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ADVANCE PROCESS CONTROL

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Lecture No – 24
LQG and MPC

Sub – topics
Linear Quadratic Gaussian
(LQG) Controller Design

So last lecture we looked at linear quadratic optimal control which actually forms the basis of model predictive control, so in this lecture I want to complete some part of this quadratic optimal control and I am move on to the main control scheme that I going to talk about is model predictive control. So hopefully we will be able to do it by end of this week that is the next lecture, so I talked about linear quadratic optimal control mainly because it forms the background for model predictive control.

Model quadratic control will see that how in terms of concepts when your smooth transition from one idea to the other idea, many of the books do not introducing this way many of the books chose to introduce it meditative control not connected with the LQG, because the way developing the industry was not exactly the way I am presenting okay. So today where historical view point then connections between LQG and predictive control where established somewhere later in the literature.

But the connection already exists it is not that they were not there, they were not so appearing in the initial works that appear anyway, so let us start looking at this LQ controller linear quadratic controller now I had at one more term here linear quadratic Gaussian regulator initially we are develop controller only for assisting innovation error to disturbances now state disturbances you know un-measure no measurement noise it was a clean system.

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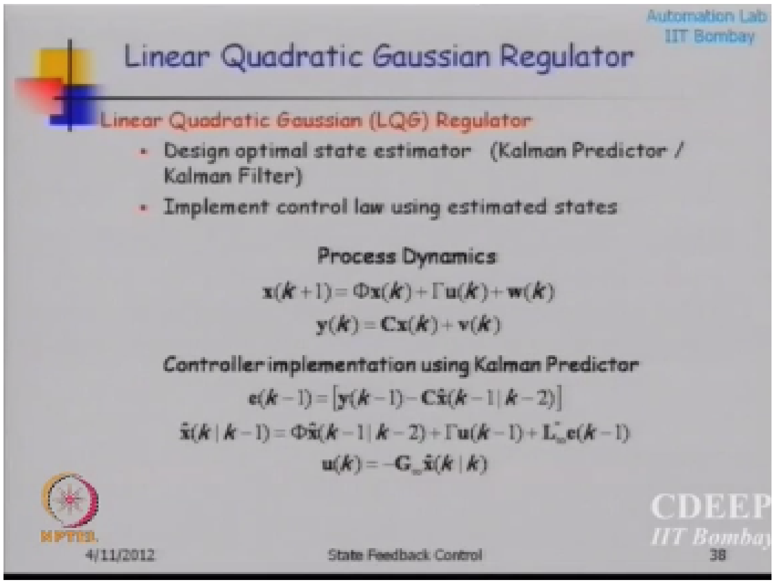

Most ideal case we looked at a problem where it is perturb from origin and you want to bring the system back to the origin that is only limited from the very simplified problem that we looked at, actually what you do in reality there are state disturbances even if you relax these two small constrains that is our conditions that is there are no state disturbances and there are no measurement noise then we have to make some modifications.

Now moment you have this kind of a model okay in which you do not neglect the state disturbance now would you neglect the measurement noise and you let us say you have correct characterization of these two signals this is 0 mean white noise this 0 mean white noise you know covariance's okay. Then how do you implement the controller well the way you implement the controller he is not using the actual state feedback but using the output feedback what you do is you construct a state estimator here I have constructed predictor okay.

Just to keep my development simple it is possible to do same thing using Kalman predictor okay, I could have done everything using Kalman predictor I am just choosing to develop entire notes using kalman predictor that is only to keep mathematics or algebra simple okay it has nothing to do with you know like this is ∂ observer nothing like that okay you can develop this controller using kalman predictor as well okay.

In kalman predictor have to write one more equation okay prediction correction here I have one equation, so my subsequent algebra becomes simple apart from that there is in a development once you understand the concept extending it to from kalman predictor to kalman predictor is minor modification so it is not a great thing okay. But if you see here what I got for linear quadratic optimal control work quartic equations then I will not interested in the dynamic Riccati equations I want interested in the steady state Riccati equations to steady state secant equation I got the gain matrix right I got this g infinity matrix this is my controller gain okay and what is this l* here? Is the observer gain how do you get the observer gain?

You have metal of designing the observer through kalman's approach that is kalman predictor you have Riccati equation for observer okay, so actually you have to solve when you design this feedback controller you have to solve two sets of Riccati equations one for observer the other one for controller okay. One to obtain l infinity other is to obtain g infinity okay when you are doing our assignment now okay I am expecting you to implement this control law okay, I am not expecting you to write now to implement observer which is time vary time varying gain.

Forget about time varying gain you can directly get steady state solution of the Riccati equation both for the controller and the observer just find out l infinity g infinity okay implement a control law like this that is what I am expect you how do you solve the algebra Riccati equations? Okay that is the complex business okay you leave to mat lap there is a sub routing in mat lap called ARE is it for discrete time is it DRE yeah for discrete time it is BARE discrete algebra Riccati equations okay.

That will solve that will give you the controller settings controller matrix okay the work for kalman predictor there is a sub routing call d kalman okay, you just give system matrices to d kalman okay, it will find this gains for you it will give you both d kalman will give you predictor gain d kalman will give you filter gain you have to chose whichever one you want to implement you have to chose idea filter gain or predictor gain and implement the control law okay.

Just become the steady state gain and implemented okay that when it comes to close loop control when it comes to state estimation and when you are doing kalman filter okay at that point you the b Riccati equations see how the pk changes of the function of time I how lkh in the set of function of time, after sometime lk will go to lk * or l* okay so what will happen? So that you can see there so these two are different exercise I will upload today that it tells the instructions from how to submit this assignment okay.

So address it I want to include one more thing in this, so the 3 easy components of this assignment is one is systematic dedication then observer programming and then you know this lq controller linear quadratic optimal control. So these three you do and then the forth one is you implement manipulative control okay that is what I am expect. So that will have higher vector is done these thing so suppose you put 25 marks so first three is 555 and npc is about 10 marks okay so do predictive control implemented that is the main thing.

So this is this idea clear, what is happening here? So right now again we are looking at very limited problem you are looking at a problem of moving system from somewhere to origin okay what about the stability okay you can show that you can construct the way we constructed the close loop equation last time okay for the Luien Bab observer in a same way you can construct a close loop equation here you have this is a true evolution of the plant.

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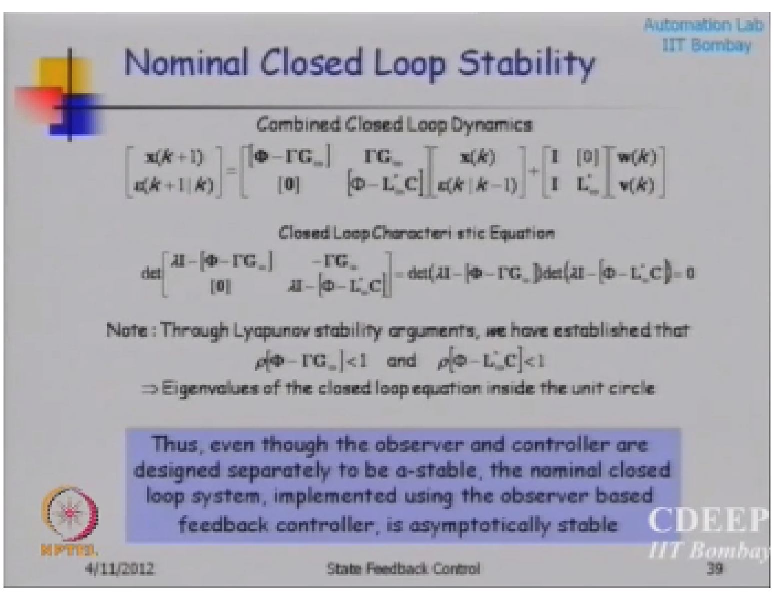

This is the error estimation error okay you get a same thing here that is one technical problem in directly applying stability condition to this equation is because this now we have included this w k and vk okay. Now the trouble with wk and vk is that because we have made an assumption that these are Gaussian okay they are Gaussian distribution, a Gaussian signal is an idealization it is see Gaussian distribution is define for in say if you take some random variable x okay Gaussian distribution is define for x going from – infinity to + infinity okay.

Now the Gaussian distribution will make sure that large values never occur they are very low probability never less okay you have defined a domain which is –m infinity to + infinity okay. Now once you have turn back this system has inputs technically which are unbounded technically in reality disturbances are not going to unbounded in reality wk state noise and measurement noise is going to be bounded it is never going to be unbounded model use for it is saying that it is unbounded okay.

That create some technical difficulty in talking about bounded input bounded output stability because this inputs as we have defined right now that is Gaussian distributions are not bounded inputs so there is a technical difficulty, if you view this equation at a deterministic equation were w and v are some deterministic inputs to this difference equation which are bounded you make that approximation then you can talk about bounded input bounded output stability by looking at the Eigen values of this matrix see this is the augmented state vector okay this is the and this is the matrix whose characteristic equation will determine whether a system is going to look stable now this argument we have done in the case of earlier for the state feedback control law, yeah.

No, so when you define see how will the Gaussian define otherwise what you have to do is you have to define something call truncated Gaussian distributions okay so which means that w can take just take a scalar space yeah so x time see if you take or if you take a Gaussian distribution of a random variable x okay it is define from x going from – infinity to + infinity it is not it if you want to take you cannot define a Gaussian distribution for a bounded x if you want to define a Gaussian distribution for a bounded x then there is something called truncated Gaussian distribution which means let us say x varies between + or -3 suppose okay you want to say that.

Then the distribution is go on the Gaussian it is almost Gaussian with truncation, but it is an input you know with the system it is an input to the system so is it a input point of a bounded domain it is not moment it says a Gaussian random variable okay it is not input depend on a bounded domain that is technical difficult okay. If you ask me whether so actually speaking creating as model for a real world problem in which an input can take unbounded values is not realistic but it is mathematically convenient.

Why we do it in because it is you know mathematically much more convenient if I were to work with those truncated distribution is the life will become I mean the algebra with become quite basic u and you cannot get nice okay, so technically it is an unbounded it defined an unbounded domain do not in reality no sub-stream at unbounded the input exist right. So basically we have seen through Lyapunav argument that φ – γ g infinity this is inside the unit circle and φ – nc infinity this is inside the unit circle independently observer and a controller are stable.

Then you can just combine them the combine system will be close loop stable this is what this roughly the idea behind the separation principle you can separately design a observer you can separately design the controller. Under the nominal case does it we jointly stable there will be a system which is join to stable okay.

