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ADVANCE PROCESS CONTROL

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Lecture No. 18

Topic:
Soft Sensing and State Estimation

Sub-Topics

Development of Luenberger Obvserver (contd.)
And Introduction to Kalman Filtering

So I am looking at SOS single output systems and I am going to design a Luenberger observer so there is only one measurement okay idea was to come with a observer game l such that poles of this matrix.

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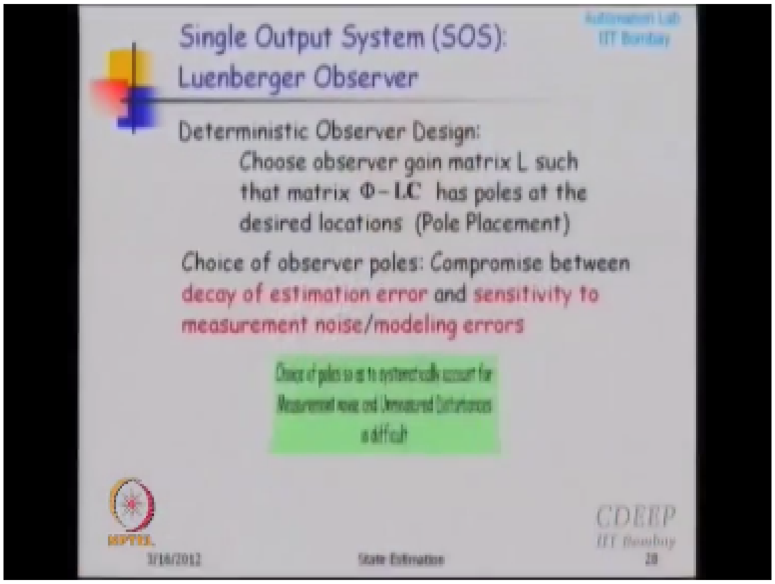

Φ-LC are at the desired location this problem is called as pole placement problem and it was solved first time by Luenberger sometime in 1964 or 62 for single input single output system and later he extended it to multiple input multiple output systems we are not going to look at the multiple input multiple output case I am going to describe something else or multiple output multiple input systems.

But conception ally this actually marks a landmark because you are trying to decide how fast or how slowly should the poles of the error dynamics how dynamics of error should evolve by choosing the poles by locating the poles at the desired location now, there is a comprise to be struck here because if you choose the poles very close to 0 the error will decay very fast but then your observer will become sensitive to noise okay right now we are not taking into consideration noise in any way okay we.

And if you placed it close to 1 the error dynamics will be slow okay it is slowly converts to zero it will not be so sensitive to noise but it will slowly convert to 0 so there is a balance to be struck and how do you choose poles this balance is not easy task okay so one needs to look at some other ways of solving this problem never less it is an important landmark development and there are many developments after that which actually can be do that extension of these ideas so this is one of the so when there are Socratic disturbances this luenberger observer might give you some optimal behaviors some optimal you know performance.

But then, it dependence upon the view point the way the way you want to design your controller or observer so I am not saying that one should not go by this approach this is one very nice way of designing an observer and if you get comfortable if you start getting feel of how to place poles you get good disturbances and then you can use this off course okay

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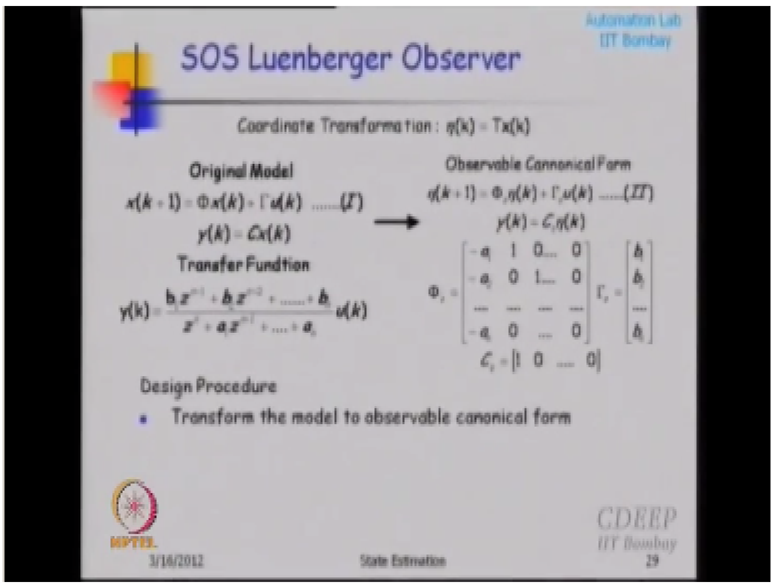

So quick over view of what we have done we did a transformation from the original state place to a new state place this is the power of we are doing everything in state place you can just reorient your state place do a design in a oriented state place and come back okay so that is qualitatively similar to many times what we do in Laplace transform we go from time domain to Laplace domain do some manipulations there come back and come up with the solution in time.

So philosophically you can look at this is the same thing you are transferring from one state place able to another which is convenient what is what does not change when you reorient the state place the transfer function between input and output does not change that is very, very important okay so you do this transformation to this observable canonical form we already know how to arrive at observe canonical form in some different context so you transfer in a transfer qualities to do a design you places the poles where decided location and then recover the observer gain matrix in the original domain.

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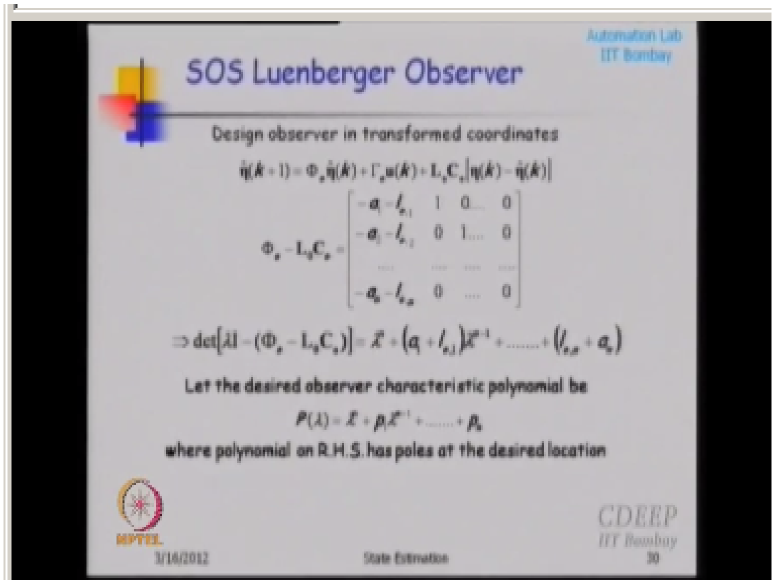

So the transformed system looks like this and together with then observer the poles of this ϕo-loco or lace so these are nothing but roots of the characteristics equation which appears in the first column so just looking at the observable canonical form is advantage looking at the first column you can tell what is the characteristics equation so if you choose a specific characteristics equation then you can just map the coefficients you can just equate the coefficients.

And then come with the design in the transform domain okay so this is what you would get if you have to use an observer you want this characteristics equation to be equal to this equation just equate the poles just equates the coefficients once you equates the coefficient see what is known to here you are specifying this polynomial so you know this you are specifying this polynomial you have chosen certain roots of the closed loop that will give rights to this polynomial.

So you know the coefficients here you know a1 to an you do not know l1 to l that this can be found by equating the coefficients this is the characteristic polynomial of the close loop with the observer this is the decided character polynomial you just equate and you will get L0 once you get L0 you will just use the inverse transformation and come back to the original state.

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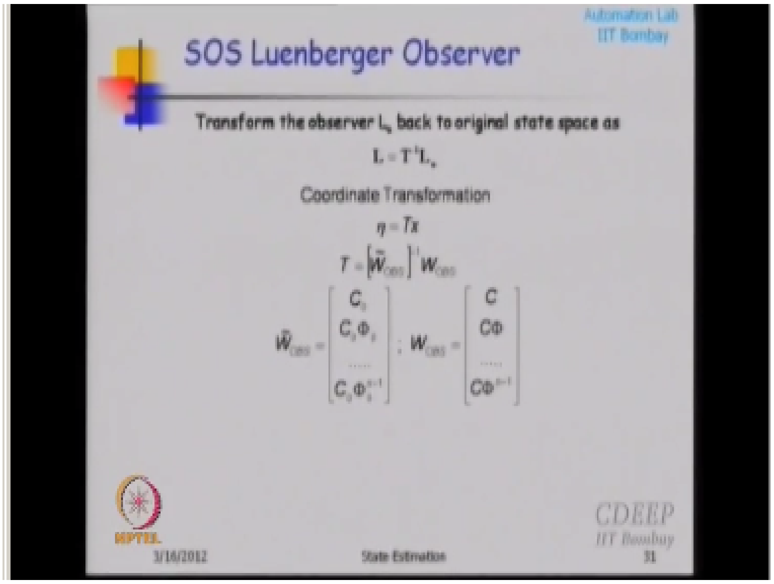

So you got L0 in the transform space you get L in the original space just by doing the inverse transformation which is t^{-1} and how do you get this t matrix is nothing but, you know it is obtained by multiplication of observantly matrix inverse of transformed matrix into observably matrix of the original system both of these are known because of the special form of the transform system you know this by the way we are talking about single output system.

So there observantly matrix here is the square matrix okay single output system the observably matrix will be n.t okay if it is system will observable the rank is equal to n otherwise the system is not observable okay so this actually expression tells you that you can place the poles at whatever location you want provided this system is log the system is not observable this t matrix cannot be computed okay.

So rank of the observable matrix is very, very crucial okay it is very crucial because T has to exist and t inverse also have to exist for t inverse to exist this matrix observably matrix of the original system should be invertible right both of them are enclosed matrixes okay this t is given by this multiplication of these two matrices t inverse will involve inverse of observantly matrix that is possible only when rank is equal to f.

