NPTEL

**NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING**

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ADVANCE PROCESS CONTROL

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Lecture No. 02

**Topic:
Development of Control
Relevant Linear Perturbation
Models (Part 1)**

**Sub-Topic:
Linearization of Mechanistic Models'**

Let us begin the development of control relevant models, last time I gave you the overview of what we are going to do.

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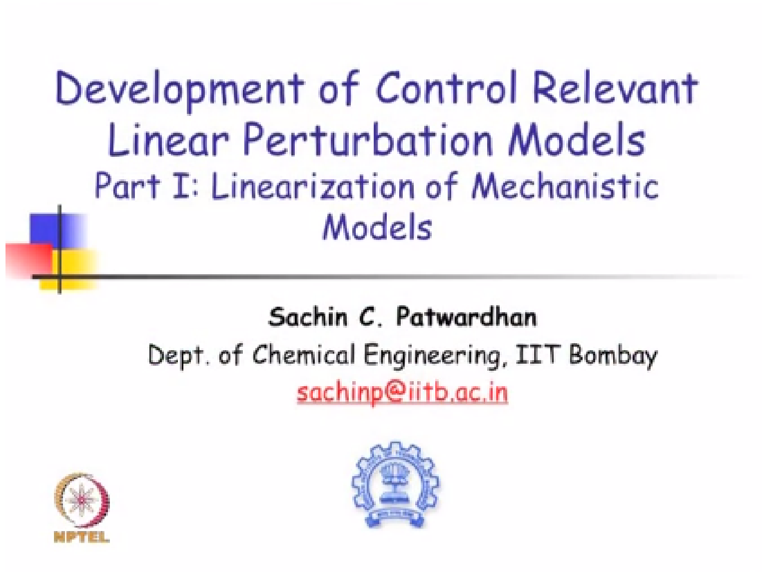

Now I am going to start looking at each component. We will begin with modeling control relevant modeling. My course, this particular course is going to be limited to linear controller synthesis or it will be very precise controller synthesize using linear dynamic models. The resulting controller be turnout to be non linear in some cases, but the – which is used for designing controller is a linear dynamic model.

So the question is how do I develop a linear dynamic model which is controlled relevant. I am going to do it two ways, one is start with models that are developed from physics, I am calling them as mechanistic models. The second class is developing models directly from data, and then both of them finally will have same structure, but probably interpretation of variables might be different.

Okay, so let us begin the development of linearization of mechanistic model, this is my first module. Well the way I want to run this course this time is mostly I am going to rely on my slides even for the intermediate steps. We have time and anywhere you want to stop me, ask me some doubts you can do that, I will explain you. If something is not clear from the slides I will explain you on the sheet by writing.
So please stop me okay. I will also upload overall schedule for the entire course how the sequences of lectures is going to be.

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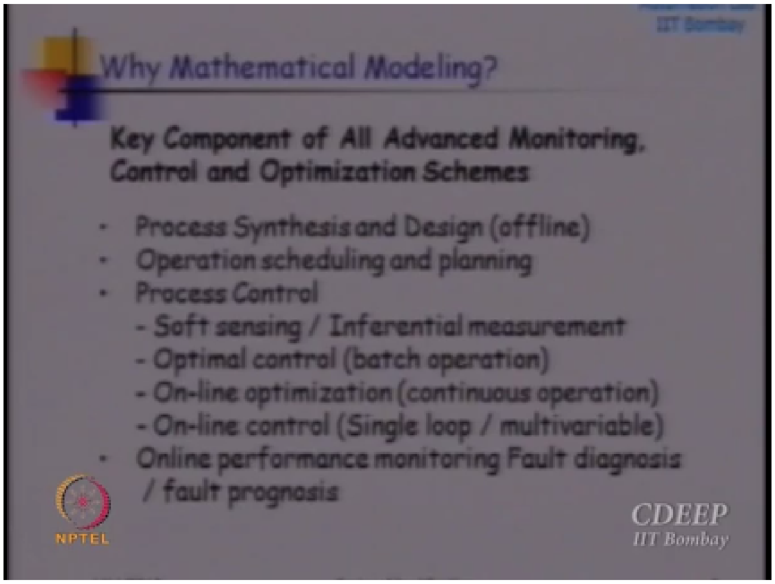

So this is something which I had earlier talked about. Why do we need mathematical modeling? Mathematical modeling is required in controls of variety of reasons. We probably have to do design of a system, so we need mathematical modeling, that is not concerned in this course. The concern in this course is the next part which is operation. Operation on minute to minute, second to second basis.

So process control that is what it is our control, the current objective. Well in control there are variety of things for example, there is something called soft sensing, I will be talking about it a little later or estimating variables which are not directly measurable. Optimal control, because you want to move a system from one state to other state in some optimal fashion. You may want to optimize the operation of the plant online.

Or you may want to implement single loop or multi loop controllers. Sometimes you need models for doing online fault diagnosis, the system start behaving malfunctioning, you want to find out what is the reason, what is the route cause? Through the route cause analysis we can use dynamic models that is one of the major uses of dynamic modeling.

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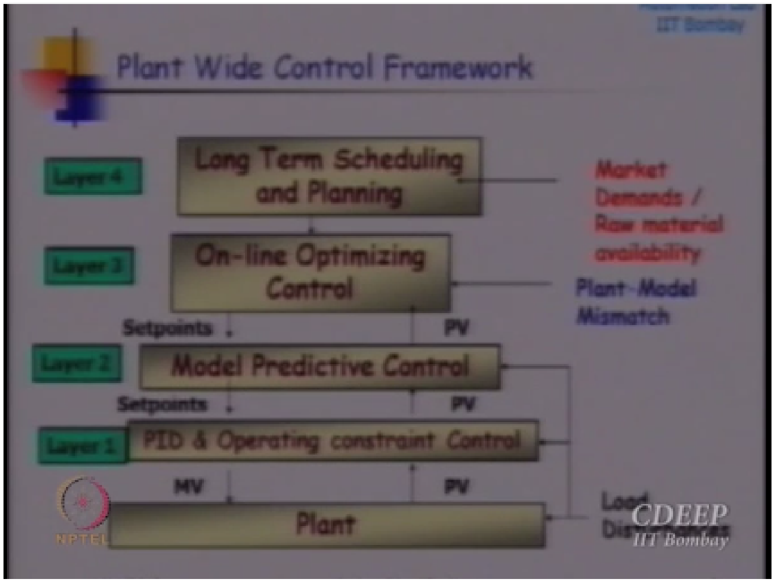

I had put this figure earlier, I am just going to connect again. There are different labels of models that you need for different tasks. These, as I said there are four or five layers and we are going to concentrate mainly on this layer, model predictive control, this particular layer or multivariable control for a plant. I am not going to talk about the layers about, and I am not going to talk too much about PID controllers or single loop controllers.

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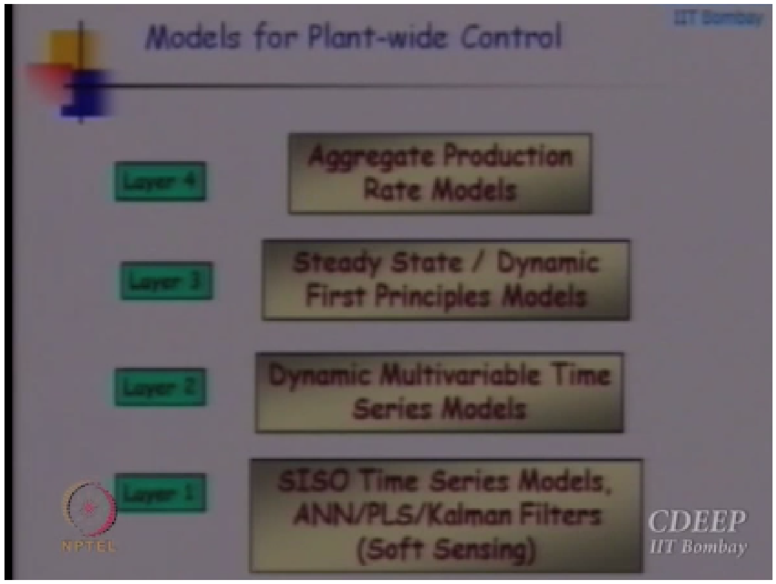

So which kind of models I am gaping to develop, I am going to develop models for layer 2, layer 2 is dynamic multivariable time series models. Well, probably you understand right now only word dynamic, dynamic means the model that capture the dynamics, the remaining part will become clear as we go along.

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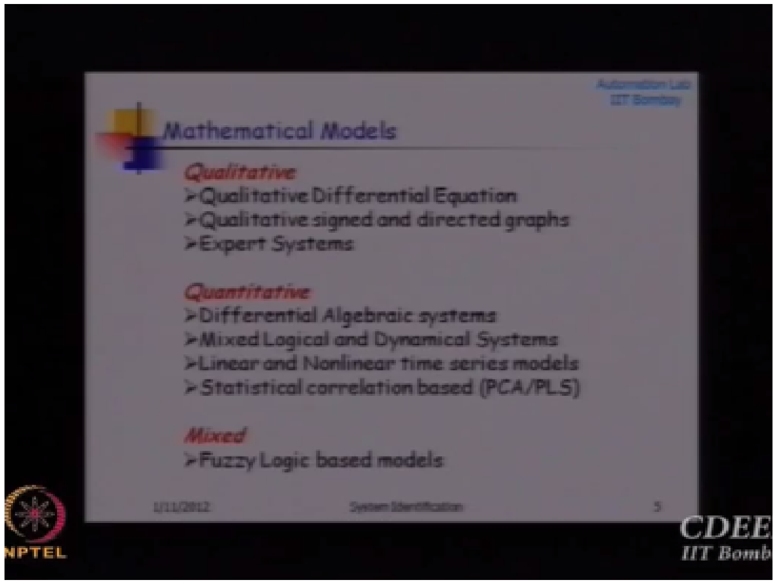

So what kind of mathematical models one can think of? Okay, one kind of mathematical model is qualitative models. There are different kinds of qualitative models which are used in control, whether we are going to use them in this course is a different story, initially I just want to, what kind of models exist, what kind of models can be used for control. And one of them is, one class is qualitative models.

Well you have qualitative differential equations there is whole lot of your theory associated with this in the period of AI. Then you can use find directed graphs that is also comes from AI tools, expert systems. Other class of models which as engineers we use much more are quantitative models. So the classic form here is differential algebraic equations okay, differential equations or algebraic equations or differential algebraic equations coupled, this is what is typically the form of models that you come from physics, that you develop from basic underlying physics of a particular system.

There are other class of models in which you have to deal with logical variables okay. and then you can have a system which is mixed logical dynamical system. There are condition if this happens, then take certain action or if temperature is high shut off your steam ball for example these kind of actions are there in, where you take and design and controller. And these kind of conditions can be modeled using mixed logical dynamical systems.

Other class of models are these time series models which are essentially developed from data. And we will look at in detail these kind of models. There are statistical correlation based models, PLS or principle component analysis and so on. And then finally there are mixed models which are semi quantitative, semi qualitative. The word Fuzzy systems you might have heard is for these systems are semi quantitative and semi qualitative models.

Well people a word Fuzzy control is because of washing machines, you will have some sales literature saying that this washing machine is on Fuzzy control. Well there are Fuzzy models, so this is the entire gamut of models we are going to look at only one small aspect of modeling.

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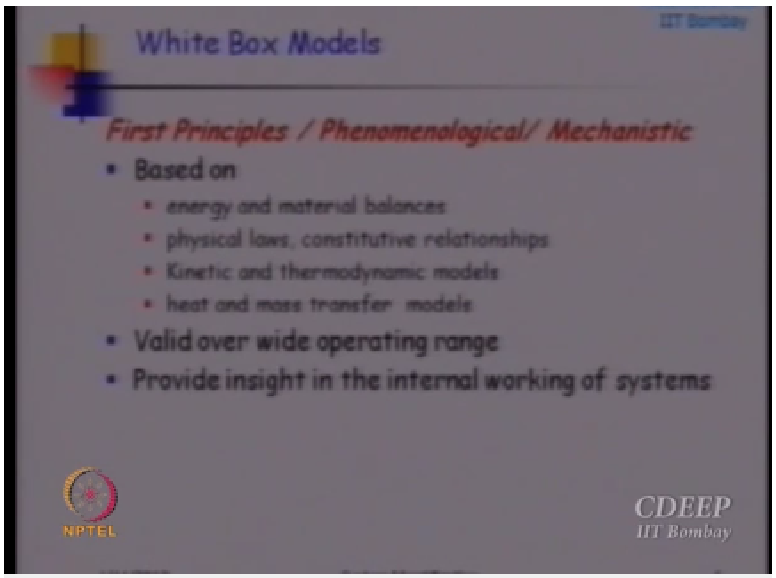

Well the models as engineers we know are white box models or models coming from first principles. What are they based on? They are based on energy balance, material balance, charge balance, they are based on physical laws, constitutive relationships, thermodynamic relationships, kinetics in chemical case reaction engineering you have reaction kinetics, you have heat balance.

