constant over the horizon, this does not change.

I made this assumption when I talked about LQ controller same assumption I am making here, disturbance does not change over the future okay. Estimate of the disturbance, what is this LS bringing in, it is estimate of unmeasured disturbance, it estimate of plant model mismatch, everything kept together is contained in this signal EF as the times F and EF here, both of them are bringing this information about unmeasured disturbances, plant model mismatch, everything is contained in this okay.

So my z(k+2) is going to look like ϕ^2 z(k+1) and so, here this is $\phi\gamma$u(k), u(k+1) given K is this clear in this equation algebra clear here, here algebra is pretty simple here okay. What about (k+3)?

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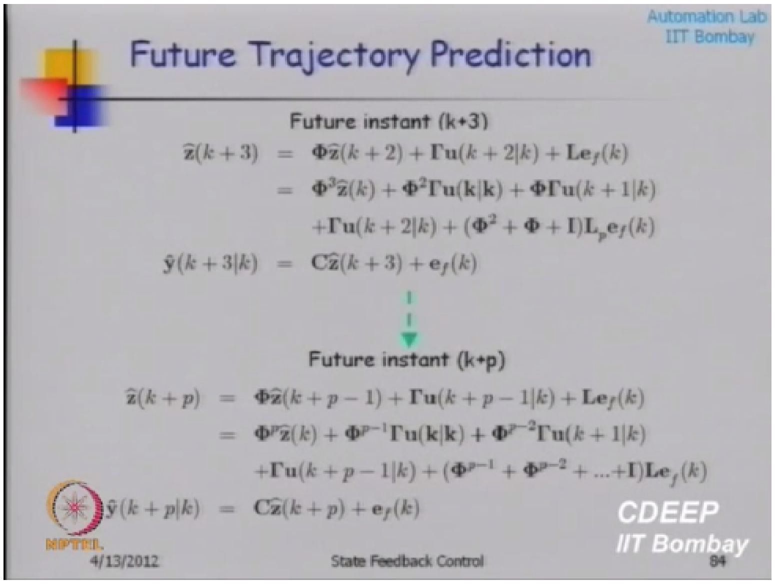

I just go and write in our difference equation. See I have a dynamic model, I can explicitly forecast, so these Z here and Y here are forecast. This is how the system will behave in future, provided I happen to implement, I happen to implement u(k) given K, u(k+1) given K, u(k+2) given K, I can just go on doing this till u(k+p), I can do P step, I had predictions using my difference equation model.

This prediction are not open loop prediction, they are close loop prediction. In what sense? I am feeling that information about unknown disturbances, plant model mismatch through EF okay, and through EF appearing here in the output. So I am correcting the output, I am correcting the state dynamics to compensate for the fact that the model may not be exact, there could be unmeasured disturbances in the state and in the output.

And what is the impose there and what is used as a representative of these unknown inputs EF, LF times EF and EF, they are used as representatives of the unknown inputs okay. So this is my forecast okay, this is how the system will behave if I happen to implement one particular future input sequence right. So it is like, you put, what this, if you have, if you are doing it very, very fast okay, you can actually for every input you think you can plot what will happen in future right.

This is the future trajectory, so I actually created one future trajectory okay the LF interpret this very carefully. See what is the z(k) here, z(k) connects with current state okay. So there are very, very nice interpretation.

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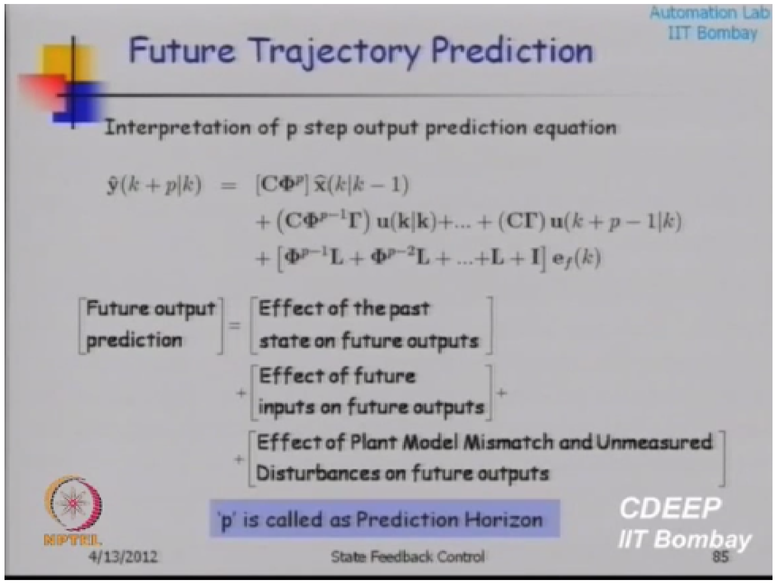

And I want you to understand this very carefully. This prediction P step higher predictions of what is going to happen okay, consist of two components one component is effect of the past on the future, this is the effect of the past on the future, and this is effect of the future on the future. A dynamic system has the memory okay. Let us go back to this painful example of your studies. So x(k) given (k-1) is the current state okay.

