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TECHNOLOGY ENHANCED LEARNING

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ADVANCE PROCESS CONTROL

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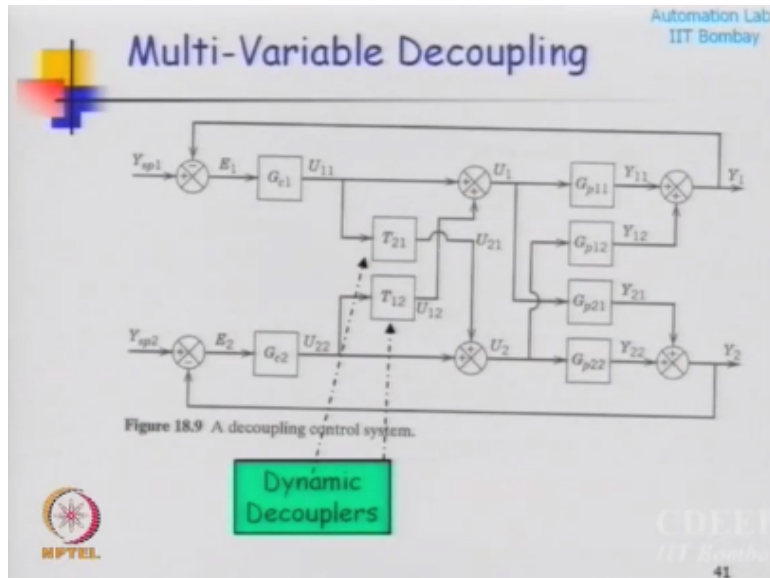
Lecture No – 16

Topic 1:
Multivariable Decoupling Control

Topic 2:
Soft Sensing and State Estimation

So in our last lecture we were discussing about multi variable control multi variable decoupling.

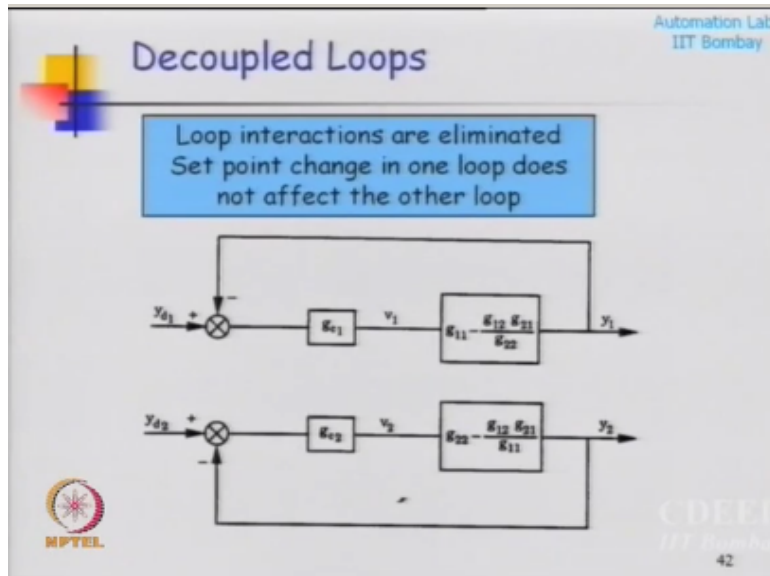
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And t talked about introducing two new elements dynamic elements into my controller, so in this T_{21} and this T_{12} these two are two new elements that I want to introduce into my dynamics of controller, so now controller block will be 1 2 3 and 4 there are four elements in the controller just like your process has four elements which interact interaction specifying interaction between both the inputs and both the outputs likewise the error which is for which respect to the first measurement.

And a error with respect to the second measurement used through two different blocks okay is used to two different blocks, so it is a multi variable controller and the idea is to introduce these two blocks T_{12} and T_{21} in such a way that effectively you have to decoupled loops.

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Okay I want to do it in such a way that the two loops are not connected effectively okay, so this is called dynamic decoupling and we that achieved the decoupling then you can have a easier or better control of the process because now how will you design a decoupler.

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Design of Decouplers

Choose T_{21} such that

$$(G_{p21} + G_{p22}T_{21})U_{11} = 0$$

or $T_{21}(s) = -\frac{G_{p21}(s)}{G_{p22}(s)}$

Choose T_{12} such that


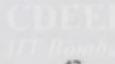
$$(G_{p12} + G_{p11}T_{12})U_{22} = 0$$

or $T_{12}(s) = -\frac{G_{p12}(s)}{G_{p11}(s)}$

Difficulty: Model should be perfectly known
Decoupling elements may be unrealizable if time-delays are present