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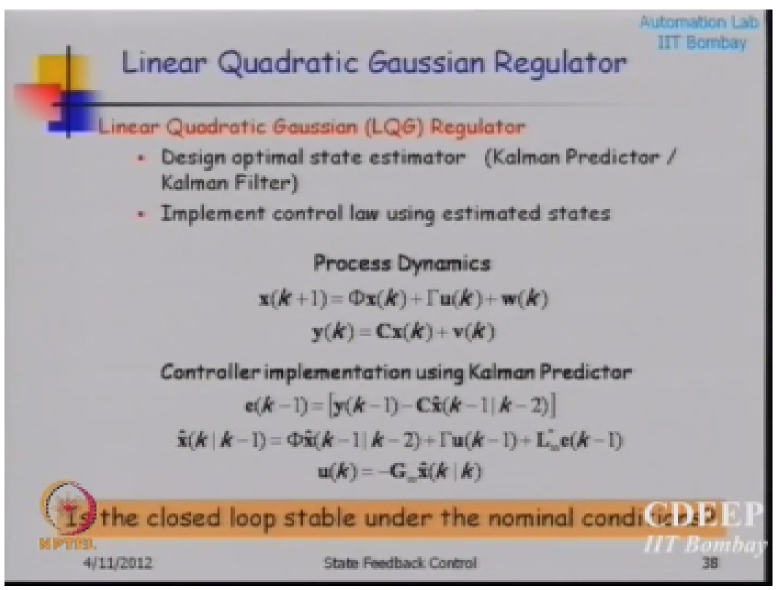

So this let us move on to more realistic formulation yeah, 38 I can use a kalman filter here also okay, so I am using kalman predictor here just to keep my algebra's in to this because this whole thing becomes easier to develop it is not there I cannot do a kalman filter I can do own thing with kalman filter I can use why I am using yk – 1 here? Oh! Sorry it should be k – r this one right yeah thanks.

It should not be k given k it should be k u and k – 1 okay, so the first thing is that we made an simplifying assumption but the disturbances in the state okay are white noise it is a very you know simplistic assumption okay there could be disturbances which are influencing the state dynamics that are colored are drifting okay. Assuming that they are whiter noise is the simplification but helped us to do some mathematics okay, now let us go the real problem that it is in reality it is not a white noise okay.

It could be color okay now how to deal with this problem will see that second thing is that when I did all these derivations okay I made an assumption but φ γ are c matrix which are used for the plant and which are used in the model are identical okay this is also just a simplifying assumption to you know get some incites in to ideal behavior but is reality yeah, go for this we want yeah so I am going to do that.

So right now I am just in this particular controller I am doing in it from non 0 initial condition to 0 initial condition so 0 straight for non 0 initial state to finally I want to control at 00 this particular regulator when you say regulator you are only regulating at 00 okay this uk = g infinity will only ensure that x k will go to 00 0 as time as k go to infinity I am just going to origin right now okay.

So I have designed a controller to move system to origin from a state which is not at the origin okay now I want to use this controller see why I did all this because I want to derive all these Riccati equations.

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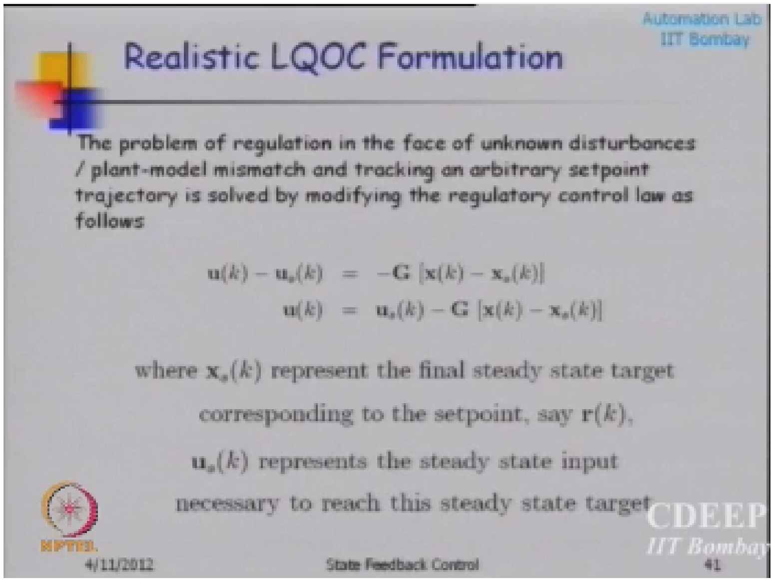

I wanted to derive all these Riccati equations that is why I have done this simplification and now I know how to compute g infinity okay right now I know how to compute using this Riccati equations I am going to form this algebraic Riccati equations I am going to estimate g infinity okay and instead of implementing controller like this because I do not have true statement domain okay I am construct an observer see since I cannot implement the controller using the true statement domain.

Because they are not available I am going to construct an observer here and using the estimator state I am going to implement a control log what is this control log going to achieve move the system from at time 0 the state is not at 00 it will move to 00 this is very limited problem okay. Now I want to make it in to a realistic problem of moving the system from anywhere to anywhere okay, how do I modify this control log okay, so that I move from anywhere to anywhere be that what you want to ask?

This is achieve in a limited of this case moving from anywhere to 00 okay, so now how do I change this and right now I have said that under ideal conditions okay the observer in a controller together will give your stable close loop behavior so it is fine if you implement the control log using estimated state instead of the true states okay.

So observer controller is a thing which is sort of married together you cannot implement state feedback control or without an observer you have to have an observer, observer will give you state estimates or a sob sensor will give you estimates of the unmeasured states using those states you implement a controller okay. So other thing is that there can be mismatch between the model and the plant if there is a mismatch between the model and the plant then this control log may not take you to the decide location 00 that is one the problem.

So we have add some kind of integral actions what is done it what will happen is if you do not if you just use this control log blindly okay when there is a model plant dis match or there is the disturbance you will get in offset you will not the system will not move to 00 it will settle at some other point okay. So now what is done in PID controller you all of you know that if you just have proportional controller okay then what happens? Is that you may get offset okay.
So you need to introduce some way of integral action in to this system okay, now there are several ways of introducing integral action in to the system I am going to talk of two different ways of which I will emphasize one and the second one also I will talk about but, so what if I want to move the system to some non 0 initial state what I want to move the system to some set point okay which are specify I want to move the system to a given set point okay not to 00, see you take this guardable tank setup in guardable tank setup when you have quarterb the system and develop the state space model 00 means the initial steady state okay.

Now what is I want to change the level set points I do not want to keep system only at the initial steady state all the time I want to move to some other set point okay, so that is a possibility second thing is you know my model at the plant need not be identical you know that real system is non linear you are identify the linear state space there is a approximation so wanted in the plant or not exact so that is going to clear some problems okay.

The third problem is that there could be drift in disturbance which all are accounted for when I model okay so all these three things are going to clear the problem. So I am going to modify now my control log using these two additional terms here okay, this xs is the steady state target okay, and us is the steady state input now what is this? I am modifying this control log if you see here if you see this I am trying to keep the form of the control log same state feedback except I am trying to apply some corrections here okay.

So the input and to the state okay these corrections how do I compute this correction we come to that okay, how do I compute this but finally realistic lq controller which I am going to implement is going to be uk = usk – I have removed here g infinity this is g infinity that is why I am writing g infinity every time or this is the g here x – xs okay. Now the question is how do I find out us and xs okay.

Such that I reach the desire set point arbitrary set point I reject the entire disturbances okay my controller will work even if there is a model plant dis match okay this is my target now. So now I am moving from a unrealistic but mathematically convenient formulation to a realistic formulation okay where I am going to use the results of my previous part where my results of previous parts are going to come, g infinity okay.

I am going to compute g infinity using algebraic Riccati equations those are started out earlier okay. Now I know how to compute gain now given this gain I want to only sort of the problem of okay non 0 initial state or non 0 final state disturbances okay model plant dis match always these I want to forgot out now. So these are called as target states okay, now let us move a little bit to this just look at this equation.

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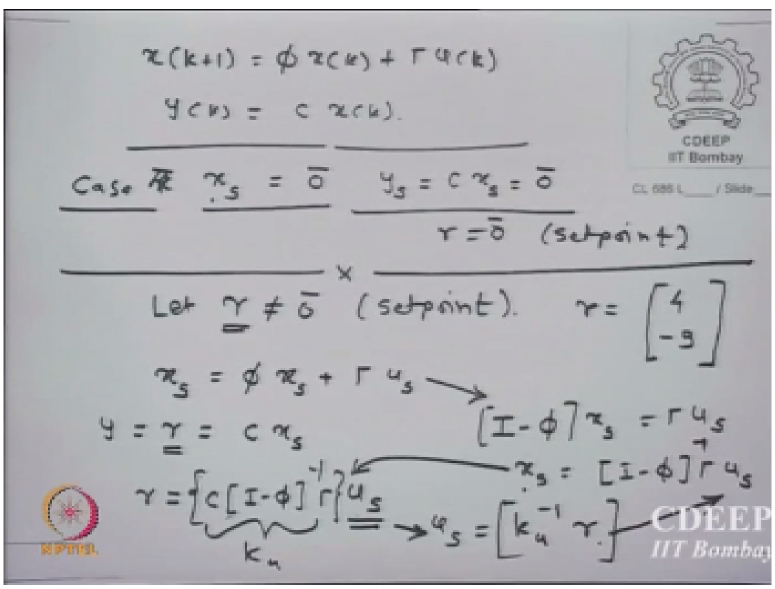

This is my model okay, right now do not worry about observer controller all that this is the model okay now when you say that your final steady state let us call the final steady state it for the time been as x okay let us see the same time model xs if xs = 0 0 vector what will be y? y is c xs 0 vector dimensions will be different this will be n x 1 this will be r x1 there are r outputs there are n state okay.

So the dimensions of these two vectors will be different but these are to 0 vectors okay, now so which means what is the final set point here output set point 0 when you say that I want to reach the target state of 00 the final state is 0 okay. Now just keep this as a side so this is the part now let us say I want to reach the set point okay which is given by r okay, so this is my in this particular case so when initial at case xs = 0 ys = 0 so r = 0 so this is my so r also becomes equal to 0 vector so r is the set point okay.

What is I want to reach r set point which is non 0 okay, so let r is not equal to 0 now this is my set point okay. Now what I am going to do I am going to find out the value of the steady state okay I am going to find out value of the steady state xs okay which corresponds to r which is not equal to 0 okay r is suppose you know we get this power level pole time problem and then r you have given as you know 4 cm and -3 cm, I want to reach these are deviation variables we have derivation variables we represent in the initial state I want to go to + 4 and -3 okay in tank one and tank two I want to raise the level of tank one I want to reduce the level of tank two okay.

So this is my order so what is the steady state that will give me this r can I find this out I can find this out because I have this model so what is the steady state let me call this as xs = φ xs + γ us I still do not know what is this xs and us I have to find out okay and r = c xs where I want reach? I want to reach y = r okay, so I want to reach y = r = this okay r is given to me r is this. How can I find out xs from these two equations can I find out.

So from this equation I know that i – φ x xs = γ us okay so xs = I – φ inverse γ x us okay if I substitute this here I will get an equation r = c x I – φ inverse x γ us correct, is it okay everyone with me on this okay, so what should be us? Okay which will give me this r? if this matrix in the square matrix if the number of inputs is equal to number of set points if this matrix is the square matrix then number of measurements equal to it will happen then number of inputs equal to number of output see in the guardable time setup okay we have two measurements level one level two we have two inputs two wall positions okay.

This matrix actually this would be steady state gain matrix if you look carefully this is the steady state gain matrix okay this matrix let me call this matrix as ku okay then us = ku inverse x r okay, is everyone with me on this us = ku inverse in to r right if us = ku inverse in to r you substitute this here it will get xs okay. See what I have done is that I will found out this now this I substituted here then from this equation I got us and if I substitute us I will get is this clear is this equation clear?