So all these are used to construct dynamic models coming from physics. Here we have understanding of this physical system, and this is what as engineers we often are introduced to in the courses. Typically, we are introduced to models that are static, we tend to ignore dynamics in many disciplines, that is because we are more concerned about long term behavior called design of the systems, and then we look at only static part.

In control we are actually we were concerned about managing the transients and we have to work with models that are dynamic okay. So the rate of change of variables cannot be ignored and that is where, what is the advantage of these models, these models are valid over a wide operating range. You take a system, you develop a good model from coming from physics and if your model is tuned to the plant the parameters are correct, the sizes and everything is correct this model will help you to predict behavior over a wide range okay.

So these are good models, very good models. They can be used for, they provide lot of insight into what is happening inside the system okay. So if you have the models coming from physics okay, they are always very useful. You might, I mean now that we have beginning you might wonder what do you mean, I only know models coming from physics, where are the models not coming from physics okay.

We are going to learn that in this course, how to develop models not coming from physics, but those are dynamic models okay. I will be talking about them soon, maybe after two or three lectures. So, but I want to begin with models coming from physics, because all of you know models coming from physics, that is where let us begin, let us see what are the difficulties in developing those models, and then move on to models that are not coming from physics okay.

So that will be a easy transition from what you know to something that is relevant for control but if you have these models nothing like if you have a good first principle model for some simple systems like induction motor or some circuit you may have a good model coming from physics okay or for some simple small reactor may have a good model coming from physics and then it is worth using that model if that model okay.

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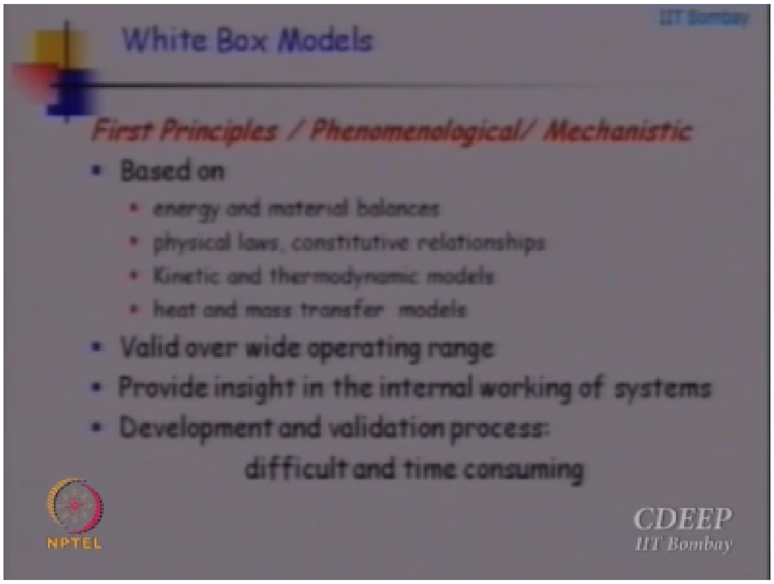

Well that trouble is developing these models is not an easy task typically to develop these models you need an expert, if I have to develop a model for a automobile coming from physics I need a person who understands modeling of automobile you should be a mechanical engineer or a automobile engineer should have done so many courses that you will need to know for doping this modeling.

And if you have time and money to do that well please remember it is not just that time when it goes to industrial control it cost money if you have to employee an expert who knows lot about automobile engineering how to developed a mathematical model for it and you know you actually get a model which is quite close to the real system well it takes times so you have to wait for a longer times you have to pay him salary for a longer time and you know it might deal your process of developing a controller.

Is it worth doing those models yes it is worth doing those models yes it is worth doing those models in some situations it depends on those equations depends up on the context if you are developing a simulator okay for example in training pilots you develop a slight simulator okay those models are what is of light simulator it is set of differential algebraic equations that actually simulate a flying conditions.

You better have a person who knows aerodynamics and you know who can actually write differential equations algebraic equations partial differential equations that really similar that conditions and then you know that simulator is running on the background and makes the trainee feel that actually he is fine, okay you better have model so there is a situation where you need those models but developing a controller if you start using those kind of models it can become very complex. It can be a long dawn process we want to do you know short cut.

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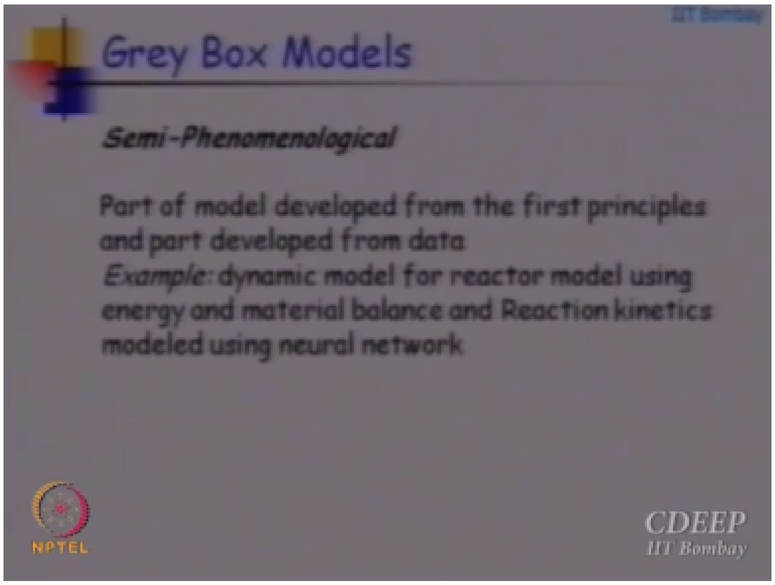

So there is one more class of models which are in between we call them as Gary box models, now what are these Gary box models, Gary box models are something in between know it is a model between a it is some part of it is developed from full physics some part of it is developed by some colorations you know so they are sometimes called semi phenomenological models. Part of it is coming from data or some kind of heuristics I will give you an example here I have stated an example in which you have reactor.

Heat and mass balance are developed from physics but I do not the reaction how the reaction occurs so I am putting a neural network to develop the reaction model and I combine the 2 I get a model which is a grey box model partly coming from physics partly coming from it is a good model if you can develop that.

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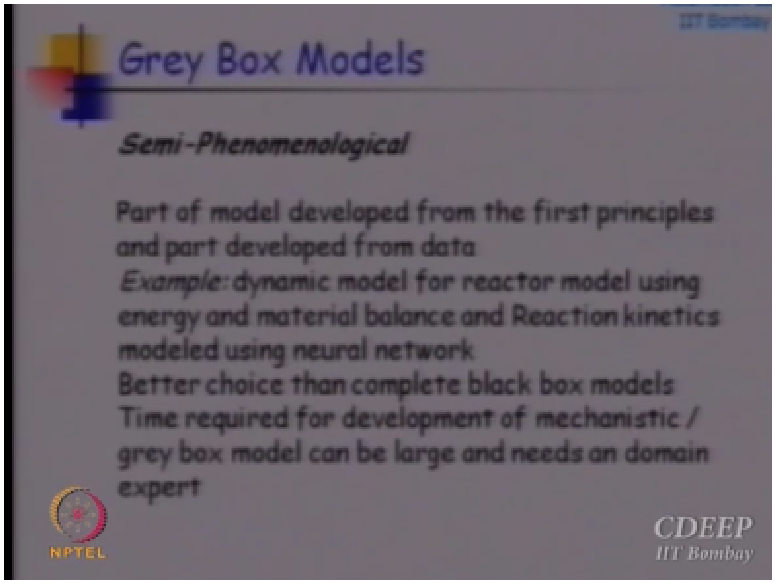

It is a cuts down the development time and it is better choice then completely black box models but again has a I said the last point is important developing this grey box models are developing white box first principle models mechanistic models you need an expert okay and needing need an expert you need lot of time which means translates lot of money so some time it is possible to do it sometimes it is not possible to it.

So let us look at both the scenarios sometimes if possible to do it and we can developed those models what to do how to use them from control what if you are not able to do those you do not have that kind of model how to develop a control deriving model.

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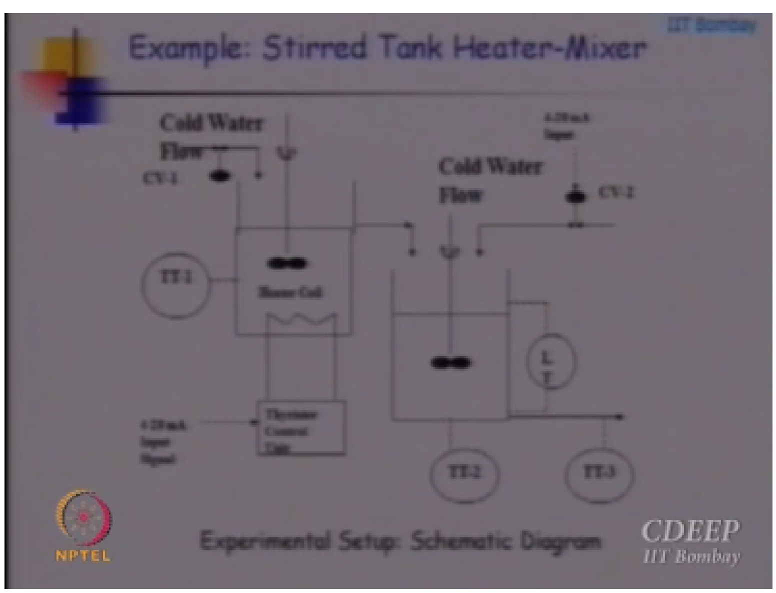

We look at both the scenarios I will just give you an example of a model which is developed for a laboratory system in my lab automation lab we have a very simple system here if you see there are two tanks in series the first tank is used to create hot water okay this is a stirrer here there is a stirrer in this tank cold water comes in into the first tank okay this is a heater electrical heater which is used to heat the water raise it to certain temperature in the second tank is a mixing tank hot water and cold water is mixed into the second tank from the first tank to second tank there is a you know over flow.

So whatever water comes in that over flows into second tank in the second tank the level changes oaky as a function of time also temperature changes a function of time because your mixing hot fluid and cold fluid okay this is a toy system here but this is what you have you know in your bath room and you have a water heater and then you know in your bucket you mix a hot water and cold water okay exactly the same system simple system.

Well my problem control problem is to control level inside the tank and control temperature inside a tank if you ask me a control engineer what I would like to control level inside a tank and temperature inside a tank okay so I am measuring 2 temperatures, temperature in tank 1, temperature in tank 2 and level in tank 2 I can write differential equations for rate of change of temperate in the first tank rate of change of temperature in the second tank, rate of change of level in the second tank.

I can do this using knowledge of physics so in minus flow out heat in minus heat out okay I am not going to develop this model here in this class right now I am assuming that all of you know how to develop these balances and I am just going to show you the final model but there is one tricky part here well if you see here there are 3 inputs there are 2 walls is a control wall here cold water control wall.

There is a cold water control wall here and there is a high resister control element here is just like your you know speed regulator your fan okay I can change the correct into to the high resister power controller the heat output will change okay it is it takes input between 4 to 20 million it gives me heat you know between some minimum to some maximum okay if I put 4 mille amps it will give me 0 if I put 20 mille amps it will give me some you know maxiuamum heat whatever it is.

Now how do I develop models for this high resister power controller one way is of course no I can call some electronic engineer and say that well why do not you develop model which relates you know the 4 to 20 mille amp current and the heat input so it is a very complex model I do not want to do that okay it will take lot of time it is not worth it particularly just if want to do control same thing about wall the wall is a very complex system control wall and if I want to develop a model for it is dynamics statics it is a difficult things.

So I have 3 inputs here I have a input at I have a input current input this to this wall I have current input to this wall and I have current to this high resister power controller which are coming from a computer okay I can manipulate these walls now instead of developing models coming from physics for these 3 components I am going to develop some kind of you kiwi correlations. And I am going to mix this correlation with my physics model that will give a grey box model.

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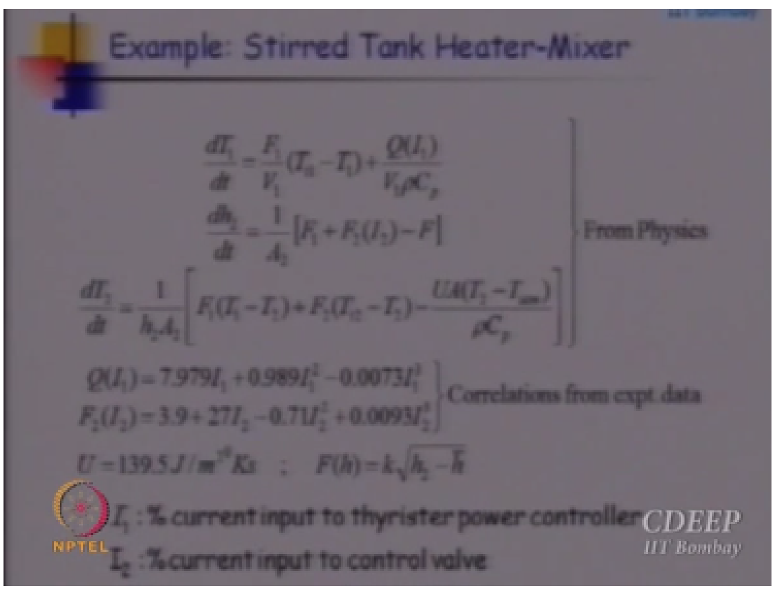

Look at here if you see this model it has 2 components 1st component is 3 differential equations we feel differential equations are coming from physics from energy balance and material balance flow in flow out energy in energy out okay these you can refer to any standard book on heat transfer will get this model I am not going to develop this 3 differential equations here if you look at for example if you look at the 2nd equation dH2/ dT it says that f1 is flow in + flow from tank 1 + flow from tank 2 – flow out f is the rate of change of level is related to these 3 flows.