Will it not have effect on what is going to happen on the ancient, it will have, it will have overall grades, it will have an effect okay. But is everything lost no, you still have project to do, you still have, you know, on ancient too write. So these are all future inputs, and then you can study okay. So these are all future inputs that will go into the system. So what will happen after sometime, is that cumulative effect of two things.

One is past having effect on the future and future having effect on the future okay. So in any dynamical systems okay, there are two components that influence the future. One is past history influences the future okay. And you have degrees of freedom, you can change the future okay. You can change the future by injecting input move. If you make them carefully you can change the future okay.

So this is what is the message here, that there are three things here, there are three components, there is one more component here which is effect of plant model mismatch and unmeasured disturbances on the future. So that is captured through this EF okay. So prediction a P step higher prediction using this observer has three components.

One is effect of the past behavior onto future, effect of future inputs on the future and effect of past knowledge about model plant mismatch and disturbances onto the future okay. So these three things which – so in some sense this model also reflects lot in here we keep saying. What is going to happen in future is your – so always old karma comes to, you know exact K given (k-1), but everything is not lost.

You have a degree of freedom to change the future. So that is given, so these two things here this and this, well bring in the effect of the past and this is the future which you can manipulate. So P here is called as prediction horizon okay.

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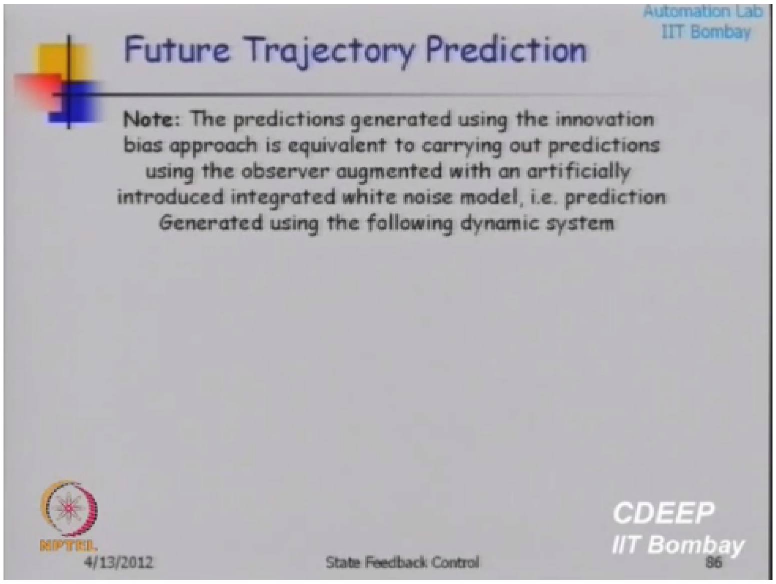

Okay, now how do I want to move from the current point to the final point? See I am not at the said point, currently I am not at the said point okay. So let us say you are driving or you are piloting the plane and you want to go from the height 10,000 meters to 15,000 meters okay. What should be your check point trajectory, should it be, you know within one instant 15,000, 15,000, 15,000, 15,000 or it should go gradually.

Depends if you are a civilian pilot or whether you are a fighter pilot, if you are a fighter pilot you want to go from 10,000 to 15,000 in the next seconds that because maybe some, if I am behind you and then you want to move at faster, because you touch point. And you know mind if it over shoots and, but of course if you are a civilian pilot you want to go gradually to said point okay. How do you achieve this okay.

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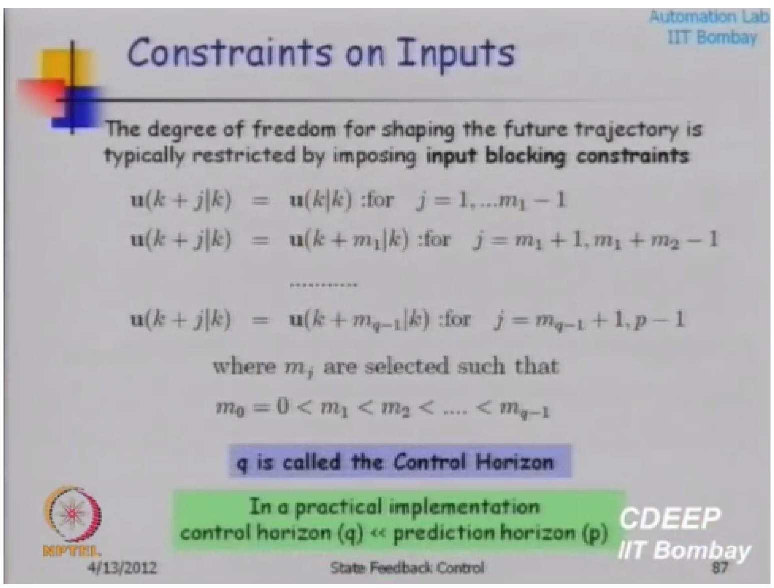

Let me before I do the maths, let me show this visually.