Just try to derive it yourself it very simple okay and given see I am given a set point for the given set point I want to find out the steady state corresponding steady state I have modeled equation using the model equations I can actually find this particular steady state I can actually find the steady state okay is this clear, so let us not bother about it right now you understand this simplify thing if it is not a square matrix you can use the sever us okay.

You can use appropriate silo inverse okay right now take the simplify case it is square matrix if it is not a square matrix you can use silo inverse of this gain matrix, it is not always square but like we initially made lot of assumptions right now make a assumptions that it is square okay. So if you want to control a system okay you have a situation if you want to control a system at a specified set points then what is the condition normally input should be there and how many if you have some if you have say two outputs you have to have minimum two inputs you can have more than two inputs okay.

So it can be square in a set that the number of inputs have always should be always be equal to or more than the number of outputs if they are not then you cannot solve this problem okay. so if they are not equal or they are not more than the number of outputs then you cannot reach the decide set point is the fundamental see what is the number of just look at this algebraic equation if the number of inputs or less then the number of outputs can you unequally define the inputs which you will take the system to the decide outputs.

The input space is smaller than the outputs space then can you take system from anywhere to anywhere just linear algebra problem when you unequally solve this ask this question see I will put it in a abstract form.

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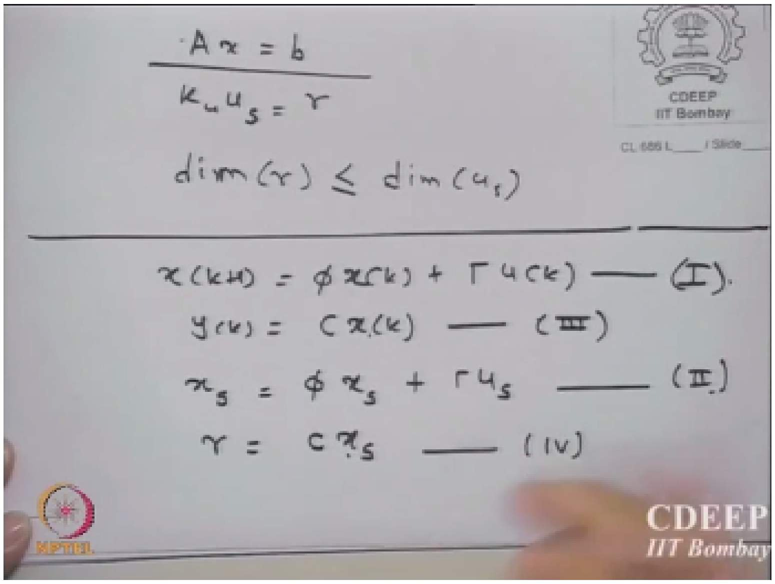

If I have a x = b okay when can you have a unique solution see this problem is not different k u x us = r these two problems are same map them and then see if the dimension of r okay is larger than dimension of us okay you cannot take system from you cannot manipulate input in take it from any set point to any set point the dimension of r so dimension of r should be less than or equal to dimension of us okay if that is the case I can always take situation like this is be equal know I may not want to manipulate more inputs I want to manipulate equal to the number of which are controllable they may not reachable they might be controllable to 00, but they may not be reachable okay.

Reach ability is the stronger condition so if you have example you show me we can look at it. So under a domain system there is always a problem okay so we are talking of well condition problem where system is you know number of inputs have more than the number of outputs and then we are able to derive the system from anywhere to anywhere that well, so if you have a situation for the dimension of us is larger than the dimension of r you can use seldom inverse to the power so you get one solution.

Okay let us go back here so how do I get this xs and us knowing r is clear from here at least for the ideal case where number of inputs is equal to number of set points or number of outputs okay. See what I am trying to do here is something like this continue on this page okay so you have found out you have the system x k + 1 = φ xk + γ uk and y k = c xk okay, and I have this another I have this steady state equation xs = φ xs + γ us r = c xs right I know how to find out xs and us now given r okay.

So all that I am going to do is to subtract equation one and equation two and equation three and equation four I am going to subtract this and this and this from this okay if I subtract what I will get is.

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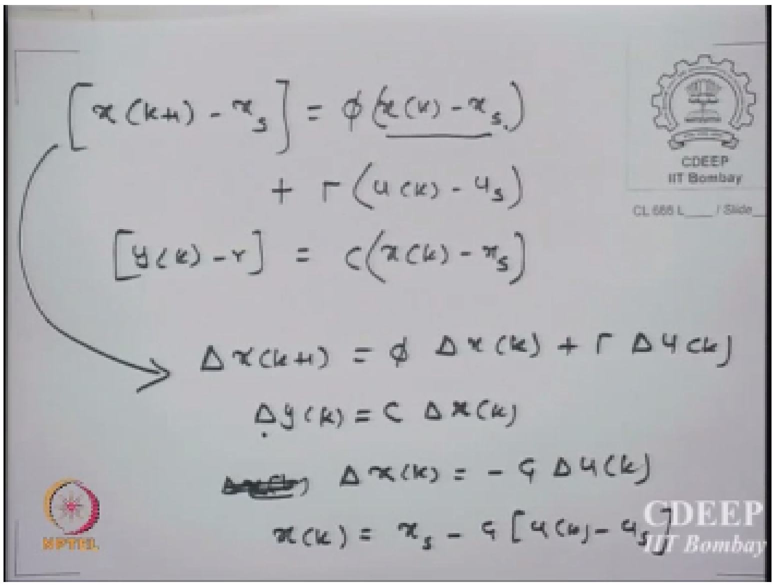

Is it okay and yk – r = c xk – xs everyone with me on this what you are consider I will consider yar you want to consider everything in one short is not possible see we develop things in pieces and the entire thing will become clear only after you do not jump okay just be patient we will consider noise we will consider all kinds of noise okay. I have to explain a concept by removing certain complications then you know see I can show you the final expression which looks very complex okay.

So to explain that I am removing certain you know things and then if 0 being going to noise going to change xs you tell me it is the 0 mean noise it is going to change xs now this is a non 0 mean noise x going to change how to deal with, I will coming to that okay, just wait just have some patients. So now this is this can be this equation you know I can write as Δ x k + 1 = φ Δ xk + γ Δ uk and Δ yk = c Δ xk is this fine.

Now this is a perturbation model perturbation around develop around this steady state okay this steady state will take to me this set point r okay, so the trick we do is to design a LQ controller for this perturb system okay what is the origin of this perturb system xs okay see origin of this perturb system is when you reach 00 where did you reach xs what you mean by reaching xs reaching xs means you are reaching set point r okay, see this is a trick I am going to do to give the same old control log in the new context I want to reach a set point which is non 0 okay.

I found out a steady state corresponding to that non 0 set point I subtracted created a new system which is this perturbation system okay, for this perturbation system I design a control log that is Δ xk – g Δg uk which is nothing but xk = xs – g uk – us you see what I am doing, I am just reacting this back in to the original form okay, so did you see this now what is the reason why I am doing this, yeah.

So the reason why I am modifying this control log okay is to find out that steady state which will be take me to the decide location okay.

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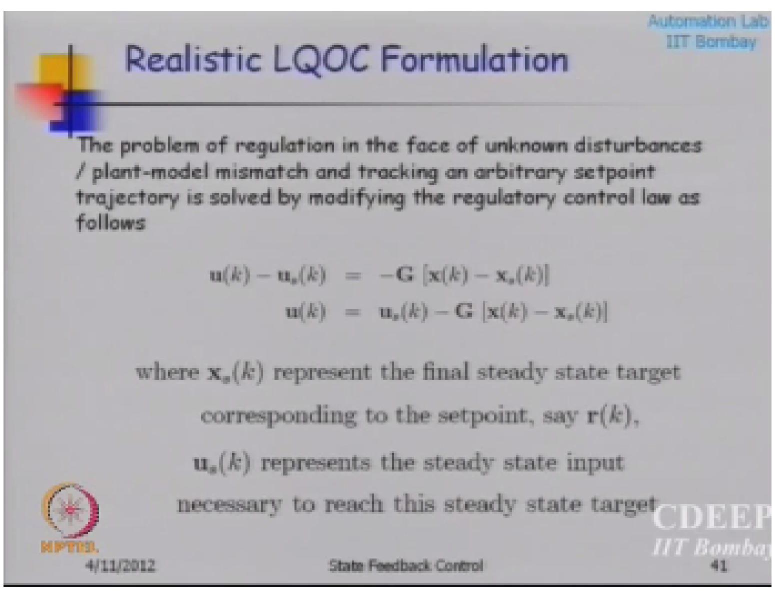

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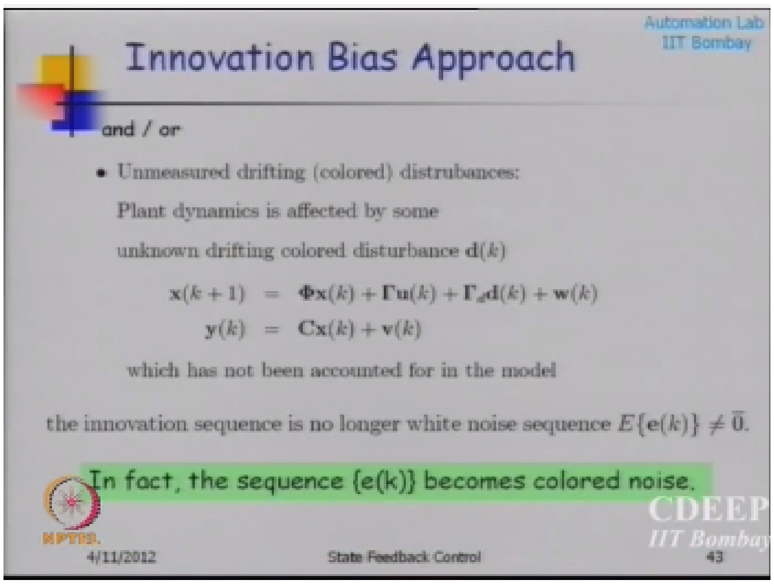

So I am going to talk about two approaches of dealing with model plan dis match noise color noise and all kinds of disturbances that can occur the first approach is I am going to talk about a implementation right now you will get more and more understanding about this as you actually work with it okay. So now observer is based on this okay, when the model is perfect what we know is that ek is a 0 mean signal it is like a uncorrelated white noise sequence it is a perfect white noise signal moment there is a dis match between the plant and the model moment it happens that the true plant evolves according to some π bar γ bar c bar which are different from φ γ c okay.

This ek here this ek here is no longer a white noise sequence okay is no longer a white noise sequence so this can happen under two situations one is that this φ bar γ bar and c bar are different from φ γ c it can happen when your model the true plant evolves let us say according to this there is another term here okay this is the drifting disturbance which you have not accounted for in your model when you develop the Kalman filter you never knew about this okay.

So you are not accounted for this guy here b but in the plant it is there okay, when these two things happen what will happen is that the innovation sequence is no longer a white noise okay this innovation sequence okay this innovation sequence becomes a colored noise I am skipping the proof if you want I can give you some of the reading material on this that why it becomes a color noise but right now trust me that it becomes a color noise.

So what is the meaning of it becoming a color noise if there is no plan model mismatch okay there is no plan model mismatch then ek would be something you know this is my ek, ek would be something like 0 means, so this is 0 okay so ek will be something like this.