Gain Decoupling

$$T_{21} = -\frac{k_{21}}{k_{22}} ; T_{12} = -\frac{k_{12}}{k_{11}}$$

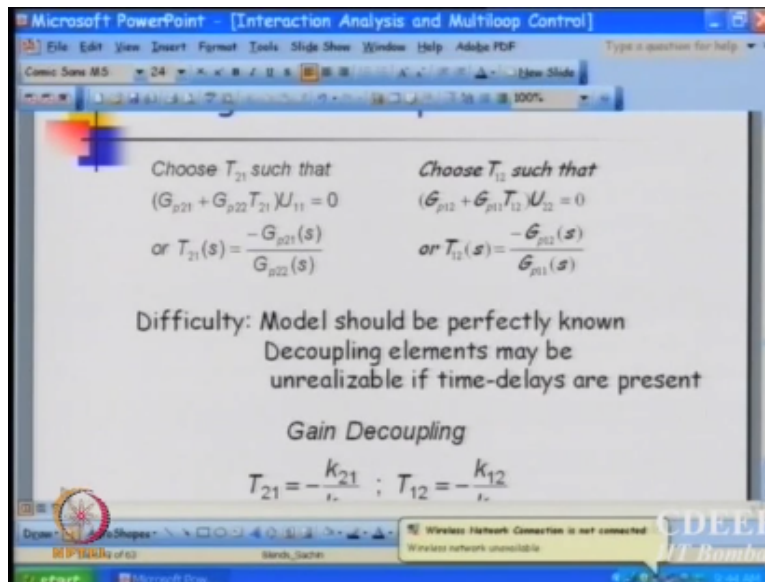
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So I am going to do this in very simple case for a 2/2 case there to we will see that will under ins some problems but more or less this our good step before going to really advance control if you can realize these kind of decouplers through conventional digital control hardware like PLC or DCS then you can improve okay, so now what I want to do is to introduce this new element I want to introduce this new element in such a way that let us go back and see what is U_{11} here U_{11} is influencing through.

T_{21} its adding for you to okay it is adding you to and adding to U_1 okay so this U_{11} + additional element which is coming here, that is my U_1 and this U_{11} which is coming out of this first controller that goes through here it adds to this I mean U_{22} which is coming from here and then that together forms that so what I want to do is to introduce this element in such a way that as if this is a separate loop so this coupling I want to remove okay I want to remove this coupler, so if you do the algebra okay.

You want to see to that effect of cross linking or that is effect of U_{22} U_{11} should be so I want to choose this element in such that the effect which is travelling here back on U_{11} that should be 0 and effect which is travelling here, the effect of the first controller on the second loop should b 0 effect of second controller in the first loop should be and that gives me these two equations I want to choose T_{11} and T_{12} so that you cancel the dynamics the cross dynamics okay, I want to choose T_{11} and T_{12} so that you cancel the cross dynamics now T this should be T_{12} .

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Okay so you I want to have these two decouplers designed in such a way that they effectively to its become decoupled we have to do algebra to come up with these equations you can do that by substituting and trying to find out that the close loop for equation I am stepping the in between steps it is just simple algebra, so what that the trouble here is that I am designing these two decoupling elements T_{12} and T_{21} so which are functions of the process model okay so it is very, very important.

That if I want perfect decoupling between the loops that model is perfect okay, now perfect model is actually only a concept in a in book there is no perfect model okay, only if you are doing computer civilization you will have perfect model because the plant is perfectly simulated we can have perfect decoupling so there is a trouble when you want to do this implement this kind of decoupling but nevertheless it is even if you have approximate models it can do good amount of decoupling.

If you employ this simple approach yeah if there write as 0 so I just come to that so this is not so this is I am introducing this more as before I go to multi variable control I am going to talk about something which has been done historically this has been step in between okay, so I will talk about the fact that this is many times not possible to this so what you say is right if you it as right hand 0s then this becomes if it is as dead line I will show you that it become unrealized if there is a dead time.

We will get when you divided two transfer functions you will get sometimes dead time which is positive which means the present value will depend upon the future value, so it is not realizable okay, so many times like the problem which is talking about is what is when you do this inversion here that is G_{21} / G_{22} G_{21} / G_{22} it may happen that the this T_{21} has some poles which are unstable okay if this happens then that is not possible to realize a unstable transfer function to unstable means you know.

The behavior can become one more dead output can become unbounded it is not possible to really like this, so there are very trouble when you have to actually physically cancel the dynamics is like this okay, we are trying to introduce to additional elements which will delify the cross dynamics that for you have to do and that makes thing difficult because it is not possible to cancel dynamics always it is difficult to consider dynamics, now one intermediate wave which is which I would is.

Not perfect decoupling but you know you can do something called gain decoupling which means do not worry about a dynamics okay, just look at the study state gain of this two elements you can study state gain of these two elements and then do decoupling then it is not going to then if you do gain decoupling you are not going to get into the situation it is not possible you will not going to perfect decoupling but it is something is better than nothing, so you know this gain decoupling is something.

Which you can try is this T_{21} and T_{12} or not realizable or to do not have perfect models or whatever reasons.

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Example: Wood-Berry Column

Pilot scale distillation column

$$\begin{bmatrix} x_D(s) \\ x_B(s) \end{bmatrix} = \begin{bmatrix} 12.8e^{-s} & -18.9e^{-2s} \\ 16.7s+1 & 21s+1 \\ 6.6e^{-1.2s} & -19.4e^{-s} \\ 10.9s+1 & 14.4s+1 \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix}$$

PI Controller Pairing
 $x_D - R$ and $x_B - S$

Steady State Decoupler
 $T_{11}(s) = 0.34$; $T_{12}(s) = 1.477$

Dynamic Decoupler Design

$$T_{21}(s) = \frac{-G_{21}(s)}{G_{11}(s)} = \frac{6.6e^{-1.2s}}{10.9s+1} \times \begin{bmatrix} 19.4e^{-s} \\ 14.4s+1 \end{bmatrix}^{-1} = 0.34 \frac{14.4s+1}{10.9s+1} e^{-1.2s}$$

$$T_{12}(s) = \frac{-G_{12}(s)}{G_{22}(s)} = 1.477 \frac{16.7s+1}{12.8s+1} e^{-2s}$$

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Let us go back to this Wood-Berry Column just other than example in this case if I do study state decoupling okay I just how to look at the gains okay and then I can introduce to clause elements T_{11} and T_{12} and then I get these two decoupling gains okay these decoupling gains will try to reduce the interaction they will not eliminate interactions okay w will not really eliminate interactions, so this is only a little bit you can adopt way of getting over the problem of but nevertheless you can do this also you can in this particular case you can actually design to decoupling elements.