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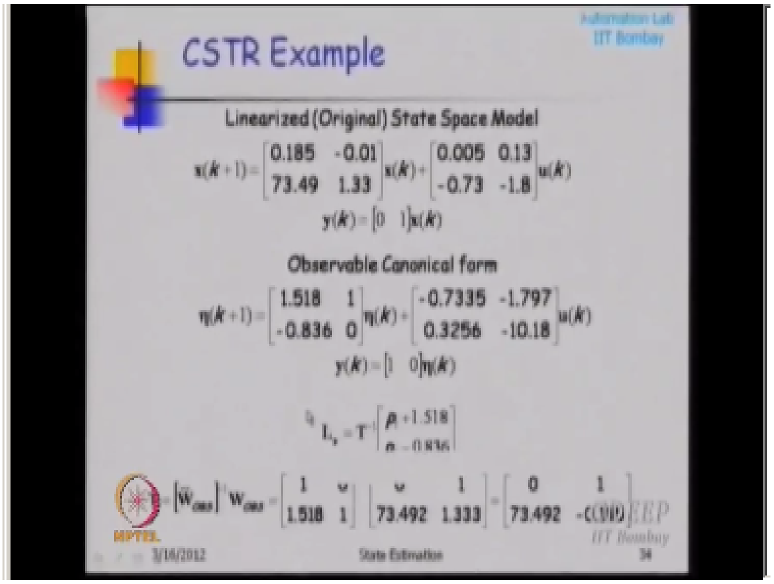

So okay and then I just showed a specific example where I took this CSTR the reactant example where we have two states concentration and temperature I am only measuring temperature so y equal only 01 x you have seen that in this system is observable so I can place the poles where ever I want okay if I do a canonical transformation the observable canonical form will look like this is the these are the coefficients of the characteristics equation which appear in the first column.

And if you do design you get this L0 which is per well why I am calling this LP right now will become clear soon but the observer you know equation comes out to be just T^{-1} where T^{-1} I will show you what is the t^{-1} can be computed very easily so this is how you design the observer.

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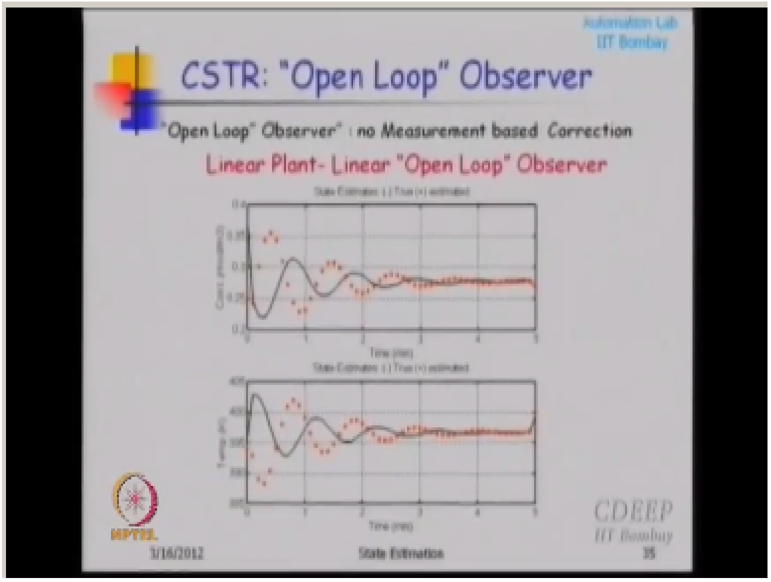

And what is the effect this is open loop stimulations okay open loop observer know feedback okay module running in parallel to the plant same input given to the model and then plant okay only problem is the initial state is wrong so initial state which is known to the observer is wrong the plant is somewhere else okay this plant is open loop stable okay observer error goes to zero very slowly okay.

Now you can see here it takes about five minutes for the observer error to go to zero where as when I put this observer even if I choose poles at 0.5 the observer goes to 0 very quickly okay within a one minute within ten samples okay the time scale here are different this is two minutes where as this is 5 minutes okay so this is expanded here in a very, very short time 0.5 minutes in 5 samples or 6 samples the error between the true and estimate goes to 0 when you are using the feedback gain okay so the observer does help you feedback correction does help you to quickly reduce the error to 0 okay.

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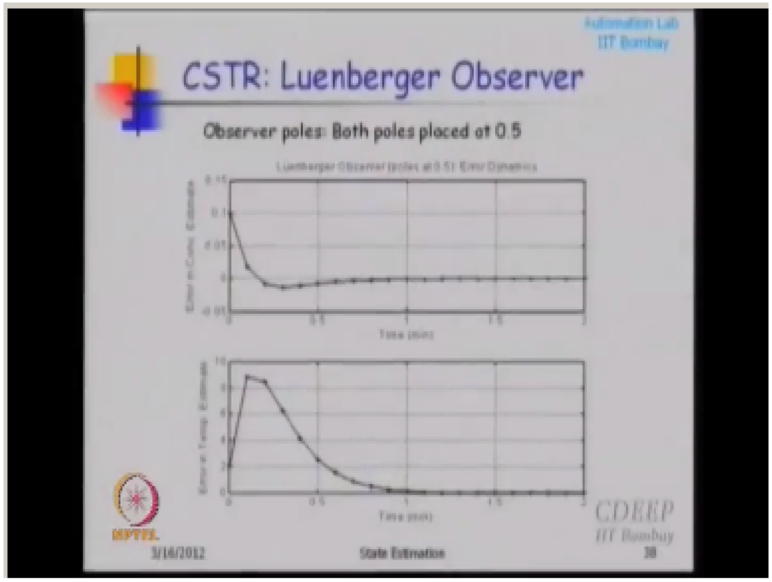

This is the plots of the errors when both poles are place step well we can check what happens when the plant is linear similarity model ling is not linear simulation so all those things we can check both of them are linear off course the error goes to 0 very fast if you take the realistic situation where the plant true plant is non linear simulation.

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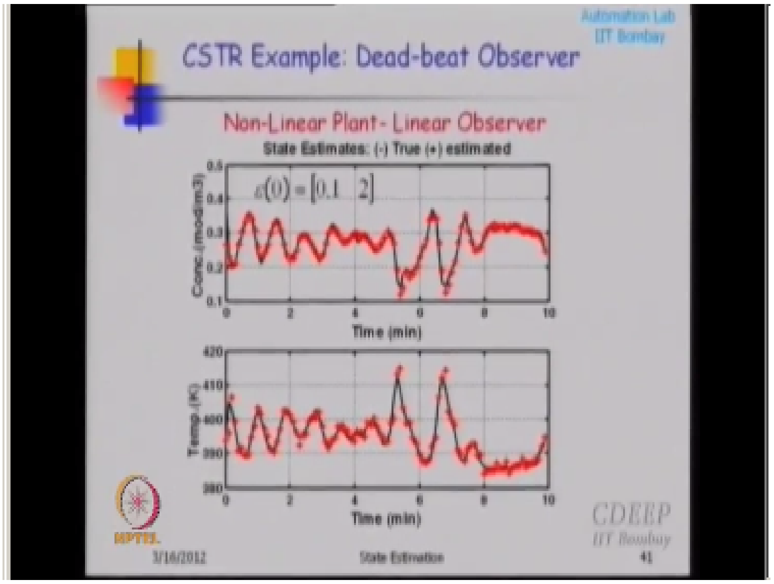

And you know my observer is given by the observer which is near observer then also it works by 12 it is not that as long as the range of as long as the operation range of your system is close to the point of linearization okay you are using a linear observer the plant is non linear they will behave like each other as long as your closed to the operating point where you liberalized okay the model and the plant become too different then this will not work but, you can see here that I am only measuring temperature in am not measuring concentration okay I am plotting through versus estimated just for your reference in reality I am never going to know.

The true value I am only going to measure temperature okay but since you are doing computer stimulation you can compare the true versus estimated okay and you can see that estimates are very close to the truth so I can use this observer as a soft sensor see I want to control concentration okay so I can put now PIT controller that takes the concentration as a measurement.

You can give a concentration set point okay this is many times called as infinite control that you are inferring concentration from temperature measurements through a model okay and then using it for the further control right now we are not going to the control we are just talking about the software sensor where I get an estimate of a concentration you know with

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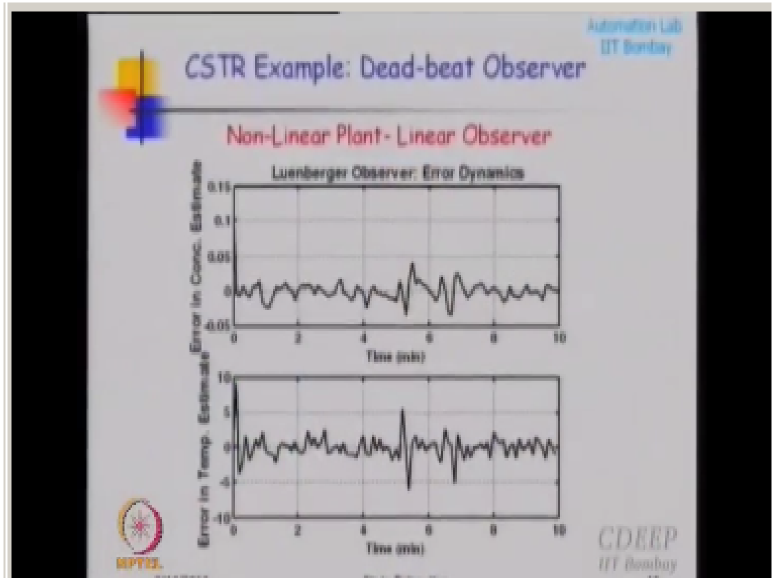

And you know this is the error how error behaves like it is pretty close to 0

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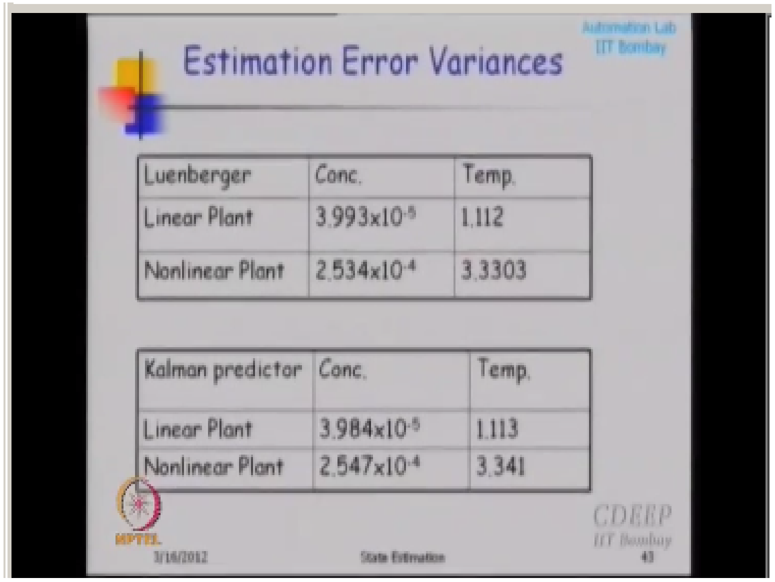