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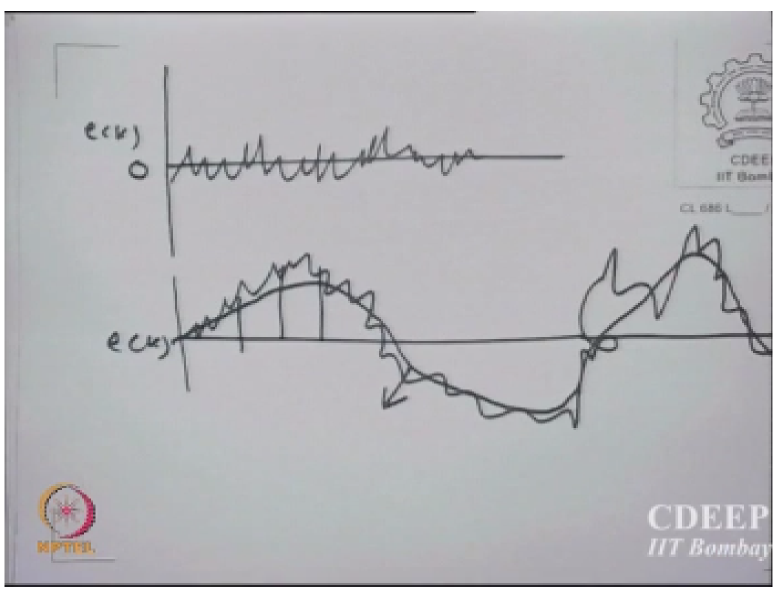

But moment there is a model plant mismatch this ek will be a drifting signal okay ek can be a drifting signal which is there is a time correlation between k and k – 1 k and k – φ you take any difference there will be time correlation it is no longer a 0 mean signal okay this is the 0 mean signal means actually if you take a moving average it will come out close to 0 okay here it will not come out to close to 0 here you will see that there is a trend okay.

Now one can of course do modeling of this online using all the time series method is that we have studied I am not going to get in to that right now I am going to have a simplified you know fix to deal with this particular problem that it becomes colored okay. What I am going to do is I am going to find out what is the drifting mean of this signal okay what is the mean of the signal which means I am right now interested in finding out if this is changing like this I am just interested in knocking of this high frequency noise I want to find out this trend okay.

I want to find out this dominant low frequency trend okay, and then I am going to take this dominant low frequency trend as an indicator of unmeasured disturbances okay if unmeasured disturbances where not there ek will be like this if it is there it will be sub drift, what is the drift? How do I get that drift from the data of ek which is coming okay so what I am going to do here for that is I am going to filter this signal okay this is the first order difference equation what is the filter do, what is the filter?

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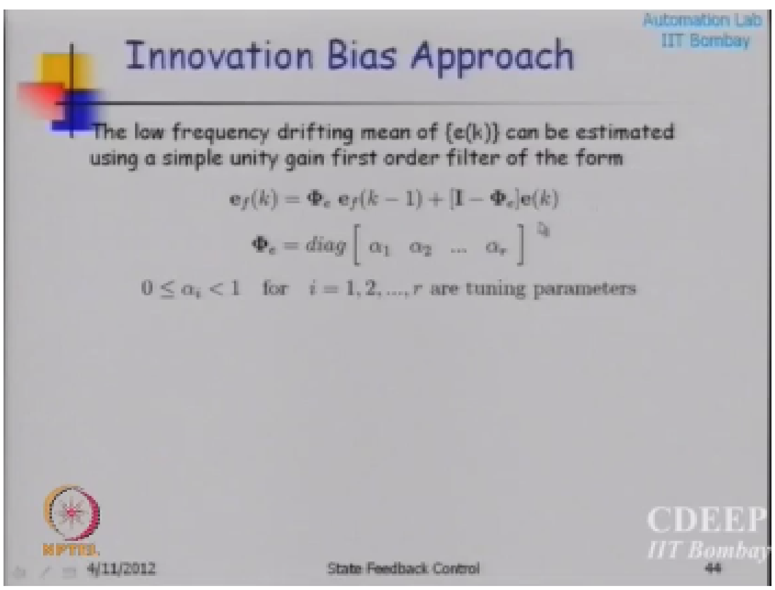

It filters certain frequency and it gives you a signal of low frequencies okay, filter is when I use the word filter okay particularly all those chemical engineers will start thinking in terms of something in you know electronics are engineering domain and we do not know what is this it is not like that filter is anything that filters high frequency signal in chemical engineering you know if I have a flow and if I want to remove high frequency oscillations from the flow what I will do is I will put a tank in between okay and the flow comes in to the tank and out flow of the tank is given to the reactor.

I have put a filter in between to falter out the high frequency noise okay the level in the tank will fluctuate but the flow out will not fluctuate okay, depending upon how broad or how small the tank is you will be filtering different frequencies in the flow signal okay. So a filter essentially when it comes to compute a programming a filter is nothing but a differential equation or a difference equation.

A difference equation is a filter okay a difference equation will filter input given to the difference equation the output of the difference equation will be a filter the output okay. So what will decide the filtering ability the time constant okay the time constant what is in the case of discrete time where does the time constant this is come in to picture sampling time and φ Eigen values of φ okay.

So I am going to specify this φ matrix okay it should be stable filter of course, so I have to chose this φ matrix okay who is I am going to chose this φ matrix to be a diagonal matrix okay α 1 α2 α3 α r so I am going to filter this using a signal which is you know this is the simple first order filter with 1 – φ e this will make sure that the gain of the filter is unity, how do you find out gain of the system?

The steady state gain, what is the input to this? The difference equation, what is the input? E is the input okay e filtered is the output okay e is the input okay and then this particular difference equation will give me a filtered output maybe let us do let us see whether in mat lab I can show you this okay.

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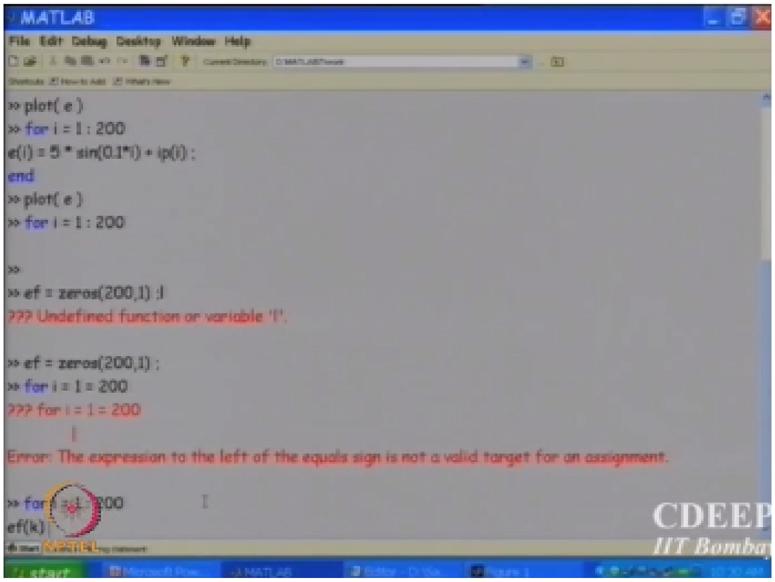

So I have created a signal which is colored noise which has some kind of this thing and when just say plot this ek e okay, I think let us do a little bit let us write this single dominate I going to reduce the, okay so this a drifting signal okay now I want to filter this signal okay so I am going to write a difference equation and then filter this okay. So for I = 1 to say 200 okay or I have to say ef e filtered let us create a dummy vector initially ef = 0 200, 1 and now for I = 1 to 200 e filtered I = let us take filtering constant to be 0.8 okay.

I want to create a unity en-filter so this is if this is 0.8 see this is filtering is *ef I + 1 = ef I + see this is the last filter value new filtered value is 0.8 times the last filter value okay plus 1 – 0.8 this will make sure that the gain is unity okay * ef ei okay, so this ef is a filtered signal now let us we have this signal here let us say hold on and I will say plot ef on the same graph you see here.

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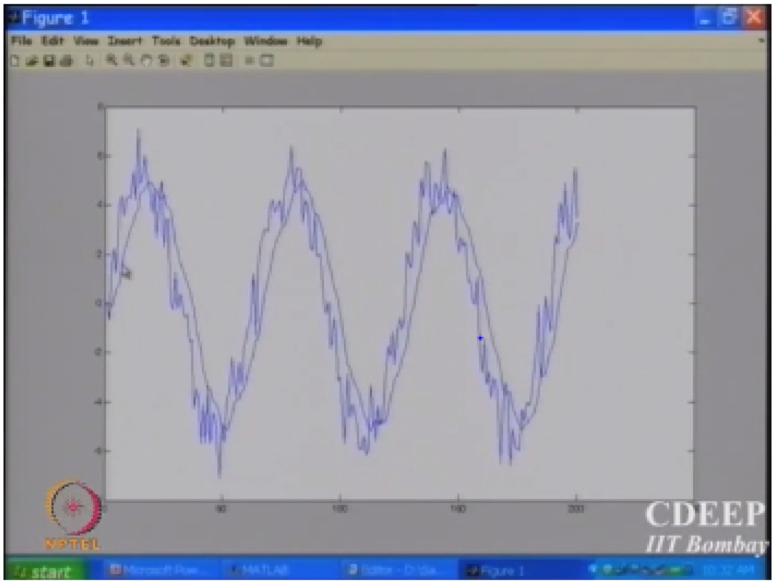

This is the time shift, this is my filter signal this is my original signal what have happen when I moved from filter, there is a shift like this that is okay that is fine okay what about smoothing if I were to see there is one time shift I should actually account for that but that I have not done here right now I can do that but what is happened here is this is smooth signal this is the smooth signal as compare to the so what is happen when I moved from here to here the high frequency component has been muffed off have I so moved see it is time shifted but have I recovered changing mean I have recovered changing mean see suppose I were to spurring pose this or this what will happen?

The high frequency is not off the low frequency drift is captured okay, see you can make it even less oscillatory by changing this see instead of 0.8 I can have this is 0.9 all make it 0.95 is 0.95 and this is 0.95 okay I have to make this equal to 1 – 0.95 so that the gain of the filter is unity steady state gain on the filtering unity my I do not want to change the gain of the signal I just want to knock off high frequency component okay.

And let me say again plot ef and I will say red color so I will see this well when I increased it there is a trouble it reduced it is not able to capture the full height okay, but the smoothness what about the smoothness? The smoothness is there okay, I could actually move other direction, so this is 0.6 see now 0.6 this black one is 0.6 it has less oscillation but it is trying to capture the mean pretty well as compare to 0.9 or this signal okay.

So there is way of recovering drifting mean and knocking of high frequency signal just by doing simple unity gain filtering okay, that is what I want to emphasize here is this clear from this picture, what I am trying to do.