And if you implement this decouplers together with to PID controllers then you will get perfect separation between two loops which means what is a meaning of that if you change one set point okay the other loop will not get disturbed at all okay only one, one if I want to change the distill competition only distill composition will change nothing will happen to the loop for the bottom competition that is what it that is what it means, so for this particular case it is possible to realize these realize this particular decoupler.

Now I just want to point out the same concern which one of you had what if the decoupler is not realizable that an happen.
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Dynamic Decoupling: Difficulty

Difficulty: De-coupler T. F. can be unrealizable

$$\begin{bmatrix} X_D(s) \\ X_B(s) \end{bmatrix} = \begin{bmatrix} -18.9e^{-s} & 12.8e^{-s} \\ 21s+1 & 16.7s+1 \\ -19.4e^{-s} & 6.6e^{-7s} \\ 14.4s+1 & 10.9s+1 \end{bmatrix} \begin{bmatrix} S(s) \\ R(s) \end{bmatrix} \quad \text{Let us assume PI Controller Pairing}$$

$x_D - S$ and $x_B - R$

Dynamic Decoupler Design for above pairing

$$\bar{T}_{11}(s) = \frac{-\bar{G}_{21}(s)}{\bar{G}_{22}(s)} = \frac{19.4e^{-s}}{14.4s+1} \times \left[\frac{6.6e^{-7s}}{10.9s+1} \right]^{-1} = 2.94 \frac{10.9s+1}{14.4s+1} e^{6s}$$

$$\bar{T}_{12}(s) = \frac{-\bar{G}_{12}(s)}{\bar{G}_{11}(s)} = 0.677 \frac{12.8s+1}{16.7s+1} e^{2s}$$

Decouplers are physically unrealizable as they include terms of type e^{+ds} with $d > 0$, which would require current decoupler output to depend on the future inp it ;

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Okay so now all that I have done is I have just shuffled the rows here okay, so now the input is S or here it was RS okay the first input here our second input was this now I decided to do no the decoupling the other way of which means I will look at this transfer function now, so my T_1 what is T_{11} T_{12} changes now okay and I decide to do controller pairing a other X_D and S and X_S X_B and R that means bottom competition control using top our reflex flow rate and steam flow rate used to control the top competition.

This is in reality you are not going to do this okay but I am just giving just want you to give an example where things can become not I mean the decoupler can be unrealizable, if I take this particular consideration and then design the decoupler using this formula which we have arrived at you get this here decoupler here which is if you notice here okay here you have to $e + 6 S$ right we have $+6$ what does it mean it means that you know the current output will dependent upon the future input which is not possible.

Which is not possible to compute okay we cannot compute something which is now based on something that is going to happen in future, so well as human beings okay bring that and we will be looking at something cooperative controller later but we realize a differential equation which depends upon you know the present behavior depends upon the future behavior is difficult is not possible, so these two couplers re not realizable the previous case that decouplers are realizable.

In this particular case the decouplers are not realizable because you get here positive dead time and which would mean that the current value will depend upon the future if you convert this into differential equation you will realize that current value will depend upon the future value and then you can also implement this decoupler okay, so decoupling I just do not want to get too much into detail we probably have some problems we solve in the assignment that and you will get some idea but decoupling probably you can use for some small system..

To input to output are mainly we input but these decoupling is a limited it has limited power because the model has to be perfect you can do this gain decoupling in simple situations and then you can implement this the main power of this is that you do not need any special software or doing advance control we can implement this ideas of decoupling to a normal distributed digital control system hardware DCS hardware or programming logic controllers of the shelf which you get in the market.

You can use them and actually implemented the decoupling ideas okay that is nice about this and you can read with some small dimension systems in terms of multiple input multiple output 2/2 or 3/3 but beyond the point this result help us.

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CSTR Example

Consider non-isothermal CSTR dynamics

$$\frac{dC_A}{dt} = f_1(C_A, T, F, F_c, C_{A0}, T_{ref})$$

$$\frac{dT}{dt} = f_2(C_A, T, F, F_c, C_{A0}, T_{ref})$$

feed flow rate
coolant flow rate

States (X) $\equiv [C_A \ T]^T$ Measured Output (Y) $\equiv [T]$

Manipulated Inputs (U) $\equiv [F \ F_c]^T$ Feed conc.

Unmeasured Disturbances (D_u) $\equiv [C_{A0}]$

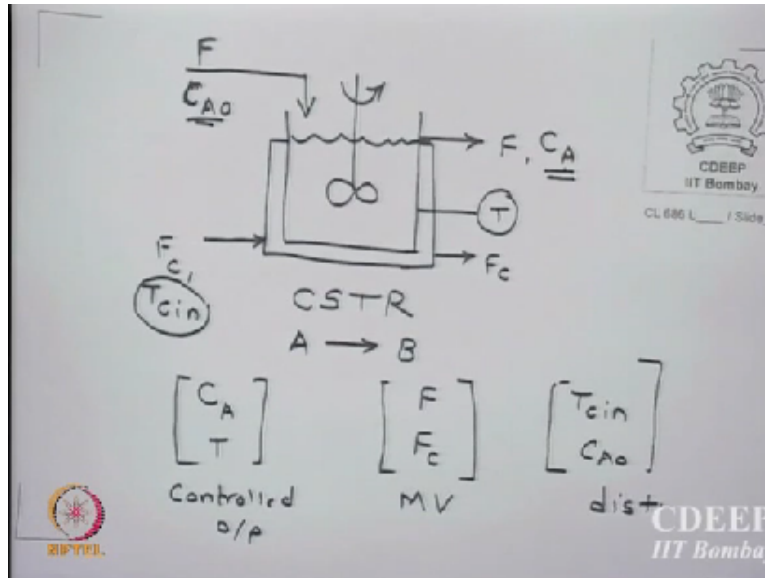
Measured Disturbances (D_m) $\equiv [T_{ref}]$ Cooling water Temp.

