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ADVANCE PROCESS CONTROL

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Lecture No – 17

Topic: Soft Sensing and State Estimation

Sub-Topic: Development of Luenberger Observer

We have data which is input and output data you have inputs u known we have output measurements coming from the plant and we have the model so we know ϕγC matrix you have data, now you can appreciate that it is sufficient to estimate x(0) that is state at time 0 because if you have the model, if you know the inputs then you know you can one x1, x1 and u1 will give you x2 and so on, you can just go recursively or using this model online to construct subsequent.

So the key thing is what is the initial state just imagine that you have some real plant okay, and then you want to start estimating some variable you are going to start using a model online okay. Now you have everything the data is coming to you okay, why data is you are connected with data equation system so why data is coming to you inputs manipulated inputs are something that operators sets or a controller sets, so the computer knows about it, okay.

So now what is known to me when I want to tick of my state estimator is xs0 initial state I do not know what is initial state, so I have to give a guess okay, what should happen is when my model starts running online in parallel with the plant I am going to run a model parallel with the plant in real time okay, so k, k+1, k+1 let us say my sampling instant is one second okay when the plant advance is in time by one second the model will advance in time in real time.

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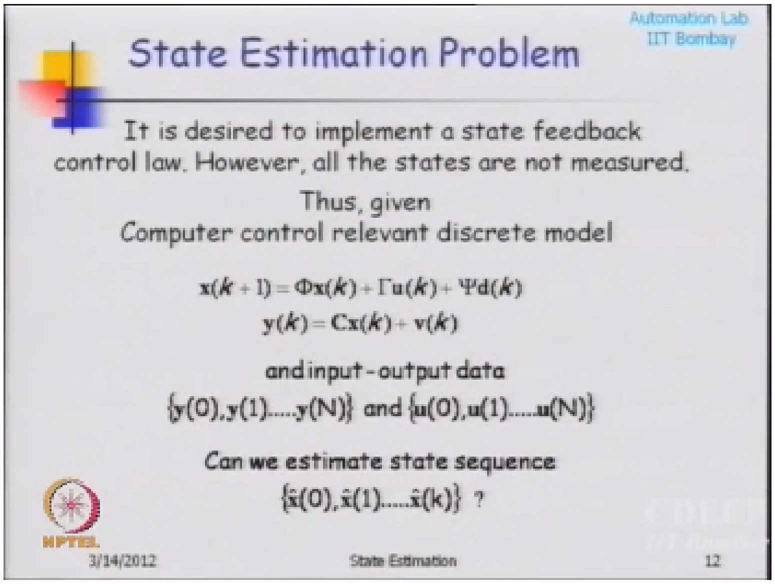

On my computer in one second okay, now if I want I would want ideally model and a plant should you know at some point at least they should become identically. The model output should become close to the plant output, model state should become close to the plant state then only I can use the model as a state estimator otherwise I cannot, okay. Now the problem is when I am using this model online let us take a simplest case where d is 0 you have only u there are no unknown inputs.

So you are giving this u okay, let us assume that measurement noise is 0 there is no disturbance most ideal case even then there is a problem because you do not know x(0) okay.

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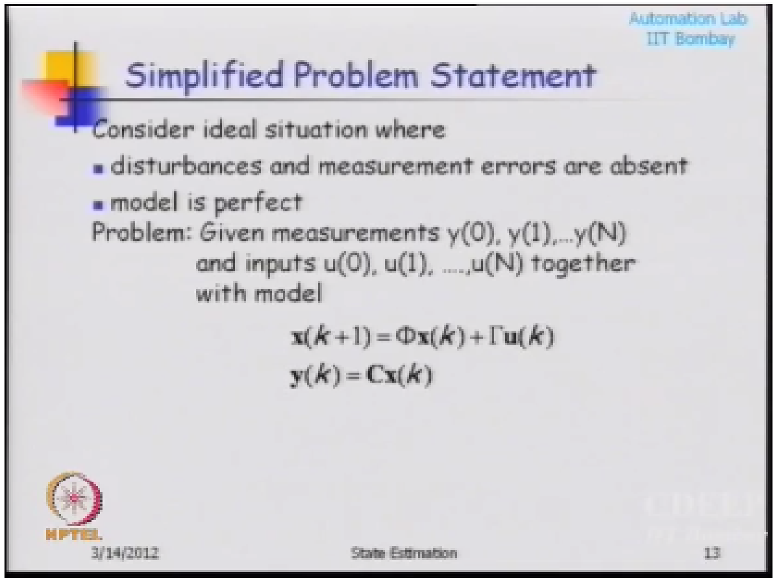

So I have a perfect model there is no disturbance I am taking a situation where there is no measurement error okay, only possible error now I can commit is in the initial state x(0) , see because the true plant will have some initial state x(0) and I have to guess because x is not measured so I have to give a guess, I give a calculated guess I give a good guess even then it is a guess, okay. So when I am guessing what is the guarantee that a model predictions will be close to the true value, okay.

So just imagine the scenario now this is something different and what we have done earlier, earlier we were trying to modeled a plant now I am going to run these model online okay, in real time so as time progresses the model also advances in time so model time under real time are matched okay. So I want to estimate this sequence and then you will see that it is sufficient to estimate x(0) and then we started formulating this problem of estimating x(0) .

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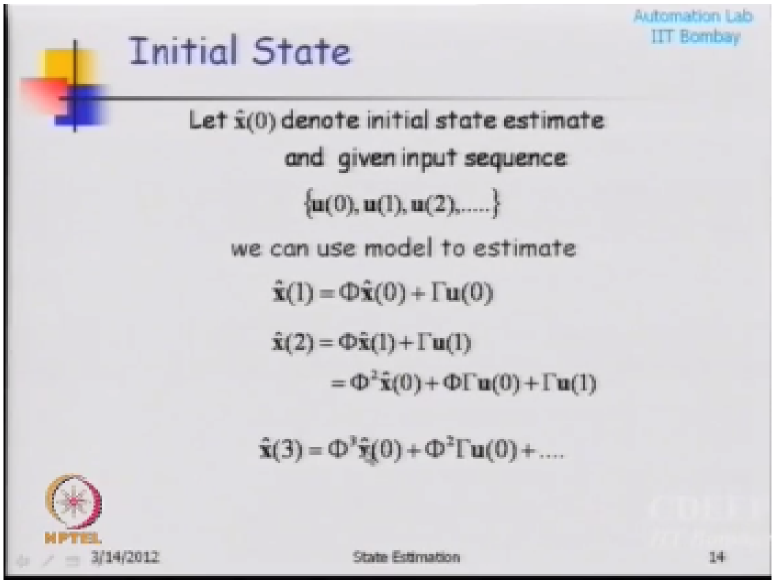

Let is assume that x(0) is the $\hat{x}(0)$ is the initial guess that you are going to give and you know the input sequence okay, you know the input sequence you have the model so I can use x(0) if I have a guess for x(0) I can use it to estimate $\hat{x}(1)$ because I know u(0) I know γ I know ϕ okay, so in my computer I advance in time I use I estimate $\hat{x}(1)$, I can estimate $\hat{x}(2)$, $\hat{x}(2)$ will be function of u(0) and u(1) okay, but it also function of x(0) okay.

Initial guess that you make okay, also has a influence on how you estimate x(2) okay, you can go on doing this and you can show that actually you know x(3), x(3) will be function of ϕ^3x(0) y^2u(0)+ there will be a u(1) term and u(2) term okay, so with reference to time 0 what is going to happen in future okay, consist of two components one component which comes because of the inputs that are given after a time 0, u(0), u(1), u(2), u(3).

Other thing is you know information about the system passed in the system that is x(0) okay, there are two things which influence how the system behaves so do I find out x(0) okay.

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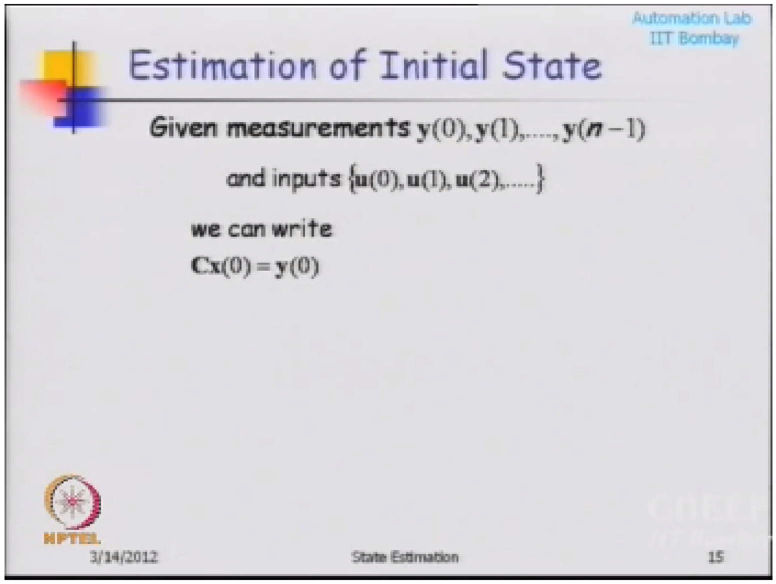

So what I have done last time that I want o estimate x(0) using the measurements I have measurements y(0), y(1), y(2) I am taking measurements okay, as a time advances I am taking measurements. Now I want to use this measurements I want to use the model okay, and then reconstruct x(0) from these measurements and the model okay, so I know the inputs my first equation is this Cx(0)=y(0) you might wonder you have this equation why cannot you estimate x(0) from this equation what is the trouble not those who are already done a control course others should tell you, what is the problem why cannot I use this one equation.

How many states do you measure typically in a real plant I have distillation column okay, I might be having some ten sensor there are 100 states okay, what will be dimension of C matrix, C matrix there are 10 measurements and there are 100 states C will be 10x100 can you solve this equation you cannot, there are more unknowns then the number of equations there are 10 equations 100 unknowns you cannot solve this equation, you cannot use only one equation.

See let us take the simplest case CSTR problem, in the CSTR problem we said we have two state what are the two states temperature and concentration what is the measurement temperature okay, using one temperature measurement can I estimate concentration and temperature see only I will have one equation.

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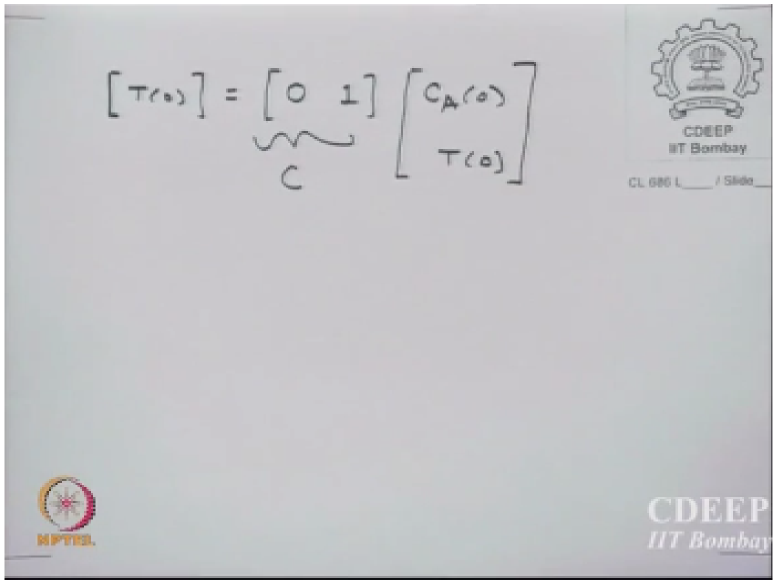

See I have one temperature measurement T(0) this is equal to 01 this is my C matrix into $C_A(0)$ T(0) using this one equation I have just one equation can I estimate $C_A(0)$ from one equation I cannot okay, so in general the number of measurements are in general very, very small compare to number of states in a real plant, okay. So using one equation two equations you cannot estimate x(0) one equation is not sufficient okay.