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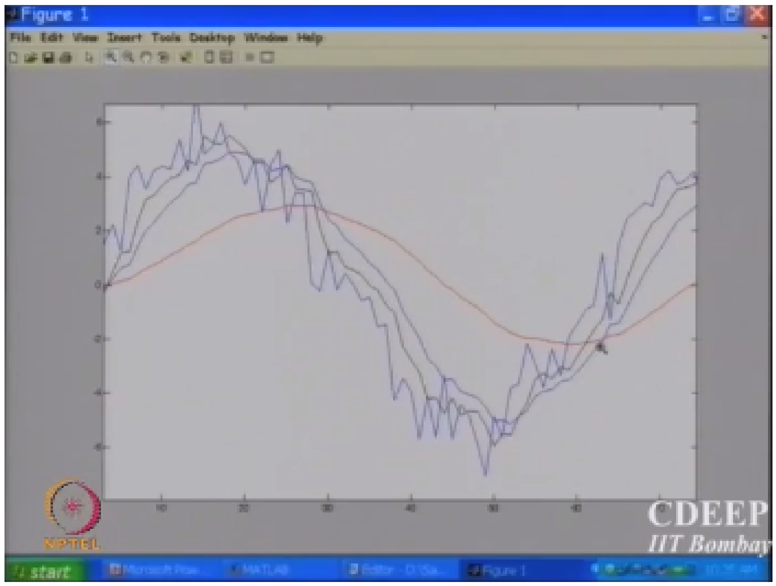

So this parameter which I am talking about here this parameter which so this ef is the signal which is containing the drifting mean for how do you chose this α is a golden question it is tuning parameter right now we have to chose it between say 0.6 and 0.9 we have to tune it to see how the close loop behaves okay typical value is 0.9 I would say but it depends upon what is you frequency contained and you have to have some experience on tuning this.

So I can use this filter signal okay and now I am going to so what is this filtered signal contained okay this filtered signal let us suppose that there is some optimal way of choosing this and knocking of the high frequency part high frequency part is like white noise low frequency part is like a drift okay. What is this drifting signal contain it contains everything that is not explain by the model what is that model plan mismatch it may have drifted disturbances okay.

Model plan mismatch drifted disturbances color disturbances non white disturbances everything is contained in this ek if the model was perfect if the true plan noise was perfectly wise then ek would be white noise but ek is not a white noise in reality ek is not white noise if it is not a white noise we try to find out its mean okay this mea signal contains the information about model plan mismatch unmeasured disturbances everything that is not expend by my model oaky fine.

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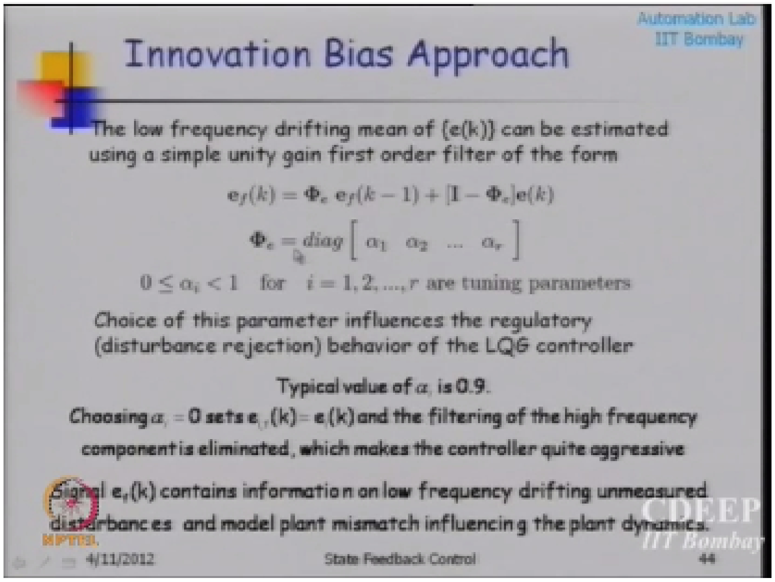

So now I am not making any when I move to this point I am not making any assumption on what kind of disturbances exist disturbances can be of any type okay colored non stationary. Now what I do is my controller is now modified like this xs and us and then what I do is I solve for this equation now okay I am going to find out steady state xs okay I am going to find out steady state xs for the specified rk, rk is my set point okay and putting this r not a s a constant earlier when I read the development I said there were the constant set point quart by the set point trajectory okay, so I am saying this to be rk you know the set point is changing at the function of time.

Now what is this ef here is the filtered innovation we created this filtered innovations here if you remember I have create this filtered innovations okay.

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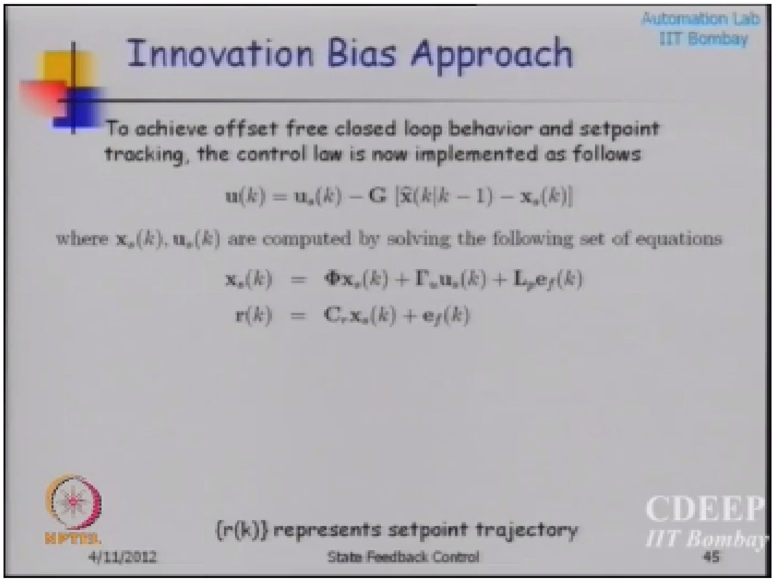

This filtered innovations are used here in this equation okay what is this component bringing in information unmeasured disturbances un- model dynamics everything is captured as an effect through this e filtered okay, this e filtered is changing as a function of time that means I cannot talk of one steady state target okay I have to talk of time varying steady state target okay. See that is why I am doing this so now I am going to solve this equation just like I solved a equation right now here okay.

In the same way we did this here except now the signals are time varying okay which means now this rk here is not constant r is not constant I am looking at rk okay, I am also looking at a correction in the state I am also looking at a correction in the output this is com through filter denotations okay yeah, so here we are so now I am going to solve for the study state here.

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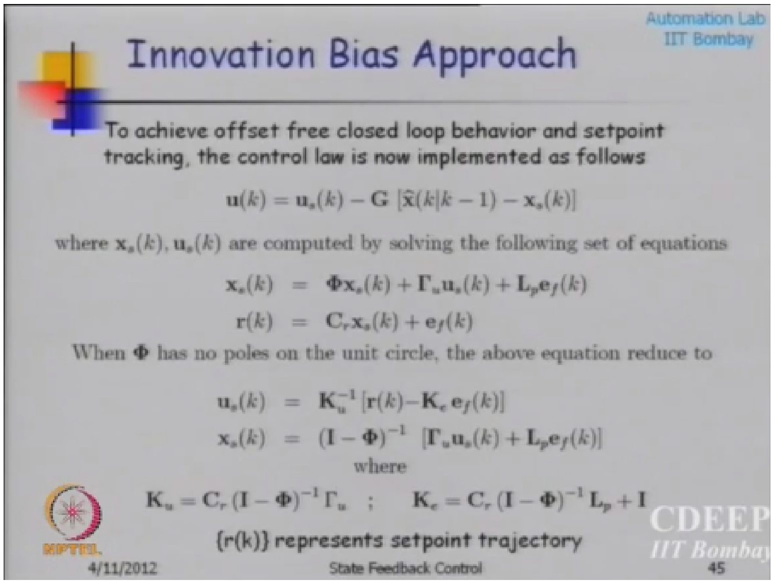

Okay and then this will give me of course the way I solved earlier I am going to solve for this okay this will give me U study state solving for this equation and then substituting for excess I will get x study state, so this is a time during excess Us which I get when I solve this okay and for this close system of course when the system is square you an write this as Ku inverse okay the system is not square you replace Ku invert that this is inverse the okay the same thing will work okay so this is my either my moving targets okay at this moving target I am going to.

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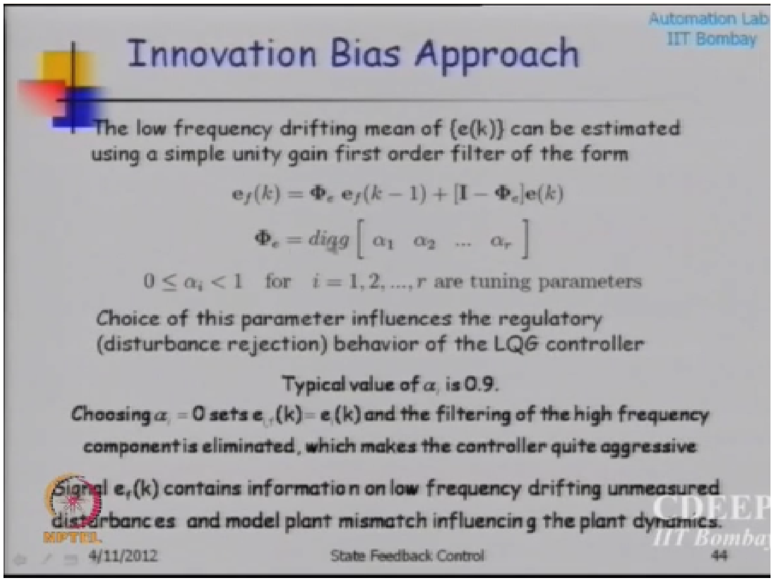

And with this moving targets I am going to implement this control law okay now this control law this modified control law will take care of model plant mismatch drifting disturbances will take care of everything okay even though I have done development in bit some cases initially assumed that there is no disturbances okay to most ideal case I just use that situation to find out g once I know how to fix g okay I will fix for in the disturbances model plant is more okay that is through there is tools.

Okay this is a fix which will help you to deal with model plan miss match and well your disturbances so actually when I implement the control law when you are going to do in your course at okay you are actually going to do this you are going to do this filtering of innovations we write the observer for the observer we will take innovations filter them.

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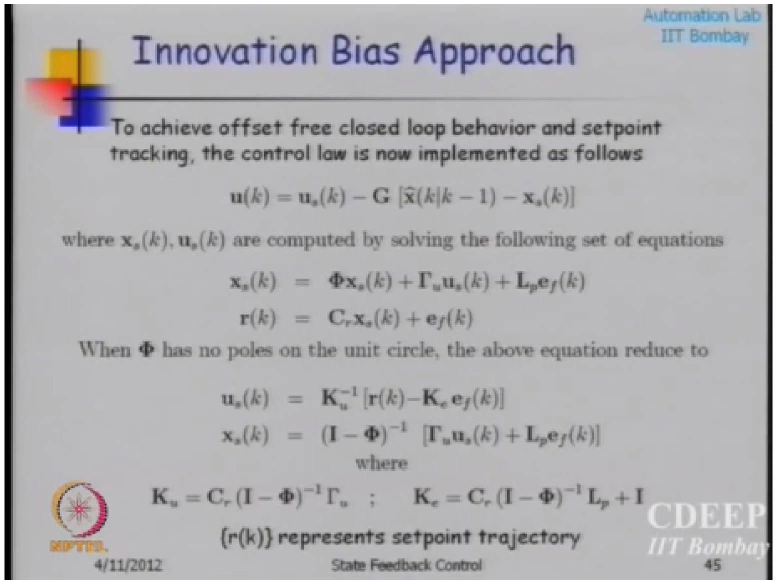

From the filter innovation at every time you spent at any time instant we are going to find out the new target state and the control law is going to be like this okay this control law will take care of everything okay this step control I will show an example of all this work and when you actually implement you will see how it works okay.