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Okay I just want to give an example I want to move now to multi variable control before I move to multi variable control I want to give a motivating example there is to what is the difference between signal loop controllers and multi variable controllers does it pay of going for some advance control we are going to look at a set of advance control logarithms which are very, very popular in the industry but I am trying to do is either after we do all this I am going to call one of my hostel mate who.

Works with yokogawa and he will present case studies of whatever we study here in the class, so we will present industrial case studies one or two industrial case study depending upon the time where has actually implemented these model base controllers or controllers chemical plans, so to set the background let us look at a very simple problem this is a problem of a reactor and do not bother receive if you are modern chemical engineer you can appreciate the control problem varies in, in this particular system you have reactor in which some feed is continuously being pre turn we are drawing the product okay we are drawing the product let me draw a simple diagram here.

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So the CSPR stands for continuously stand tank reactor so this is a system which is like this, this is a cooling jacket, now I fed in some okay now this is my this is my CSTR is the time reactor so we already shows carrier which indicates that all the contents in this reactant mixture or well mixed and a properties are uniform, so temperature is temperature of entire mixture is same concentration inside then dimensional same what I am doing is I am pumping in some fluid here at the rate of S and whatever over flow.

In this tank goes out so flow which comes in S it goes let us assume that it is a very reaction here simplest of the reaction will be A going to be as summarization okay and then you have this cooling jacket there is a cool which is being pumped here okay this cool is being pumped here and the cool and temperature is T_{cin} there is a this invert flow rate is inlet flow rate is of component A whose concentration is A_0 and the reaction occurs here it is gets converted into B and what you out this concentration A.

Okay so this is my simple system a very, very idealize representation of what happens in the chemical reactor and what I can major here is temperature inside this in the this reacting chamber, so I can measure temperature okay here okay the level is constant actually I would like to know what is concentration of the product this is not product which is coming out okay and I would know what is the concentration of the product okay that is my key variable but a trouble is many times.

It is very difficult to have on line measurement of concentrations it can be very, very costly temperature can be measured very easily it is very cheap you buy a good commercial temperature system inside about 20,000rps I mean I am talking about from a very you know good company if you, if your application is does not demand very accuracy or you can even get 5000 rupee you can get a good temperature measurement system whereas the cost for concentration analyzer just concentration another or so line would go to something like 4 to 5 lakhs, and if you want to go to online it could be 10 lakhs or 15 lakhs I mean very, very high and that too is possible only for certain variables okay.

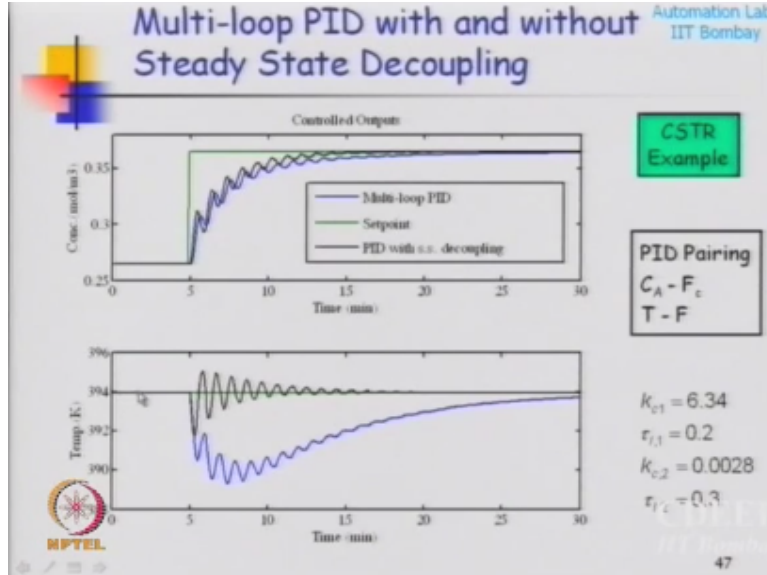
So I have temperature measurement here and then this in this particular system you can you know you need to there are control outputs are to that is control outputs are well the simplest caser is that controlled output is temperature but ideally I would like to control concentration and temperature we have the two controlled outputs, let us not write now bother about the fact that concentration measurement is very costly temperature measurement is very cheap let us assume that you know both of them are.

Measured and available to us somehow, how they are measured that is not worry okay so these are controlled outputs there are two manipulated variables this inlet flow rate here and I cooling water flow rate this FC which is here, so these are manipulated variables and then the disturbance is or so these d is disturbance variables okay, so the concentration of the feed which is coming in is a disturbance which can be fluctuating the inlet temperature of cooling fluid this could be just tank water.

Over a time and it could be fluctuating it could be changing with time as in a during the data tank water is want to be constant, so this could be changing so these two are disturbances I can manipulate by inlet flow rate I can manipulate the cooling water flow rate okay and I want to control concentration and temperature is a reactor very, very simple problem okay so now I decide to put two controllers okay so this is my system CSTR system given by two differential equations.

Rate of change of concentration and need freeze of temperature I have listed here the variables that we need to typically we will have only temperature measurement but right now we will not worry about that.