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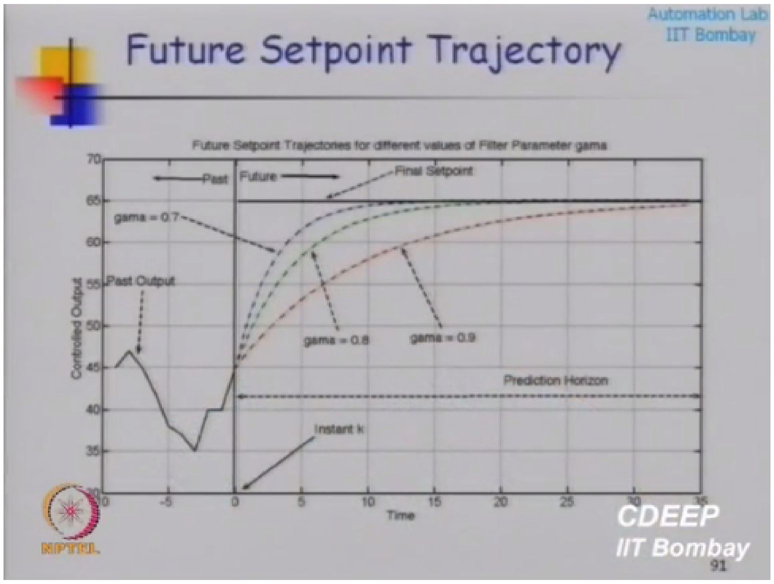

Here in my picture has got blue. So the question is, when I am moving from here to here it is my final said point, this is my 15,000 feet, I am currently at 10,000 feet okay. You have inclusive near that 10,000 feet and suddenly you want to go to 15,000 feet, I am just giving here in terms of percentage okay. You want to go to this particular, so should my trajectory be like a step with the trajectory, or should it be gradual okay.

Should I move very, very slowly here, should I move by this trajectory, should I move by this trajectory, you know I have an option. I can create a future trajectory to move from current point to the final point okay. And then what I can say is that the prediction should be as close as possible to this trajectory, is that okay? See I am predicting okay, how do you choose the moves, you choose the moves such that, you know the predicted behavior is very close to this desired behavior.

So I am giving a desired behavior from the current operating point to the final operating point okay. So this is done through this trajectory generation.

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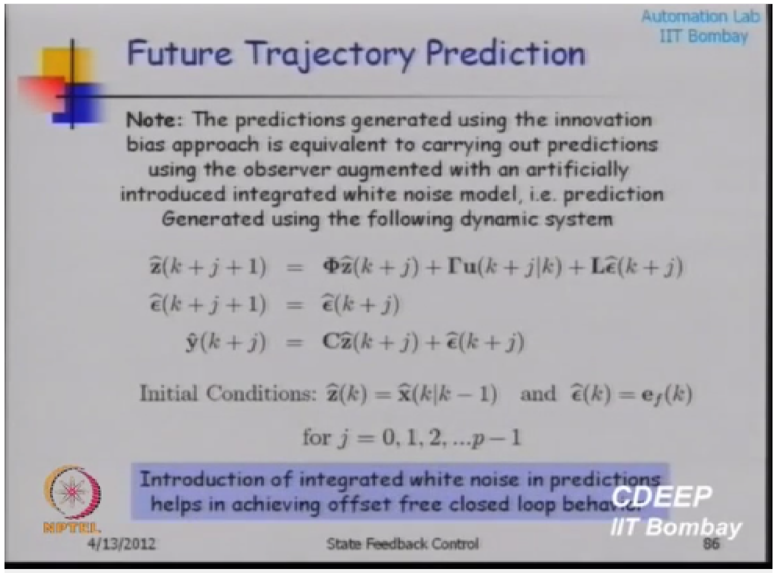

In this figure that have been, figure has looped your wrong point okay. So what I am going to do now is I am going to do – okay, this one slide is just to show that this particular model is equivalent to something okay.

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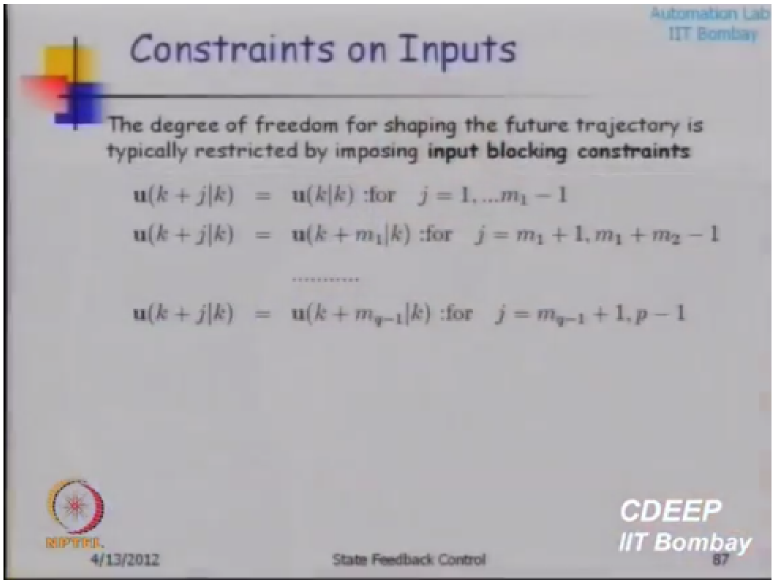

I will come here. So how do I choose the inputs, when you give me this inputs you should not give me an arbitrary input, you should not say that, you know input can go from 0 to some infinity, you cannot do that. You can change the input only at certain rates, you can change the input only within certain bounds. So I am selecting these inputs, it should be chosen properly by the optimizer.