Like and so on so we have these models which are coming from physics then I have to colorations heat has a function of current input what I have done is I have conducted lot of experiments for different current inputs I have found out what is the steady state heat input I have developed a correlation between mille amp current and heat input okay.

Without asking a electronic engineer to model at high resister power controller I have just developed a cured model between the current input and the heat output okay has some polynomial and I a have merged okay some part of it is coming through correlations which is not coming from physics it just says that if this is the current this is the heat this is the current this will flow out okay, I am merging these modules I get a module which is a day box model.

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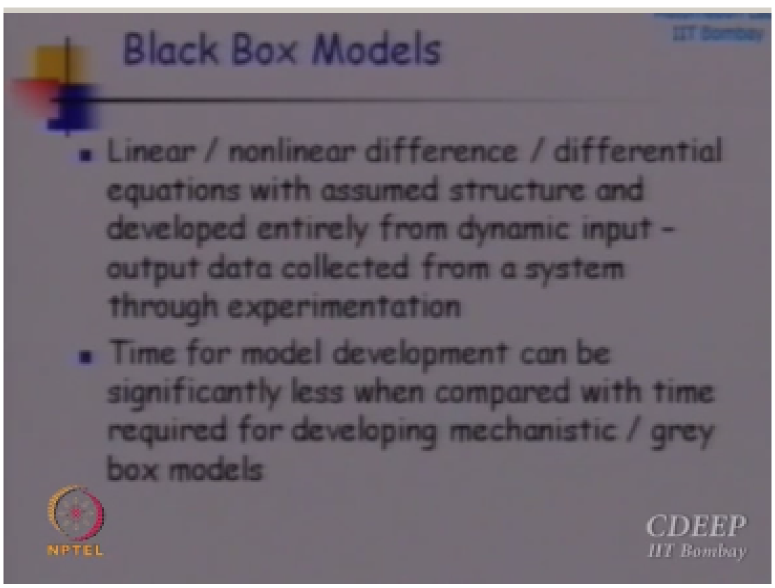

Okay the third category is black box models black box models are non linear difference equations or differential equations with assumed structure what is a structure of this model we have to assume and we will talk about this in great detail how do you develop this models, these models are developed entirely from data we are not using too much of physics are any physics at that for doing this, so this animal is due to u and we will talk about it little later the advantage of this modeling.

Is there you can develop them very quickly I will give you the time required for example the force the grey box model which I showed you my students took one month to develop that model that you know just two times is these we want to do get reliable model that actually capture the dynamics okay, it took appears the student about a month to conduct series of experiments to get all the parameters right, the time consuming the second and then you know developed a model which are control relevant starting from that model.

Then we did control to the time concern okay the other part is just develop models from data I will talk about it and that too probably 4 or 5 hours coming 1 month to 4 or 5 hours is significant relation in time okay, so this data driven models will talk about the problem in data driven models like the correlation which I showed you is that they do not explain you the physics they do not give you any insight you know what is happening inside thy cannot used beyond the range in which you are conducted by experiment.

Okay they have very limited that useful but they are useful only for the data on which you are trained them beyond that they cannot predict anything, okay so all kinds of models are required for control let us now assume that we have a somehow you know you go to a plant where or to a company where you are given from model which is coming from physics mechanistic model or some kind of a grey box model, somebody else they were fir for you.

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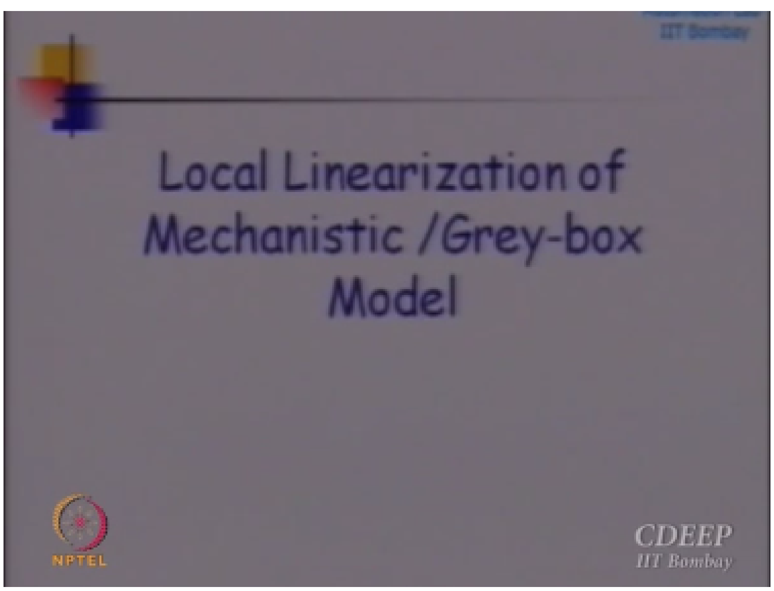

Okay or it is a simple system and you had time to develop it yourself okay.

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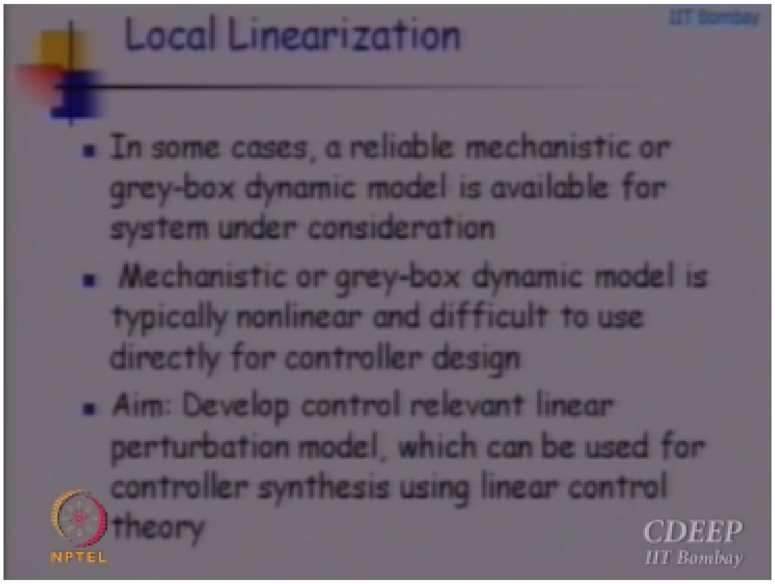

Now so I have model which is available okay now the problem is the model is a non linear differential equation how do I design a controller based on the non linear differential equation it is possible very much possible okay, but the design procedures are very, very cost or very, very difficult as compared to designing based on linear differential equations okay, it is not that we cannot develop the controller using non linear well it becomes very difficult so as a engineer my you know approach.

Is to see if some simple method of designing controller works if you does not work okay of a simple linear controller is not going to work then I will spend time on developing from more controller which is based on non linear differential equations okay, so as far possible I would know the simple things first if simple things do not work I can show that super things are not working for somebody else then there is a need to go for from you know non liner controller and so on.

Let us assume that simple things work okay we want to develop models which are controller level and we want to use the so called linear controls here okay I have already use linear control theory which means control where synthesis methods that are based on linear differential or liner difference equations I want to concentrate on liner difference equations in this particular course by liner difference equations because their computer relevant and you will be talking about computer based control so in the course.

So there difficulty use and what is my aim my aim is to developed control relevant perturbation models okay I want to develop I want to start from a model which is given from physics I want to come up with simplified models okay, starting from models from physics I want to come up with simplified models okay which are in linear differential or difference equations which I can use for controller in this that is the a okay, so let us we clear about what is the overall aim overall aim is developing a linear perturbation models.

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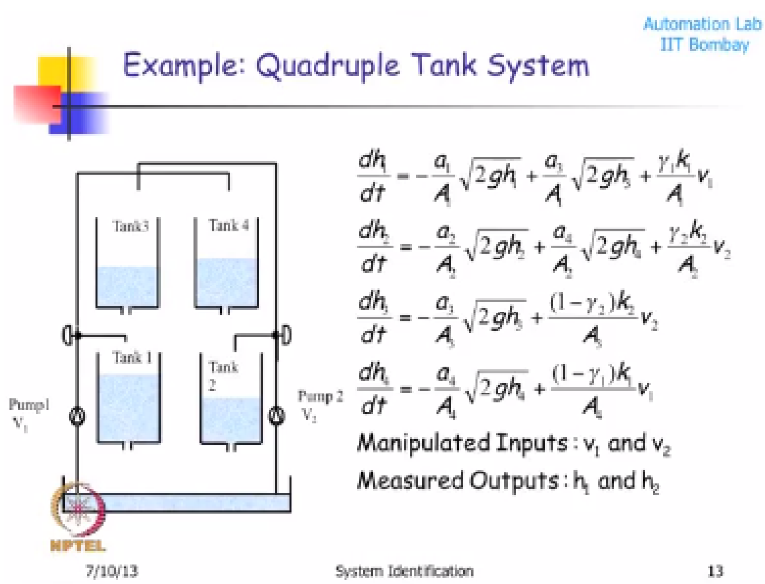

If you look at it has 4 tanks okay here there is a control wall here there is a control wall there are two pumps, the flow here is split okay part of the flow goes to tank 1 part of the flow goes tank 4 okay here the same way on this side part of the flow goes to tank part of the flow goes to tank 3 okay, so this is the interacting systems this is just a toy example that actually illustrates you know what is called as multi variable interactions.

In the real system in the real it never happens that one manipulated variable affects only one output, here if you start changing valve 1 you cannot do it without disturbing 3 levels okay, if you start changing valve 2 you cannot do it without disturbing other three levels okay, so it is a interacting system you cannot separate whatever we try to do you know changing valve and will have effect on level in tank 1 and there where in tank 2 changing valve two positions with that effect 1 and 2.

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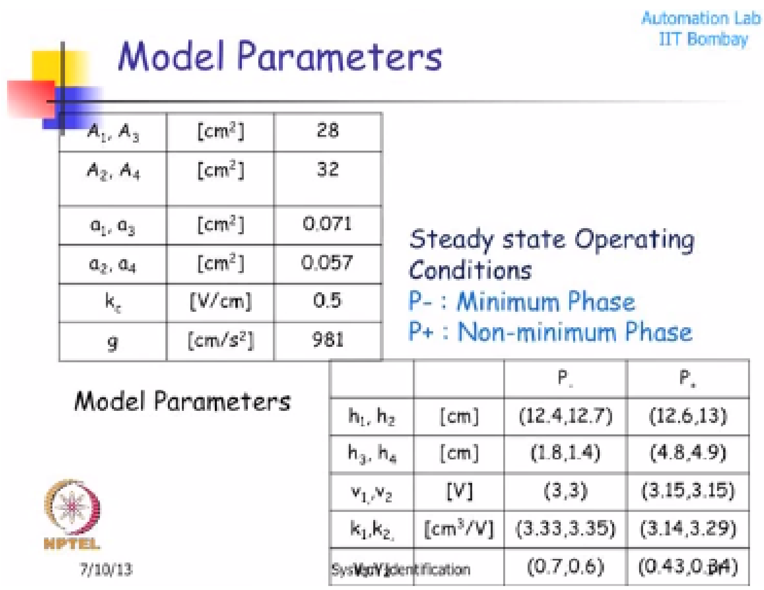

A_1, A_3	[cm^2]	28
A_2, A_4	[cm^2]	32
a_1, a_3	[cm^2]	0.071
a_2, a_4	[cm^2]	0.057
k_c	[V/cm]	0.5
g	[cm/s^2]	981

		P_-	P_+
h_1, h_2	[cm]	(12.4,12.7)	(12.6,13)
h_3, h_4	[cm]	(1.8,1.4)	(4.8,4.9)
v_1, v_2	[V]	(3,3)	(3.15,3.15)
k_1, k_2	[cm^3/V]	(3.33,3.35)	(3.14,3.29)
[illegible]		(0.7,0.6)	(0.43,0.34)

Okay these are the operating conditions set parameters so do not worry about these are some system parameters like area and so on, which I given let us look at this model can somebody explain me what is this model what are the terms here what do you expect to be this is a simple differential equation model okay rate of change of level from okay, so will break up you tell me what is how do you explain this how do you explain full terms yeah, correct so what are the three equation that I showing up here.