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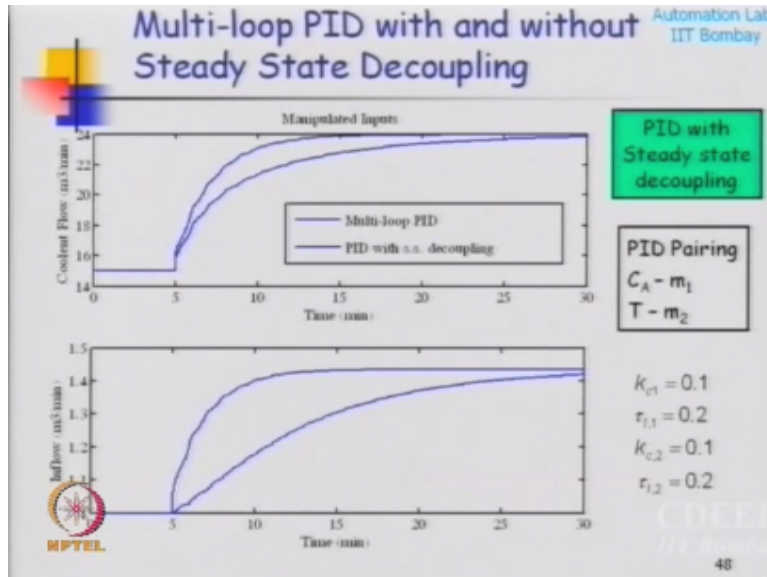


Right now we will worry about we will say that both concentration and temperature has somehow available for doing a close loop control well what is temperature measurement is only what is available and concentration measurement is not available what do you do I am going to answer that question soon, I will be starting on something called soft sensors model base senses and that will start today itself, so will be going to back but for the time assuming that both the measurements are somehow available.

I decide to put two PID controllers okay I have chosen one configurations CA concentration is controlled by manipulating cooling water flow rate temperature is controlled by manipulating by inlet flow rate, I could have done the other where I have chosen one third particular configuration okay, now I am just showing you what happens I am just what happens if I give a step change first a fall what is a meaning of two loops are coupled se here I am comparing two different performances one performance is when.

I am not taking care of anything for decoupling between the two loops okay that is this blue line okay and this black line is when I have implemented decoupling study state decoupling dead decoupling I will given okay, so I just want to show that implementing drained decoupling gives you use improvement in the closed performance, now this is concentration profile which is temperature profile what I have done is I have kept I have put to PID controllers now to PI controller to be emphasize.

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We have a drilling parameters for PI controllers and if you see here now given a set point change a time 5 minutes concentration in concentration, but I want to keep the temperature constant I want to keep a temperature constant okay I want to manage this change I want to manage the transition in such a way that Ideally I want to manage transition such that temperature should remain constant and only concentration should change that is my ideal behavior okay now that of course.

When the two loops are interacting that is not going to happen because of the interactions there are oscillations also the system has some of the reasons to have oscillations, so this particular system you see that the time it requires to settle when there is no decoupling is very large okay it takes about 25 minutes 30 minutes to settle for the temperature perturbation concentration for perturbation also settle some for about 25 or 20 minutes of tell you in a such modified introduced decoupling.

If I introduced decoupling steady stat decoupling okay only they did coupling then you see there is significant improvement in the performance the departure from the temperature from 94 to 6⁰ set point oscillations thy die down which is faster okay, so just doing this gain decoupling simple measure which you can implement through DCS or to programmer logic controller will improve

your performance to some extent okay now this is how the two if you introduce decoupling and do not introducing coupling.

This is in this one with decoupling and so without decoupling I should have color coded this so we are the manipulated input variables.

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Control of CSTR

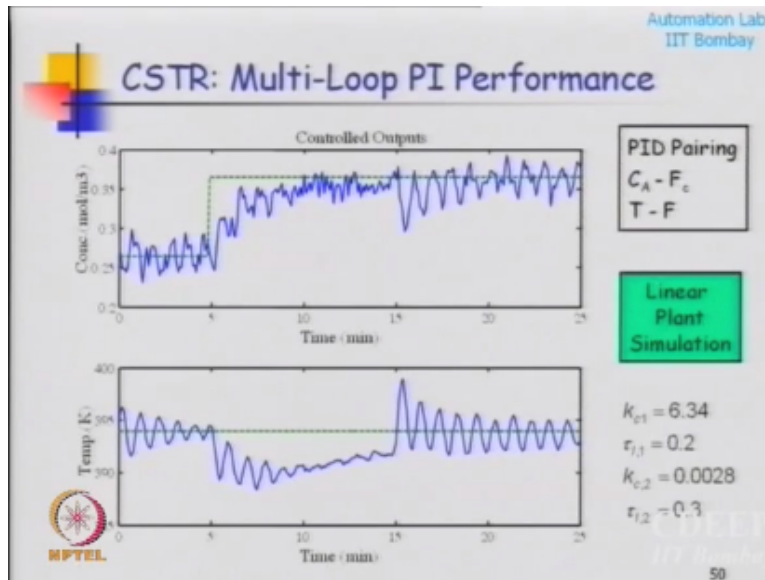
- Control schemes
 - Measure only temperature
 - Estimate concentration and temperature using state observer
 - Develop MIMO / Multi-loop control schemes to control estimated concentration and temperature
- Controllers developed
 - Multi-loop PI
 - Multi-loop PI with gain decoupling
 - Multi-variable Controller (Linear Quadratic Optimal Control)
- Control problem: Compare performances for step change in conc. setpoint at $k=50$ followed by step disturbance at $t = 150$

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And how okay now I want to control I want to actually I want to compare multi variable controllers and show that you know how much improvement we can get to multi variable controllers, so it is like you know telling you the end of the normal write in the beginning that this is where we want to reach okay, this is the kind of improvements that you get to implement and then we will again start looking at the story from beginning, so I want to implement the multi variable controller.