So I have just listed here some of the square of errors and they are pretty small so right now these numbers may not mix much

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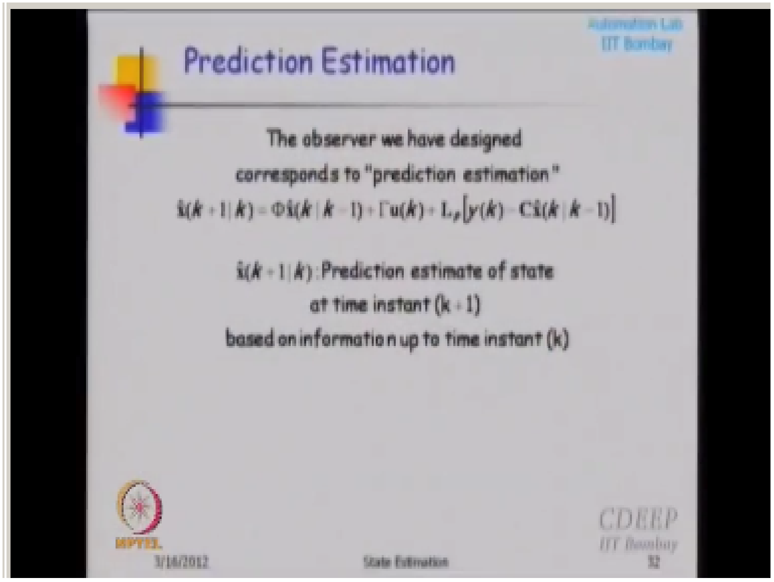

Okay so now what we designed right now is this observer okay now this is called as the prediction observer this is not there is one more way of designing observer I am going to talk about it now why I am calling it prediction that is because the state at k+1 has been estimated using measurement at point k okay or if you take k as the current point if you take k as the current point okay estimate at point k you will be constructed using yk-1 okay there is one delay between the estimate and yk okay.

So you might now wonder why I put a delay if I am getting a measurement now okay I can correct the state at the current time point you know I can correct estimate generate an estimate which is corrected at the current so this is called as prediction estimator and in concentrate when I talk about now current state estimate the difference will become clear okay so there is one lack between the estimate and the measurement and sometime you need to use this observer that is because you want to do very fast computation you do not have time to you know when you get a measurement okay.

Let us say I am doing I am writing a observer for induction motor okay now for induction motor the sampling time will be you know may be 100mikli seconds something like 40 to 100 milliseconds at the sampling time okay to do computation in 100 mills seconds depending upon what kind of method you are using it can be very, very costly okay so what I can do in that case is that I get a measurement okay and then I can using the previous measurement I can keep an estimate ready when I time point k.

Between the two samples okay I can do the calculations I have some time to do the calculations I can keep the prediction estimate ready and use it for control instead of calculating estimate at that point and using it immediately okay I may not have time to do computation but if your system is slow then you do not have to use this system you can do something better

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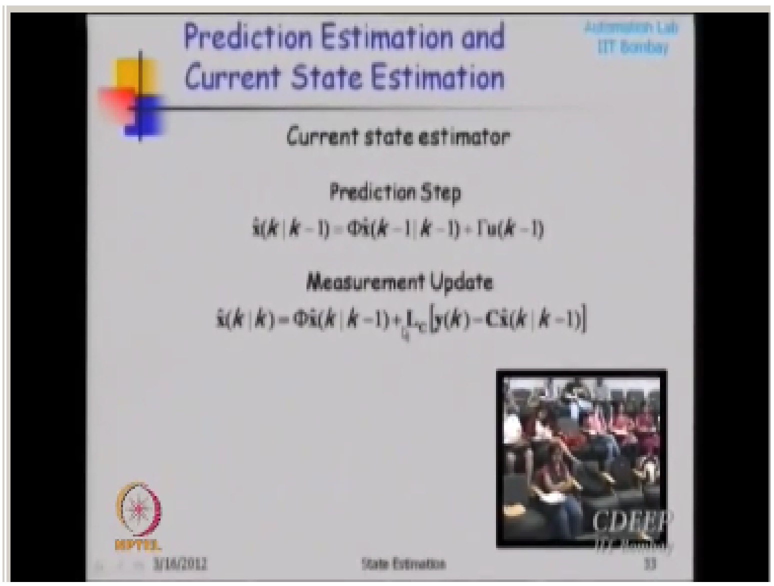

Okay so what is that thing? It is current state estimator is we modify the prediction step like this okay I do a prediction okay well the situation where you are not able to use this converted estimator and you have to use prediction estimator are kind of disappearing because computing is becoming faster and faster okay so that is somewhat you can say in many situation not quite relevant now so I do a prediction.

So what I am doing here what I am doing here is this is my previous estimate now just understand a notation xi is estimate of x okay k-1 means it is at time instead k-1 okay using information up to k-1 using measurements up to k-1 that is the notation here so, right now before the measurement have arrived I can do this computation of x at k given k-1 which means I can do a prediction of the current state 0kay using the previous information that is and the input that is gone at time k-1 okay so this is the prediction step and then I can do a correct there is a error here just correct your notes okay so what I do then is then do a update here okay I have this, prediction of x using information up to k-1 okay.

I can use that to predict y c times x at k u and k -1 is prediction of y what is this quantity this is quantity this quantity I prediction of y using the state predicted state okay this is my predicted state this is prediction of y see because my model is y=c(k) right. So I can predict y using information up to k-1 and that is this, this difference yk-is the error prediction error is also called as innovation okay and this gain observer gin times this error is added to this prediction okay I do a correction so what I m doing is here those of few to familiar with numerical methods prediction correction algorithms okay.

But integration you do a prediction using explicit method you do correction using implicit method okay so somewhat similar flow prediction correction okay so you do a prediction using model and then few data with the models see what is happening here why is the real data this, this calculation is happening in that computer I am running the model parallel to the plant the input which is given to the pint is given to the model oaky.

I did the prediction of what, what is the expected value of y based on what I know from the past that you can get from this model okay this difference will tell you okay what is actually y that you got this y here is the true measurement which is coming from the plant okay this difference is the error between estimated y and true y okay and this error is used to correct the state okay what I want is that the model should be in sync with the plant.

And this is done through the correction okay this is done to the feedback correction so I will see is the feedback gain and if you do little bit of calculations you can see that the error dynamics gets slightly modify now okay you can, you can just do this algebra is very, very simple to do this algebra we just take the truth and estimate under subtract truth and estimate and then you can derive this equation just like did earlier for prediction case you can do this here.

And obviously the what is the, what is the criteria that criteria is that course of this matrix Φ*i-LC okay I am calling this LC here okay I am calling this matrix LC here this is current state estimator okay so see here x^ k okay is estimate of k using measurements up to time k okay because I will use yk to estimate xk okay whereas this contrast with this previous one.

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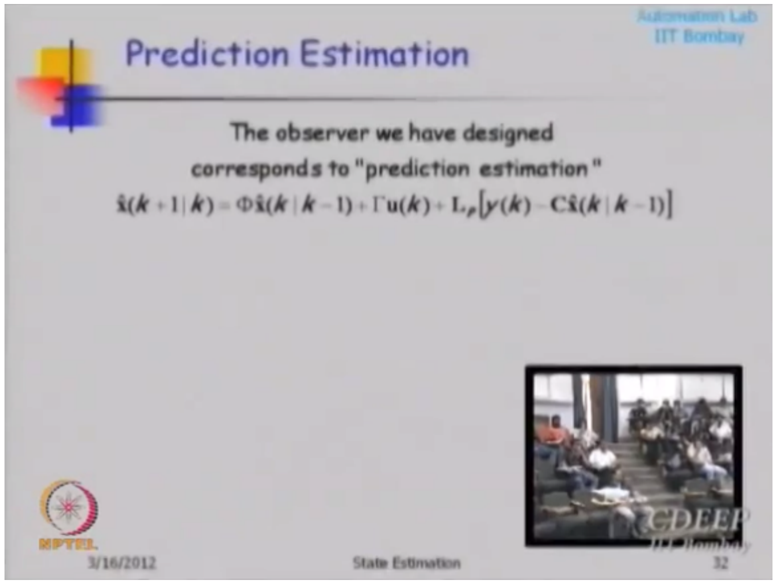

Look here there is a delay here between delay here between estimate and y see this is for k+1 you are suing yk so t k you will be using yk-1 if you use the same inversion same difference equation if you want to estimate x^ k given k- 1 will be using here yk-1 okay so there is the difference between this, this prediction estimator and current state estimator okay.

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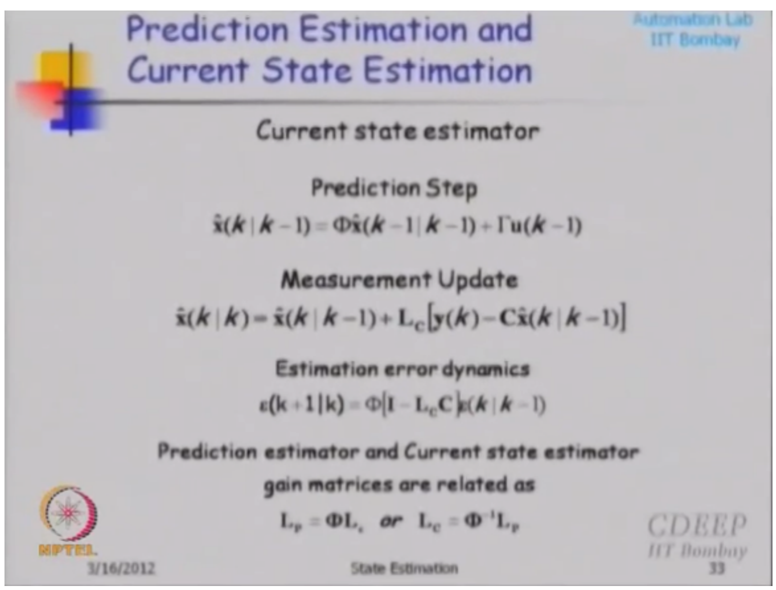

So the error dynamics is slightly different and then you can do all kinds of things here you can show what is the relationship between so the earlier one, earlier gain I have called LP that is for prediction gin this LC stands for current state estimator again okay so that is certain different between the two and their related through this relationship Φ is of course invertible okay Φ is not invertible there is the problem but if Φ is invertible these two are related through this equation.