Optimizer should know the limits on the inputs for optimizer that can be done by specifying the constraints okay. So I can actually give the constraints, I can also say that well after all I am not going to implement all the future input moves okay, I am not going to implement all the future input, I am going to implement only one of them. I am going to discard the future input pole and then, you know I am going to redo the optimization problem again okay.

Then why compute next, suppose I am predicting over next two hours, why compute a future trajectory over next two h ours. If I am going to use only first one minute part of it okay. So what we do is, what is called as input blocking, so instead of using I will just show you the equations look complex, I will show you picture that we show okay.

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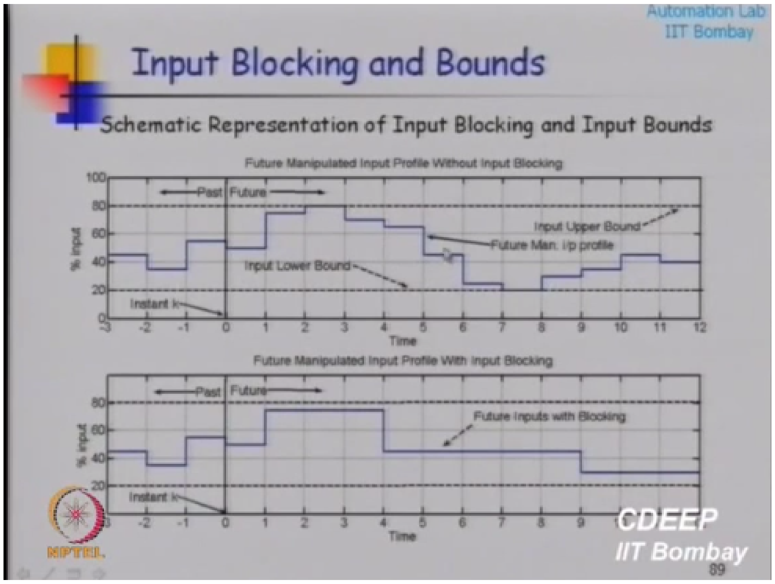

Instead of saying that I am going to manipulate over the entire future horizon, I am going to say that, I am going to manipulate over one input to the next. Then say, I am going to keep the input constant over next three instants, I am going to keep input constant over next five instants, I am going to keep input constant over the prediction okay, instead of changing every input in the future, I am going to constraint the movement in the future.

That is because even if I compute all these inputs optimally, I am not going to use them again okay I am just going to through it uses only one move and move a head okay. So what I typically do is I do not give I do not use entire degrees of freedom in to the future I constrain it.

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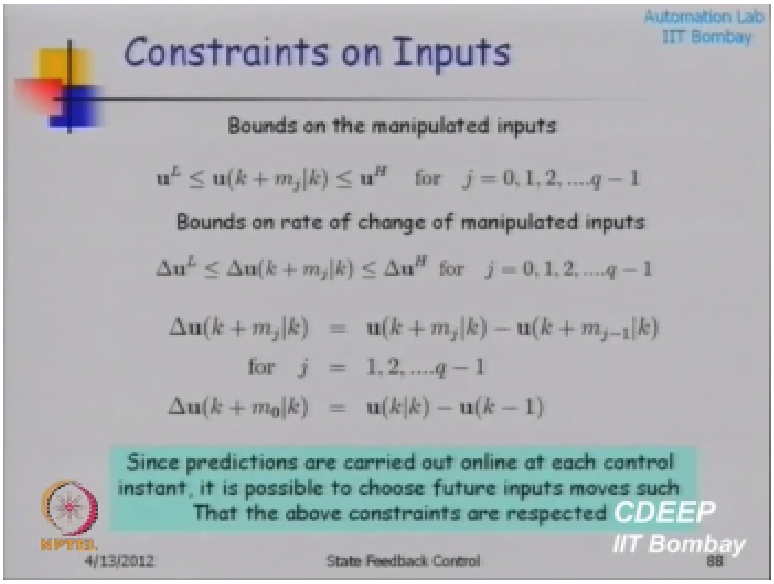

Second thing is I should chose this future input moves in such a way that they are within the bounds right this u low is the lower bound okay u high is the higher bound upper bound I should chose them in such a way that difference between two succession moves is also constrain I cannot excessively increase between two sampling instants I cannot excessively decrease between two sampling instants these are the real problems if you have control boll you cannot take it from 50% to 0% in one second okay you can probably take it from 50% to 49%.