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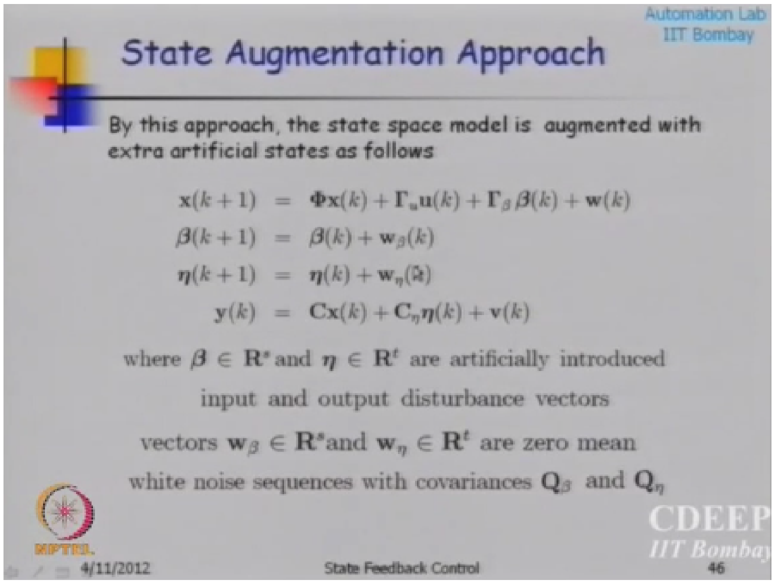

Now this is the only way of doing this there are other ways of doing this one is this is called a state augmentation approach what you can do is I am going to get too much into details of this or this is the similar approach what you do here is that you add extra states β and η your outgoing system using artificial extra states okay and artificial extra states are we get, so these are integrator so if you see here these are pure integrators so you add pure intetagtrors into your system model equation.

Arbitrary okay and then you develop a controller for this out bounded system okay and develop a observer in the outgoing system you develop a controller of outgoing system and the you take care of so these augmented this white nice here and the white nice here these are treated as loop tuning parameters and then on can then has to find out the co variances only is to tune in the co variance of this or particularly find the previous approach which I discuss this is the simpler approach this strict to this.

When you study come of first thing okay when you become an advance in your if you want to strict in the other approach you can do that this is the easier approach to get re like the color nice on model is miss match moving to only set point everything that the actually doing in this particular formulation.

Same thing can we actually through alternatively through what I have discussed here you can this go or go through this and I am not going to go just see here what we have done there is something like a state correction there is something like a output correction okay these are arbitrarily added augmented state first, how to add these how to choose these γ β and see η are given some.

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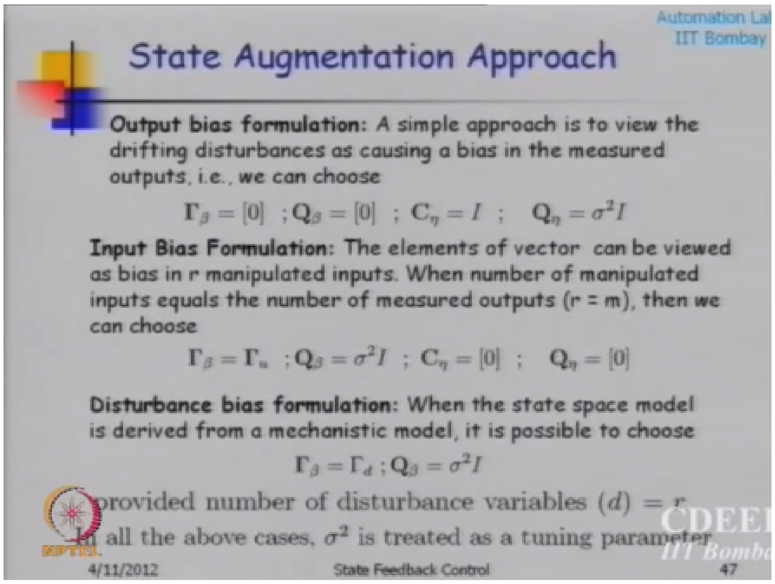

Some guidelines here there is a different ways of doing something output bias formulation is something called input bias formulation you can choose these drifting bias to be in the outputs you can choose it to the inputs you choose to be input and output then above combinations you can do, so let us provided the number of disturbance and you can outgoing the state place model.

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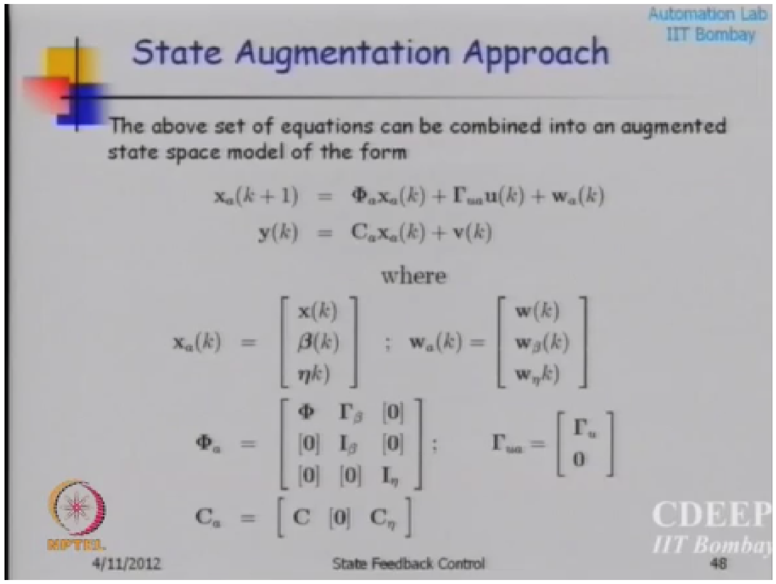

Then you work with augmented state place model and then develop your controller to the augmented systems and so I work with this augmented system okay I have given the methods of augmentation we work with the augmented controller augmented model develop our UT controller artificial introduction of integrating states with remove this model plan miss match w will remove the offset sorry to remove the offset okay we will not remove the model plan miss match because you arbitrary adding some integrators w are making some fixes to a model to make sure.

And this is just to make sure all this is just to make sure that there is no offset okay this is not this a fix you should remember that you know fix.

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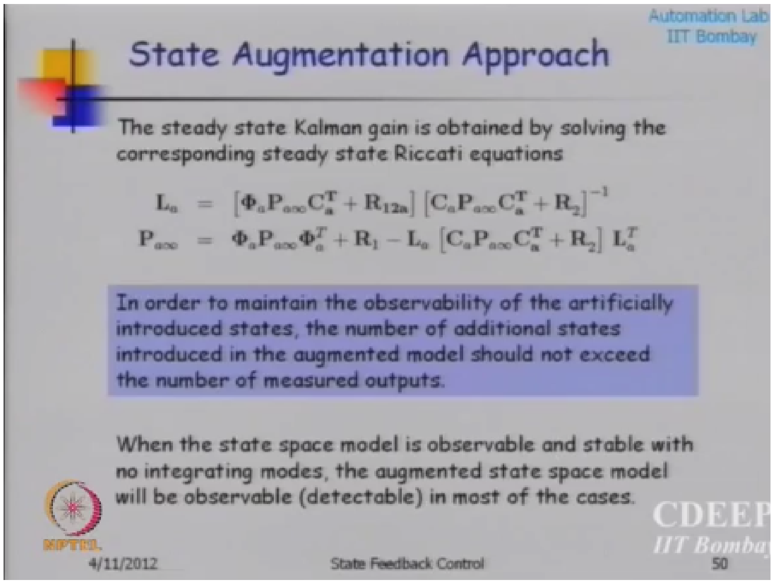

Yeah this is the equation so there is the fundamental limitation that you cannot add to the last statement to maintain the conservative of the additional states you cannot add more than number of outputs and there are 20 states and 5 outputs measurement measured outputs you can add utmost 5 artificial variations okay, so those could be out of that all 5 can be in the input all 5 can be in the output 3 in the input 2 in the output whatever you cannot add more than 5 to the fundamental limitations.

As to okay actually the innovation bias approach which I talked about is also adding artificial variables and it is not very clear here you have to work out some little bit then it is exactly adding equal to number of outputs because innovations number of innovations is equal to number of outputs okay, so that condition is perfectly here we have to make sure that you should not add more than in wise you choose the observability then you cannot design the observer and then you get into problem.

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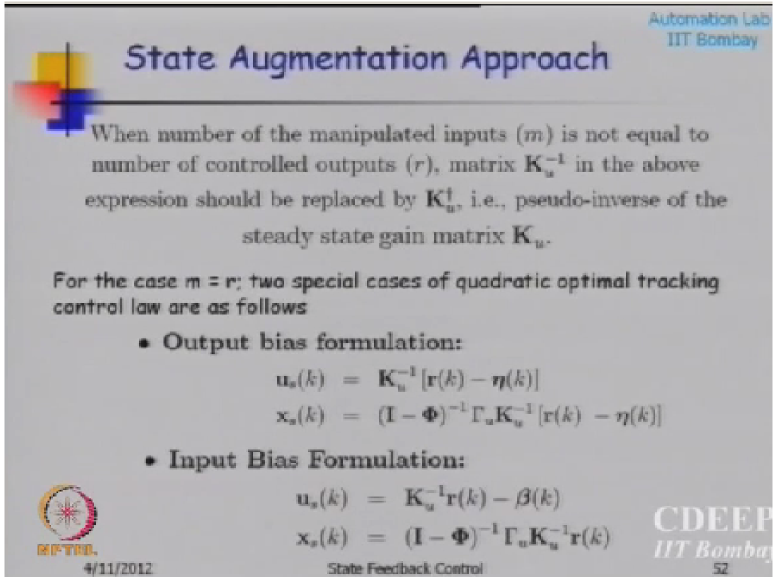

So I just show you an example here we have to well we have to same things we have to do when is your augmentation there to find out the targets state you modify the controller and in the same way then you implement the controller does not equals everything is set and then you know well I have mentioned this here that in case Ku is not in multiple in use you know should inverse so all add that is.

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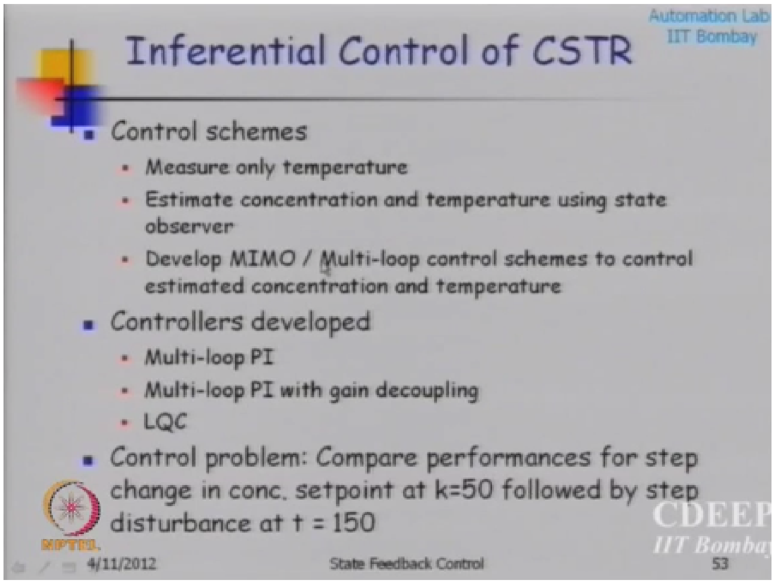

I am just keeping through this because this is another approach you can strict to one of them for the completeness I have put the other approach in my loops okay, so further if you go and if you want to use if you are not convince with the innovation bias approach you can use the other one like okay so what I have done here is I am going to compare I am going to show to that I am going to develop our control scheme for a CSTR the reactor for which we have been looking at from a quite some time.