Height 1 so that is flow out that is negative please notice that, that is negative it means is the flow out is proportional to level to in tank 1 square root comes from simple Bernol's equation okay is you probably done in your great well first year of engineering what is a second term second term is flow in from tank 3 okay flow in from tank 3 that is actually flow out from tank 3 which is coming as a input to tank 1the flow out of tank 3 is proportional to level in fact tank 3, so you have √ of H3 coming there.

Right what is the third term is for the flow in due to the wall okay the fraction of flow that is coming in into to tank 1 that is given by γ1 times k / area okay, now what is coming in is proportional to what is that a1 a2 here A1 A2 or cross sectional areas of the tank okay what a small events a2 opening areas of the openings see H3 and H4 where is only there is only one inflow and there is only one outflow okay.

That is why there are only two terms here if you look here there are two terms one term is out flow okay, proportional to h3 and this is the fraction of the flow which is coming in okay, the same thing about h4 there is this is the out flow from tank 4 and this is the in flow to tank 4 okay, so this is simple balances simple flow balances flow in – flow out for each tank, okay.

And flow out is proportional to the square root of the height coming from Bernoulli equation okay, the flow out will reduce as the level decreases flow out will increase as the level increases but what is the proportion square root and the proportionality constant g here is the gravitational constant okay, so this model is coming from physics that model is simply coming from physics v1, v2 we have to interchange.

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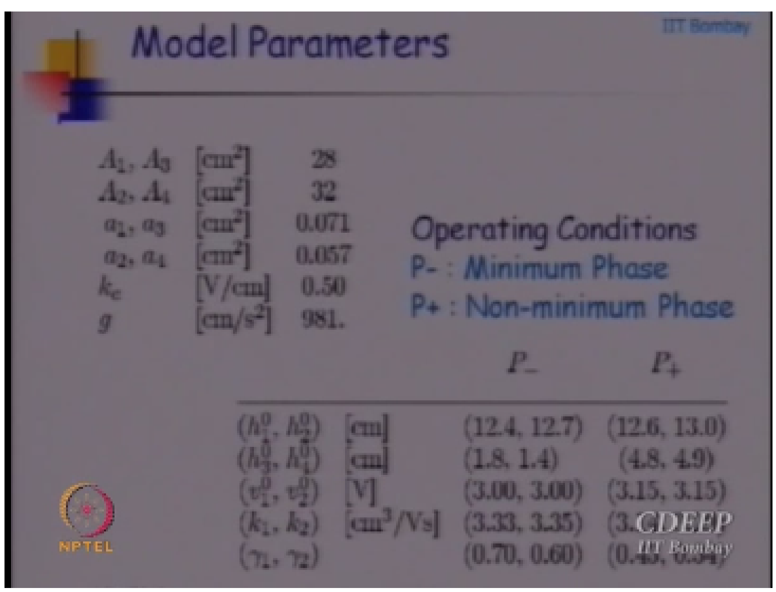

So these areas and you know specific areas of the out flow those are given here these I have taken from the paper I am just reporting them here.

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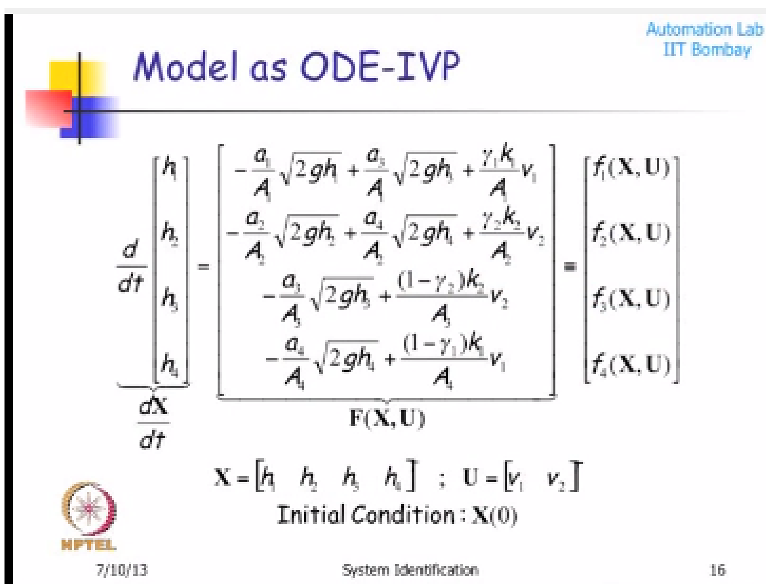

Now what I want to do is I want to develop a linear quadruple machine model starting from this particular model. First of all I am going to classify the variables associated with this model there are two variables or two vectors I am finding out here one set of vectors I am going to call as x okay, this x are dependent variables or states. Here what are the dependent variables, dependent variables how many equations are there, there are four equations, four differential equations.

How many variables are associated with this as still six there are four levels and two voltages right, there are four levels and two voltages and these two voltages are v1 and v2 okay, so a six variables four equations two variables are specified independently v1 and v2 are the voltages which are specified by an operator or by a controller if you when you design a controller but these are the two degrees of freedom we have to control the system, okay to control the system are two degrees of freedom voltage 1 and voltage 2 to the two pumps, okay.

Dependent variables are h1, h2,h3,h4 four levels okay, I have put this now into an abstract form dx/dt okay, dx/dt where x is a vector okay so this is a vector differential equation. On the right hand side you have a function vector f1X, f2X, f3X, f4X actually the function of X and U both not just X so what is this first element of this function vector this entire equation, okay. So here do you get this four elements of the vector equation so finally what I have is a vector differential equation dx/dt X and U okay, this F is nothing but this function vector entire function vector I am now calling as F here you here okay, so this is the vector differential equation dex/dt= function vector F which has two arguments X and U dependent variable and independent variables is everyone get me all this, okay let us move to the next part.

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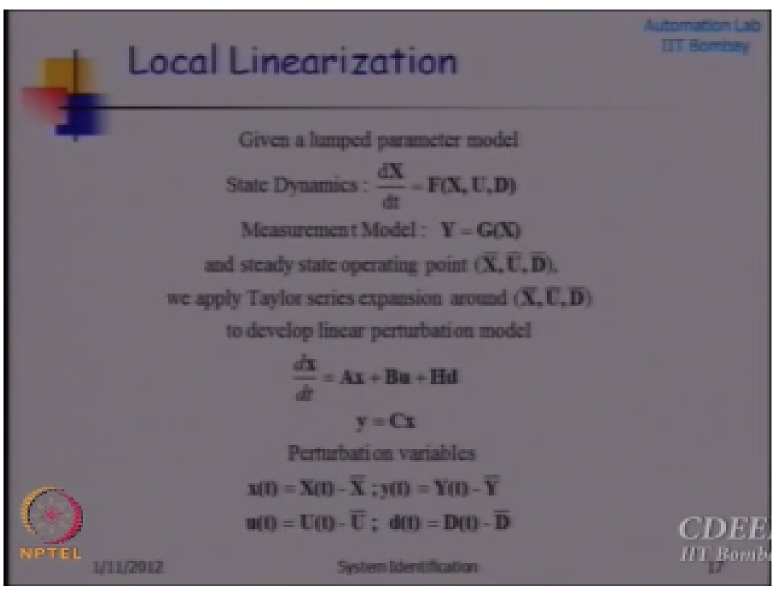

Now well you may have further classification of inputs form a control view point you might say that you know the inputs some of them are manipulated, some of them are disturbances I think all of you are done one course in control right, so disturbances are those inputs to the system which we are not manipulating okay, in the heater setup that I showed you a typical disturbance would be L temperature surrounding the heater system, okay that would be a disturbance because that changes according to some other factors we are not able to manipulate that , okay.

But that will have an effect the way heat is transferred from each of this vessels to the air okay that will have effect on the dynamics. So here I have a model which is dx/dt which is function of X, U and D, U and D are basically inputs I have just sub classified them as disturbances and manipulated inputs does not matter if you club them or what I want to do now is I want to linearize this differential equation okay, I want to linearize this differential equation. What is the basis for linearization differential equation, in many situations you are operating a system in the neighborhood of some steady state point operating point okay. I somehow have bought the system let us go back to the our four level tanks I have bought the system into four you know steady state levels okay, I will write now let us not bother about how do I bring it there may be a as an operator you have done some manipulations bought us as into a steady state.

Now in automatic mode I am worried about maintaining the level at those four points okay, so what is this $\bar{X}$, $\bar{U}$, $\bar{D}$ these are some steady state level operating levels for this particular system.

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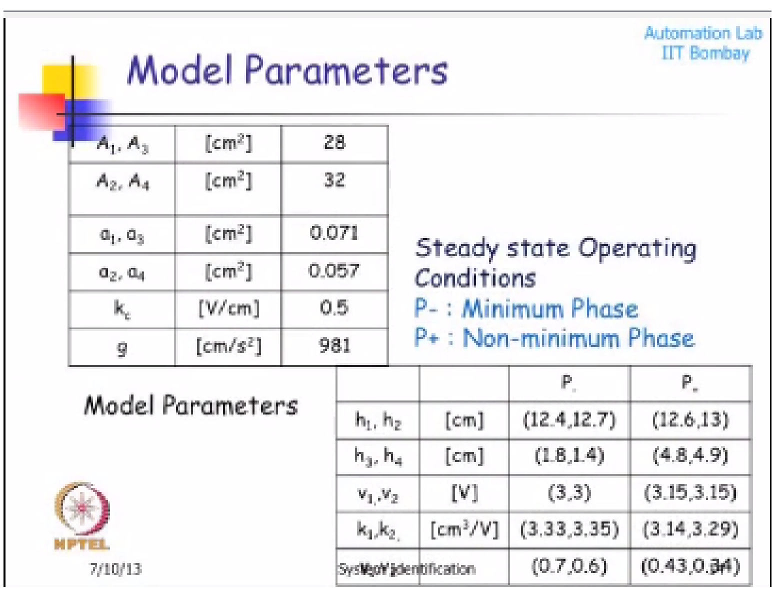

Steady state operating levels under different conditions are given here you know for there are different operating conditions what are they will, we will look it later. But let us look at P- under this operating condition the steady state level is 15, 12.4cm like 12.7 cm this is in lower two tanks and 1.8 and 1.4 in the upper two tanks okay. The inputs steady state inputs are three volts and three volts okay, so this 3 volts and 3 volts is my Ū and this 12.4, 12.7, 1.8, 1.4 is my X̄ steady state levels, steady state inputs, okay.

I have bought a system to the steady state somehow, now I am worried about modeling developing a model which is control relevant which is valid in the small labor hood of the steady state okay, I want to models small perturbation around the steady state, okay I am not worried about the global model that talks about variation over the entire range.

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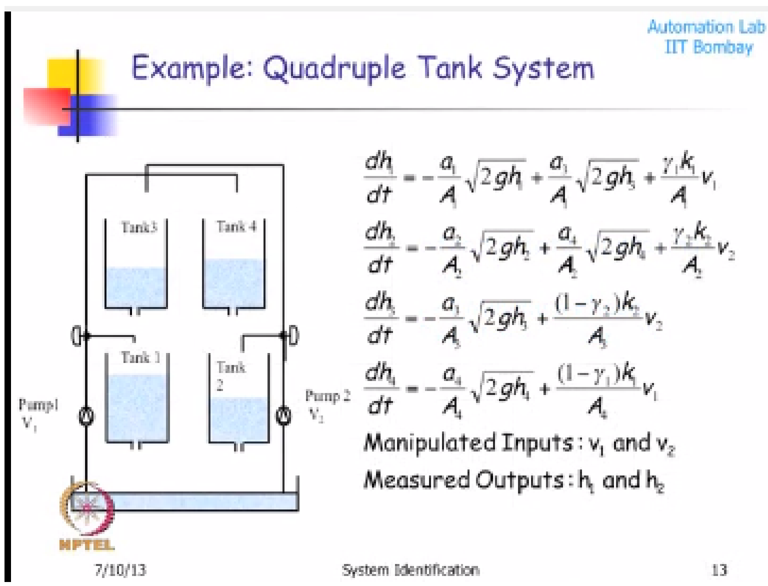

Which is the global model this is my global model okay, but I am not going to use that I am to use a local model okay yeah.

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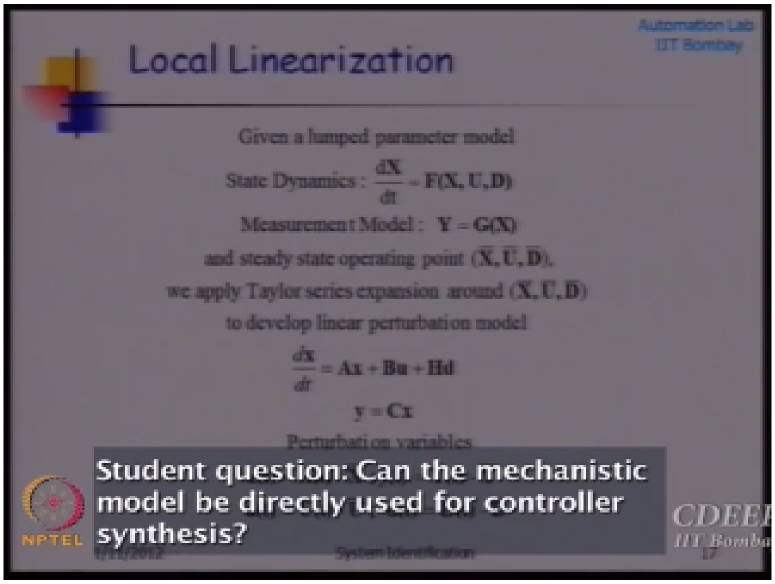

You can that is done in typically in non linear control where you do linearization on the fly okay, as you move along in the dynamic space you will linearize on the fly. But beyond the scope of this course that is done probably you might do it in the part of your project but not in this course, okay.