And putting down a little unknown name here linear quadratic optimal control which is what I'm going to cover as the part of the course, so right now we saw two things we saw the comparison of two multi controller no decoupling and to multi loop controllers with decoupling with gain decoupling we got some improvement in the close loop performance okay, now this is one more experiment.

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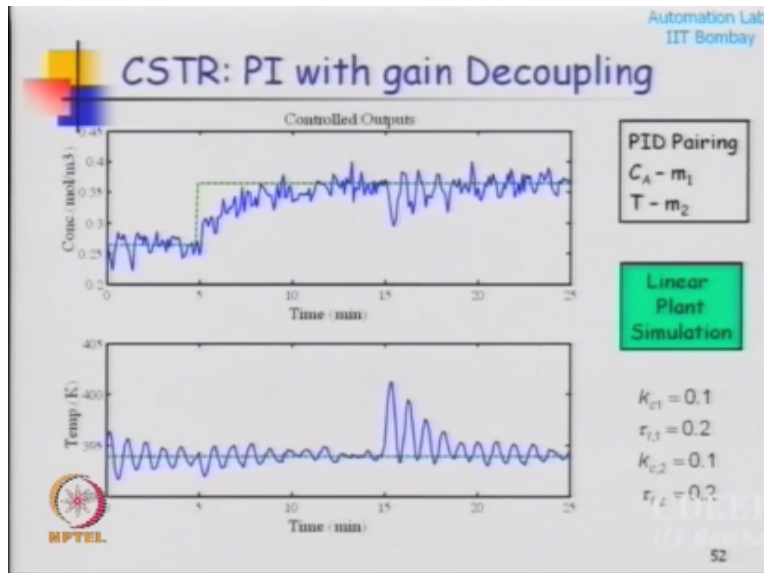


In which what I have done is again you know right now first time what I am looking at is multi loop PI controller no decoupling okay I have given have decided upon one experiment, experiment is give a set point change at $K = 50$ and give a step disturbance in one of the disturbance variables at $K = 150$ okay, so there are two things the controller we expect to do reject the disturbance and move the system to new set point okay, so this is the point where I introduced a step change in the set point.

And at this point when this concentration is about to settle okay I have introduced disturbance now this simulation which I have done here or have been I have try to make more realistic by I putting measurement noise okay fluctuations that can occurred in the flow, so I have to do realistic simulation okay because a measurements which you get I already corrupted with some noise then there are always some unknown inputs in the plant, so I introduced all that to make it realistic and see how it looks okay so you can see that when the disturbance is introduced in lot of fluctuations here.

And obviously just looking at this one if you not planned and you will say that I do not like this response because first a fall here it takes long time to come to settled to a value and then disturbance reaction gets into some kind of oscillatory behavior you do not like this controller.

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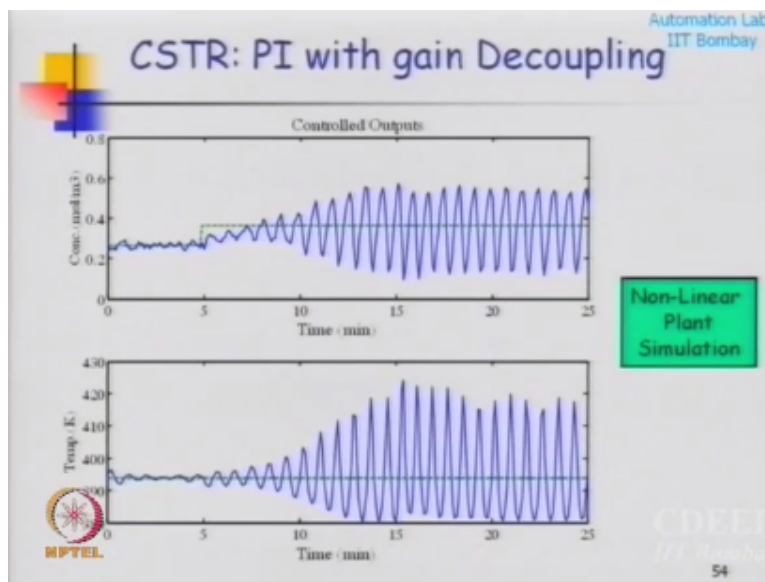


Okay we have the input which are what if I do decoupling if I just do gain decoupling there is some improvement okay if I do the same experiment if I gained then you know it is reaching here faster just compare here it is reaching faster and then this oscillation seems to reduced to some extent, so this oscillations dis oscillations saw little less right and here to this done in faster okay, so if you just do gain decoupling you have some advantage over what you know just having two loops.

So it definitely helps and if you do not have a resource to implement a very complex multi variable controller gain decoupling is about solution, so I do not want to reject gain decoupling but I want to improve called gained decoupling but when I go to multi variable controllers okay those are the inputs which are, so this is the disturbance which are given at 15 minutes or sampling is goes to 150 and well what I have done here till now I simulated the plant using linear differential equations.

I took non linear differential equations linearize them to the linear differential equations and called you the plant okay and then I did all this to PI controllers PI controllers with gain decoupling or what a simple gain decouplers things seems to work the real plant is non linear okay in my simulation what I have done next is I took the same decoupling controller and change the plant to non linear simulations more realistic and trying to be more and more realistic okay if I try to be more and more realistic in that simulations well.