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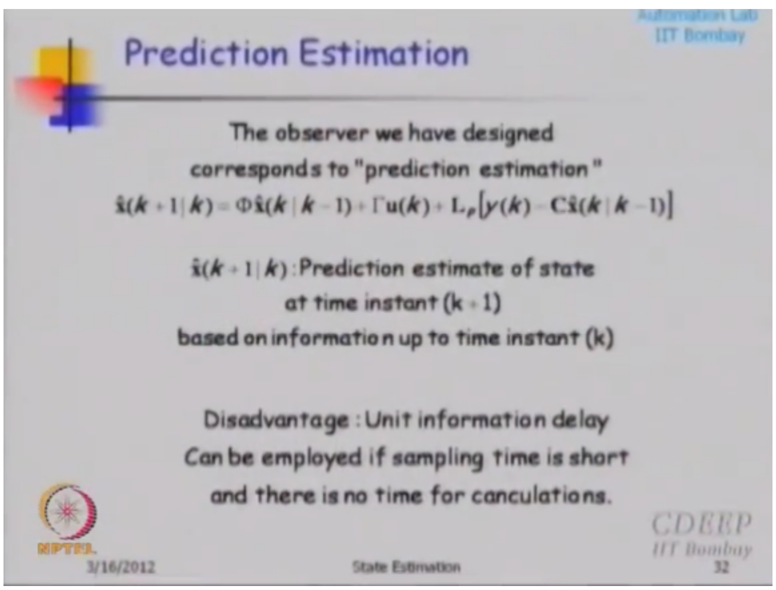

Yeah, prediction estimator I would use actually see you know you want to let us say you are doing control of concentration based on temperature measurement okay and see.

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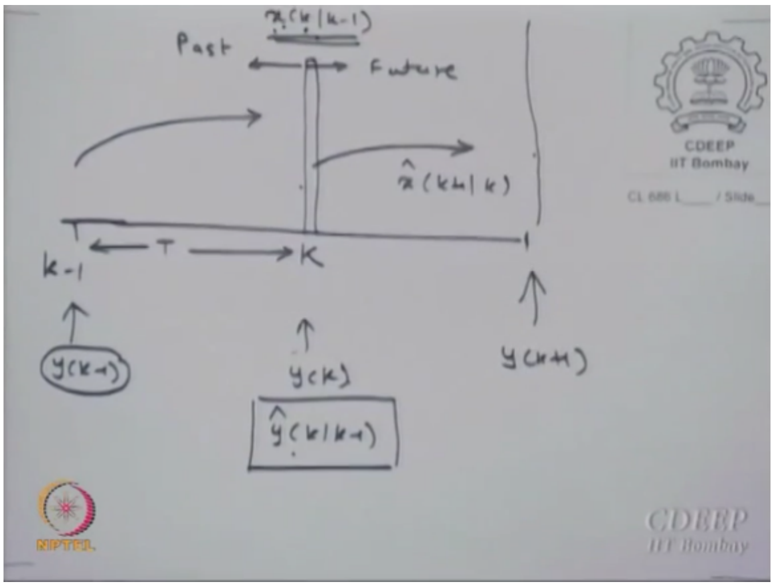

This is my current time okay so this is future and this is past k is my current time okay now see I am going to get a measurement here which is yk-1 I am going to get the measurement here yk okay now there is the time gap between this, see this kind gap is my sampling interval okay now see using yk-1 I can predict using this yk-1 I can predict here x^k k-1 I can okay so I can predict x^k given k-1 and if have this prediction then using this I can create y^k given k-1 okay.

So then what I can do is to let us say I want I am implementing controller okay I can use this y ^ without waiting for measurement to take place okay see because I can use this y^ or I can sue this x^ for my control see suppose I m only measuring temperature okay and I want to control concentration in might be controller so I need this x^ okay.

So I have this x^ available with me using the measurement which show the obtain at k-1 okay when you do this calculations I have time to do that calculations in this gap okay and I am ready with the concentration estimate here okay so without waiting for yk okay without witting for doing observer calculations I just use this concentration estimate and do control I collect the measurement but do not do calculations.

Now I can do calculations in between here and then in between here I can compute x^k+1 given k okay and the new instant I will get another measurement here which is y k+1 but I have this estimates so I will use the estimate for control okay when you will do this you know when you know the time required the time gap see if you know you do not have too much time to do computations.

If your system is very, very fast okay then I will do some calculations in the inter sample period keep it ready and use it okay so that I the advantage between predictions estimator okay now see this you will not relies unless you place a situation where the computation time is so small or the gap between two sampling interval is so small.

That you know you are not able to do computations but between two samples what you do you have time to do computations right so I can keep some background computation which is you know doing this see this calculation can go in the background between the two samples when I have yk-1 I can use it to estimate x^ k given k-1 okay and at instant k without writing for the measurement to come do 0berver calculations I just use the estimator predicted value and don control okay.

Now here you are saying that after the measurement arrives at instant k you re first doing the observer calculations okay and then, then the observer I will use this estimate for control you see the difference see now this these two steps okay might require some you know 20 minutes seconds if you do not have this 20 minutes seconds see suppose you are sampling interval is 15 minutes seconds if you spend 20 minutes seconds in doing calculation for observer okay.

Then you know there is the problem we should instanteously do your calculation that is what is expected when you led a control instanteously means the time required for computation is very, very small compare to the sampling interval if it is not then what you do is use the predicted value from the which you re generate between two samples okay do control collect the measurement okay in between next two samples do again the calculations keep it ready for the next sample.

So this more way implementation okay and then why I am still talking about prediction estimators when you can when you know computation times of becoming smaller and smaller will become clear end of the lectures because I m going to connect thee observers to time series model situation and I will show that the time series model that we are developed t the thing but prediction estimators so that is the reason why I am prediction estimators even though you know they might appear outdated.

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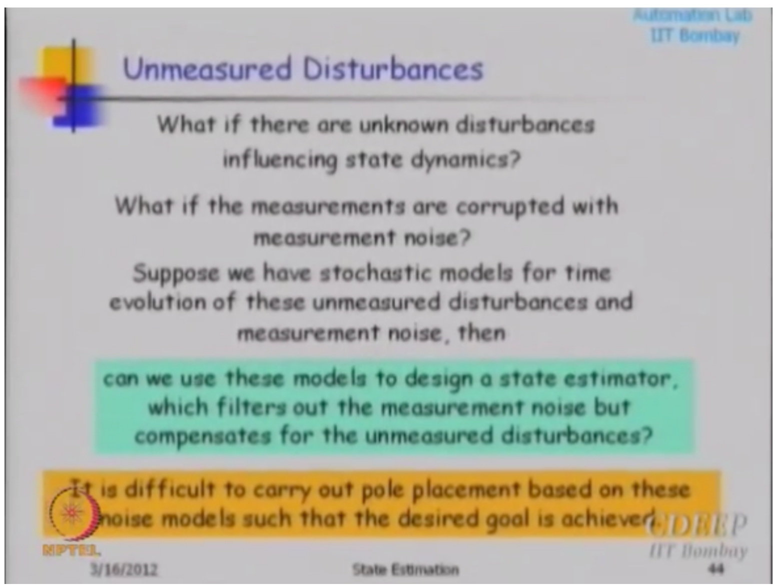

Okay now we get an ideal world design low measurement noise low unmeasured disturbances model is perfect and we said only possible error is in the initial state of the observer plant state is something initial state of the observer is something and we were worried about whether the estimators good through the truth under the ideal conditions but the real world is not like this they always measurements which are corrupted with noise.

You will never get a perfect measurement okay you always have some kind of unknown input affecting the sensor measurement now we know how to do stochastic modeling right we know white noise we know colored noise we know will kind of things so if I have a sensor okay I can try to develop a model for the noise I can try to develop a model which is a white noise or colored noise typically sensor noise is a white noise okay you can if you do this experiment of taking a sensor keeping the true value of the plant constant and just collect data you will find that the noise in the measurement is typically something like a white noise okay.

So we can develop a model how do you develop a model for white noise mean and variance particularly if it Gaussian white noise life is very easy Gaussian distribution is characterized by first two movements mean and variance so just characterize just find out mean and variance and you have model for how the noise behaves that mean even if you do not know exactly what value the noise will take you know that it behaves like white noise like Gaussian distribution with mean equal to 0 expected value is 0 okay.

So this you can you have model okay you can construct the model for the noise okay then if you have a model for noise can you use it for improve your estimates that is our first question the problem is even if you do this modeling how do you choose pole okay how would you pole placement said that you know the noise is effective and rejected is very difficult question to answer okay.

The root that we have taken of pole placement okay is said that it does not what I will have to do is I have to actually develop a optimal gain which rejects the noise is to do trial and error you tries different values of poles and how many values of poles you can try so how do you try there is no systematic way of you know choosing pole location sets that the noise is rejected optimally so knowing the noise model does not help in know placing the models okay.

So you know how the model and noise you have something more now about the noise does not help you chasing the models so you need some other methods for these also one more possibility that there are errors in the measurement okay there could be unknown inputs influencing the dynamics right.

We assume that input is only u which is not true that could be something else okay which is influencing the dynamics I do not know and we have already seen that in times is modeling just effect of u does not explain everything happens in y there is something else so there is some unknown input and then we need to model the unknown input okay.

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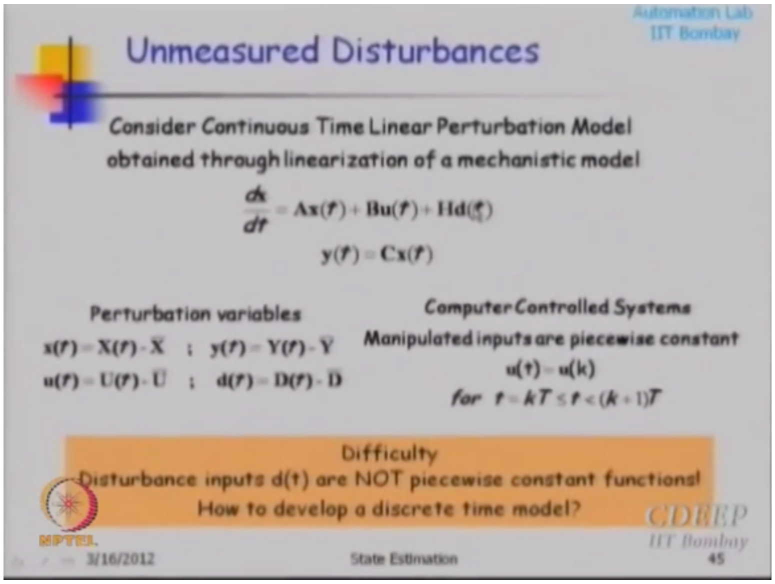

So let us check the good old linearization okay remember is continuous time model which we developed okay we had this model which is ax T+Bu P+ this is the disturbances term for long time we will never talked about it okay.