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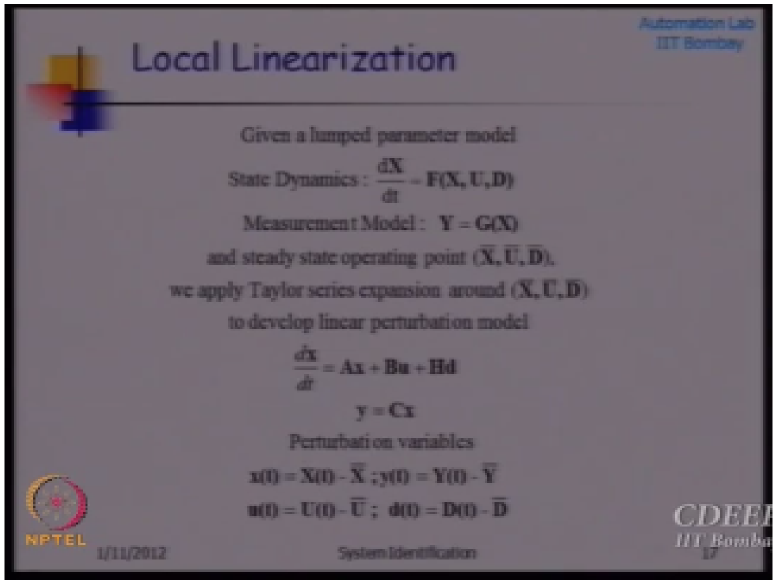

So right now I am worried about developing a model which is local for small perturbation okay, so I am going to define these perturbation here, x(t)is X(t), X(t) is the physical actual variable minus the steady state okay, so this small x(t) is the perturbation in the levels okay, y here are the measured values I am not going to measure everything typically in your real system you do not measure all the states I am not going to measure all four levels.

I am going to measure only two levels h1 and h2 it is caused you know putting a level measurement cost me I am not going to measure every level only two levels I have made. So here measurement some function of x what I am going to measure is some function of x u are the perturbation inputs small u are perturbation inputs so I have this steady state inputs and I am worried about small perturbation surround the steady state okay.

I want to develop this models that starting from this model which is non linear differential equation, I want to develop this linear differential equitation this is the linear differential equation okay, it is says that rate of change of x which are small perturbations in level okay in the neighborhood of the steady state okay is related to the perturbation itself x vector through a matrix a okay, big another matrix b relates the rate of change to perturbations in the manipulate inputs okay.

And this third matrix here relates change to a disturbances in this particular system we do not have any disturbances okay, or we have not considering right now we are not modeling any disturbances so forget about disturbances part look at only the x and u okay. So I want to find out two constant matrices a and b I want to find out to constant matrices a and b okay which will approximately capture local dynamics in the neighborhood of the steady state okay. The way this is done is through tailor series expansion.

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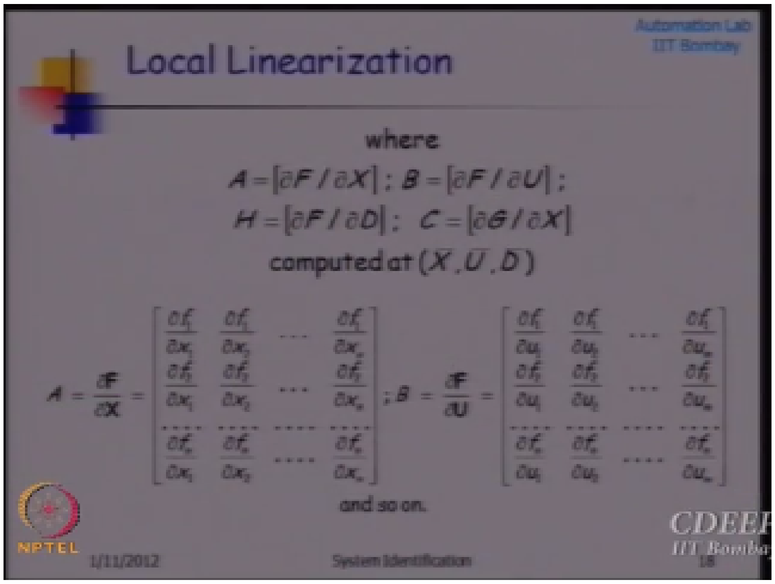

The way this is done is ways to tailor series expansion this a b matrices are actually computed by binding out partial derivatives of the function vector f with respect to x partial derivative of function vector f with respect to u, okay if you look at these matrix this is called a Jacobean matrix, so this as function, vector element one differentiate with respect to x1 with respect to x2 with respect to so what is this f1 f2 we have put them here.

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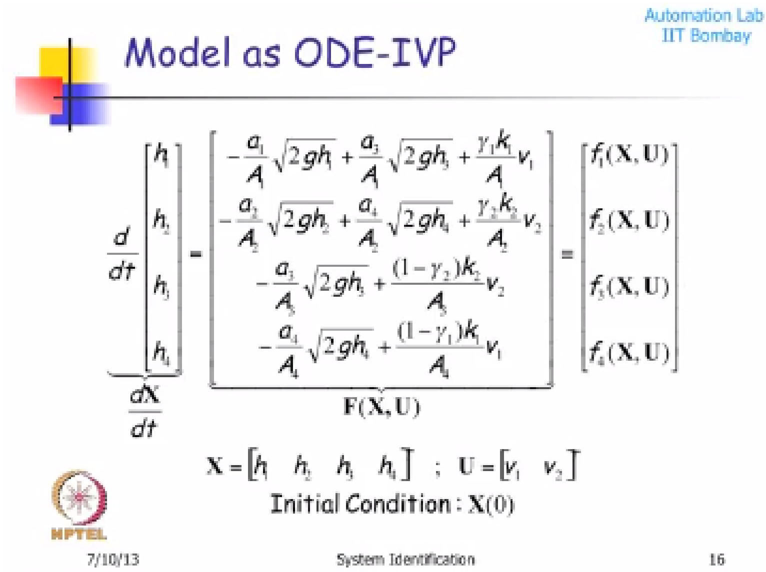

This is f1 this is my f2 this is my f3 this is my f4 I am going to find out perturbations okay I am going to find out partial derivatives of each one of them, with respect to h1 h2 h3 h4 also with respect to v1 and v2 okay. So this is the way I am going to construct my local matrices yeah.

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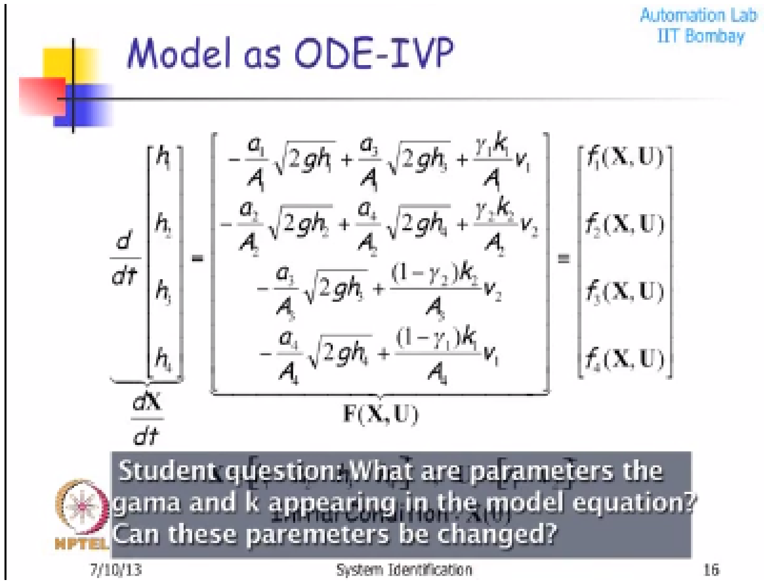

So γ 1 and k 1 are the two fix parameters which we can chose the split γ 1 talks about the split from to tank 1 and tank 3 yeah, so once you fix it is cost okay.

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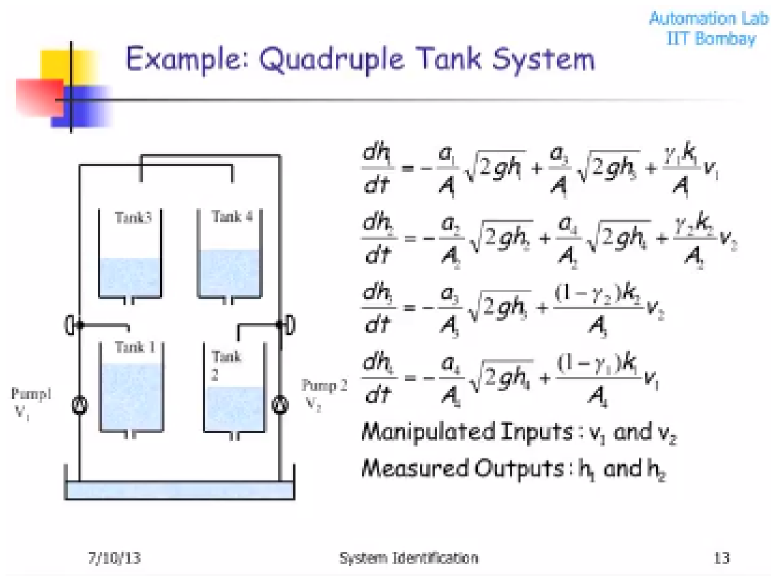

So there is a in a real system there is a way of fixing the ratio flow ratios between the split, so γ 1 and γ 2 are splits close splits okay, what portion goes to the top time what portion goes to the bottom, yeah.

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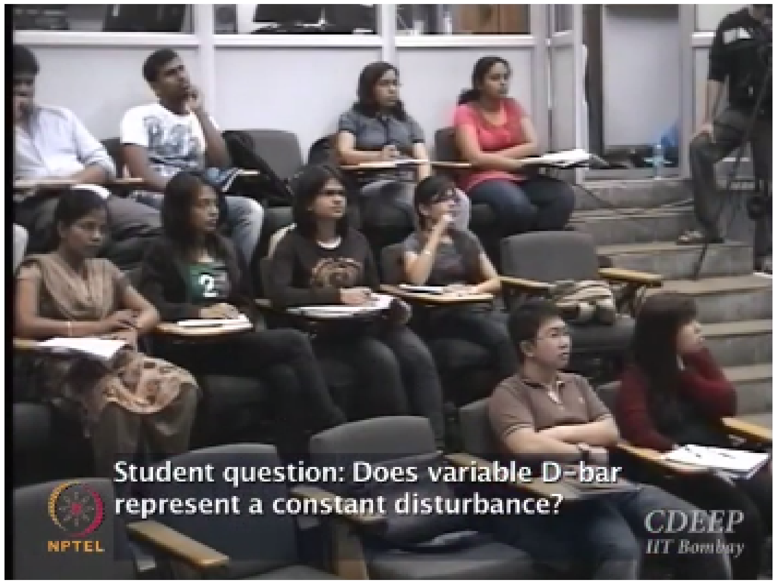

Not constant disturbance d bar is the constant disturbance level at the operating point which you have.

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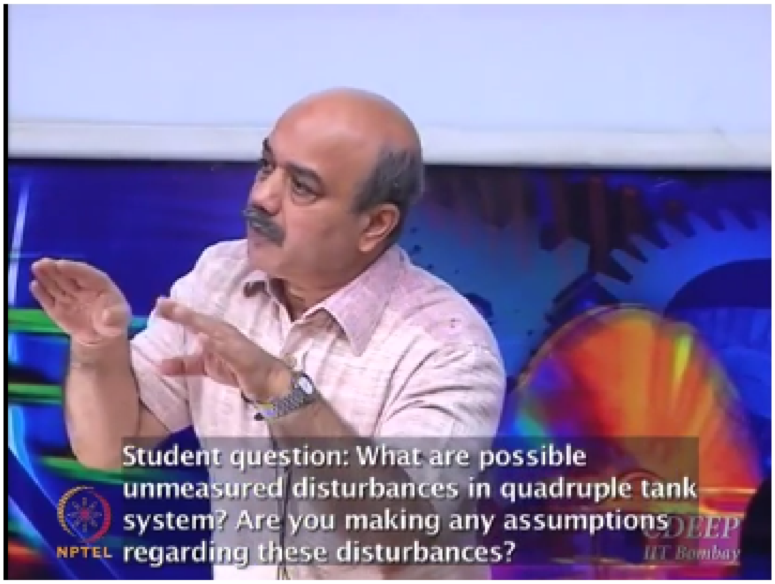

No in this particular system a disturbance could be for example, I will you an example of disturbance in this system. A disturbance could be I have two pumps okay the voltage input to this pump is fluctuating okay. yeah so write on assuming that there are no disturbances o voltage is constant okay, voltage is constant, so if I tell you know if I give certain current input I will get a fix flow from the pump okay.