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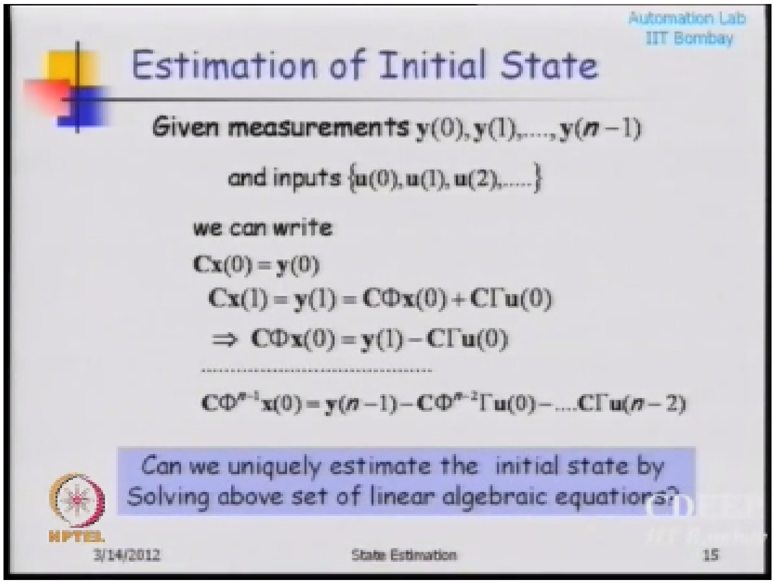

So I can add I have, I can use the redundancy in measurements to create more equations okay, one is not sufficient I can create two equations three equations, four equations, okay and then I can collect enough number of equations so that I can uniquely estimate initial state okay, so what I want to do now is I want to collect more equations one equation is not sufficient to estimate x(0) so I am collecting one more equation I am writing Cx(1)=y(1) is everyone with me or with this yeah, so all that I have done now is I have rearranged it in such a way that unknown part is on the left hand side, okay and everything that is known y(1) is known to me okay, C is known to me γ is known to me, u(0) is known to me I will just take it on the right hand side, okay.

So likewise if I go on doing this okay, I will get these equations I have just written unknown component on the left hand side I have written known component on the right hand side. How many such equations I am going to collect I am going to collect n equations, what is n, n=number of states in this particular case for Cx(2) have problem there are two states I just need two equations okay, for a distillation column where there are 100 states and 10 measurements I would need to stack 100 equations, okay.

Yeah, you will be coming to that but before that I want to before I do data fitting okay, there is only two data fitting I will come to that right now there is no error right, see I am taking the most ideal situation y is error free okay, if y is error free okay, then I do not to take more number of equations this everything is perfect there are no errors I just take exact number of equations and solve linear algebraic equations, okay that is my game plan right now.

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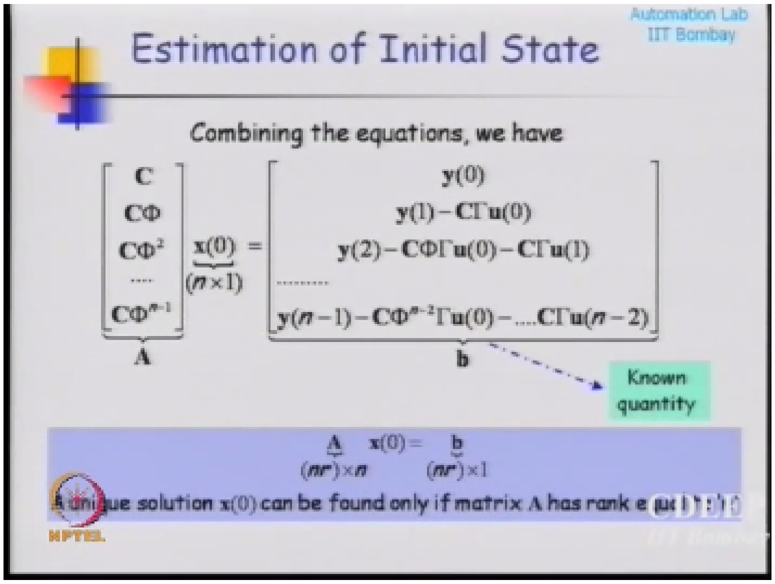

So now what I am going to do is, I am going to combine all this equations into one big equation, I am just going to combine all these equations into one big equations what is not known to me is this vector x(0) okay, this matrix is known to me okay I have measurements of y(0), y(1) up to y(n-1) I know inputs u(0), u(1) up to u(n-1) so I know all this inputs okay, so I can take all the known terms on the right hand side okay, and this is like ax=b right, this is an equation which is linear algebraic equation, okay.

What we will decide when you can uniquely find x(0) rank of A matrix what it should be, full rank what will be the dimension of A matrix. If there are 10 measurements okay, and 100 states what will be the dimension of A matrix see it will be nr x nr okay, your stack l equations okay each one of them there are r, see there are r equations corresponding to each time point you have such stack n such equations so there are nr equations okay, in n unknowns so this is a this matrix is not a square matrix this is a non square matrix, okay.

In order that you can uniquely find x(0) rank of this matrix should be equal to n, this is the fundamental condition on the dynamic model discrete dynamic model most ideal situation no noise, no disturbance everything is prefect model is perfect I do not know x(0), the equation is can I recover x(0) from the measurements of y and u okay, this problem or this question can be fundamentally answered just looking at rank of this matrix okay, and so this is the property of the linear dynamic model.

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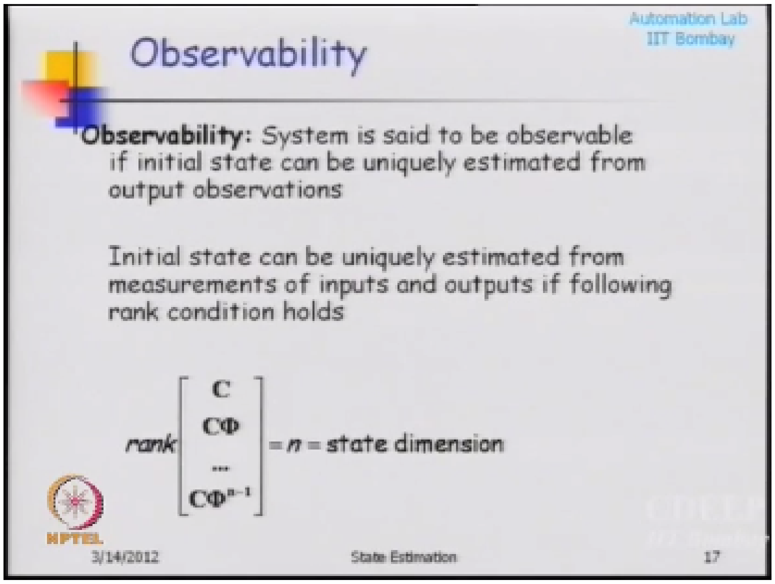

This is called as observability okay, a system is said to be observable provided this rank is equal to n equal to system dimension okay, so this is one of the fundamental properties of a linear discrete time or you can have a equivalent definition for a continuous time model this is for the discrete time model. In the continuous time model you will get C, C_A, C_A^2, C_A^{n-1} okay, you can estimate initial state uniquely from the measurements only if and only if this rank is equal to n.

If it is not equal to n whatever you do you collect more measurements you do anything you cannot uniquely estimate, uniqueness is only guaranteed by this condition okay. Yeah, PFR in the case of PFR you discretized and then you have large of states still you will have few measurements see I am going to take a PFR example towards the end okay, you will discretized it let us go to the PFR. So this is my plug flow reactor.

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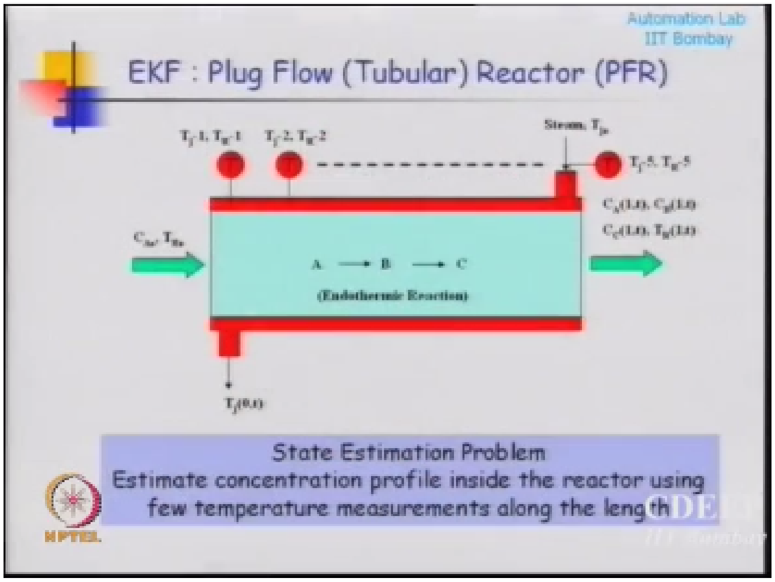

And you know what I would like to know here is the state profile temperature profile are concentration profile inside the reactor okay, how many and how many points well if you start by taking number of point infinite number of points okay. So actually see distillation column is a high dimensional system this is a infinite dimensional system okay. But for the practical purpose one can discretized okay, and create compartments pseudo compartments even so corresponding to that see there will be we have done this is the model.

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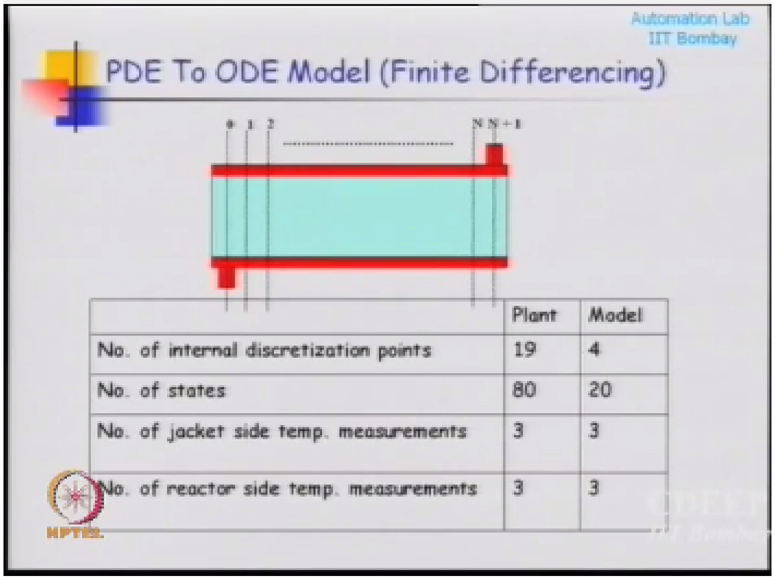

And then you know we have done these compartments of 19 compartments we have done and there are 80 states, but we have only 5 temperature measurements. The question is can I estimate concentration profile and temperature profile inside the reactor using just five temperature measurements okay, or six temperature measurements.