Otherwise the mechanism will break, so there is a limit on which you know you can actually move a system. So those limits will get told to the optimal so the limits on the move at a time okay we are called as rate of change of moves okay and then there is a constrain on look here I have drawn this constrain boundary this is the upper limit this is the lower limit you cannot chose input beyond this point.

There is a limit of how much I can move at a time okay, so I should actually obey all these input constrains when I podcast okay so that is impress it in optimization problem.

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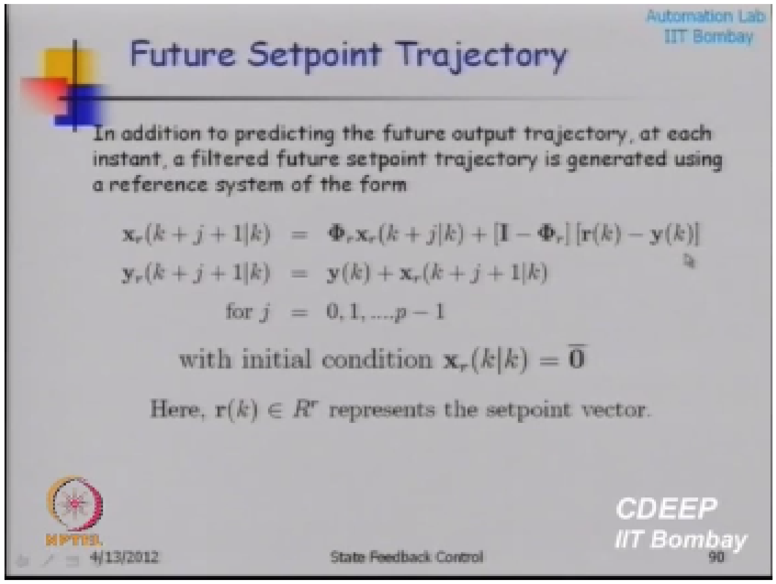

And of course I talked about a future set point trajectory future set point trajectory generated by filtering technique okay you actually filter the current set point – the current output through a difference equation a linear difference equation will actually create a filtered signal in to the future okay again just go back and think about what we have in doing till now just look equation look little complex yeah, input blocking.

See I can formulate the problem over the future by taking degrees of freedom can I go back can I go hidden and come back again go in to blocking I will do that they will become more clear why I am saying that okay.

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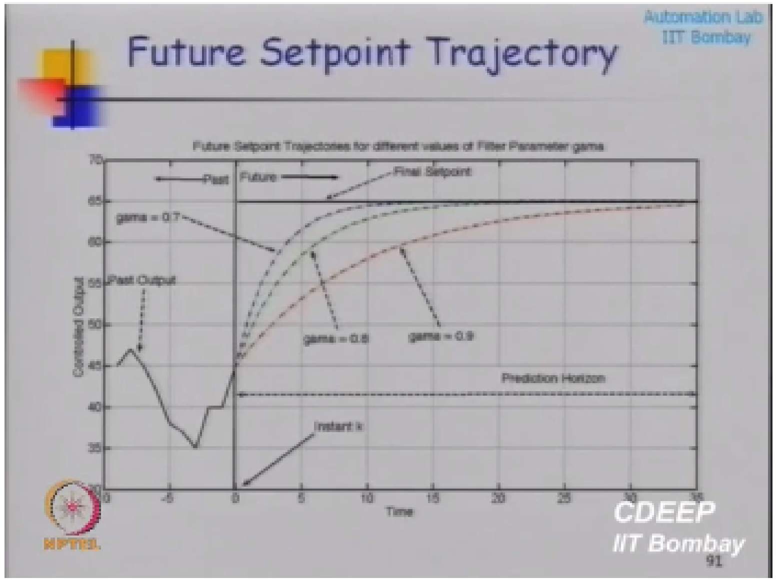

So there is mathematical way of generating future set point trajectory and that is given here let us assume that there is a way of generating the future set point trajectory and then how fast or how slowly you take it from the current point to the next point and we decided by a tuning parameter γ so that parameter γ have given here this is nothing but linear difference equation again this is again a linear difference equation I am moving then I am generating a set point trajectory from one point to the other point this is a tuning parameter I actually do once okay. So every time I am act a current point I generate a move trajectory from her to okay.

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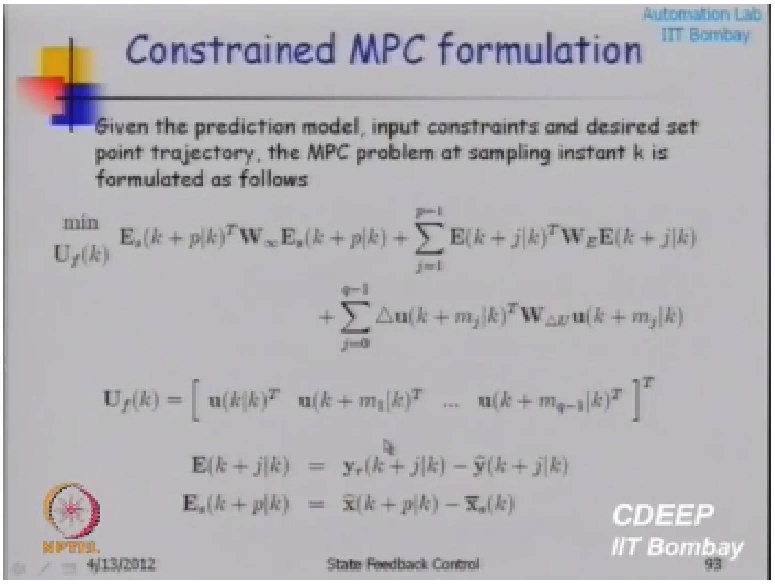