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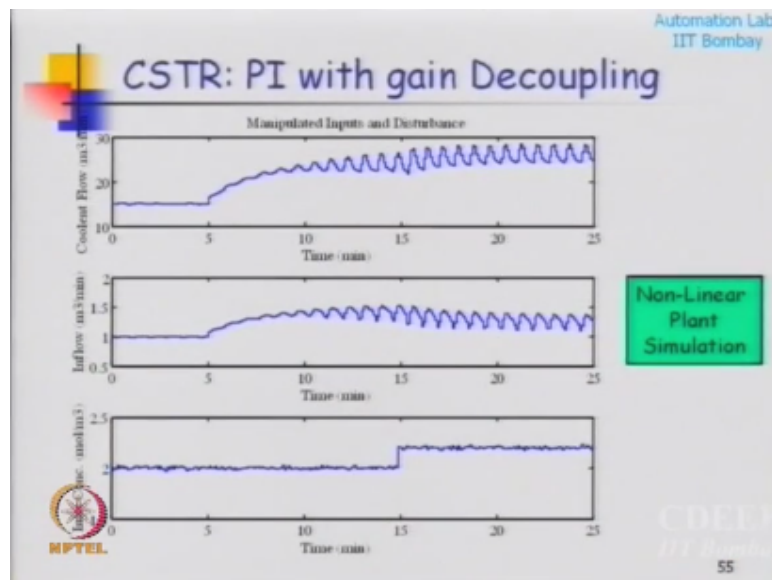
Gain decoupling which seems to work in the ideal world of linear models does not seem to work when I go to non linear plant, okay see when I am doing a simulations will be doing now simulations and putting that whole thing so we have a choice how do you simulate the plant behavior in the computer I can simulate the plant behavior using linear differential equations I can perturbation model I can simulate the plant here using original non linear differential equations which is much more realistic that actually simulating the plant with linear differential equations.

So linear differential equations if you do everything your algebra is nice everything was fine, okay we enter to all differential equations and a controller is still linear control is not non linear your still PID controller is with some simple gain decoupling okay, you have trouble where you want to implement is, so what it means see when I design a controller what I would is first I

simulate the plant using linear differential equations perturbation model design the controller this simulations after that before I go to the real system.

What I will do is I will remove the linear simulation put non linear simulation that is more realistic and see whether this controller which is based on linear system theory does it work if it works it gives me confidence to go to the real plant okay that is how I would step by step check my controller but if you can do this for the decoupling gives into the work okay.

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So I have trouble here PI with.

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Multivariable State Feedback Controller

Discrete time State Space Model

$$\mathbf{x}(k+1) = \Phi \mathbf{x}(k) + \Gamma \mathbf{u}(k)$$

$$\mathbf{y}(k) = \mathbf{C} \mathbf{x}(k)$$

State feedback multivariable control law

$$\mathbf{u}(k) = \mathbf{G}(\mathbf{x}_d(k) - \mathbf{x}(k))$$

- Step 1: Assume the states are measurable and design a stable control law / controller
- Step 2: Design a state estimator which constructs estimates of states by fusing measurements with model predictions
- Step 3: Implement the controller using the estimated states

Design Method: Linear Quadratic Optimal Control

Advantage: Multi-variable systems can be controlled relatively easily

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Gain decoupling just do not even think about to decoupled to PI controllers which are independent and not talking to each other if I go to non linear plant that it becomes a it starts oscillating okay, so now just the theory is that if I design to independent PI controllers without then talking to each other you can have trouble is a real plant okay of this simple example just two variables to control and two variables manipulate we can imagine what happens when you have large number of inputs and large number of outputs.

I want to move to this state feedback controllers that is what I am going to start doing from today so by that time you reach state in the controller design it will take probably end up this month we have to do lot of things in between, so what is the basic idea here look at what I am going to do here well this is my plant this is model which I got you know this model I could have got some physics linearization discretization for I could I have got this model from data times in this modeling state realization.

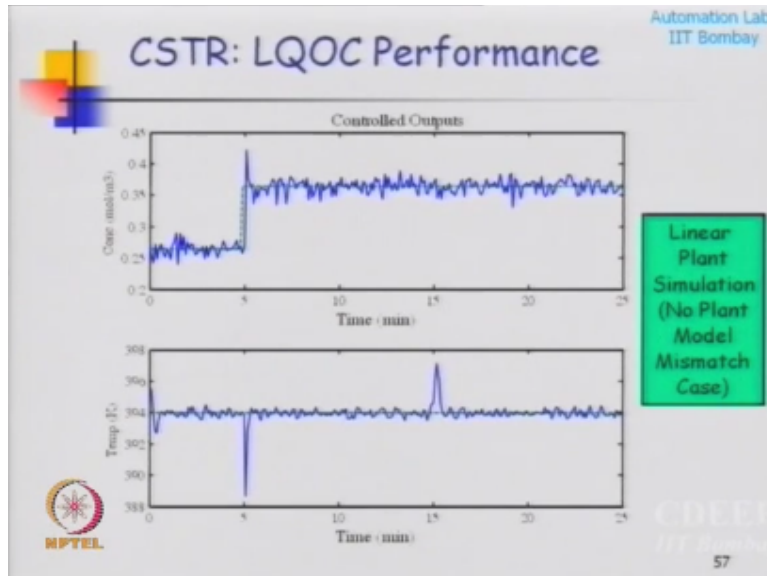
And then I go this model whichever way I got this model I have this model now I want to design what is called the state feedback controllers state feedback controller is this is the gain matrix okay this is the target state where I want to reach and I want to put a negative feedback for the state so I am assuming here for the time being that a state is perfectly measured okay now we will have to relax assumption because in reality the state is not measured only why is we go, but for the time being just lead other side.