Now what is critical why this is important is suppose I were to choose see here we have taken six temperature measurements the question is our six temperature measurements enough to reconstruct the state can I reconstruct 80 states using six measurements is a fundamental question. If you depend up on the rank of that particular matrix linear as matrix, now is this relevant it is not just algebraic condition suppose the rank is not equal to n you should add more measurements, so that rank of that new matrix so you should change C matrix, you should the number of measurements that you need to obtain to get a unique reconstruction, are you getting my point.

See what is the use of this, what is the use this so what is the use of this condition okay, suppose you have chosen certain number of measurements for a given system you will get a corresponding C matrix for that okay, with that C matrix you actually computer rank of this Cϕ and all that, you will get rank of this matrix. If the rank is not equal to n to make it equal to n you will have to add more measurements okay, when you add more measurements structural of C matrix will change, okay. So when should you stop, how many measurements to be added say this is the fundamental question then I have a plan which has large of states how many measurements I need to have to unique to be able to uniquely reconstruct the state you can add more if you want but what is the minimum number, you should make sure that this rank condition is met, okay.

Suppose you have a C matrix in which you know you add a more measurements such that this condition is met is not a problem, but if you add less and this becomes ranked efficient you cannot estimate the state uniquely which is what you have to remember, okay. So this is the something this is a tool which can be used at a design of a sensor network very, very important tool when you design a sensor network.

How do you choose, how many minimum number of sensors are required to estimate the state uniquely that is the fundamental question that can be answered for linear dynamic models that can be answered using analytical tools linear algebra, okay so your linear algebra understanding is deeply linked with how do you design your sensor network, okay so this is called the observability matrix

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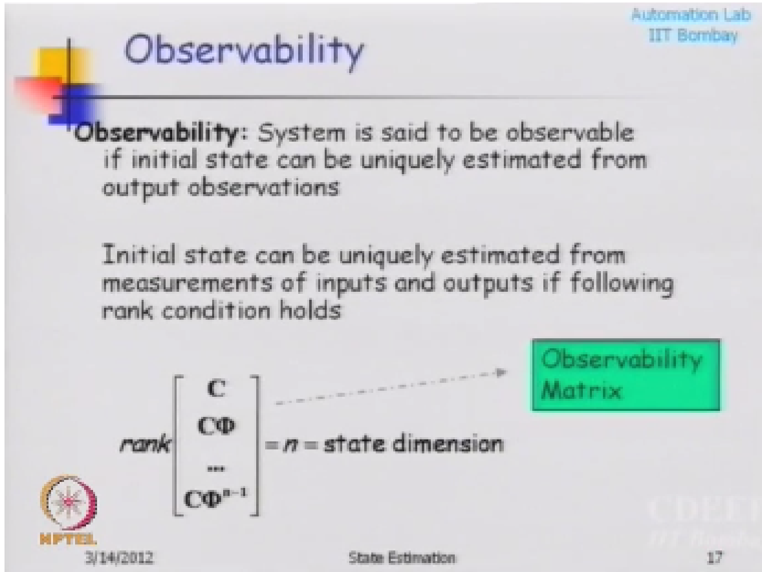

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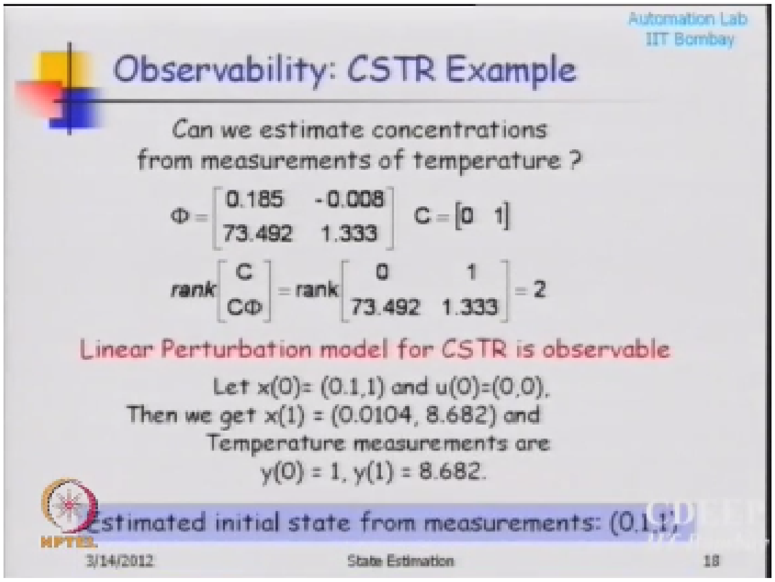

And this is one of the fundamental properties of the system, now let us go back to our CSTR example this will my ϕ okay, I wanted to estimate concentration using temperature so my C matrix is this okay, in the state the second state is temperature first state is concentration so if I consider this matrix C and Cϕ what is n, n=2 okay, look at this matrix it should be C, Cϕ up to $C\phi^{n-1}$ when there are two states you just need, you know you just need two things okay, rank of this matrix is two which means you are guaranteed to get estimates of concentration just from temperature mega bytes, okay just from temperature mega byte.

You are guaranteed to get so this is a fundamental property of the linear dynamic model okay, so the system is also variable and then I am just showing you one particular example if I, my initial state is 0.1 and 1 and you know u(0) is this then you can uniquely estimate these are all dimensionless variables so you can uniquely estimate it is not just solving to linear algebraic equations you can leak over whatever you want to it is not a problem.

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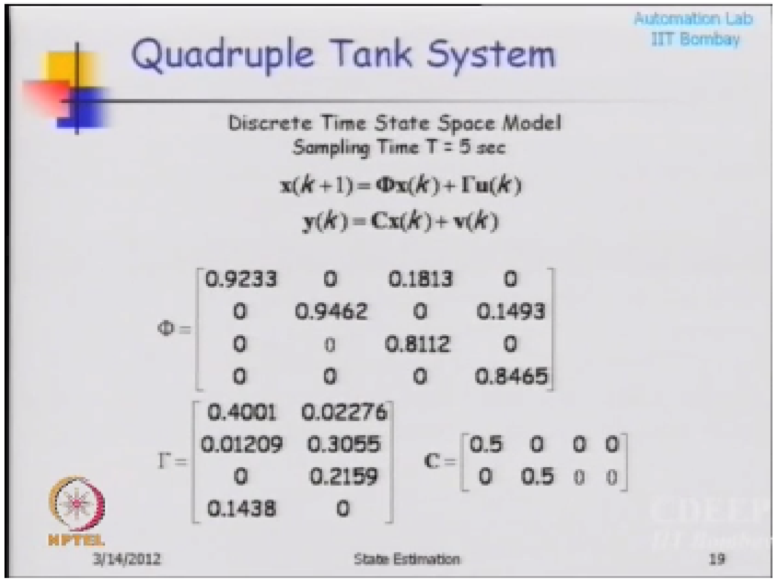

Okay, now let us go back to the quadruple tank model okay, quadruple tank model we have four states right, four levels that two inputs and there are two measurements only two, question is can I reconstruct all four, can I estimate all for levels using only two level measurements okay, this is the first question that I should answer. So these are my matrices ϕ matrix is this, γ matrix is here this we have done earlier when we did linearization.

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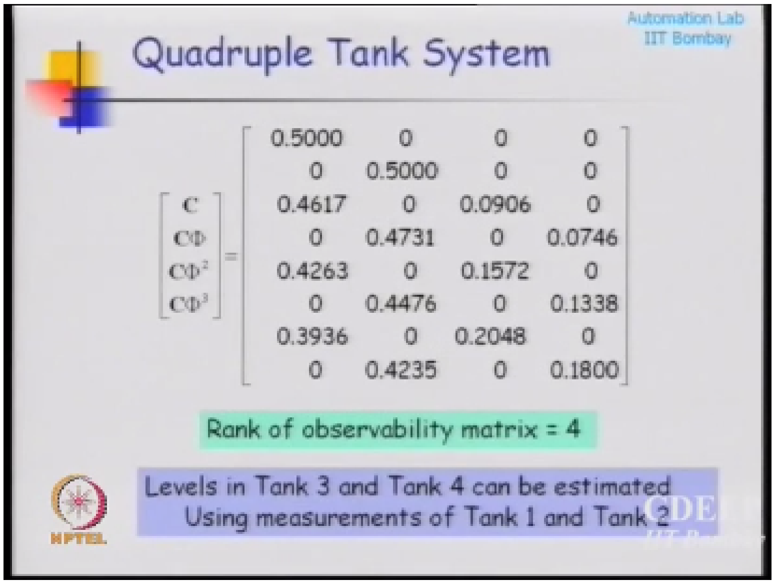

And if I actually see now my observability matrix what is n is 4 okay, so I have to check C, Cϕ, Cϕ^2 okay, in MATLAP there is a command called OBSV observability OBSV okay, you just give ϕ matrix and c matrix it will reconstruct this whole matrix and give you back, it will stack and it will give you back this matrix, so you can check the rank for this particular matrix the rank happens to be four which means I can fundamentally estimate. You can do one experiment here you just try what happens if you take only one level measurement out of four and see whether the rank comes out to be four.

Can you estimate all four states just using one level measurements why do you want to make some decisions well, level measurement cost each level measurement is 50,000 okay, so I want to add minimal number of level sensors why should I add too many right, that is where I need to decide what is the minimum number, so if I put 2 I get this rank to be 4 which means I can estimate, so I am guaranteed, okay. So this is the most ideal scenario no noise only error is the initial state okay. Let us slowly start relaxing these conditions one by one, okay.

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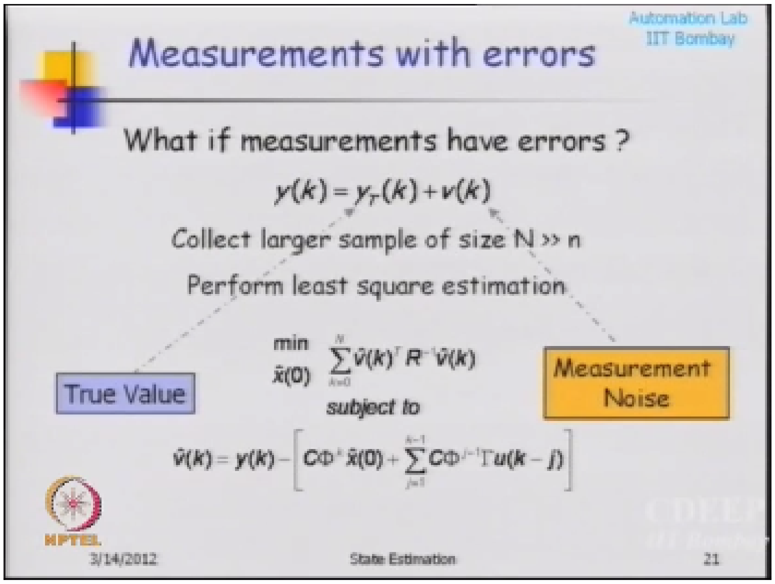

Now what if there are errors in the measurement next question, see this perfect recovery of perfect recovery was possible because there are no errors in the measurement okay, so only error was only thing there was unknown was x(0) but if there was an erroneous measurement then you cannot use just n equations and solve okay, then you have to pose the problem as a least squares problem. What is the best estimate I can, you can consider because every measurement has an error okay, so then I cannot just go back and use n equations.