We assume that you have a stabilizer your eliminated any disturbance Interviewer: he voltage you have a stable no disturbance okay but even if you have a fluctuation of the voltage let us say between say 2240 they might be some steady state you know typically it is up to 20 so that d bar would be 220 okay. so I have to find out these two perturbations will actually do this enthuses for some real system as a part of you know problem solving, so you will get more fell for it.

Now if I actually do this for the quarter pull tank setup if I find out partial derivatives what do I get? Okay I will get this matrices a and b all that I have done is I have just taken partial derivatives of the first element with respect to so this is partial derivative with respect to first element $\partial f1 / \partial h1$ this is $\partial f1 / \partial h2$ see if you go here h2 does not appear here, correspondingly you have 0 here x4 does not appear here you have a 0 here and h1 h3 appear so there are two partial derivatives associated with that. Likewise d x1 / dt x2 h1 and h0 still do not appear so there are 0's here okay and there are two partial derivatives that coming here the two equations these two equations are particularly simple h3 and h4 because they do not involve h1 h2 they just have h3 and h4 separately, so if you go back here and see this equation third equation as only h3 it does not have h1 h2 or h4 and this has only h4 it does not have h1 h2 h3.

So correspondingly here you have 3 0's and you have 3 0's only two elements non 0 okay, the same way I have taken partial derivative for my inputs with respect to my inputs, okay so this is a linear perturbation model this is a linear perturbation yeah.

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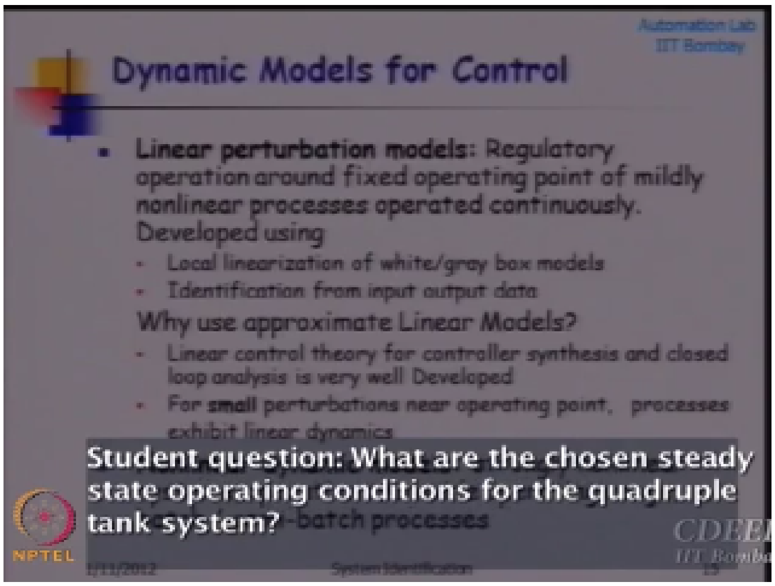

Yeah x bar and u bar yeah, so this is around this p minus conditions this around –p condition h bar is 12.4, 12.7, 1.8, 1.4, u bar is 3 words 3 words okay this voltage is v1 and v2 vary between 1 to 5 or 0 to 5 I have to check in the paper so this is at some middle point okay, the flow ratios have been fixed 2.7 and 0.6 which means 0.7 70% goes to tank one 30% goes to tank 3 and 60% goes to tank 2 and 40% goes to tank 4 so far this condition this particular condition p – conditions okay.

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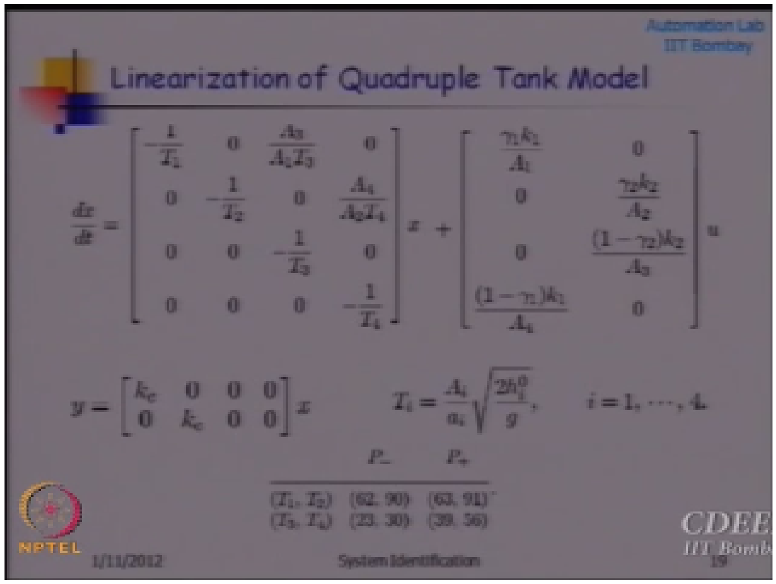

Well you have these new variable ti defined here they are defined in this particular case you have to go back and read the paper you probably have the second do one or two partial derivatives you will understand how this is done, is everyone with me on this you understand how this matrix yeah. we have only two measurement available that is why this matrix here see this is multiplied by x there are x as 4 elements h1 h2 h3 h4 I am going to measure only level in the two lower tanks okay.

That is why this model is a measurement model why perturbations in the level which are measured are only h1 and h2 okay, now what this kc kc is coming here this kc represents relationship between actual measurement actual measurements is in the voltages. I have a transmitter which does not give me a centimeter which use me the voltage reading that is okay and it is liberate to the actual centimeters through this relationship voltage level measured actual and voltage level measured h2 is liberate to the centimeter level through this is everyone clear about this.

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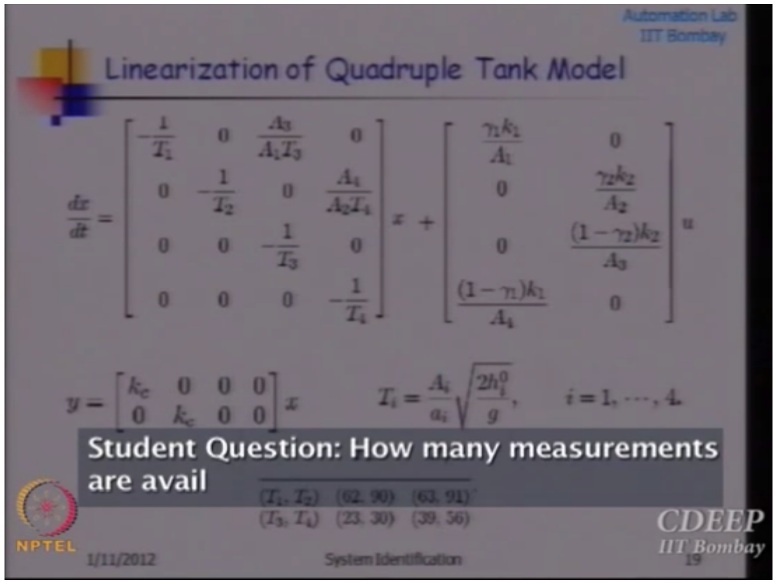

How will derive or should I just do it anyone has a doubt I will do it, I am just measured in two levels I have to pick up level 1 and level 2 from the vector so this matrix if you see here this matrix will pick up the first element and 0,0,0 appear here what is this Kc? Kc is the conversion factor between centimeter levels to voltage level, voltage level means transmitter output which is voltage okay.

And the same thing is here there are identical transmitters so the relationship between centimeter to you know level in voltage is, is this true same relationship that is why same kc, kc appears suppose they was some weight of directly measured in level of centimeters this kc would be one okay kc would be 1 if there was directly way of measuring level in centimeters kc would be 1 okay.

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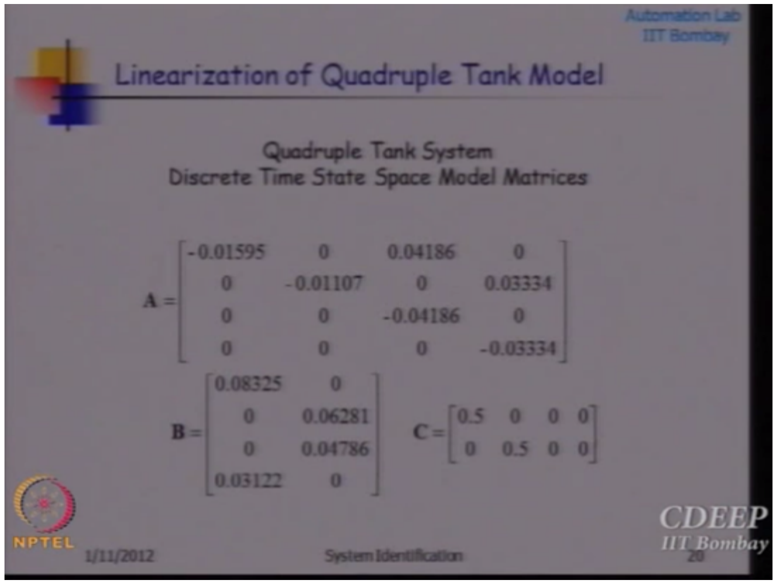

Then that case this matrix will reduce to just 1,0,0,0 and 0,1,0,0 okay, okay now if I actually do this linearization for this particular process okay you might find I am going little too fast here but when we do this is just oaky next week will do some we actually take some simple systems linearized put in matrices get the matrices you will get better feel of what is happening okay.

The actually put in the numbers for the area of the steady state for everything that I am talking about you will get these matrices A matrix will look like this B matrix side I just put the numbers and then so at the end of this exercise what do I have I have a model that relates rate of change of level okay to inputs quadruple inputs okay it also tells me how quadruple inputs are elate to quadruple measurements of quadruple states are relate to quadruple measurements.

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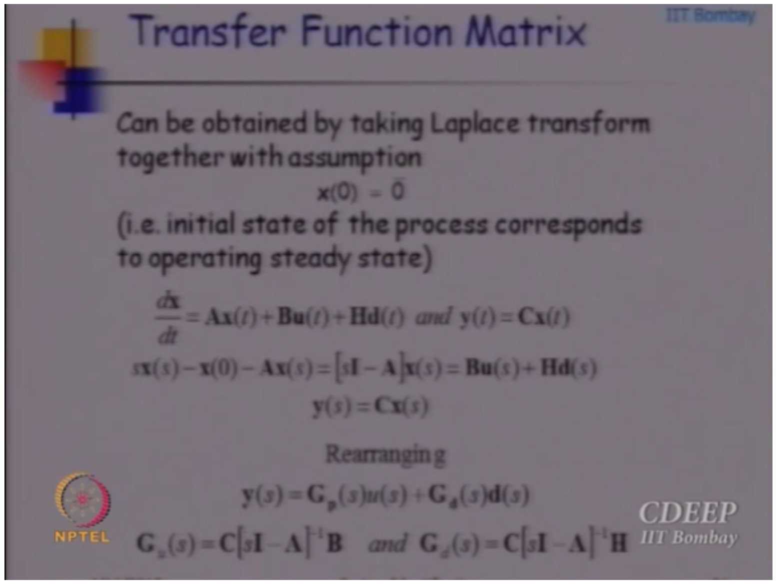

Why is a measured output? X are the states and C matrix will tell me so .5, .5 is the conversion factor kc okay well when you do your first course in control you always study the Laplace transform right now what is the relationship of this module with the model which you get from Laplace transform I all do connect the quote okay so you feel comfortable something that you are studied already Laplace transforms.

And this model okay that way easier to relates when you did Laplace transforms you only looked at one input and one output system now we have a trouble we have two inputs and two outputs okay you have two inputs and two outputs so I need to now get not one transformer function how many transform function to be there may be four transforms.

Actually what I am going to get is the transform function matrix okay that relates output vector with the input vector measurement vector and input vector that is what I want to get okay how do I do this look t this step okay I am starting with this differential equation here I am starting with differential equation this is my differential equation okay.

If I take Laplace transform to the left hand side what do I get simple you know S times Xs remember Xs is a vector okay and now the manipulations that you do have to be consistent with the vector and the matrix A is a matrix on the right hand side which has to be consistent okay I am going to assured that my initial condition is 0 what is the meaning of 0 here?

Your starting point is exactly perfectly the steady state okay this is typically a simple kind assumption when you develop a transform function model they assure that the steady state your initial point is 0,0 0 does not mean level with 0 what does it mean quadrupation in the level gets 0 the level is not 0 quadrupation from the steady state is 0.

Important to keep this in mind we are developing only quadrupation models okay so this X_0 is actually 0 okay all that I have done is this A times Laplace transform of this I can take on the left hand side this notices here okay and because this gets 0 now I am going to collapse this term and this term into this term here.