But now I want to consider this disturbances term BP okay so this could be in real plant this could be some input which is actually entering the system which you have not measuring we do not have control over it okay so for example if we just have a simple tank and you have a flow coming in the flow may not be constant will be fluctuating okay.

So the fluctuations in that flow will be actually d here okay so many, many situations you have inputs everything that affects the dynamics you cannot measured okay so you are the manipulated variables d are the inputs a disturbances inputs okay now in y effect of d is going to be represent okay.

Whether you like it or you do not like it okay the effect of disturbances is there in y okay where of course we have this quadrapations variables and e normally assume that inputs the manipulated inputs are p^2 constant why do we assume that because we implement digital control or computer control through a 0 order old that is digital to analog converter in which we hold the inputs in p^2 constant okay.

Now there is the trouble here what about this guy pt that is not going through 0 order old that is actually entering the system continuously this changing continuously within the sampling also it is changing okay I am going to make a simplify in assumption okay I am going to assume that the sampling period is small enough okay that so small that I can approximate this dt using p^2 constant function okay.

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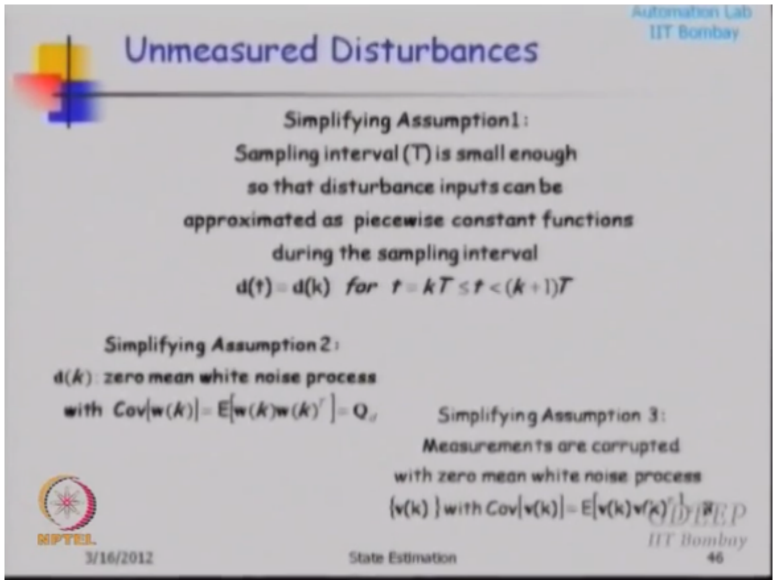

So just like the manipulated a variables are p^2 constant I m going to assume that inputs disturbances inputs okay are also are can be model has can be adequately represented as okay p^2 constant functions this does not mean there actually entering the p^2 constant functions the disturbances which are entering real by disturbances are entering or not entering the p^2 constant manner.

It is simplifying modeling assumption that we make you simplify the mathematics okay I am going to make one more further assumptions right now because you know not only that p^2 constant I am going to assume it is like white noise okay what is the white noise we can solve the problem later.

Let us take a you know idealized problem where disturbances are entering as if there white noise p^2 constant white noise okay that is what is our assumption okay so it is white noise 0 mean there assume white noise so what you can know characterizing white noise mean and co variance okay there is no mean and co variance so I have a model for disturbances so the disturbances are entering as 0 mean white noise process with co variance equal to d.

Well to do my algebra I do not need Gaussian anywhere so I will come to Gaussian little later okay I will be using Gaussian has some other part so now I have this three simplifying assumptions just note them first simplifying assumption is that I am assuming the disturbances that p^2 constant okay or can be model as p^2 constant this is simplifying assumption 2 is that not only p2 constant there like white noise 0 mean and co variance I know the co variance I know the meaning 0 okay.

I will worry about there not white noise better okay there are waste of something will see it later my third assumption is that the measurements re corrupted with another white noise my measurements re corrected with another white noise who is mean is 0 and so this is the errors in measurement are random okay I know the mean I know there variance typically this matrix are here will be diagonal matrix.

If there are five measurements five senses then this will be diagonal matrix with all diagonal element is equal to 0 diagonal elements will be variances of each sensor okay error variances of each sensor that you can find out you can find out the constant temperatures okay I know the true temperature of water I can take the measurement temperature by not difference some of the square divided by N will give me variance right.

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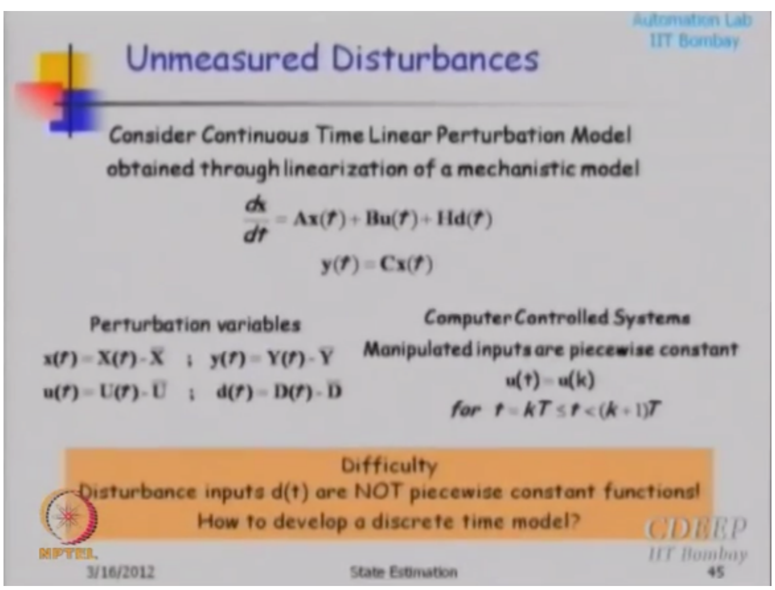

So yeah, actually I just write here no, no see this is the wire unit like this because this I what we have got through linearization of non-linearization differential equation right now we are not talking about noise now we will add a noise model through this okay so I am composing a model okay through this linearization plus I m attaching to it noise models okay.

So now if I do discretize of that model and assumptions okay then I get this model okay with a little bit of algebra between what I have done here is this term here okay I will get this term okay I will just do it here.

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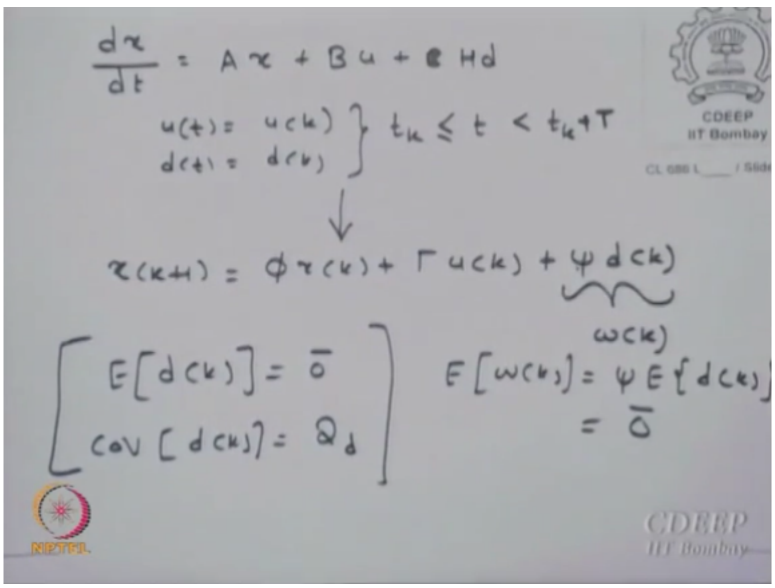

See I have this model dx/dt=Ax+Bu+Hd when I discretize this and then we are assuming that ut=uk and dt=dk this is for sampling t_k and capital T is the sampling time okay during this intervals there are assuming that the manipulate variables are p2 constant and the disturbances are p2 constant okay if I do the discretization of this system okay I will get x k+1= Øxk+γuk+ ψ dk okay.

I will get ψ dk not a ψ dk I want to call this as wk okay I want to call this ψdx as wx and then we assume that we had assume that expected value of dx=0 and co variance of dx=qd this is what we assume right this is what we have assume now if I define this new variable wk which is ψ times dk okay what is expected value of wk=ψ expected value of dk which is equal to 0 because expected value of dk is 0.

So expected value of ψ times dk also 0 okay and then you can how with little bit of algebra that co variance of wk which is expected value of ψdk,ψdk transpose okay this will this quantity here will turn out to be ψqd, ψ transpose okay so this is, this is what I done here just look at this okay I am defining this wk to be ψdlk my expected value of wk will be ψ times expected value of dk see the expected value of dk=0 I get 0 here.

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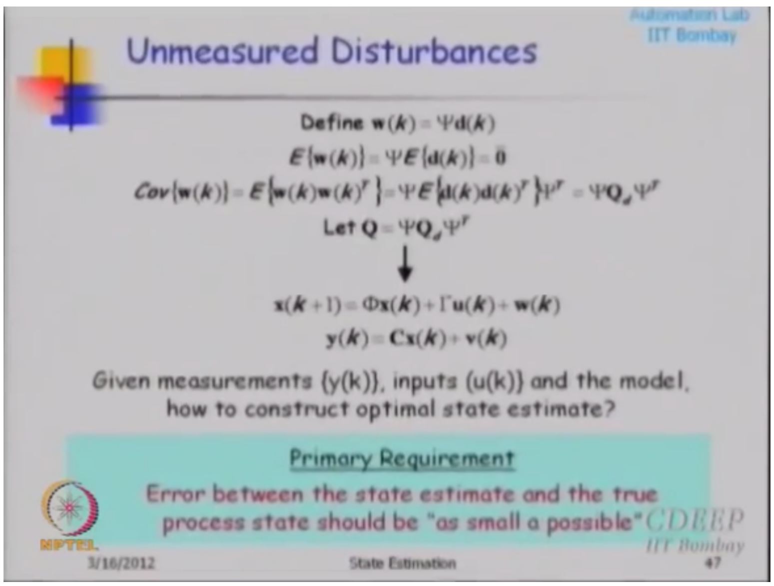

A covariance of wk will be the expected value of wk, wk transpose that will be turn out to be is it okay so I am calling this quantity I am calling this quantity ψqd, ψ transpose t okay no, it is not colored, colored with happen that is a good question it is co related within excel but it is not related time you said this color only when time is correlated okay.