Because if I use you know further CSTR problem if there is error in the measurement if you use first two equations you will get one estimate of x(0), if you use next two equations you will get another estimate, you use third equation you will get third estimate that is because there are errors in the measurement, measurements are not perfect and that is the reality every time you never get a perfect measurement you will always get erroneous measurements. So what is y is something like this y true plus some error, okay.

And this is my measurement noise then what you are saying should be done you know we should collect more number of measurements and then try to construct a least square estimate of x(0) okay , that is the best way of going about so what we can do here is we can minimize some of the square of errors okay, what is done here is v is a vector of measurement errors, r is covariance of this measurement errors I am just normalizing these errors using covariance inverse, okay.

I am normalizing this errors with covariance inverse I do not know the true value of y that is the problem I only have the measurement measured value of y, so I can only construct a best estimate okay, you will never know in this case but this is the model proposed what is the model true plus error okay. Now if you have if now we can look at v error stochastic process you can say it is a white noise, 0 means white noise with some covariance, covariance r okay.

In that case what I can do is now here I am trying to construct an objective function which is sum of the square of errors okay, let me just write down this what is this.

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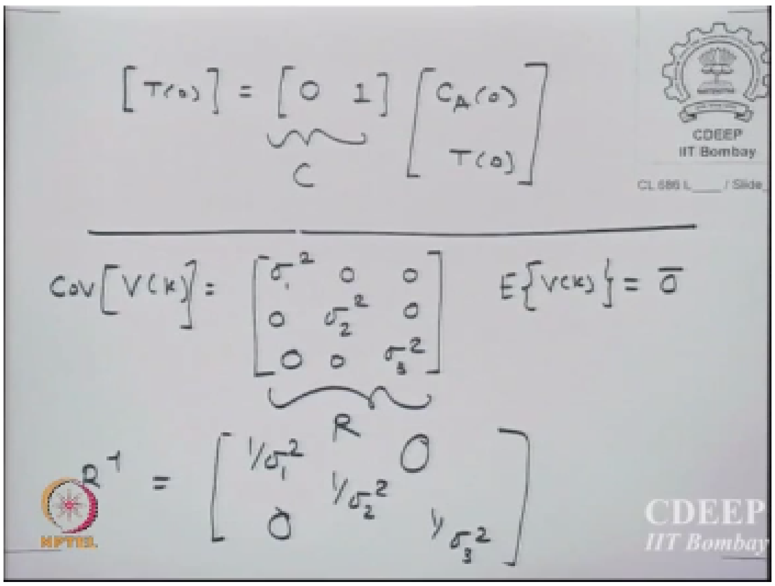

See this v(k) okay, is let us assume it to be a white noise okay, suppose there are three sensors okay, there are three sensors then I will say that covariance of v(k) $\sigma 1^{2}0$ 0, $0\sigma 2^{2}$, $\sigma 3^{2}$ let us assume that v(k) is a white noise okay, such that expected value of v(k)=0 expected value of v(k)=0 okay and each one of them has variance equal to σi^{2} okay, so what will be this matrix see this is my R matrix what is the R^{-1} $1/\sigma 1^{2}$ $1/\sigma 2^{2}$,k $1/\sigma 3^{2}$ 0 0 right, okay and what will be this.
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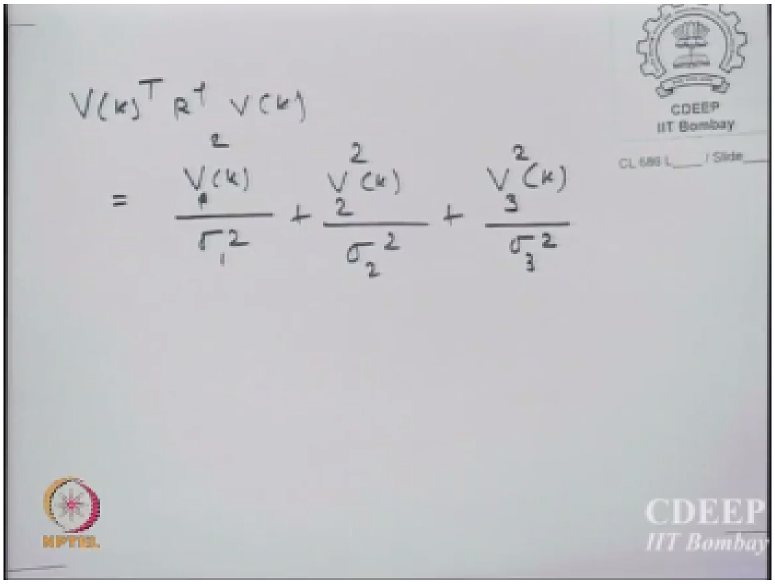

$V(k)^T R^{-1} v(k)$ this will be just take this $v_1^2(k)$ first component square by $\sigma_1^2 + v_2^2(k)/\sigma_2^2 + v_3^2(k)/\sigma_3^2$ so what I am doing here I am normalizing each one of the errors using it is own standard deviation or variance okay, why this kind of a thing is done that is because each measurement may have different accuracy also each measurement might be different type you know and you may have pressure measurement, level measurements see v2 could be pressure measurement, v2 could be level measurement, v3 could be temperature measurement and they have different units, different variability.

So I want to minimize some kind of normalized objective function and that is the reason I use this information of variance to normalize my objective function, it should not happen that one of these variables because of it is numerically large value it should not dominate my objective my optimization problem, okay that is why I need some kind of normalization and that is why I am using information about the variance, okay.

Remember we are again revisiting stochastic processes here v(k) is modeled as a stochastic process 0 mean white noise it covariance equal to R it is a model proposed for measurement errors okay, white noise means random error okay, you can if you want to distribution you can say normally distributed random error okay, and then you are using that to normalize your objective function. Now you can solve this problem as a least square estimation problem that means I want to estimate $\hat{x}$ (0) okay, subject to this equation if you look at here what I have done is this part actually.

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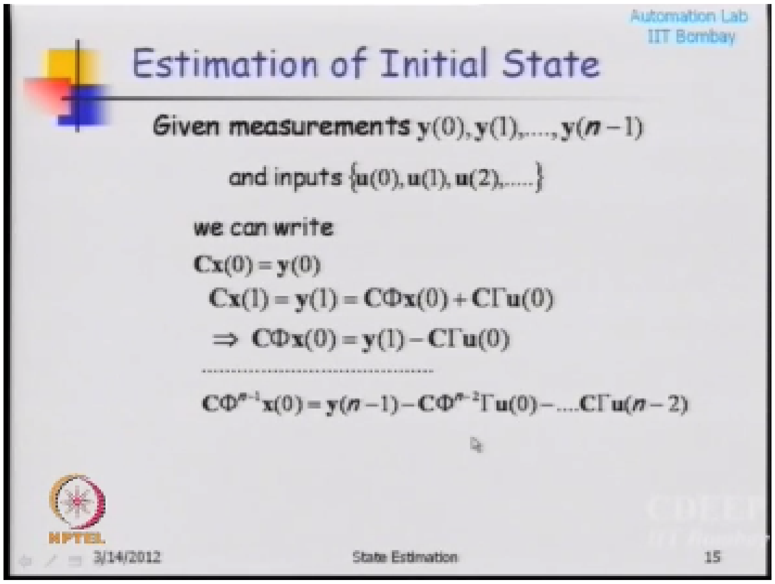

This part comes from here I have written y at any time k in terms of y(k)-u okay, that is what I have done I have just used that particular information so this kind of recursive relationship I have used and then I have some to this okay, I have just expressed my y true in terms of initial guess for x(0) and all the known inputs that is all I have done, are you clear about this. y true is if see if you knew $\hat{x}$ true y true will be obtained through in the model equations right.

Model is giving this y true, giving an estimate of y true actually when you get data you will never come to know what is y true, you will only data will only give you y true plus error, because you will never come to know you can only estimate error you can estimate y true okay, we will actually talk about this more detail. You can only say that statically you know if you collect large number of samples you might approach the truth, you will never be able to recover from a sample the truth, okay.

This difference is nothing but y(k)- y true k, y true k is expressed using the model equations recursive use of model equations, but a model y true will depend up on x(0) and u and I have shown you that okay, so I am just using it is as a construct optimization problem, yeah. no, jumping right now model is perfect, what happens to model mismatch we will see you do not ask a question in the 10th standard when you start looking at a first standard, okay.

Suppose there is no model mismatch right now yeah, usually it is the case, usually the measurement noise is y I can show you lab data I showed you one lab data you go back and look at temperature data which I have put in the system identification notes. Each one of them will have whiteners errors in the measurement are arising from see the fundamentally when you have randomness coming from large number of sources, each having a different distribution the random error that occurs tends to be with a random noise with Gaussian distribution this is central limit theorem, okay.

So when you have a measurement noise it is arising from large number of things electrical fluctuations, conversation error between analog to digital you know it is something it is picking up on the way transmission line disturbances everything put together what you get is a random disturbance which is like Gaussian in fact it turns out to be Gaussian white noise. Measurement noise Gaussian white noise model is many times more than sufficient for large number of cases.

There might be some 5% cases which are methodological but we are right now worried about 99% of the cases which are you know whenever noise is not you take sensors put them with by this thing to be at constant this thing measure it you can estimate if you know I am taking temperature measurement I can independently estimate what is the temperature and say the tank, and then I can take the measurements find out the difference I will get the error I can get an estimate there are different ways of doing it, it is not you can any other doubt, okay.

Now this particular formulation even though it is quite logical you go this way it is in convenient, what will happen if you start using this online the side of the problem is start growing okay, see because as time advances you have 100 measurements, 101 measurements, 103 measurements so you go on starting the problem which is of increasing size okay, and where do you stop how many you take from least square estimation theory you know that you take more measurements better it is okay. But increasing solving these problem with more and more number of measurements as time progresses if you want to solve this every time it is not convenient so you need some better ways of doing this okay, so that is why we are going to look at so called recursive estimators, okay.