Now what I am going to do is let us keep this target business okay, so what I am going to do now is I am going to form MPC problem and the constrain optimization problem constrain optimization problem which is such that this is the future set point trajectory – the future prediction the difference between the future set point trajectory and the future prediction that is given by this p okay.

What is this es? Es is the final value where you want to reach okay, es is the final value where you want to reach okay. So for the time mean forget about this ed you understand what is this yr this is the future set point usually we have generated I generate a set point trajectory here right I am currently at this point I want to move here okay and these the let us say this is my future set point trajectory this green value is what I want to use this is my future set point trajectory I want to move along this trajectory.

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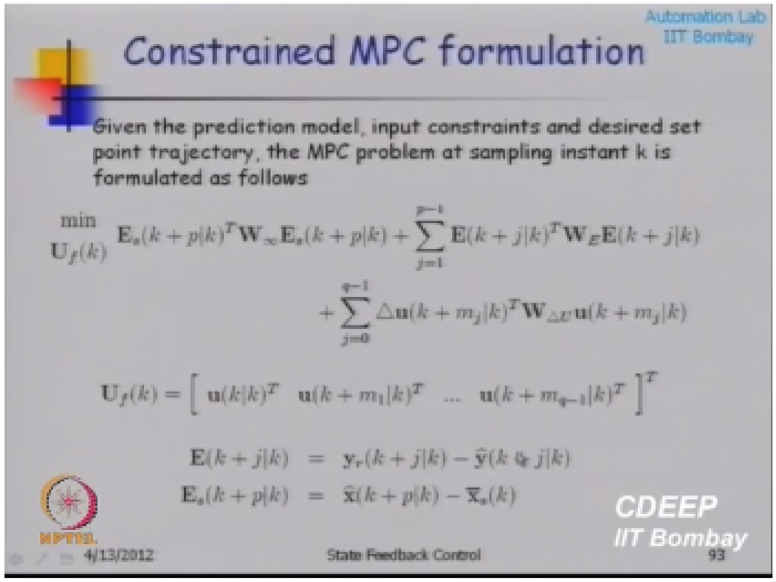

Now what I am doing here I am defining the error between the decide trajectory on pore cost okay, what is it that again change in the future, future input moves okay so these are my future input moves okay this part y^ k + k given k what does it depend upon future input moves okay so the difference between the future set point and the future behavior should be governed by future input moves okay I have defined here t transpose W_E I am defining some kind of distance measure what is this square of the distance this E transpose W so this error vector error between set point and the prediction okay.

That transpose W and E this gives you square of the distance okay then I am finalizing the ref input moves okay I am putting some weight on the future inputs okay.

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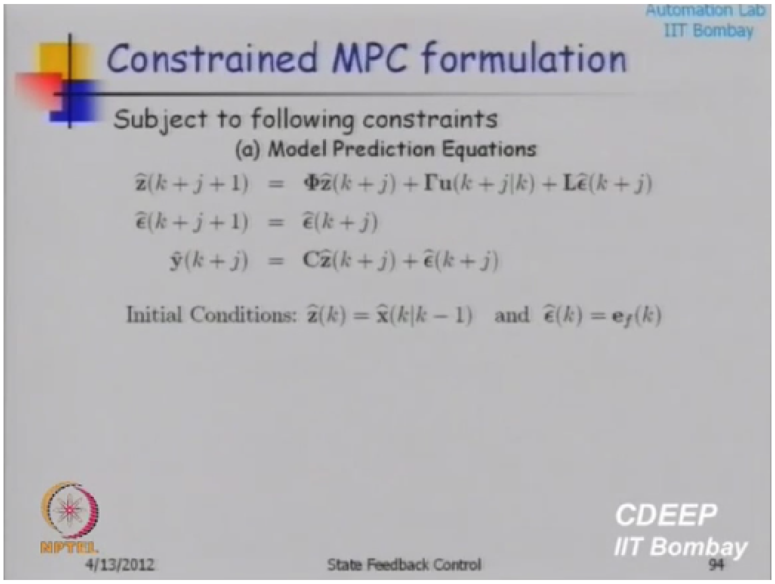

And what is my MPC formulation my formulation is minimize this objective function with respect to minimize the objective function with respect to future inputs okay minimize the objective function what is the objective function some of the squares of errors from what is this? What is this error between future behavior and decide future cut point I want to decide future predicted error behavior the difference between that should be as small as possible ideally what should happen it should actually followed at future cut point okay.