It will be correlating space okay so elements of w1 and w2 will be correlated but yeah, so time correlation difference from space correlation okay so I want to work with this simplified model okay I have done will these algebra to show the connection with physical model which you get from the linearization okay.

So this is the model which I want to work with xk+1=5xk+γuk what is wk here? wk quantifies those inputs which are affecting the dynamics and which you re not measuring okay is the effect of those inputs see what are these dk, dk unmeasured disturbances okay so this, this untidy we also call it as state uncertainty okay.

This is state noise or state uncertainty, uncertainty in the state dynamics because there are some other inputs other than u which are enforcing the dynamics do they quantify w okay and what is d? This is the measurement noise or measurement uncertainty okay I have a model for this I have model for this what is a model? 0 mean white noise you know what is the white noise right.

There is no time correlation okay so 0 mean white noise signal except earlier we looked at white noise because the clear now we are talking of white noise vector okay concept does not change there is no time correlation, there is no time correlation that is the important okay so wk, wk-1 wk-2, or uncorrelated expected value of wk, wk-j transpose it always 0 that is what meaning of white noise okay.

So this w, this w is a not quadrant time okay the same as true bout vk, vk is random error in measurement which is not quadrant time what is the meaning of white noise no time correlation okay elements of w, w1, w2, w3 they can be correlated each other but they are not time correlation okay that is critical okay now I have this model that means I know for yγc matrices I have model for w1b how do I optimally estimate the states okay what is the primary requirement error between estimate and the truth should be as small as possible.

How do you define as small as possible cross correlation some of the squares yes some of the squares, some of the squares error should be goes to 0 or some of the square error should be as small as possible but some of the squares you have to be little bit carefully with the vectors so error transpose error so two norm so you can say norm of error width you know may be one norm two norm some norm are error vectors should be small as possible okay.

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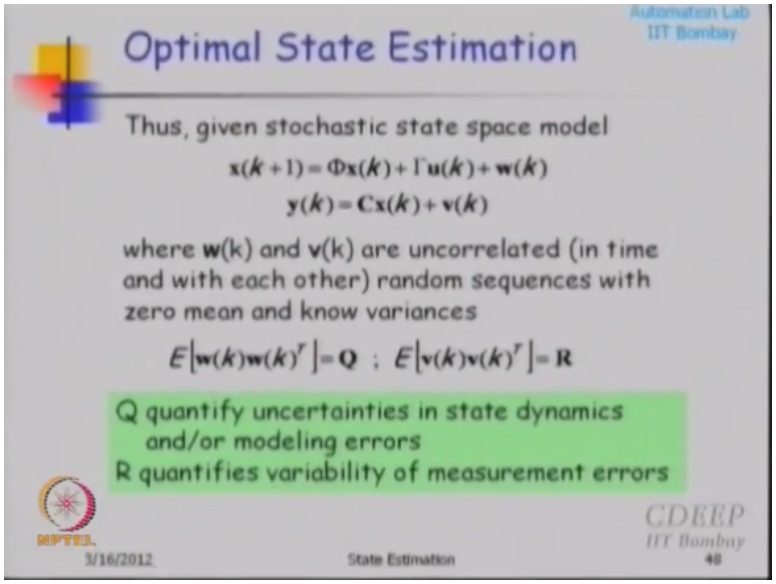

So let us see how do how Kalman's problem now stop me wherever you want if you feel sleepy just tell me because I have to change here so we have stochastic state space model okay we have a stochastic state space model why do the stochastic state space model there are deterministic inputs there are stochastic inputs what is the deterministic input? U stochastic input is w okay so actually x by virtual the fact that w is the stochastic process x also becomes stochastic process okay.

And w is a very nice stochastic process is a white noise 0 mean x is not simply you know it does not have such a simple behavior it has more complex behavior because this w is going through the dynamics okay so in some way w is white noise x will not be a white noise because current x depends upon okay so x is not be white noise w is a white noise v is a white noise and where dealing with the stochastic system stochastic difference equation.

The measurements increase trouble the measurements are also you know corrupted with noise so life is not easy because there are two sources of uncertainty one in the state dynamic other in the measurement okay and you want systematically handle this let us make this assumption that measurement noise under state noise are uncorrelated okay.

Let us make a simplify assumption okay we make a simplify assumptions so that we get nice problem which can be solved easily through the Maths that we know and then we get some insights and then you build up on it and try to solve complex problems that is the trick which is normally done so well what if wk and vk are correlated we can solve the problem okay.

But we will do that later you may have that so if I correlate on something I might be able to handle it through adjustment of covariance, co variance of w uncertainty so this is the model you have to understand that the model I not you are giving actually to write a model and say that the true system behavior like this model your results re compromise okay.

Your truth is not a model okay yeah, but you can never, you can never ever develop a model which is exactly equal to the truth that one possible any you know you can develop more complex models but does not mean there is I mean you reach the truth okay only place where the true model or true summation can be inside the computer.

When you approximate some this is the reasonable assumption that errors in the measuring device have nothing to do with the disturbance that appear in the plan this is perfectly, this is perfectly you know logical my fluctuations which are in some flow which are coming because of something happening up to you or some temperature fluctuations because I am getting a temperature from you know some storage time on the on my breeding I take it to measuring error measurement error.

So measurement error and errors in the disturbance are uncorrelated very, very logical assumptions the earlier one is simplifications okay so this model is now we are t this point we have this model we know the variances of w, we know the variance of Q okay and now I want to find out I want to get a optimal estimate of x using this model using this dynamic equations.

Together with this stochastic model yeah, how will I get q and d well I we have recently my student PH.D on how to get Q and R we have only state I can send you the paper it is not so easy to answer some of these questions so many times what people do is use r you can find out R is not difficult Q many times we are using the parameter you know that there is 5% uncertainty and then know that is one approach the other approach is actually develop identification algorithm to estimate parameters of Q that so, so see the it is like a huge possible.

And then you know I have to explain some partial isolation it is know of explaining the whole thing together okay so the end of the course probably the entire picture will become complete okay so right now assume that somehow you know Q and R okay now how you Q and R do not ask the question right now okay.

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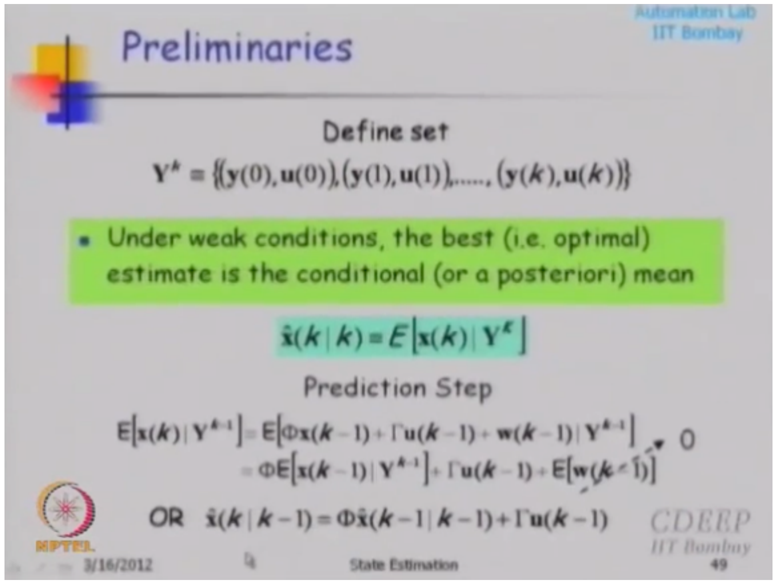

So now there are some preliminaries okay notation is going to be complex okay so just I am making you away now this set I am calling this set Yk y is super script k it is not y erase to k it is only the notation it is not wire is to k what is the meaning of this Y super script K it is set of al data collected up to point time point K okay so this data consists of u and y measurements and inputs which re gone o the system from time 0 to time K okay.

So that set is called as okay what you can show what you can show is that the best estimate the optimal estimate okay of the state is equal to that conditional beam of X condition X is a random variable because w and v are random variables so X is a random variable see let us go back here w is a random variable b is a random variable now I m going to use the measurements I am going to use the measurements to correct the state estimates observer right.

So my estimated state is going to be function of Y see the true state is not function of Y estimated state is function of Y why? Because in the observer we sue the feedback right Y-y^ okay so my estimated state is going to be function of the measurements okay so since w and v themselves are random variables X is actually stochastic process you are agree with me X is a stochastic process okay.

And you know it will have a mean to stochastic process it will have a mean value okay it will also have variance okay the variance could be time varying see here at any time point the variance is Q for this for any time point variance is R okay X is a stochastic process okay it will have it is own probability density function okay it will well so probability function okay so what I am saying is that the estimate of X best estimate of X is equl to now this proof you can referred to the book by Soderstrom I do not have time to do I have given the reference at the end and you can see why this conditional mean of X.

So have you heard of conditional density function that is or you have heard base rule you must have base rule for sometime okay so probability of event A given that B had occurred okay so you know I can talk of conditional density of a variable X given that Y has occurred okay so same thing I m trying to talk here.

I am going to talk about conditional density of X probability density of X okay given there I have this measurements collected up to time K because to generate an estimate I m going to use I will these measurements I am going to use all these make measurements to generate a estimate of X okay so let us leave this thing here let us proceed and we can come back to this particular statement this is little okay.

So now let me do let me start doing predictions okay so what is the conditional expectation of X given measurements up to Yk-1 okay I am going to do conditional expectations of X I do not have exact density function right now but I m going to use the equation okay all that time I m doing here is I am writing you see what I am doing here I am saying that Xk conditioned on measurements up to k-1=expected value of this right hand side.

What is the right hand side? Φxk-1γuk-1wuk using information up to k-1 okay now I am going to take expectation operator here inside okay so I get Φ expectation of xk-1 given yk-1 okay yeah, yeah we will come up with the notation now just wait okay so if you use this definition here what is this quantity expected expectation of x^k-1 given k-1 see this x^ of k-1 given k-1 right okay.