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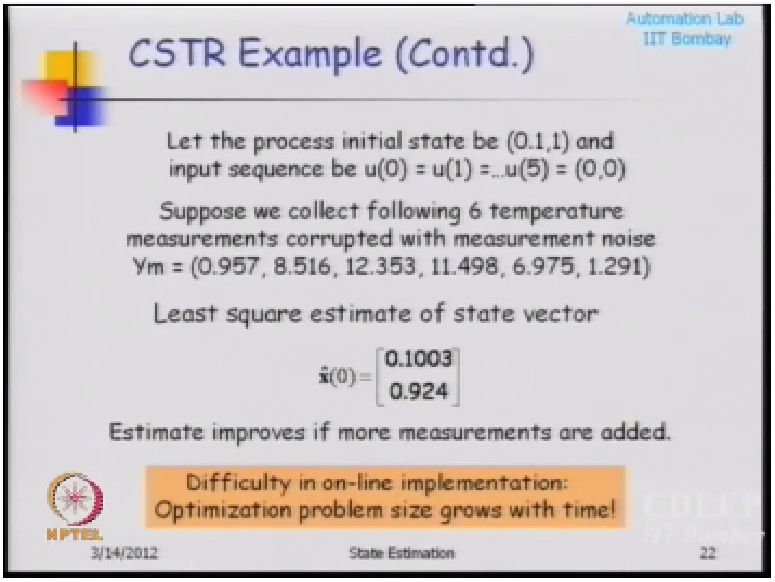

So anyway I will just give an example before I move on let us say you have this ϕ measurements which are you know sic temperature measurements which are noisy then you will get a least square estimator estimate you will not be able to recover the exact truth you will be going close to the truth so you get here 0.1 and 0.933 so the main problem with using this kind of thing which is just a demonstration using a batch of 5 data points, but 5 will become 6, 6 will become 7 in a real time if you want to use this optimization formulation it is not practical so you need some other way of doing it, okay.

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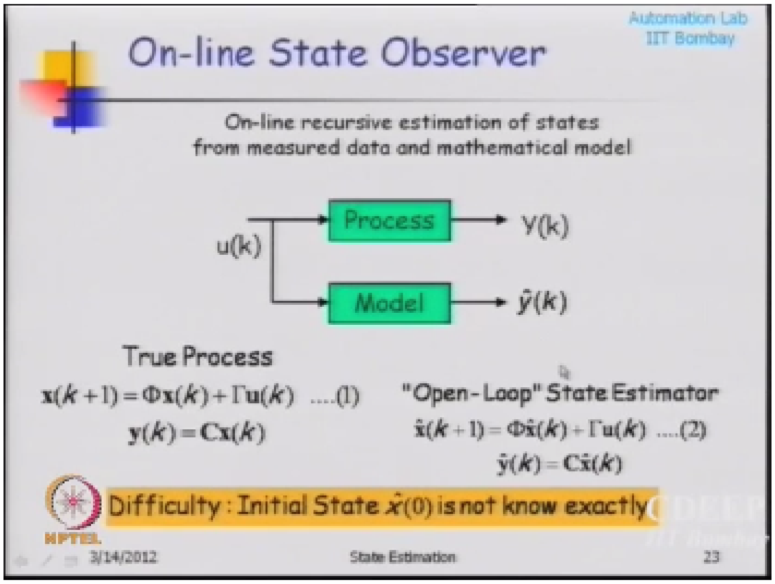

Okay, so I want a recursive estimator okay, so what I am going to do is I am going to run by model online okay, whatever input that goes to the plant I am going to fitted to my model okay. And so this is true process okay, and this is my estimator which is running in my computer online okay, so when the process starts at the same time I start running my model in parallel on the computer okay, whatever input goes to the plant I am giving it to my model, okay.

Now let us ask the first question that I forget about all that optimization formulation and all that a I am using this observer, let me call this are estimator what I am doing is I am just feeding in whatever u that goes to the plan I give it to the model okay. will the question is will this xk 10 to x true will it tend to x true okay I have given a guess for x 0, so initially at least for sometime the way this is behaving and way this is behaving is going to be different.

Eventually I wanted to merge okay let us see whether it is happens online okay so what is the error this just find out the error just go back and find out this error what is the dynamics of this error subtracted your equations just see what you get, subtract equation 2 from equation one what do you get? What mean not y right now I am just asking you to subtract 2 from 1 I just asking you to subtract 2 from 1 okay.

So if I subtract only the state dynamics okay I am defining this error x true – x hat okay this is my error then I get this difference equation linear difference equation and is everyone comfortable with this, you know at time k if you just repeatedly use this different equation you will get this solution okay. Under what condition error will go to 0 if Eigen values of φ or inside the unit circle if Eigen values are φ are indie the unit circle if you are running the model in parallel to the plan after sometime even if you are given initial guess wrong okay the error will go to 0 asymptotical.

So this is very critical but what happens if plant is marginary stable or unstable there are many situations where plant is not exactly stable, then this idea of running the model in parallel to plant will not work. This idea of running the model you need to do something else you need to make sure that, so you cannot use this idea for marginary stable or stable systems okay, this simplistic idea will work only in the neighborhood of a point where the system has stable dynamics okay.

So what about the fact, how fast or how slowly this will converge see it might take long time to converge I want convergence to occur quickly I want to start watching what is happening see my start up of the plant is a critical phase I do not want to wait for a long time till my error converges and I start see getting an estimate of what is happening inside I want a quick estimate so should somehow make it happen faster, what is the way off.

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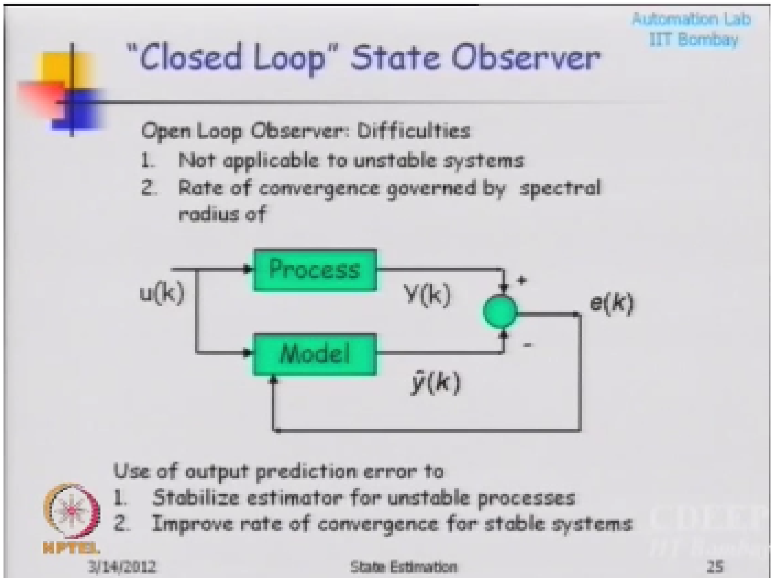

So we design what is called is a close loop observer whatever we have done till now running the model parallel to the plant I am going to call it open loop observer okay I am going to use some feedback information I am going to give the feedback from the measurements of y to my estimator okay and then accelerate the convergence that is what I want to do okay. So the idea is this I have a measurement see I am estimating states using my model which is running in parallel to the plant okay process is running okay I get measurement yk I can estimate what would be yk okay I can find the difference between the two and use it at the feedback correction so this error between y measured and y estimated this is called as innovation, this innovation is used to correct the estimator.

So I can use this idea of feedback to stabilize or do make the dynamic faster in control, why do you use the controller? What is the reason use for controller? One do one of the reasons why use controller is because see suppose you have to do things in open loop okay plant is stable and you want to take this system from one state to the other state in open loop you gives some change that will take its own time swift time to reach that okay.

I do not want that to happen I want to take it to a new state very quickly okay that is why the controller plays a role in doing it fast that is why you design a controller which is optimal which takes it fast and so on. So one of the reason why a put controller is to alter the dynamics you want to make it fast you want to make it slow you want to meddle with the dynamics, same thing

I want to do here I want to meddle with the estimator dynamics that is why I am going to put a feedback.

What is done in feedback control you put a feedback loop and you manipulate other reason why you put a feedback controller is plant is open loop unstable you put a feedback loop and you can stabilize it all of you know this right you can stabilize a plant with the feedback loop you can alter the close loop dynamics and make it faster or slower using same idea is going to be used in observer design okay.

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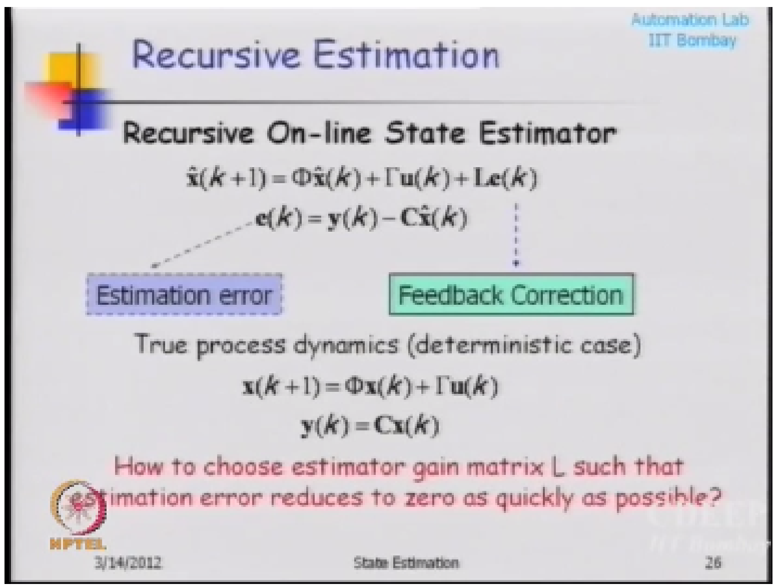

So what is going to my strategy, so my observer now is going to have one more term here okay I am adding one more term here lap l times ek what is ek? Ek is y measured – y estimated y^ k is everyone with me on this? y ^ k is c times x^ k okay y measured if I use this what is l, l is the gain just like proportional gain which you use in the feedback control loop this is the gain matrix I am going to use this gain matrix to twick the stability or performance of the observer okay.

So this is the feedback correction this is a estimation error it is also sometimes called innovation, the process is still like this okay we are taking the deterministic case right now I am still working with no noise okay no disturbances perfect measurements and I want to use this scenario now what I should do is how does error will behave now I have added one more thing here okay what is the dynamics of the error can you derive this subtract this equation from this equation just find out the state dynamics, what is it, what is a new state dynamics?

Just remember that is y can be written as cx y can be written as cx okay, when you subtract from this equation this estimator equation uk term will cancel just try doing it you do not understand unless you work it out, I will show you the result, what is the estimation error dynamics? Yeah you can put any controller, how do you chose that l matrix is going to be the you know how do I now next few lectures I will just divide to the you know problem of how do I optimally chose l okay so that I can influence the dynamics that is what we are going to look at for next four lectures.

First of all vid controller now you forget about vid controller now we are talking about general state feedback controller so l is some gain matrix it could be memo it need not be pid single input single output I am not any longer worried about single input and single output l is the gain matrix which could be non square in typically what is the dimension of l matrix tell me, x is the number of states okay y is the number of measurements what will be the dimension of l matrix?

Number of states cross number of measurements if there are 100 states and 10 measurements l will be a 100 cross 10 matrix okay, for the reactor l will be a 2 x 1 because there are 2 states one measurement okay. So I have to find out if there are 100 states and 10 measurements I have to find out 100 x 10 elements it is not same as designing a pid controller where these are three parameters okay.