I am going to measure only temperature estimate concentration and temperature using the state observer okay and then I am going to develop multi loop control scheme to PI controllers and multi variable control scheme LQC okay then I am going to compare three controllers one is multi loop PI controller multi loop PI controller with decoupling okay and LQC controller okay and my controls problem is like this I want to give a step change in the concentration set point keep the temperature set point same.

I want to ramp up the concentration set point and then I want to introduce a unmeasured disturbance my controller should be able to do reject the disturbance move the system to the new set point okay all the 3 controllers are given same task and we will see how the close loop performances.

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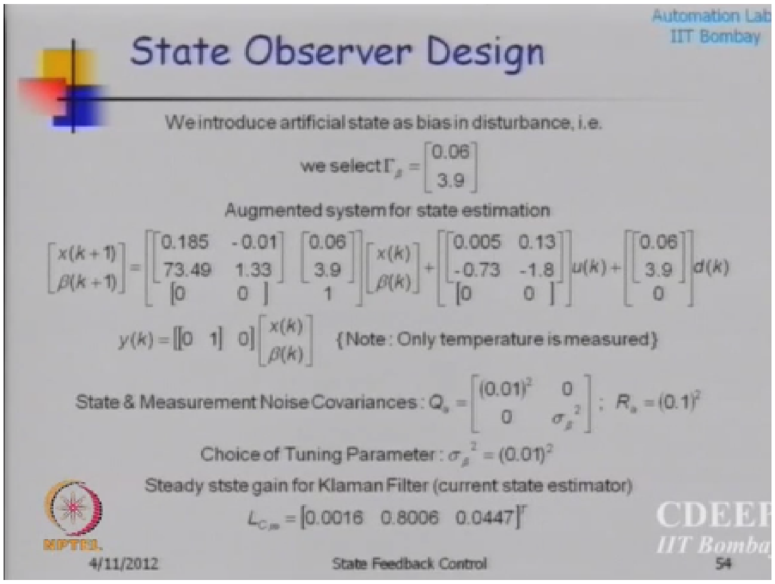

So this is I have done this using state augmentation approach I could not I am using input bias approach or the innovation bias approach research could not be different and this shown here how you design the controller, so I get here some control law which is a observer gain here and also the controller gained I the controller gain is not for the orient system and the observer gain is past on the augmented system and then you know do the controller implantation to the augmented state takes place.

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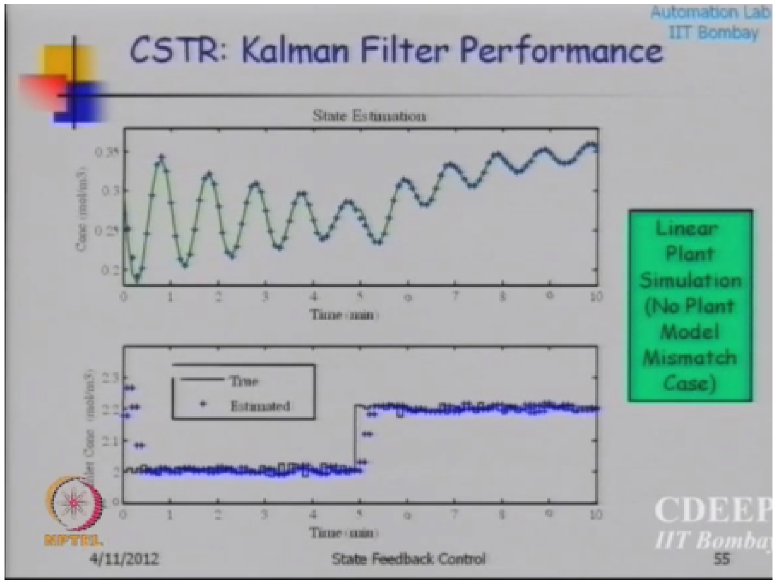

Okay so just see here first I am just showing you the observer initially I have juts design the observer okay observer designed by doing the state augmentation I am assuming that result this thing disturbance in the I am measured into detail okay and then the augmented takes place model only the temperature is measured concentration is not measured this new augmented state okay which is introduced here, but you also not measured okay so now just see here model no model miss match.

That means planted linear model is linear everything is perfect okay I gave a step change in the inlet concentration as a disturbance my observer is able to track this I am not measuring in that concentration my observer able to track inlet concentration okay, so I have a state estimator which estimates not only the mind you even the concentration is not measured only temperature is measured and estimating the reactor concentration I am also estimating inlet concentration I am also estimating change in the inlet concentration.

Through this augmentation okay these augmentation I am also trying to track that so it is able to find out the change in the okay, so it is like through the model I have disturbance observer okay if you the true disturbance is not measured but through the model line constructing and estimate which is perfect quite okay this disturbance estimated called okay I can do control which is state power control right I know I have a disturbance measurement now in direct but I have to measured I can actually so my LQ controller will get convert into a heat solver controller well I am just showing here what happens.

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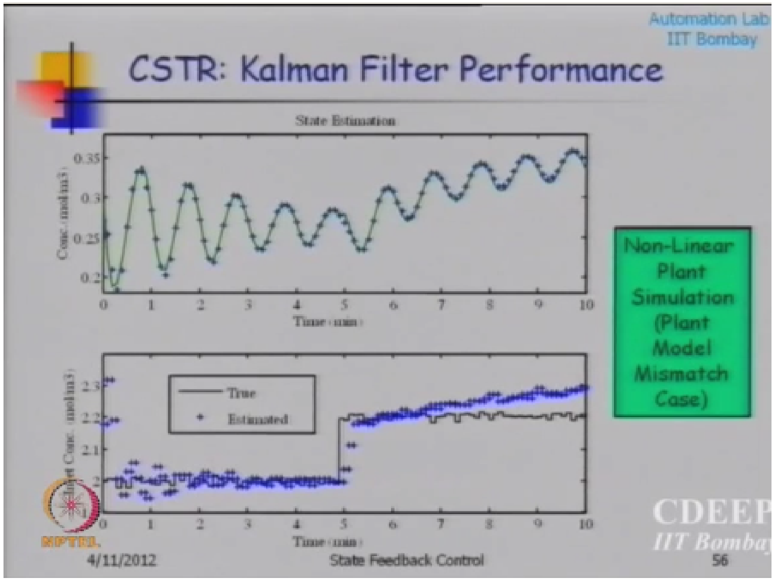

If the plant is non linear and observer linear there is a slide miss match okay but nevertheless it does capture the step changed okay, so the observer is bad now but is okay it is giving you something is better than having low information about and then this miss match will be taking care by the.

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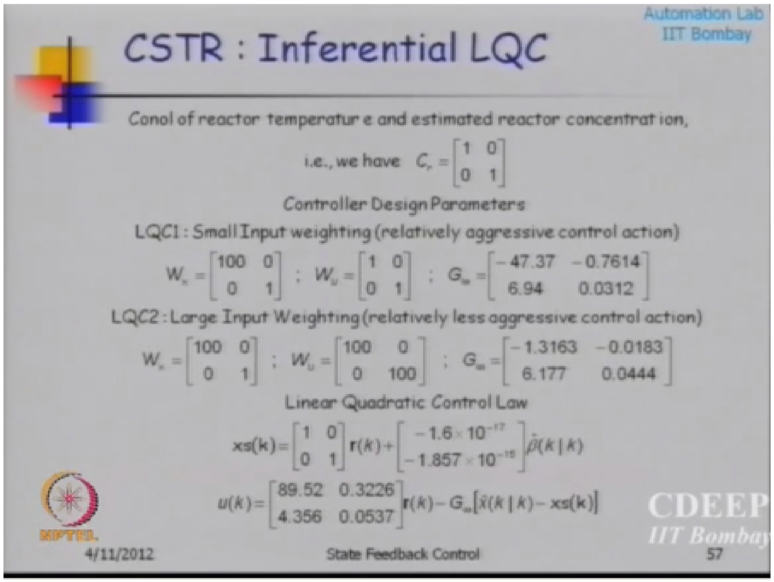

My IQ controller I choose some W_x matrix are to choose a waiting matrix for state I will choose a waiting matrix for input okay and then this g∞ is found using DLQR supportive of the metal you just give 5 γ matrix to DLQ here you give this waiting matrices okay, it will give you we were solve the equation for you give you g∞ okay we give g∞ so this is your and then with this g∞ I have just shown you two different ways of getting g∞ times once this W_x is 101 in other case so this in this case I increased by waiting here.

So here it was 11 here I need it 100, 100 okay what is your difference between this and this controller if the input waiting is more the movement of u will be smaller so the gain will be smaller okay this will be a smuglesh controller this will be a more active controller that is reflected to the gain values here you can see this is – 47 this is – 1 okay, so the values in the gain matrix have been used what is this telling what is the first x variable concentration what is the second x variable temperature.

I am saying that change in the concentration values should be multiplied by 100 give more importance to perturbations in the concentrations than in the temperature okay so my controller will bother more about controlling the concentration by controlling the temperature okay so these values have to used to reduce up the performance you know which is what is more important what is important you can tell you well the controller to be 1 okay.

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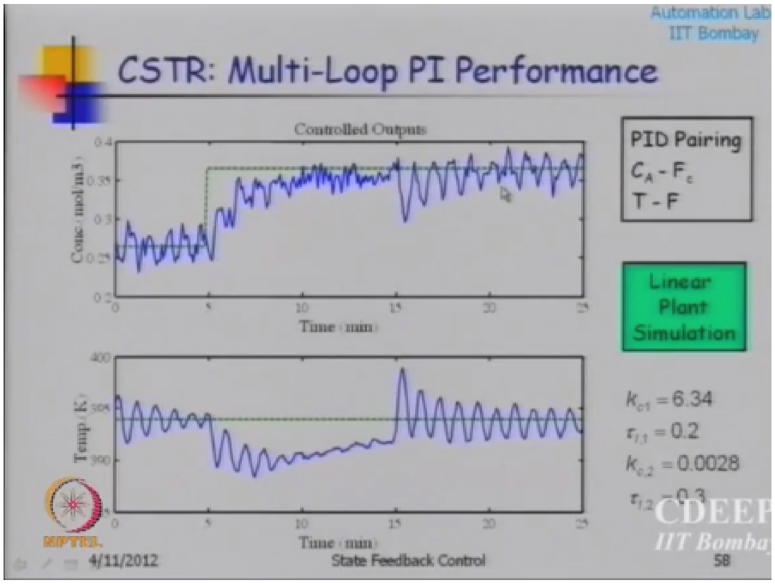

So just this will because I will show you once two controllers okay two PID controllers which are not coordinated okay it is a disaster you can see with somehow takes it here okay it somehow react the disturbance okay but what is happening is you know the system is all over which it is not a great control mind you I am not doing anything here anti I have chosen a controller PID to design which is using the text books methods I am just using them you know cool place mean method or whatever I used.