See I am going to comported this using some tricks okay so now till the trick is over just wait and see how you do the algebra okay so now what is the expected value of uk-1 uk is a deterministic value okay so this will come out of the expectation what is the expected value of wk ? So 0 mean variable okay so expected value of this is 0 okay.

So from this equation what I get is this you see what equation I got the same equation which you have written earlier okay except now interpretations re different when I talked about observer I never said anything about conditional men or anything of that solve right so that part is there were I have reinterpreting it through a different view point okay.

So now so what is this quantity conditional mean of X at time instant K using information up to k-1 okay so do not be scared that you here to do actually constructor visualize those densities right now we are got a short cut to find out the new conditional mean if you know your old conditional mean right now how do you know the old conditional mean is will answer that question later.

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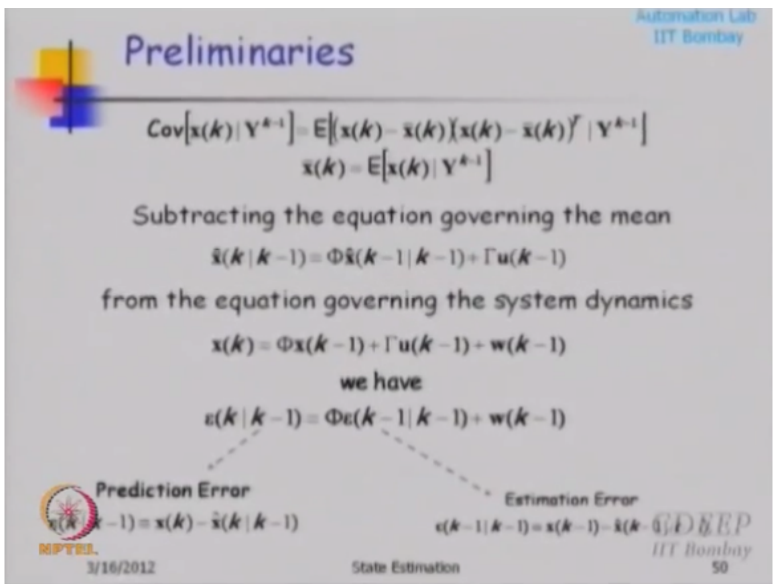

But this tells you that the new mean the new conditional mean is five times old conditional men okay plus this quantity γuk okay because expected value of uk gets 0 okay what is co variance? Is the definition of co variance correct just check I want to find out conditional co variance I want to find out conditional co-variance okay can you find out what is conditional can you worked this out that will be easier how do you find out a covariance you compute this quantity xk-x bar k, x bar k is the conditional men see we have computed this, this will be our x bar quantity right yeah, mean of xk so mean of xk conditioned on yk-1 which one R yeah, yeah thanks.

Okay so men propagation we actually found by this equation how mean propagates in time we found by this equation this is how the mean propagates for the stochastic process X the mean propagates according to this equation okay now I am going to subtract this equation okay from the dynamics of Xk okay.

And then so I am going to subtract this mean equation from this equation yeah, and then I am take the co variance okay so if I subtract this do I get this just check you subtract this equation this is the mean equation this is how the mean propagates I subtract this equation from this equation what will I get see this uk and uk will disappear okay.

You will get Φ times xk-1, -this quantity okay and I m defining an error here that two different errors k given k-1 and k1 k-1 given k-1 is very well with me on this equation this is okay what is the co variance of ε k-1 can you compute what is the mean value and what is the co variance for the time being let me tell you that it is mean value 0 I will prove it I will prove that mean value of this will be 0.

But let us say the mean value 0 how will you find a co variance so just do it see this wk and ε they not correlated wk-1 and ε k-1 they are not correlated so I have two errors here prediction error and estimation error okay I am defining two quantities prediction error and estimation error, estimation error is difference between the true X okay and the current estimate at k-1 this is true X and predicted estimate at K okay.

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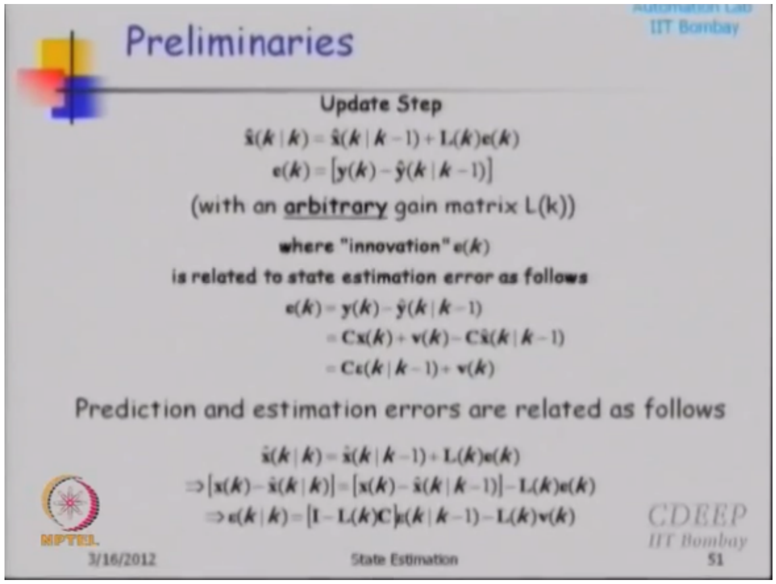

This is fine so I have this difference equation, I have this difference equation which governs the error which governs the estimation error okay so my update step is going to be like this okay now I will talk about the covariance little later are you fine with this, this definitions I am just doing some preliminaries you know finally I have to derive this Kalman's algorithm for doing optimal estimation okay.

So my update that is going to be like this okay my update is going to be like this right now I am using an arbitrary gain LK I do not know how to chose gain LK okay this is this step is you are familiar with this step you have done this earlier that is new updated estimate is equal to prediction estimate plus a correction, correction coming from the measurements.

This is y which is measured –y which is predicted y okay this difference is used to correct the current statement estimate this is the standard thing okay now well I have chosen a gain which is an arbitrary gain matrix okay so here ek is called innovation and then I have shown what is the relationship of innovation with the estimation error so this particular step is very easy to derive is not you just look at the pre equations and you know I have just substituting for y^k given k-1 C x^k given k-1 and yk is cxk+vk so you take C common here you will get xk-right is simple algebra okay.

So you can just do a little bit of some over can shown that the prediction error so the estimation error and the prediction error are elaborate through this equation this again leads a little bit of working and you can very easily prove this equality so ε of r right here now this I am trying to find out between a k given k and k given k-1 I want to find the relationship see what I am doing is I am just combining I am just using this equation I am using this equation and combining it through this I am combining it.

You can try and derive this just see okay so right now I am met a point where I have this L matrix okay and I do not know choose L matrix I have chosen there okay and I want to come up with the systematic way of choosing L matrix okay I have done some algebra kept some equations ready and then okay let is skip this for time being.

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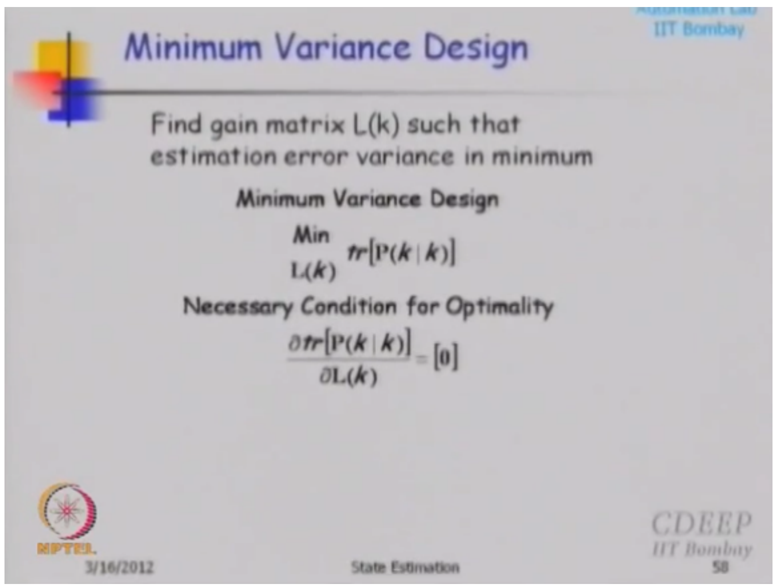

Let me tell you where is how where I want to reach and then we will do the just a minute okay where I want to reach finally after doing lot of algebra in between which will take some time to digest please bring those notes otherwise it is difficult to follow unless you have this notes and if possible you can try to read and come and then see.

I want to find out that gain matrix such that estimation error variance is minimum okay so this pk(k) is the co variance of the state at time instant k okay I am going to find out co variance matrix of this error estimation error I want to minimize the trace of this co variance matrix that is what I want to do okay with respect to L. I want to choose L in such that in such a way that trace of the co variance matrix, co variance matrix of what this p is the co variance matrix of estimation error $\varepsilon k(k)$ okay.

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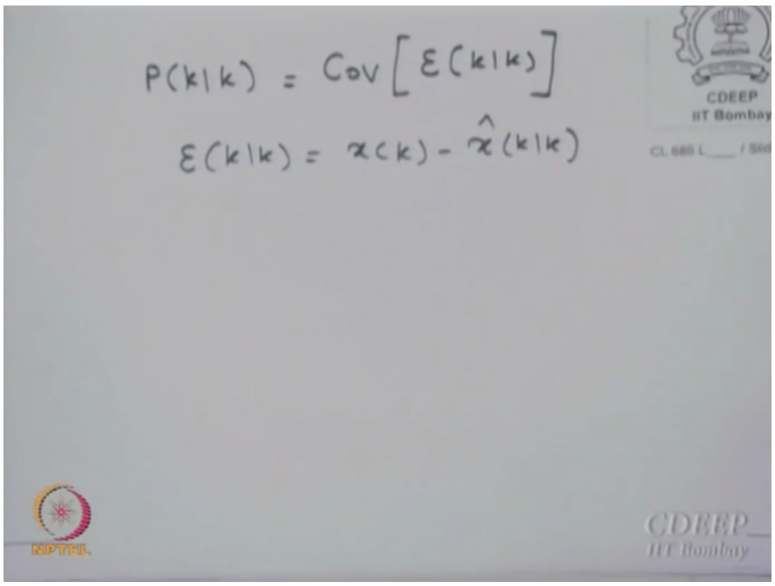

So this is okay so x^k(k) is the estimated value of x at instant k using measurements up to time k xk is the true value okay I can find the co variance of this what is the maebning of co variance what is co variance signify? If the variance you take a simple measurement if variance is large is it a good measurement no so if I want a sensor see what are you doing here you are developing an estimator of unmeasured quantities using measured quantities through a model okay what do you want to say about the possible error in the estimate so it will be small or large it should be a smallest possible okay which statistical quantity quantifies you know spread of error variance okay.