And I have to work with 100 x 10 I have to estimate 100 x 10 elements, so it is much more complex problem that pid controller design. Yeah but right now we will not worry about it, so again you now do not jump just understand basics first then you can talk about derivatives. What is the equation? 5 – lc okay.

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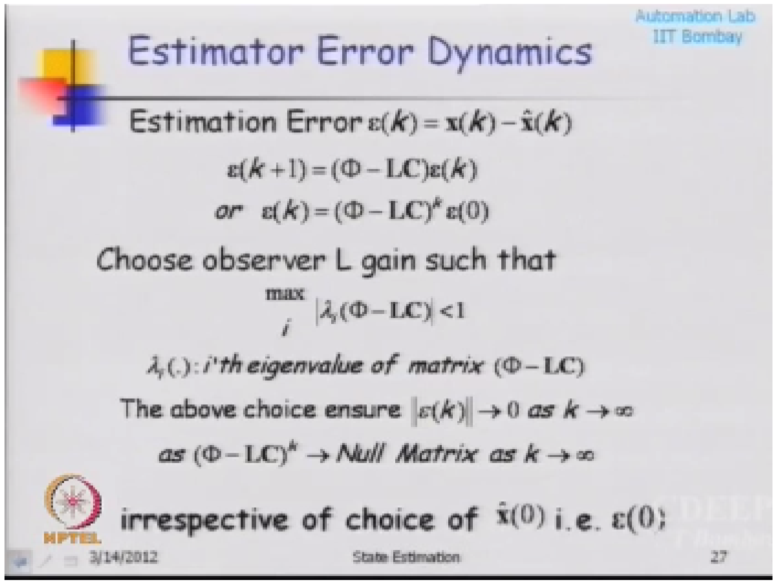

So you should get if I do I estimation error like this then I get this error equation that is error at time k + 1 will be function of error at time k through this coupling dynamics – lc okay, what is between the previous case and this case? Now in open loop I did not have any handle on how fast error changes now I have a handle to enhance the rate of convergence what is that handle I can chose l matrix okay, how should I chose l matrix?

What should be the primary criteria? Eigen values of 5 – lc should be inside the unit circle what can you say if Eigen values are close to one, if there all the Eigen values are close to one it will be slow it will take longer time for the error to go to 0 what is the Eigen values of close to 0 it will converse faster, so you have a choice now and you should be able to dictate the speed of convergence by appropriate choice of l okay, also there is one more thing if φ has φ is open loop unstable okay.
I can chose φ – lc to be inside unit circle so that even if the plant is unstable okay, this observer will not be unstable observer will be stable okay you will get estimated states to be you know the error between the true and estimate we assume totally equal to 0 even if the plant is unstable that is what you can directly through the choice of this matrix okay, yeah what is larger? Yeah so here you want the error to come to this thing very fast it may have some influence on how the close loop behaves.

Yeah, not unstable n unstable φ, right hand polls no if it is no, no what I am choosing l no I will never let a choice of l such that it is Eigen values are outside unit circle mill potter no, no but why you are only worried about degenerate cases right now that is a degenerate case why, so I can talk about fixes for those cases but not when your, not in owned right if you have that problem we can describe that later okay what you do for mill potent φ, so in general for a normal φ which is either stable or unstable if it is stable you can enhance the rate of convergence if it is unstable you can, so you are talking about thinks like which are detectable but not observable there are so many niceties which I am not going to talk in the first time introducing this concept right okay.

So this is my first criteria I want Eigen value the φ – lc to be inside the unit circle okay and if that happens then error will go to 0 now what is very important is that the error will go to 0 if you chose l such that this condition is satisfied okay error will go to 0 irrespective of what is the initial guess that you give you make a completely wrong guess does not matter okay as long as this condition is satisfied whatever initial you guess you give error will go to 0 after sometime how fast how slowly will decide upon what is this what is the Eigen values that you chose okay. Choice of Eigen value is being decide the rate of convergence of okay.

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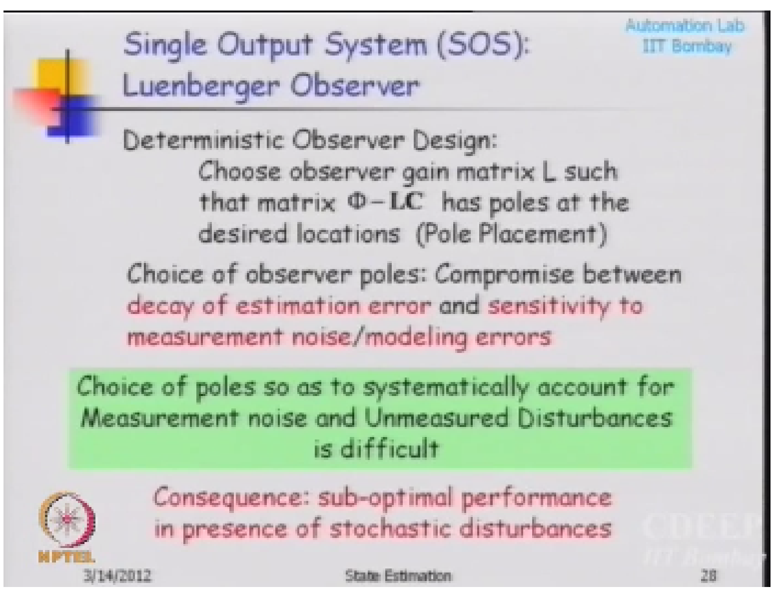

So this is this last thing is very important irrespective of choice of x0 it will go to 0 okay, so my problem is how do I chose this l matrix okay such that it has decide poll locations okay, so I am going to this is a decide problem now I am going to specify where the polls of φ – lc should lie and I want to back calculate l okay I know φ I know l okay sorry I know c I do not know l I want to estimate l given the polls okay.

Question is when can you do that okay, now if your specify polls of a systems how many polls you can specify? N what is the dimension of this matrix φ – lc? What is the dimension of just write down what is the dimension of φ matrix n cross l there are n states what is the dimension of l matrix n cross if there are r measurements n cross r and what is the dimension of c matrix r cross n okay so what is dimension of this matrix φ – lc n / n it is a n/n matrix how many Eigen value it has it had n Eigen values okay.

So if you specify n Eigen values how many equations you are giving n equations okay in general how many elements l will have n / r there are n/r elements of l and you are just giving n equations right, can you solve it? When can you solve it unequally r = he is saying r – 1, do you agree? When can you solve unequally? Unique way of number of equations = number of unknowns okay, when will it happen? R = 1 when only one measurement is there you have equations okay r will have there is only one measurement matrix l will be n cross one okay and then you can you n equations in n unknowns you can solve it unequally and you can get okay.

So how do you chose this polls is a very difficult question because if you chose them very close to 0 your you know observer brings error very close to 0 very quickly provided there are no errors in the measurement there are no disturbances, if there are errors in the measurement okay then observer which is trying to bring the observation error very close to 0 very quickly okay can give you trouble okay.

So when there are errors in the measurements how do you chose what should be the poll locations is not a easy task, but never less let us look at this problem of estimating the state the choice of polls is not a very trivial thing but now let us look at it as a mathematical problem because why we are doing this because this one of the fundamental ideas in control observer design by poll placement and even though it might be difficult to use many times in practice it still marks a very important land mark with the development of observers okay.

So how do you chose the polls is question that we will keep it aside for the time being let us say we have some way of choosing the polls I am going to say well I know that placing the polls close to one will make it very slow keeping them close to 0 will make it very fast but my it might be sensitive to noise let us take a choice you want to put it at 0.5 neither at this not that at okay, so we are going to make it place all the poll set point 5.6 somewhere in between.

So we have some way of choosing the polls and not the question is how do I design the observer okay, so let me just go back here for a moment and then.

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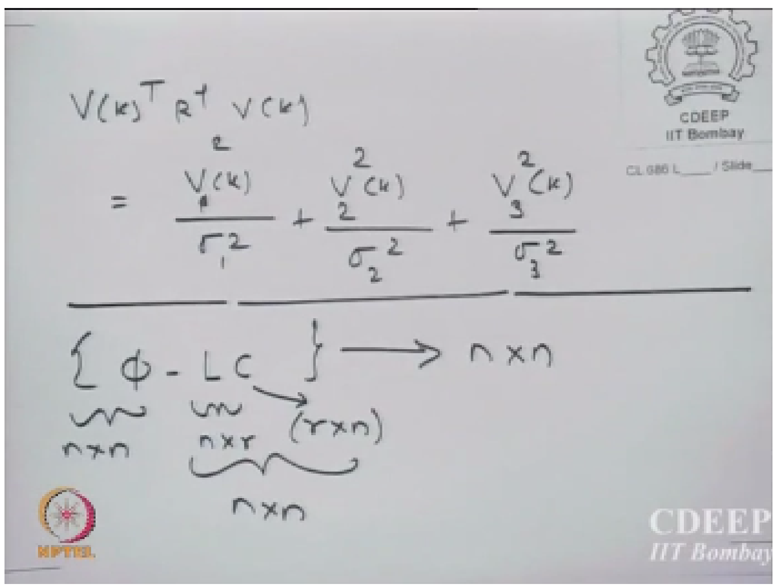

So this φ – lc so this is n x1this is n xr and this is r x n, so together this is n x n so this whole matrix is n x n okay.

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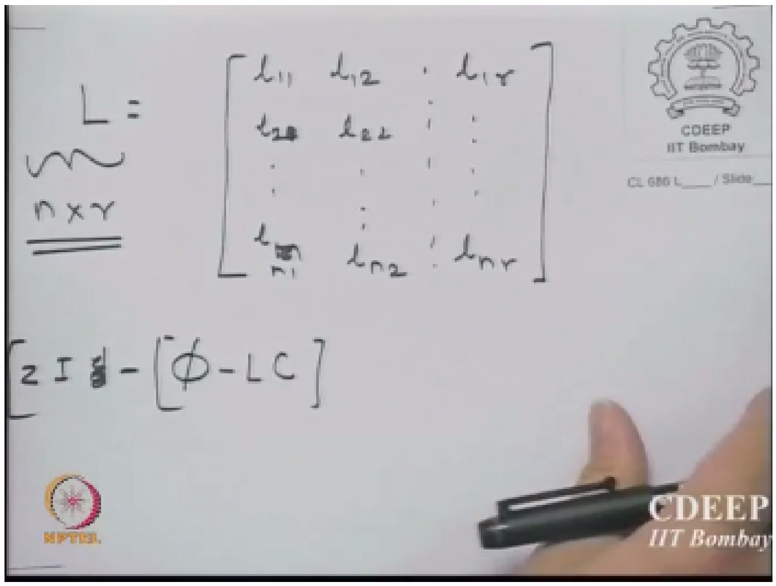

The next thing is that l is a n x r matrix okay so this will say l11 12 l 1 n l sorry l n1 this is l21 this is ln1 l 12 l22 ln2 and so on and so you will get l 1 r upo to l nr okay, there are so many unknowns in this matrix okay and what I am going to do is I am going to say that φ- lc so I am going to put zi – okay let move to the other paper.