But I want to do this I want to minimize the difference such that you know you do not take excessive inputs moves I do not change by input suddenly okay I am putting or weighting her4e such there on input moves okay I am putting weight such on input moves and so this optimization problem.

If subject to constraints what are the constraints one constraint is the prediction constraint this model is be used to predict y^k oaky prediction constraint subjected to bounds on the inputs okay and subject to bounds on the predicted outputs the predicted outputs should not go much beyond this from the constraint okay so this optimization problem input blocking constraint so all theses constraints are solved so this problem in which to minimize this objective function subject to model equations predicted equations subject to constraints on the inputs on input moves on predicted future outputs okay this problems is solved in one line okay.

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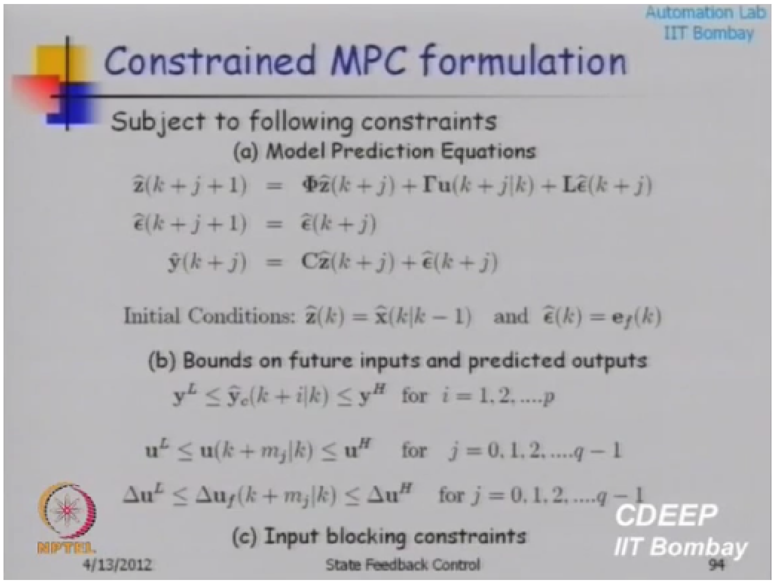

This problem is solve on one line okay so this is not so what I am going to do is I am going to I solve this online optimization problem okay and subtract after I get the optimal input I want to just use this the optimal move in an optimal input I am going to implement okay only the first move in the optimal I am going to implement I am going to reject the entire optimal.

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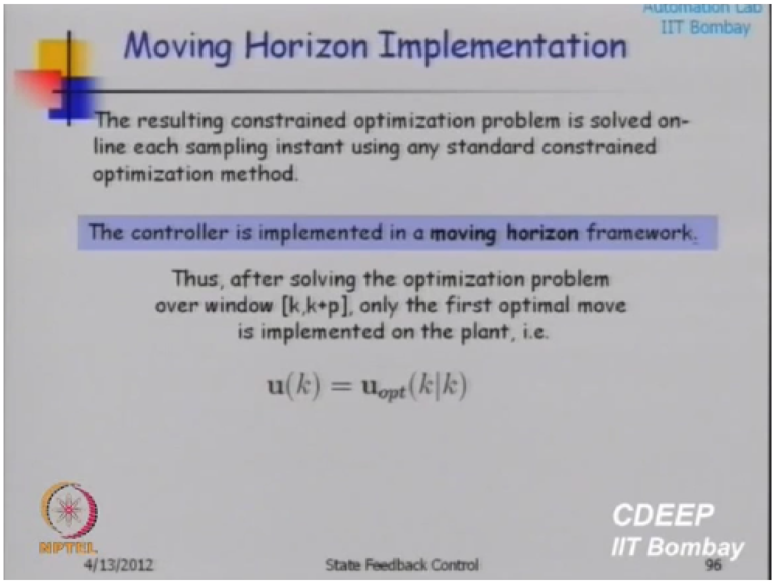

Reformulate the optimization problem and we have a next point okay and resolved it okay why do need to do this because the disturbances keep changing all the time okay where is the information about the disturbances is coming in.

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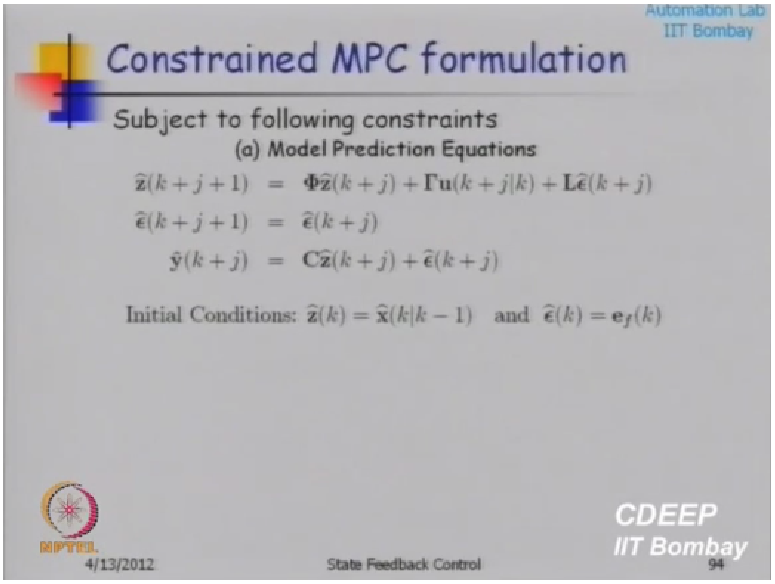

So this L times ε okay so through this L times here this ef is remaining the information about the disturbances is land model this match that can change every instant so I cannot trust my optimal solution too much I juts implement one move okay.