So I want to devise an observer which is the minimum variance observer okay I want to devise some observer which is the minimum variance observer of the estimator which gives you smallest possible variance of the now what is design variable to me is L okay but L is a matrix okay and then we have to learn a little bit bout rules of differentiating a scalar function with respect to matrix okay.

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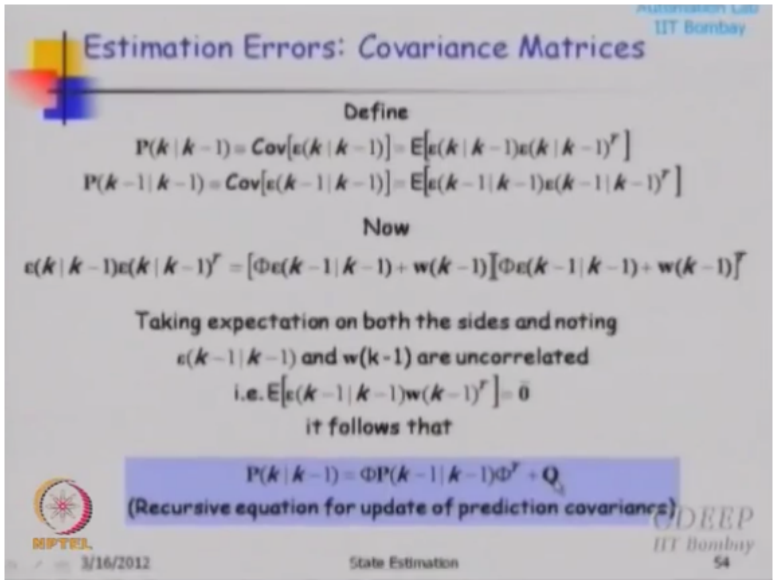

So that is why I said bring those notes okay so and then what is the relationship of this pk with all those 5 γqr you know I m going to derive the relationship which will look something like this I am going to develop recurrence relationship it looks like this that updated co variance is old co variance in to five matrix this q matrix and all that okay.

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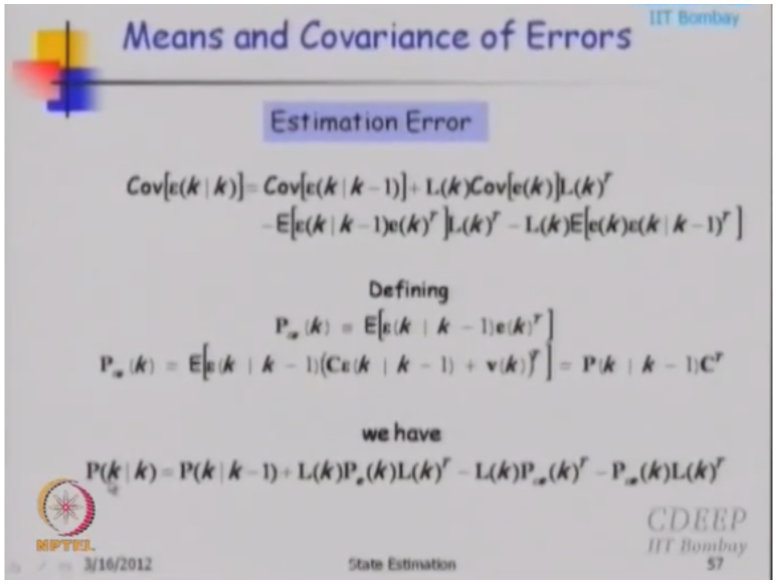

And then predicted co variance is equal to something, something is equal to something, something okay so I am going to develop all these through lot of algebra I am going to develop relationship pk(k) okay I am going to develop this relationship between pk(k) and pk-1 and then I want to minimize those functions okay with respect to matrix.

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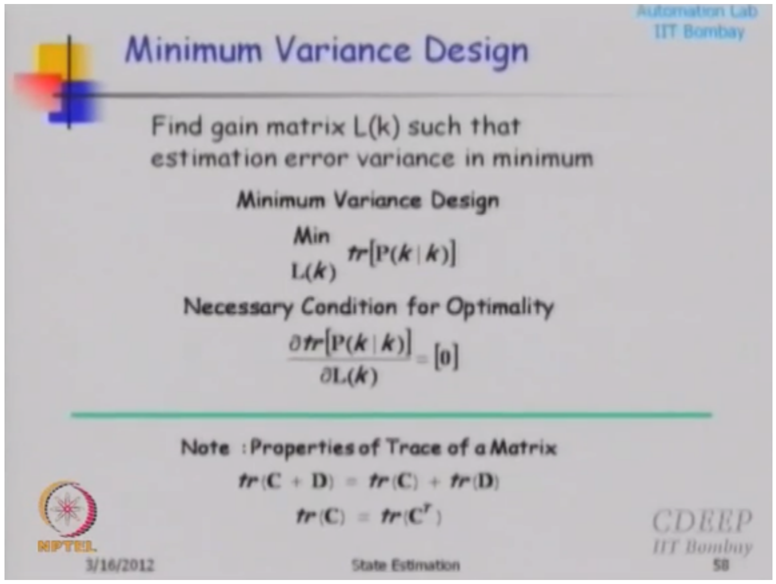

So we have to do lot of algebra to a understand how it will differentiate a scalar function with respect to matrix and then so this particular problem actually was solved by Kalman in 1964 and it relate to explore of algorithms which are used for these methods what I am talking about it is not just use in control that is used in speech recognition this algorithms re very, very generate Kalman's Filtering distribution algorithms are used in target tracking use in interpreting data from radar.

You know you want to find out see you are getting one measurements you know from this radar which is moving you want to find out the coordinates of aero plane where, where exactly it could be relevant while it is landing or while it is travelling or you have a where it is and you want to shoot whatever okay so you should know you should know the probability of you know you should hitting you should know how close you know what is the error in the estimate?

You want an estimate to be as close to the possible to that truth okay so variance should be small the way of mathematically saying this is variance should be small from the estimate value and just remember when you have measurement and when you are reconstructing the position through a model you only have an estimate. So the estimate is also a random variable you better know about it is behavior.

So that is why all this trouble, so this problem which was solved by him as lead to huge development, a rare occasion where engineer. Contributed to mathematics and which as lead to huge developments in engineering field, know we lot of things that we do in image reconstruction or all kinds of things uses this these ideas. Before I close let me take a minute to talk about we are finally uploading the tutorial problems today.

We have divided into 2 problems there are 2 reactor problems there are 12 students from chemical engineering background so we will give them those 2 reactor problems, there CSTR with exothermic reaction and so they will do this problems. There are 3 problems which are non or which can be appreciated by anyone so that is what we think. One problem of that is a formatted problem okay.

So fermentation is the usual process where you take glucose put yeast and create alcohol, it is familiar to u form or human rays for last I do not how many 1000 years, so you are giving this mathematical model to do it in a vessel okay. so there is a substrate which is coming in okay and there is the bio mass which reactants and converts into the products which is alcohol.

So this the simple model for that, 2nd model is one hitter system which we have in the lab I will take you there to show it, so that is simple to tank series there are two hitters in this two tanks and I am given you a model for that, it is very easy to understand for anyone with any back ground does not require any there is nothing special chemical engineering about it.

Third system human body problem is to measure glucose for a diabetic patient and the manipulate variables is food and insulin okay. So instead of food you can think of a person is hospitalized and you know you have two syringes one is glucose and other is insulin syringe and then you have to do dosing to control the glucose level in the blood okay.

So this 3rd problem is also a control problem and anyone can appreciate this okay, I have uploaded my programs in the I do not whether you have seen them but you can use my programs the weight for this is about 25 marks and I expect that you the groups do not copy, so if I find any copying I am going to check program line by line, if I find copying there is only grade that is 0 out 25 okay so no copying, whatever you can do.

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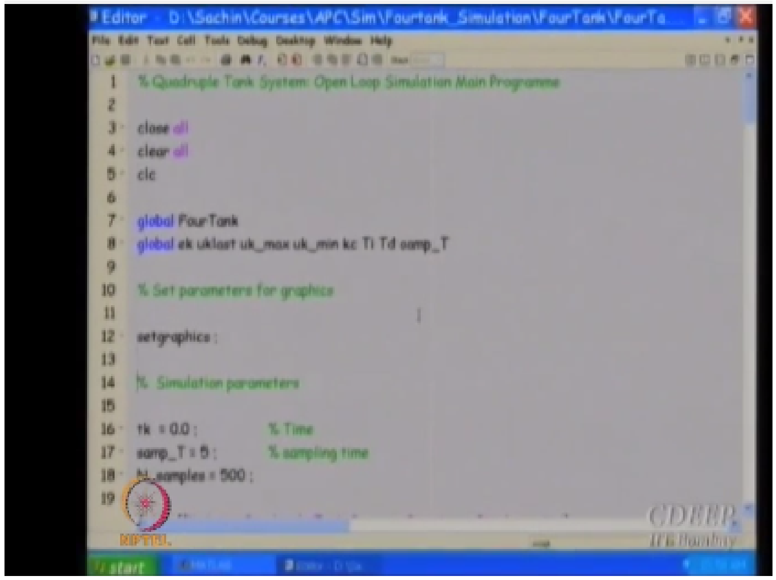

By talking to each other so this is the program which I have uploaded I have shown here how to do simulation for the system all of you know that very well okay. So you know how to linearize this in continuous time in discrete time, how to do open look simulation and you know finally find out the Jacobean matrix and how to do noise simulation everything is shown here. So this is the demo program you can hear allow to start from this program.

You can start modifying this program okay, the 1st deadline is 26th, and the first you have to do is whatever we have learnt in the course we have to do on this system okay. So first thing is system identification and linearization okay, so linear system get transfer function in continuous time, discrete time and also inject perturbations use tool box and get you know ARX, ARMAX all kinds of model and compare them.

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