And I have not derivatively found the PI controllers setting which you give me back that model is this is by the design using the design approach okay now once I move to controllers with gain decoupling okay does it help it does not okay you can see here but this performance at least visually case less oscillatory okay it is this is settling faster here so this is settling much faster whereas here it does not settled okay it is setting much longer than the settle whereas with decoupling it results to improve.

Then look as better with decoupling than without decoupling okay everything is fine is I am using linear plant simulation okay,

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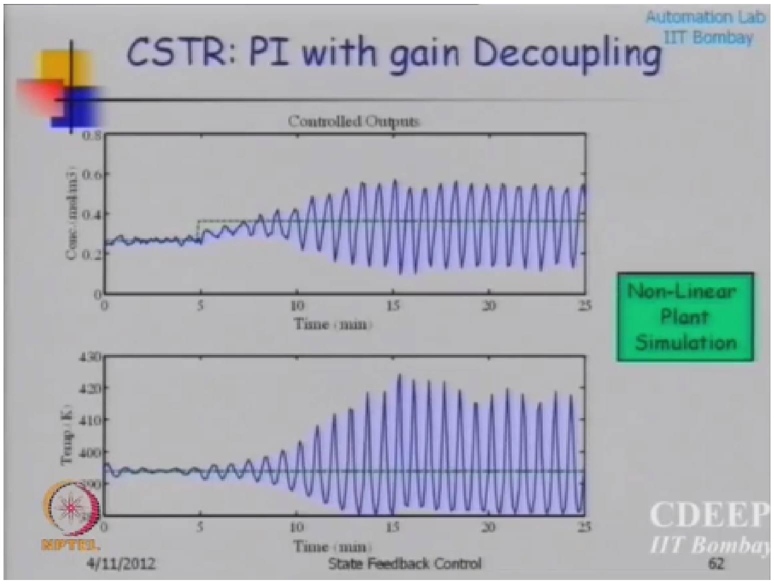

Move in or move to non linear plant simulation okay this decoupling seems to go there okay in this decoupling seems to work when planted linear model is linear everything is perfect I will just keep the controller same just change plant simulation from linear to non linear plant simulation may be decoupling seems to be work okay.

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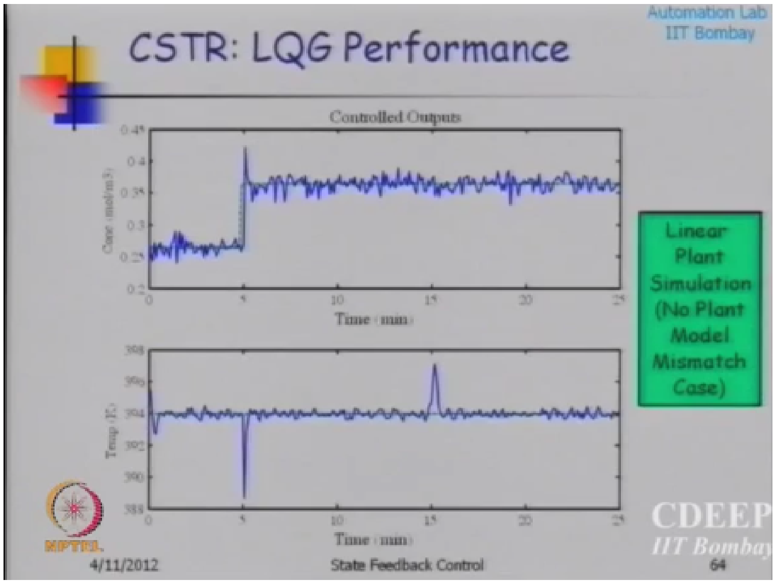

So decoupling is fine look at performance of LQG I mean just visually you can see what is the difference okay see this performance and see this performance I just gave as step change the time by you in this time 25 and this time 25 is same simulation for the same time okay here it took about 5 minutes to reach or 4 or 5 or 10 minutes to be in the set point okay look here I just gave a set point change as if you know there is no de line of the system it may come to this dual set point okay now the small blip here.

Okay the other blip which is not affected there is no loop such way we have two different loops is nothing like to 2 PID controllers starting which each other in earlier case the two controller loops where fighting with each other here is nothing is that okay.

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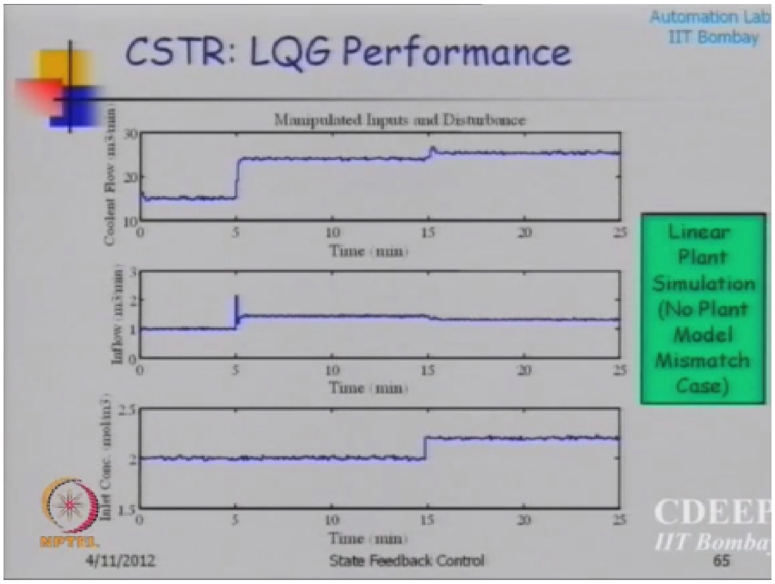

Well you will say that this is happening so nicely because of model plant miss matching not there plant is linear model is linear everything is perfect LQ controller you know we are finding out I just change to non liner plant simulation not much this has change slightly okay this disturbance is not change slightly for not much better okay they are pretty much the same comptroller behavior as you got in the earlier okay this is much more robust controller it is performance is much better than this is the multi variable controller.

It simultaneously changes both the inputs okay by using reconstructed states okay it does much better there when two PID controllers acting independent even for a such thing whether two loops imagine when they are compiler or when you are system there are multiple such loops acting together that is why you need multiple operator to our continuously watching the plant and you know it becomes and it may run a bit math because there are so many loops all of them could be fighting.

Or some of them could be helping you know it is a, so what is the problem what ar difficulties with LQG.

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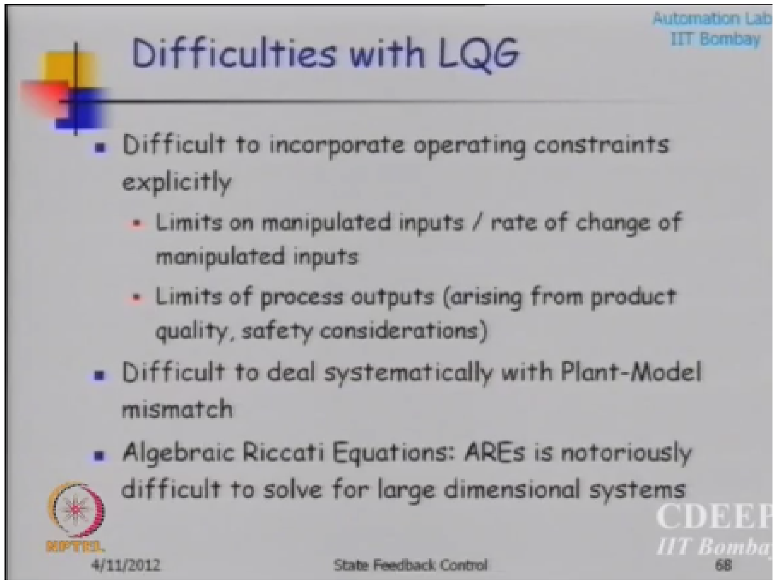

So actually seems to be a good solution very nice by people think of moving away from it what is required, so I cannot impose constraints okay for example here just I wanted to say that you know the temperature should never cross 396 and actually it is crossing 396 here okay or ti should never go below 390 okay can I say this in a LQG controller I cannot because LQG controller us gain times x estimated you know if it is estimated is large you will come out to be large it will can I say that you cannot be higher than something.

You cannot be lower than something all these constraints which are in real life there are constraints which I cannot specify to the LQG controller because it just finds on game multiplied that we are fix to deal with offset removal but you know we do not have a way of drilling with constraints okay, were also some problem estimated with using that model plant miss match I would not go into right now then difficult to this if you have a large plant okay I am going to talk about plant control by model plant control.

Which is as large as 600 outputs and 280 inputs simultaneously huge system okay for such a u system solving algebraic regard equations to get a gain matrix is a night mat you cannot do it with the reliable manner it is very, very difficult to solve ultimately with that equations even that some up new when you will start using it cross section we have a 6 states of things we have difficulties.

So can you solve crossing impulse systems more problems 5 so more difficulties when you move to plants okay it is difficult to use errors so they have they give you a good theory it is everything is found good understanding but you need something more than this okay that is where this model plant control will come in.

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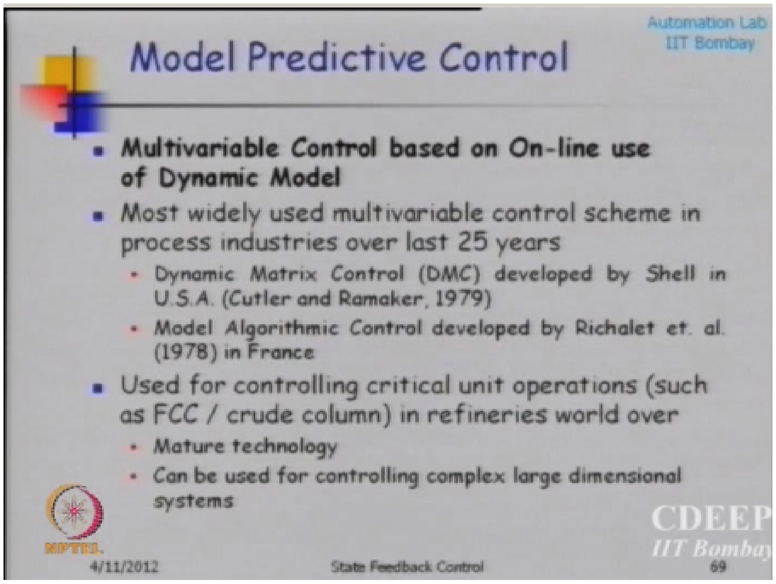

It is a multi variable controller that you just online dynamic model which is one of the most widely used multi variable control schemes in the industry well developed the something is different about this is that this particular control scheme emerged sin the industry and then move into academics more solvent control schemes where first developed in the laboratories is universities and then you know may be moved into a industrial practices this is something different this is develop by industrial practicing and then between the academics started doing there.

You know mathematics to show why it works okay so we will talk about it in the next lecture so it is a very natural ecology by known and then I will talk about it more and you can actually control where it complex larger dimensional systems okay, so with this I will close the lecture.

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