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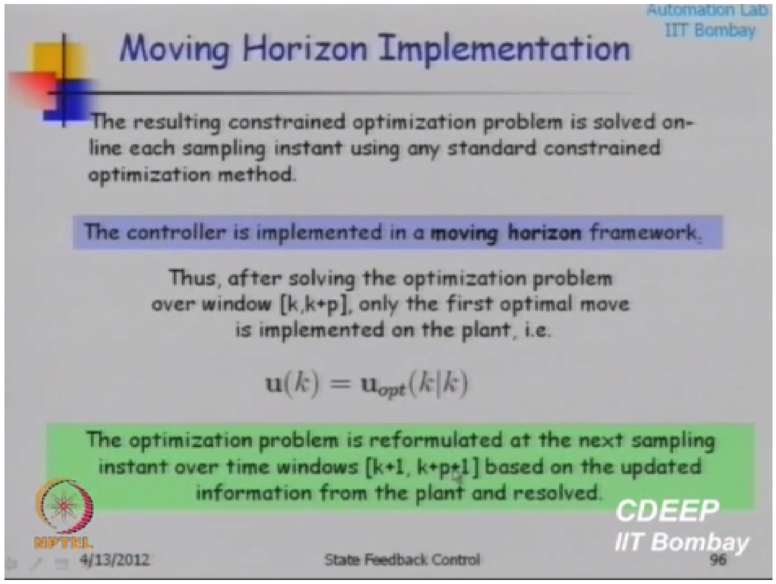

I just implement one move and then resolve the problem fork+1 to k+p*1 okay so I solving this sequence of optimization problem there is no one close function information okay while these blocking this is the very difficult to understand what else do an slightly modify my notes and upload them I want to do implement this controller on your processes okay.

So that is the real term of whether you understood what I am trying to teaching okay make it work okay so now why I am blocking I am blocking okay because you know anyway if I were to consider all future input moves and not block them see if I were not block the.

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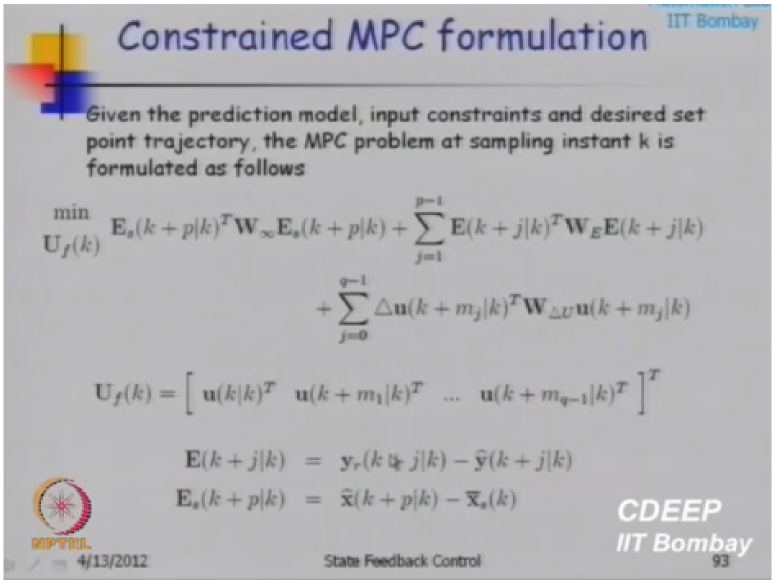

If I were to optimize entire future input moves I am just going to use one of them right I am going to discuss that should reduce the dimensional of optimization problem okay I put this constraint the dimensional of the optimization problem is reduce and then you work in a low dimensional to high dimensional.

So this input constraints use the data kind of optimal solution you compare to if you do not do this that is there but I know I am not trusting my optimal solution but I am taking one move and then I am discarding the rest so we try to do this because you want to reduce the dimensional of optimization problem so we will come back to this again so do after all that.

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NPTEL
Principal Investigator
IIT Bombay
Prof. R. K. Shevgaonkar
Prof. A. N. Chandorkar

Producer
Arun Kalwankar

Project Manager
M. Sangeeta Shrivastava

Sr. Cameraman

Tarun Negi

Sr. Online Cameraman
Sandeep Jadhav

Digital Video Editor
Tushar Deshpande

Technical Assistants
Vijay Kedare
Ravi Paswan