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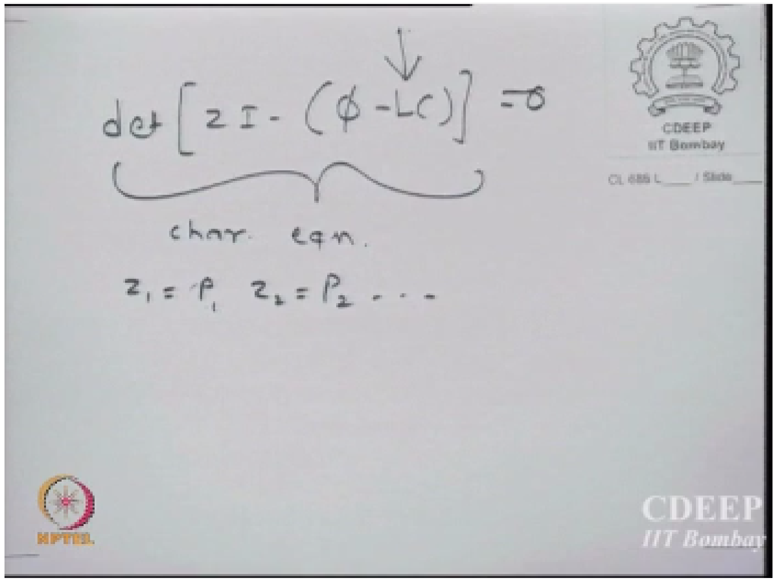

I am going to say that determinant of zi – φ – lc = 0 okay this is the characteristic equation okay and I am going to specify polls of this I am going to specify routs of this okay, how many routs I can specify? I can specify only n routs this is n^{th} order polynomial I can only specify n routs okay. So let us say I am going to say that routs of this are at z1 = p1 z 2= p2 and so on I am going to give some poll locations ay 0.5, 0.5, 0.5, simplest choice let us take very simple choice neither 0 nor 1 somewhere in between okay I have given this choice okay.

If I give this choice okay and if I write the equations these polls of this are the characteristic equation here will be function of this l right so this should be function l 11 l12 l13 and so on right. So it will be very complex function l11 l12 l13 and I can do a very you know crude job of actually expressing this in terms of l11 l12 finding out its Eigen values and saying it equal to p1 = p2 okay, if I do it very crudely okay it can become very difficult the question is if you wanted to specify the polls okay of this matrix to be equal to p1, p2, p3, p4, okay can I does some elegant algebra okay.

Such that I can do it easily I do not want to actually sit and solve this equation I will get some very complex function of l1 l11 l12 l13 and all that okay and then equating and solving those coupled equation can become very messy okay, if you try to do it for general n dimensional case okay, it can become very, very messy if you actually solve this equation raw form okay. So I want to do some very nice linear algebra such that I do not have to sit and solve this equation by hand okay, so what is this elegant algebra that you want to do?

Is everyone with me on this is there any doubt here whatever to do? I want to choose elements of l such that the polls of this routs of this equation are equal to you cannot even write routs of this equation analytically can you write you cannot write analectic expression for that okay, so saying that it is equal to something and solving it is very difficult okay, if you just try to do it as a raw problem okay it will be very difficult to you even formulate and solve this problem as a raw problem of equating you know holes finding out l11 l12 l13 and all that. So it is not very easy to do this if you just think it in a raw form okay.

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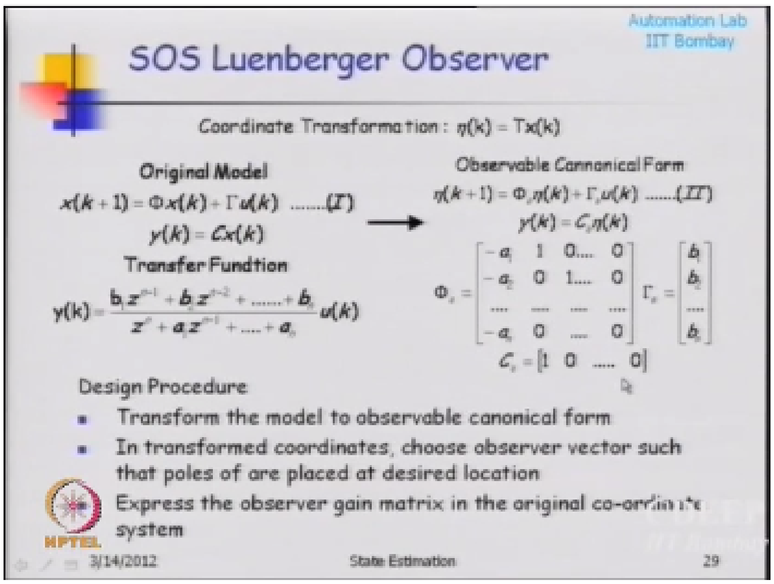

So what I am going to do is I am going to do a variable transformation this is where linear algebra will help me okay so I am going to do a variable transformation I am going to transform from x to a new state variable ε okay, first of all I am assuming that there is only one measurement okay if there is one measurement then l matrix as only n unknowns okay l matrix is a vector n x 1 vector okay I want to estimate elements of this vector okay so this is my model this is my original model okay and this is a transfer function right.

Now I am going to use the trick of sometime back we talked about some canonical form do you remember canonical forms? Controllable canonical forms observable canonical form we talked about what is canonical forms right, I am going to convert this model in to the canonical form how do you convert in to canonical form if you look at a transfer function you can write the canonical form what is the advantage of canonical form the advantage of canonical form is that the characteristic polynomial coefficients appear as the first column okay, the characteristic polynomial will appear as the first column okay see this kind of nice thing is not available when I have general φ right.

When I transform it in to this particular form okay I get a1 a2 lined up on the first column then this part is 1000010 okay. So I am going to use this particular form to simplify my poll placement poll placement means I want to chose l1 l2 l3 l4n observer gain in such a way that the error dynamics as polls at as a decide location okay. So instead of working with this I am going to work with this transformation okay.

So my trick is going to be like this I start from here I transform in to this form these are inter convertible we know that okay, these two are forms are inter convertible so this is only a different way of representing this system okay, I am going to design an observer for this system okay. And then these two are related through this transformation so I am going to come back and from the transform domain I am going to come back to the original domain and use that observer okay that is going to be the trick.

Is everyone with me on this, this transformation these we have done earlier in the class only thing is the different context and now the question is how do you go from here to here okay I will answer that question, right now just wait for five minutes to come to that point okay so my design procedure is like this I want to transform the model to the observable canonical form once I do that design a observer for the transform model okay and then express the observer in the original state space model using the inverse transformation.

So transform the problem design the observer do inverse transformation okay that is a trick why you can do this you know linear algebra allows you to play with the model in the model space you can do all kinds of things when you have you know a linear difference equation model okay. So let us see what we are going to do, this is my transformed observer I am designing an observer for the transform system okay can you derive this equation can you see whether you can derive this equation just go back here.

And try to design the observer equation for this just go back and check what will be φ a – c a see understand what I am doing, what I am doing is I am first transforming this to this form okay and then I am going to design an observer in the transform domain okay I am going to design an observer in a transform domain can you derive this matrix just check can you derive this matrix how will you get this matrix? What is c for this particular system for the transform system c ca happens to be 1000 it is very nice okay.

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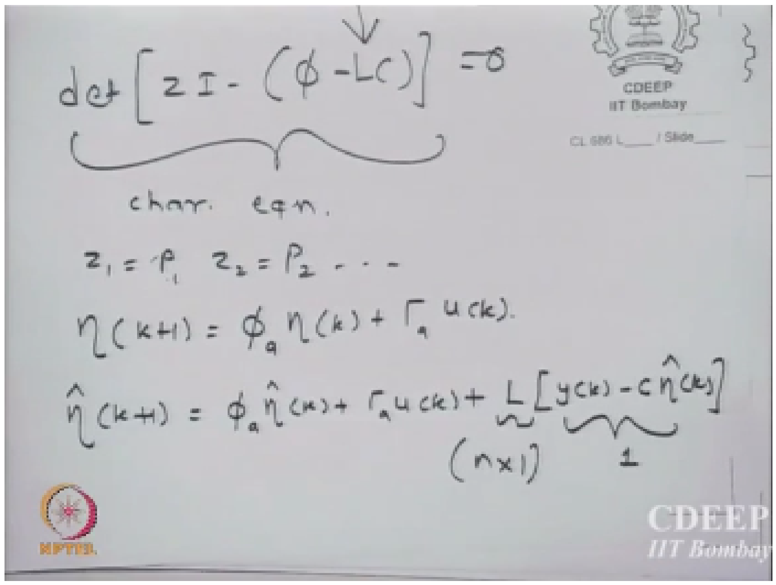

So see my transform system is, and then I want to design an observer which is I want to design an observer like this okay, observer will be in transform domain okay what is this l see there only the dimension of y is only one there is only one measurements okay what will be dimension of l? L will be n x1 okay.

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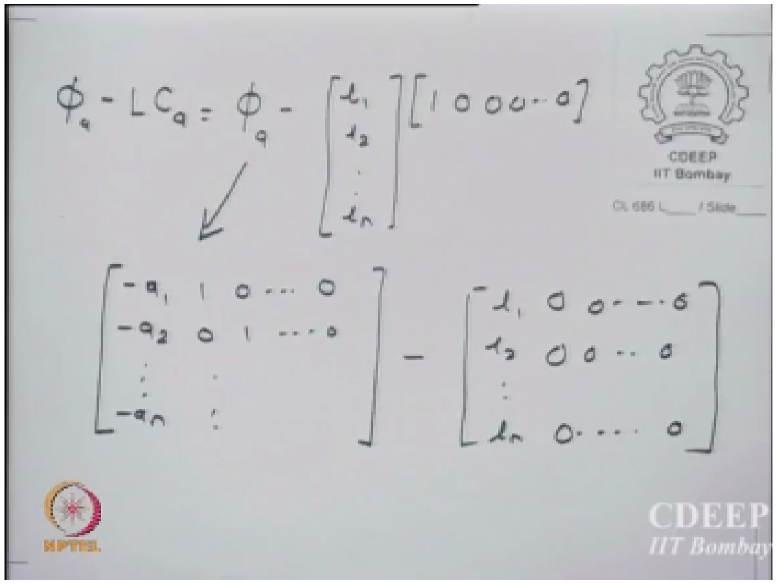

So for the transform domain φ a – L c a is nothing but φ a – what is l? L will be l1 l2 up to ln what is c 1000 okay and that is this φ a? φ a is –a1 – a2 – an 100010 and so on okay, we are working in a transform domain transform domain this transform φ is very nice only one column is there which corresponds to the characteristic equation itself all the other elements are 0 and 1 okay the transform model is very nice looking okay.

So now when I take subtract this from this see what is multiplication of this what you will get? See this will be just l1 l2 ln 0000 0000 right, it is a special form and that is why you get this in the transform domain okay.