The series come here SI-A, I have a matrix equation S is a scalar okay S is a scalar I is a two cross matrix okay –A, A is my nearest system matrix okay is equal to B times U of S okay and X times D of S okay and I have this Equation Y of X= C of X what does a transfer function do what does it relates output and inputs what are the outputs here y what re the inputs U, U is in this case V1, V2 okay y is H1, H2 okay.

So I have to somehow get read of I have to get read of this X how do I get read of this X substitute for X okay to substitute for X I have to eliminate this matrix on the left hand side SI-A you have to pre multiply both sides by universe of this matrix okay SI-A universe I am going to pre-multiply on both sides and then substitute in the second equation if I do that is it clear.

All that I have done is an eliminated X I have eliminated X and substitute for X in the second equation so pre -multiplying by SI-A inverse to this equation and then substituting Xs here I can rearrange it into two these two equations okay so y of s is gp that is transfer function reference to U okay. And this is respect to this one in our particular case there are no disturbances disturbances are 0 so we have to worry only about G of S okay if I do this or this particular system what do I get.

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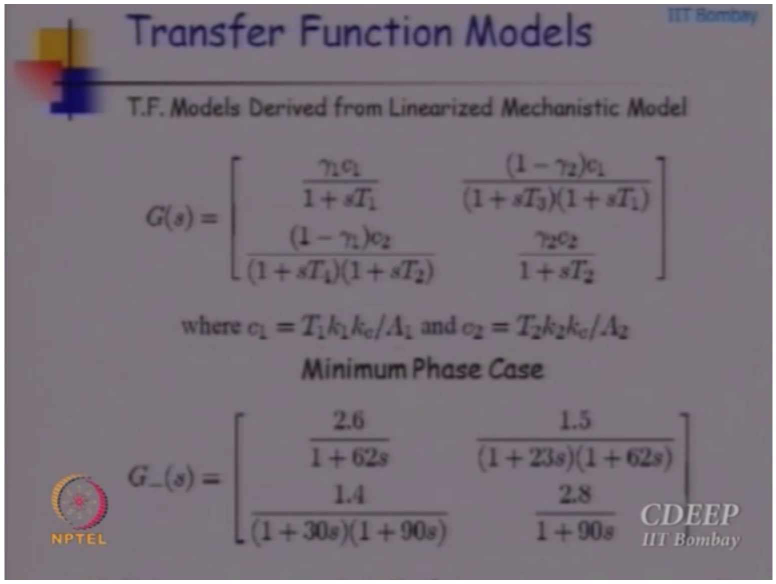

I get this transfer function matrix okay well how do you do this actually of course for simple systems in exam and all we will do it by hand but mat log as 2 you just go on defining matrices I will give you programs so do that and you know you will get all this transfer function matrices for any complexes you have 5 inputs and 10 outputs you have got an transfer function matrix which is huge there is the built in programs which will just give A, B, C, D matrixes or a, b, c matrixes, it will just pop it will give you this transfer function matrices to you okay. So and for a complex system you cannot but use a computer, so in our course we are going to use computers liberal.

So this is my transfer function matrix what does it relate? Inputs and outputs I have got rid of intermediate state vector. The state vector had 4 variables, 4 levels, now I am just worried about 2 measured levels, 2 known inputs manipulated inputs okay, and there are 4 transfer functions okay. This is between levels 1 input 1 level 1 input 2, level 2 input 2, level 2 inputs 1, level 2 input 2 okay. So now are you comfortable what we have done okay by linearizing earlier okay.
Is where to transfer function thing which you know from your okay except now we are looking at multi variable system, you have a transfer function matrix you do not have a transfer function single scalar transfer functions okay. Well now is every one with me on this upto here point is there any doubt yeah.

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I mean we are talking about minimum phase, sometimes almost after 3 or 4 weeks okay, I am going to come back to that but so right now we concentrate only on the model development.

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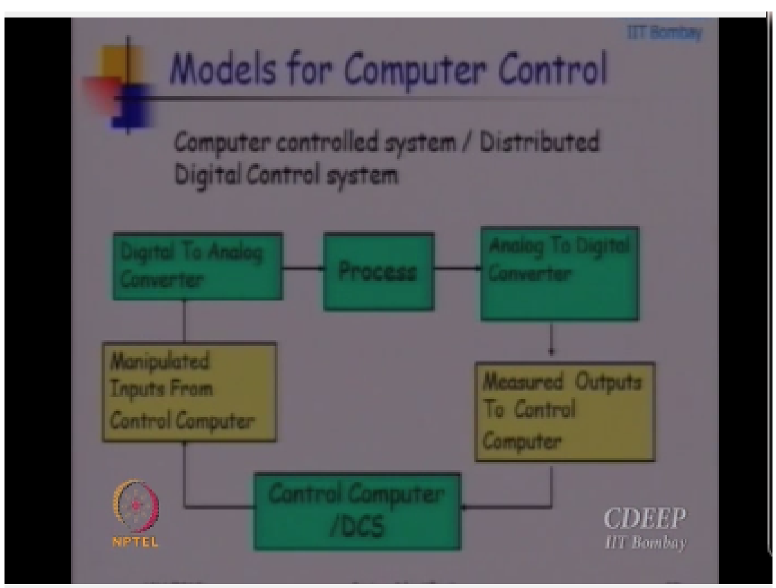

What is minimum phase what is non minimum phase those issues will start appear when we start analyzing those models, how do you interpret models, looking at certain parameters like we did the numerical method score we just looked at the diagonal values and we interpreted it in some way. So here we are going to look at values and also we are going to look at there is something called 0.

So here you worry about two things one is roots or the poles of the characteristics equation and also something called 0, so we look at that later. So well this module is fine transfer function module is fine the trouble is this transfer function module is in continuous domain. If I convert this into times domain I will get a differential equations okay. If I convert this model times domain I will get a differential equations I will get 2 couple differentiable equations.

One for h1 and other for h2 okay and I suppose if you have some idea as the how to convert a transfer function model into differential equation, this is done in this your first course. So I will get differential equations but in a computer control system okay I need something different, what is different? In a computer control system you are working with the discrete time data okay. all of you are familiar with computer control systems.'

Now you carry mobile okay in some sense it is a micro control system okay so let call it has computer control system. The data handling cannot be done in continuous time when you have a digital control system you have to handle in a discrete manner you let say you are getting temperature measurement or levels measurement from my whole time set up okay. There are two possibilities I have a analog device okay which is doing temperature measurement, second possibility is have a control computer okay.

In which I am collecting level data okay, I cannot measure every level at every time instant because when I convert what is require to measure level inside the computer, I do not want you to answer in a deep way, very simple thing what do you think is required. I will get the voltage measurement from my device, what does computer understands? Only numbers a computer will only understand a number it cannot understand a voltage.

You have to convert a voltage into a equivalent number okay, so I need a device called as analog to the digital convertor, will convert a voltage signal or a current signal into a number, because my computer understands only numbers. We are looking at one level higher, which is more of an application layer I am going to assume that in a computer control system, I will have a D2A convertor available with me.

So the measurements which are going to come to my computer are essentially sampled data okay, see when I convert a voltage into a number okay it happens.

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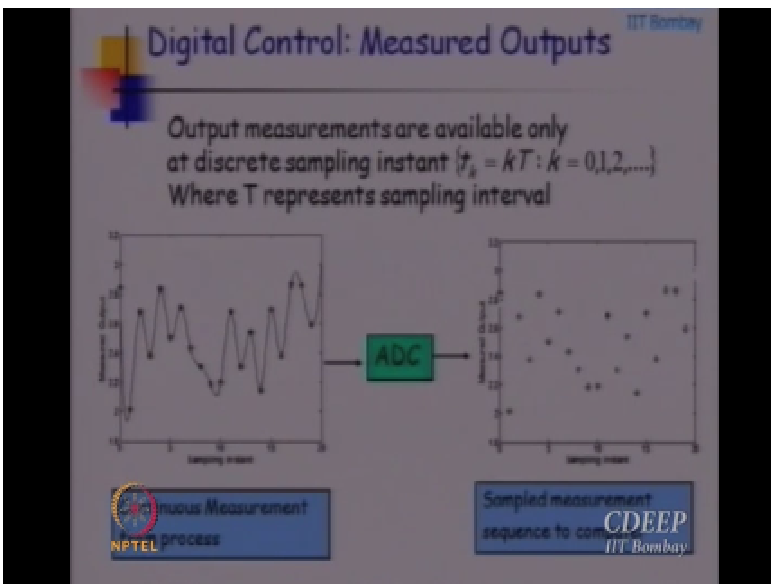

What happens is something like this because when you convert a voltage into a number okay, so suppose it takes let say it takes 100msec okay to convert 1 voltage into a number and then getting into my program that as to happen right, it will take some finite time to do this. So what about temperature variation between in this 100m/sec you cannot measure it, that data is lost okay. So what I get see this is the real continuous signal.

Let say you measure temperature at every 100m/sec so you measure temperature at every 1sec okay. so if you are setting inside the computer you will see data like this chart here, you will see this coming at discrete time okay which are proportional to level inside the tank. You are never going to see the real continuous level variation inside the computer; you will only see sequence of numbers.

So now when I do modeling I need to exist the reality that inside my computer I am never going to get continuous signals I am only going to get signals that are sequence of numbers okay at you might say why 1sec? Why not 10m/sec okay buy a better convertor you will get measurements at 10m/sec but what about 0 to1 0 m/sec, we are going to lose some data. You can make it, you can reduce the sampling interval but there is the limit okay.
You can never ever get continuous inside your computer it will always get sample signals okay, so I need modeling scheme to deal with this reality that data is not going to be continuous okay. Now let say I am controller through my computer the question is I can do controller calculations only at a finite interval; you will take some time to do calculation right. Once you get the measurement let say you do proportional controller.

You will have to multiply, you have to first calculate the difference between the set point and the measured value and multiply by the gain okay and then calculate the output and then send the output to the real world outside okay. All these operation takes finite time okay that is done using what is analog convertor D2A convertor okay. Now the problem comes is that computer only gives a sequence of numbers okay at regular interval. But the real world only accepts continuous signals.

So you have to reconstruct, when you go from a computer to the real world you have to reconstruct a continuous signal starting from an input which is discrete time okay my computer can only regenerate a sequences of numbers I have to get a voltage input to my wall which is continuous just imagine if I start giving pulses to my control valve it will start jumping okay I cannot effort to do that I need a continuous input to my control wall or to my you know, whatever I my achieved element so this d2a conversion.

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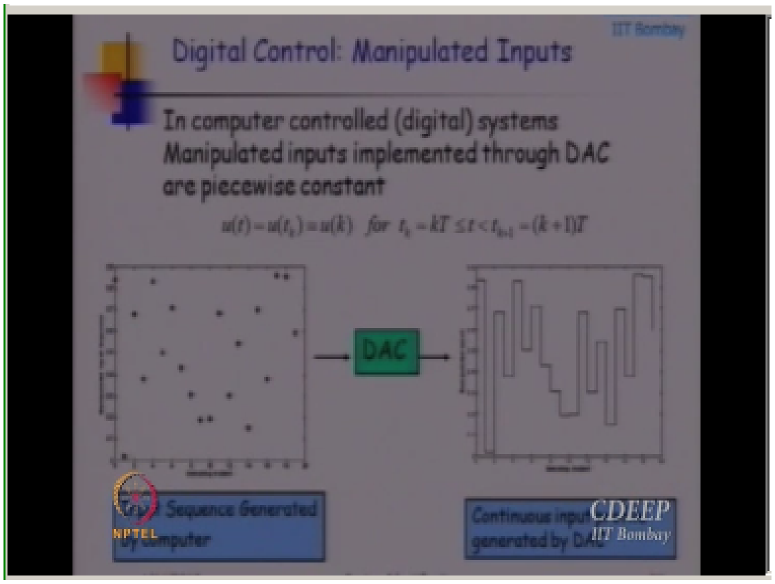

Typically using what is called as zero order hold but zero order hold is a simple device which just says that keep the last value constant in the new value arise okay this is called as piecewise reconstruction of the signal okay so typically d2a conversion that you get commercially or zero order holds okay a computer will give a signal that voltage should be you know 3 volts till the next signal comes it will holds at 3 volts the next signal will be 3.2 volts till the next signal comes it will be 3.2 volts.

And so on so how will how will the outside look like I am sending if you look at this if you are setting inside a computer you will be you know sending one bullet every one second that is one voltage value to the outside world well what the walls sees is a sequence of piece wise constant in coefficient what your activating element in this particular case the control walls are two motors will see a piece wise constant signal okay so well one can do piece wise bigger reconstruction.

And so on but all those things are very rarely done in bulk in some systems that mean we done most of the d2a convertors that will get commercially do signal reconstruction by this particular method okay now I need to adjust my models to the reality A the measurements are not continuous they are sampled B the output which goes to the manipulated variables which goes to the plant or piecewise constant they are not continuously changing within a sampling interval they are piecewise constant I need to adjust my modeling scheme to this reality how do I mathematically state this okay.

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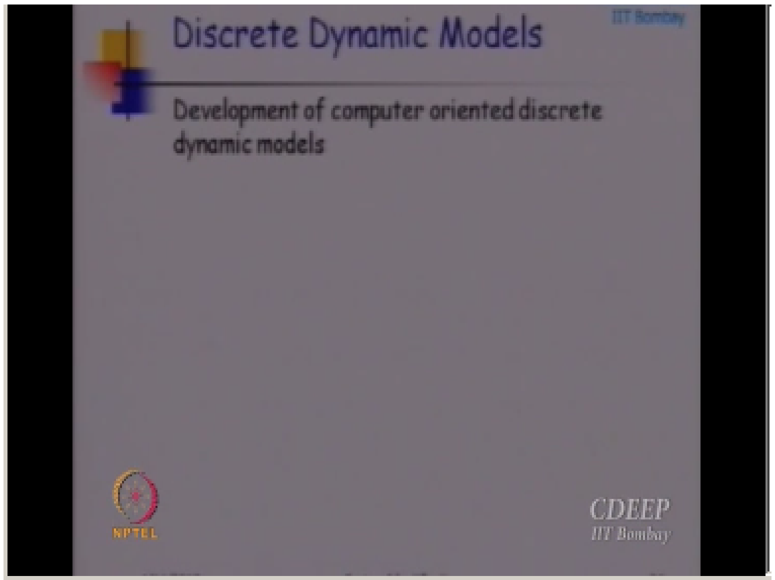

So what is want to do is develop a computer oriented discrete dynamic model okay I want to develop a computer oriented discrete dynamic model.

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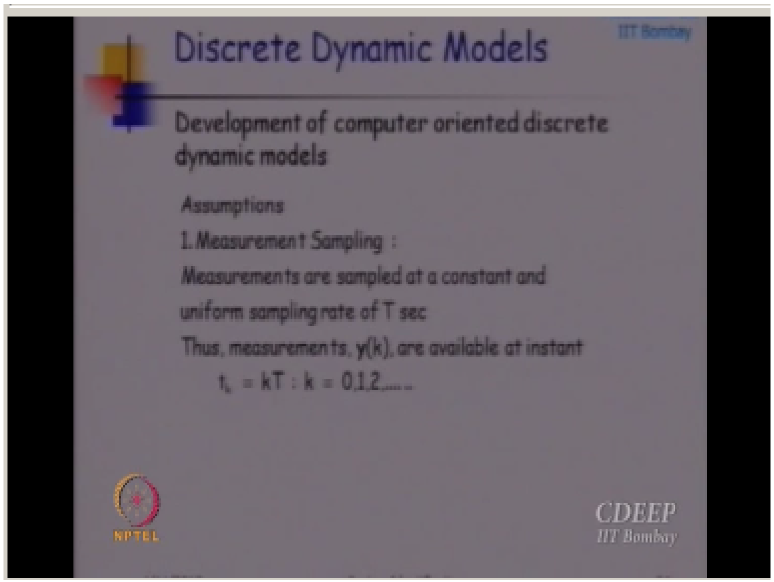

My first assumption is that the measurements are sampled let us assume for the simplicity that are measurements are sampled at constant rate there is no such restriction that one should do it one can have a variable rate sampling but we are developing the courses for the first time let us look at the simple questions okay so I am developing I am getting measurements in my computer at a constant variable every one second every one minute or every ten milliseconds what should be the sampling rate depends on the system under consideration if I am controlling a induction motor it is better be ten milliseconds if I am controlling a burners it can be ten minutes because burners have very slow dynamics yeah.

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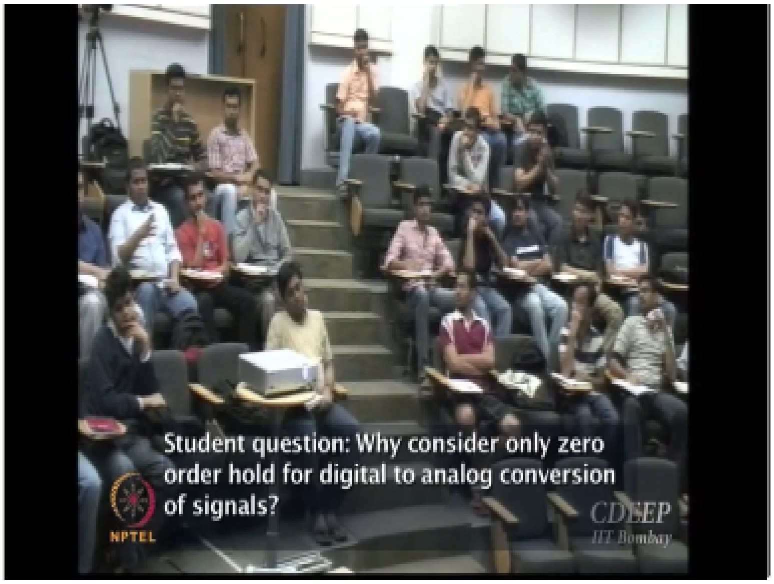

Because it is very, very common leads to it is very, very common it is 99% of the systems where you use digital control will be having a zero order okay one can adjust off course models one can develop models which are for the first order hold and so on but majority of them bulk of them are zero order hold systems but majority of the bulk of them are bulk of them are zero order okay so my one assumption that I have to adjust to or one reality that I have to adjust to is that data is coming at a regular rate okay.

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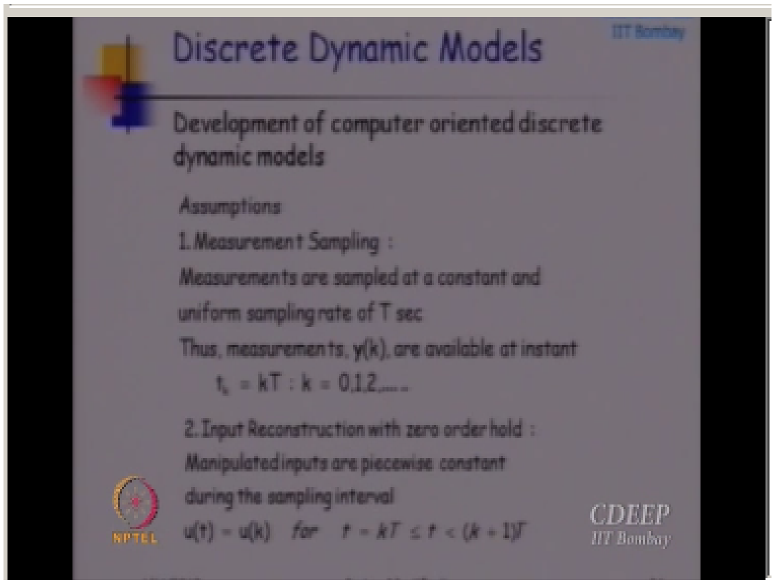

The second thing that I need to adjust to is that between two sampling instance my input is going to be held constant to the previous value so this states at for the time between see k is the sample number T is the interval between the two sample is called sampling interval so for the time between two sampling intervals my last input at k^{th} instance is going to be held constant is everyone clear on this any doubt with this okay so now I need to redevelop my models to adjust to the reality that I am going to do a computer based control okay and then linear differential equations are no longer useful we need to convert them into linear differential equations okay yeah it will take some time.

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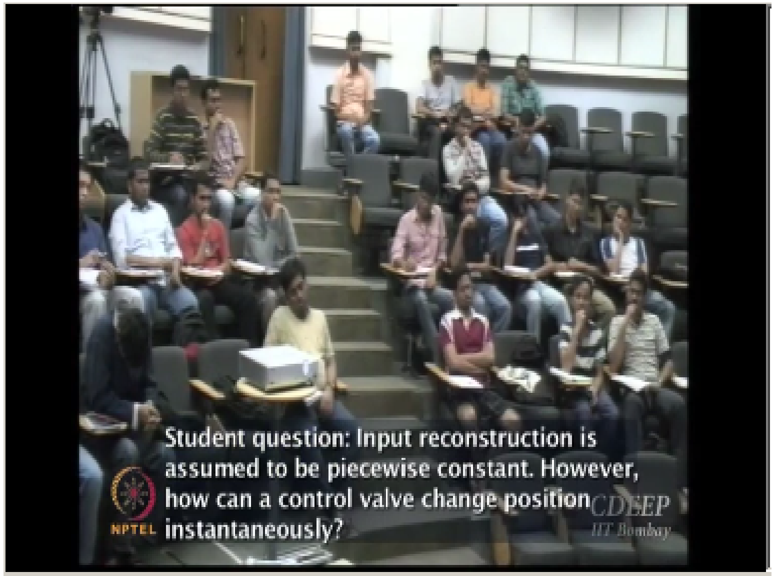

So one has to tell questions what is input to the system and what is the step? If I good question, very good question well, I will say that a my system is everything that is outside my computer okay and I will do a five in diction but when see typically we say that the wall position itself is the input let us not say that let us say that input is the voltage or current input that goes to the wall okay that is instantly changing because as far as the wall dimension is consent the dynamics between my computer and a input voltage input is instantaneous let us say okay so, in my system dynamics I should also model the dynamics of the wall.

And include when I do the modeling okay then the input for the plant is the voltage output coming from the computer okay is not the flow input with typically say that slow input is equal to voltage input which is not the base if the wall as the significant dynamics I would actually when I develop a model I will include the value dynamics in my module development and I will say that the input is just you know the current input okay now the module which is showed you or the heat system I had given relationship between the flow.

And a current input as a static algebraic relationship that is an approximation in the reality I should have modeled you know if I happened to be more accurate I should have model but dynamic differential equation between the wall position and a voltage I decided to ignore that because you know that relatively fast compared to the level dynamics or temperature dynamics okay that is the modeling okay but what you say is valid so one could view manipulate input as not the flow but whatever is going out of my computer is was manipulated.

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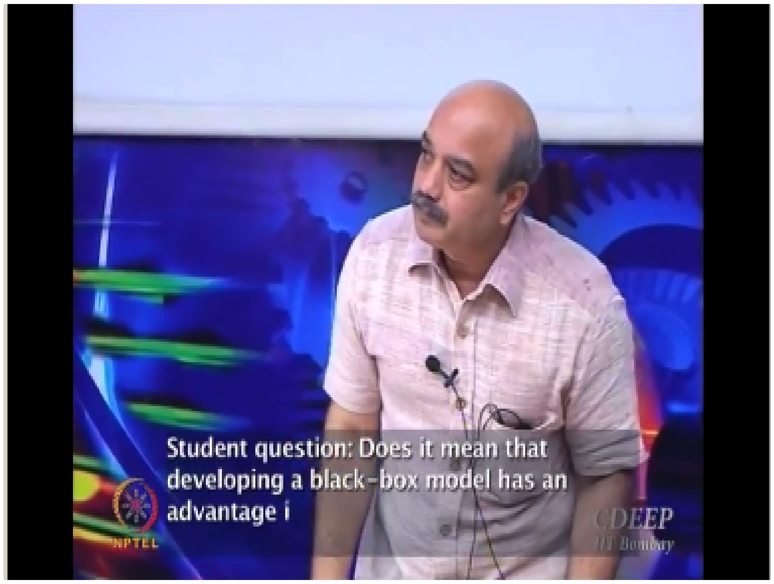

Black box modeling as the advantage that we do not have to worry about these sub components we just modules between what is coming to my computer what is going out of my computer okay that is the great advantage of the black box model we include everything that is outside okay.

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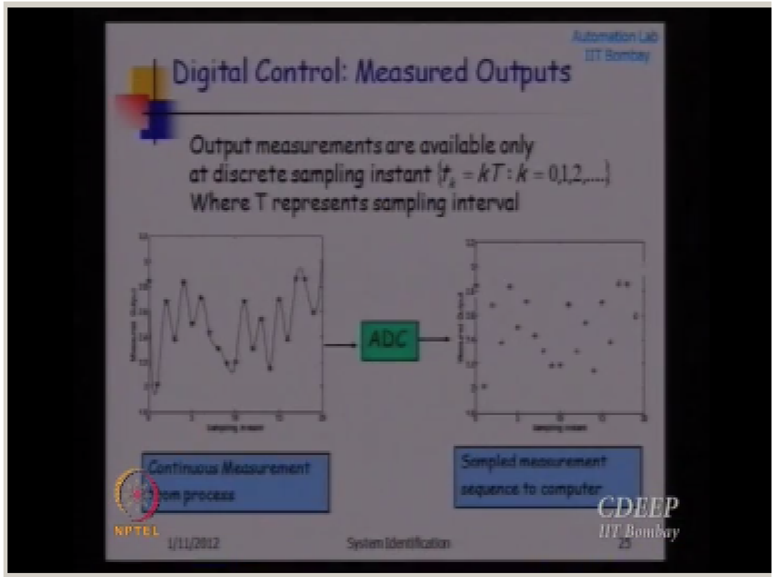

So we will stop here so I will just state this equation I started form a lonely differential equation use approximation and develop this model but then this model is not useful in a computer control system I need to change it to a difference equation model so my next class is going to be relevant to how do I change this from differential equation model to a difference equation model I am going to call it computer control relevant linear model okay that is what I will do adjust to the reality then the inputs are piecewise constant okay.

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