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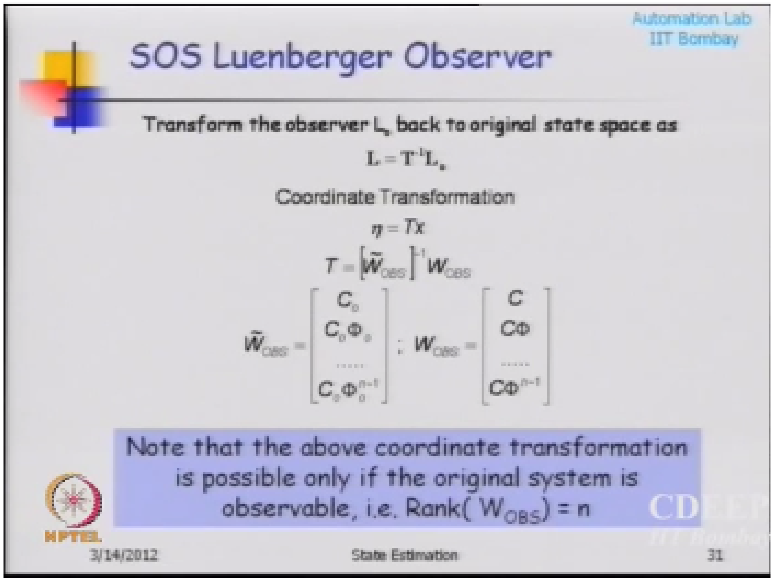

Now what is the advantage, can you tell me from here? What is the column signify? Polls of the, what is the column in observable canonical form what is the column signify? Quotient equation of which characteristic equation, so the characteristic equation of z I – l c a – φ a okay or φ a – whatever positive negative okay, for the transform part the first column will actually tell you first column will actually give you the characteristic equation for the close loop directly, just looking at this you can tell what is the characteristic equation of the close loop okay.

So this is my characteristic equation I can just look at this form and write the characteristic equation which is very nice okay and then I will just say that my close loop should be equal to okay specifying polls is equivalent to specify another characteristic equation so I am going to say that this characteristic equation should be equal to design characteristic equation this is my design characteristic equation okay this as polls at location where I want I am just going to equate the coefficients of this and this okay if I equate the coefficients I get the designs for the observer.

So if I just equate the coefficients okay I will get so this polynomial is what I have specified it has routs at a decide location where I want this is a transform polynomial I just equate the coefficient I get the observer designed in the transform domain okay I have design in the observer in the transform domain so I should come back to the original domain I have gone from x to ε I have design the observer for ε I should recover there is one more thing which we have seen earlier variable transformation it does not change polls we have seen this earlier I have shown this in the while developing the state space models just go back and look at what we have done in the state realization part okay.

Everything that we have done as a relevant okay it is not done without propose if you design in a transform domain and come back to the original domain the polls do not change okay the form changes it is not that your fundamental characteristic changes when you transform, so I am going to design the observer in the transform domain and then I am just going to come back to the original domain using the transformation matrix t, now the question is how do you get this t okay.

Not very difficult t it just multiplication of you know two observative matrixes you take the observative matrix which is original you take the observative matrix for the transform system and inverse of this times original observative matrix gives me the transformation matrix okay. This derivation is very simple I just leaving it to you I am doing this derivation it is very simple you can just show that observative matrix of the transformation can be obtain to observative matrix.

Well you know see you are given the system so you know you know c φ you know everything here, do you know this matrix? You know this matrix also because your transform se you are able to transform and you know what is the characteristic equation see this ca and φ a as specially very simple form what is the simple form for φ a and ca φ a you just find the transfer function write the coefficients of the denominator here write 100 okay what is c here? 100 so this once you find from here to here, writing this form is very easy not at all difficult okay.

So for this system if you want to find out the observative matrix it is very easy okay you can just find a transfer function construct this form write the observative matrix you are given φ and c finding the observative matrix for this system is very it is not difficult okay. so finding a transformation matrix is but then this particular equatioin has a very important message it also tells you that you can do this transformation only when the observative matrix is invertible or it has rank l okay. So obserbility and ability to place the polls at any location are linear you can place the polls of the observer at any location you want provided the system is observable otherwise you cannot do it okay.

So these are deeply link concept obserbility is not just a mathematical idea which we try to okay, so this is equal to n you are guaranteed that you can place the polls at wherever you want okay, I will just go to this CSTR example.

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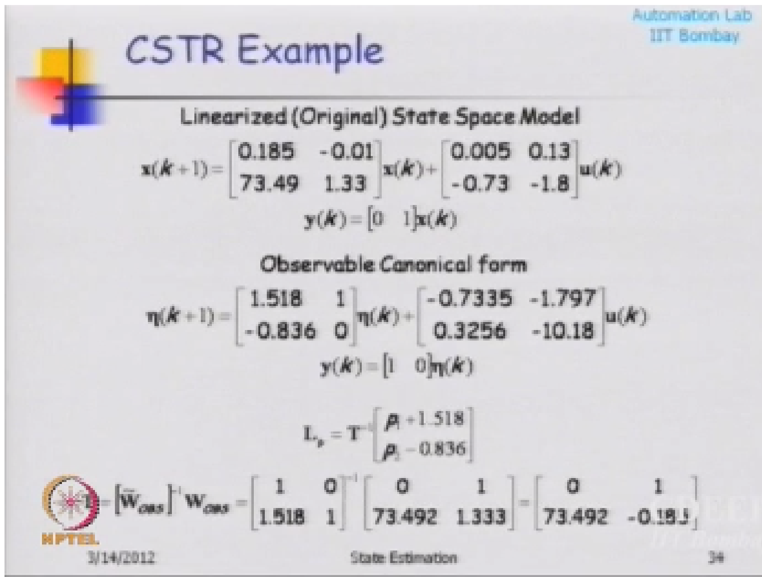

We have the system okay I converted in to this in to the observable canonical form okay formula for doing this I have already given you t matrix is constructed in the particular way you can construct t matrix without any difficulty once you point a transfer function for this system you convert in to observable canonical form you place the polls okay in the transform domain okay and recover the observer in the original domain okay.

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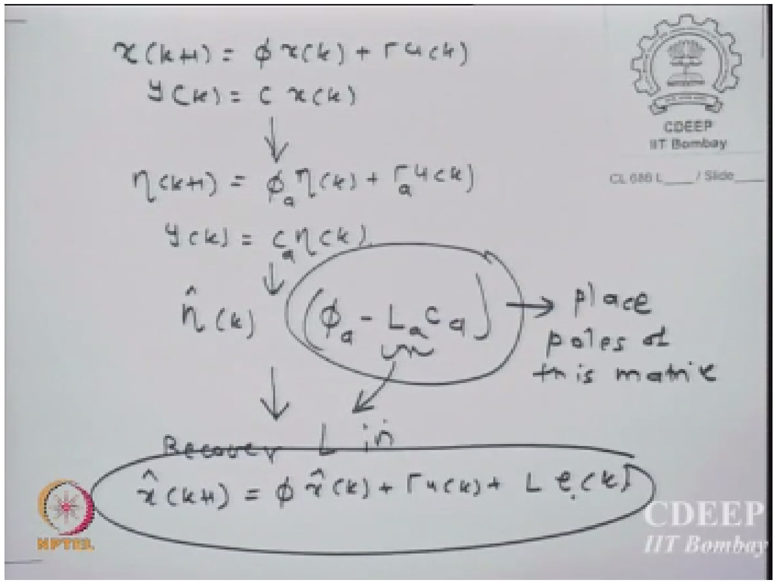

So just to summarize okay, so we started from x k + 1 = φ x k + γ uk and y k = c xk we transform this to η x k = φ a η k + γ u k yk = c η k then we design the observer for η^ k okay by placing polls for off φ a – sorry this is c a and this is γ a these are transform, so φ a – l let us call this l a c a we place the polls of this matrix and we designed this l a and then from this we recover l in this okay.

So we found out this l a from la we can recover l okay and the fact that if you move from one transformation to other transformation it does not change the Eigen value helps you in making sure that even if you are place the polls of this okay that ensures that this system also error dynamics will have same polls because transforming variables does not change the poll location that is the fundamental idea okay. So that is how this thing works okay.

So here I have gone to the transform domain in a transform domain I do the poll placement do the observer and I come back and find out recover a t using t I recover my observer in the original domain I do not want to work with the transform domain I want to work with x 1 x2 ultimately I want to estimate concentration see here x has the physical meaning concentration and temperature in a transform domain physical meaning is lost okay but it is convenient to do design so I design in the transform domain and come back to the original domain because I want o ultimately talk not in terms of η 1 η 2 I want to observer for temperature and concentration that is why I come back okay.

How good is this observer? I am just comparing the open loop observer now l with l okay for this reactor problem this is the open loop observer no l after some time obviously you know this estimate under truth converges open loop stable system no problem okay, I want to make it fast by poll placement okay the error convergence is like this if I put polls at 0.5 poll yeah.

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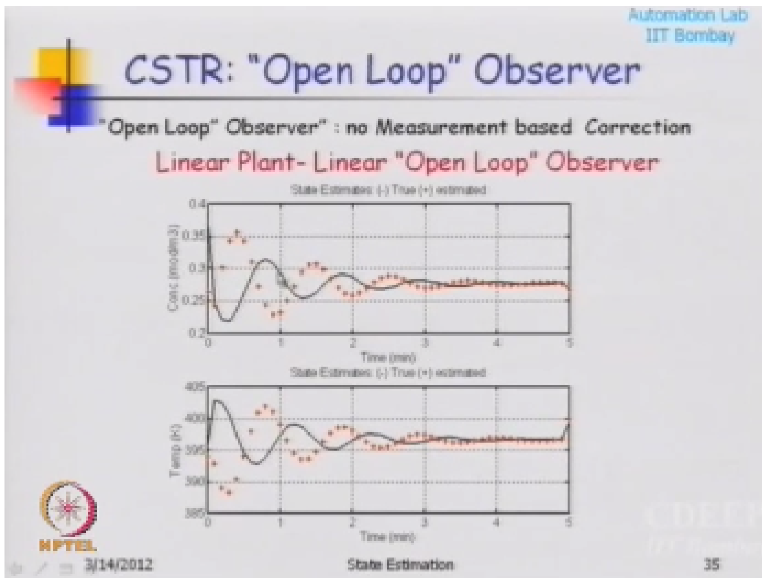

CSTR demonstration here look here this is open loop observer there is a miss match between the initial true state and estimated state okay the true state is this black line estimated state is this ++ ++ open loop observer open loop stable system after some time error between this will go to 0 okay it is not fast enough I do not want to wait for you know 10 minutes or 5 minutes for this to occur okay.

I want the error convergence very fast what if I design this is the error dynamics between true and estimated it goes like this. What if I design an observer poll placement given by the observer at 0.5 why it is called yearn Berger observer because yearn Berger develop this whole theory one of the grand role man of control theory, so just look at here okay if I place the polls at 0.5 I am able to take error to 0 very fast okay how fast just two three samples.

This figure speaks for themselves okay you place the polls error goes to 0 very fast okay and you are able to use the observer online for estimating the concentration okay using the so this is my estimated concentration ++++ using open loop observer this is my estimated concentration in reality you will never know the truth but what I know is that estimate of the concentration goes to the true value very quickly in this particular case using yearn Berger observation okay.

So that is how I can control the error dynamics by placing the polls so we will continue this right now we are not looked at noise we are not looked at disturbances and a life is going to be more complex when we start looking at noise and disturbances okay, so please bring the notes because the next 3 lectures I am going to talk about the celebrated calm on filter okay it is difficult okay to understand so please bring the notes you will need those notes to make extra.

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