Why I want to do this because this way when I do this is a model which is multiple input multiple output model if I design a controller of this form it will be a multiple input multiple output controller okay it will simultaneously change all the inputs okay when you ask him to do something it will not change one input at a time one there are it is not like n drivers is on driver one gain matrix which simultaneously modifies all the inputs simultaneously that is very, very important okay.

So there are three steps here when you do this you design this controller by assuming that x is available then you design something called the state estimator will start talking about phase distribution today the still estimator will construct estimate of state from the measurements y and then you merge that you in worry to then you take the controller you take the controller you take the state estimator merge them into one bit controller, so state a estimator provides estimate of the state a nice word for estimator would be soft sensor.

It creates and estimate of the state using measurements available and then you use that you use those estimated state to implement the control law, so this is these are method which we are going to look at is called as quadratic optimal control.

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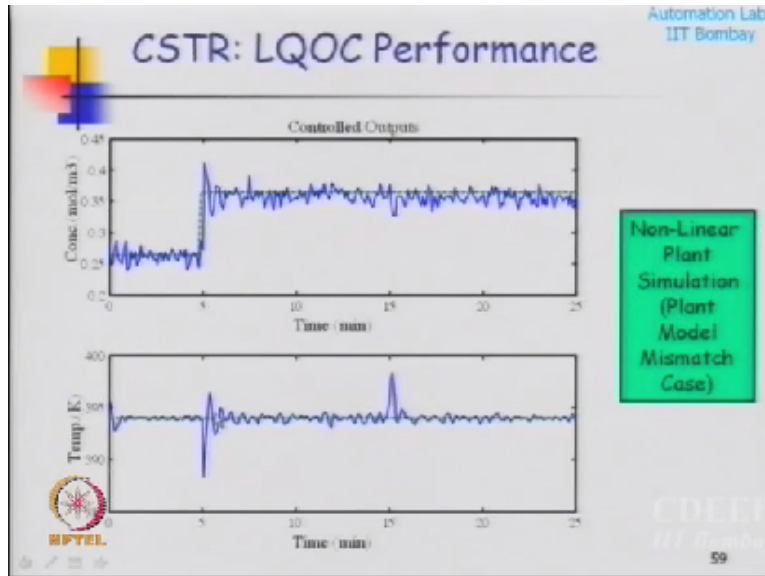


Which is been used now just as I said you know end of the novel I am just looking it in the fashion it in the beginning this is what I get if I implement a LPG controller over then I am going to a LPG controller you know that symmetry that will solve over night whatever 15 10 15 hours but you can see the difference here okay, this is perfectly decoupled kind of a response you know when I asked it to give a set point change it just went from her to here, as if it is there is no dynamics okay and there is a small blib here.

Then it returns to the state this is like ideal control what you can think of this is perfect decoupling okay if I give a step change here in loop one okay only variable on changes to the new value and this where is a small blib but it comes back okay just compare this behavior with this, this behavior or same experiment mind you same set of experiments step change followed by a disturbance okay exactly same experiment but now I have use the controller which is of this form okay I have designed a controller of this form.

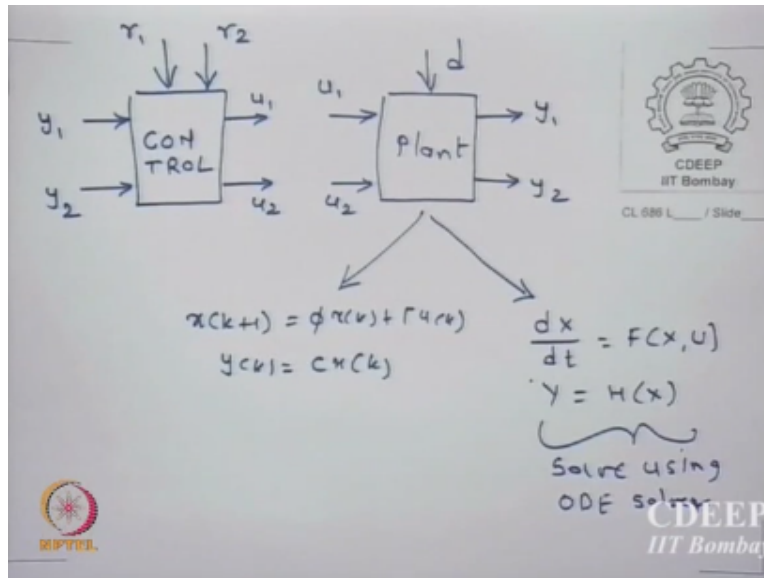
And you can see the benefits when I gave a disturbance here you can see a bib here when I introduce the disturbance with just a blib and again as if nothing happened you know a perfect control here, well you might say well your linear plant simulation linear controller everything is nice that is why it is working when if I go to non linear plant okay.

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I move to non linear plant not too much difference is steal about to hand the situation okay there is some offset you can see here looks likes there is some small of set which will take care of that, that is not an issue but you know the behavior this as slightly changed, so this has slightly changed when you moved from linear plant simulation to non linear plant simulation there is slight are you getting what I am saying about linear plant simulation and non linear plant simulation I just draw a diagram here.

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So I am simulating this close loop I am simulating this close loop this is my plant there are two inputs and I have two outputs and this is a controller block, so this is my y_1 y_2 u_1 u_2 this is u_1 u_2 this gets the disturbance d and this also gets two set points r_1 r_2 and this is y_1 y_2 , so it gets the controller gets y_1 y_2 feedback okay, I am not drawing those lines because it will like a take to the controller my controller is kept constant how do I simulate this plant how do I simulate plant here I have a choice okay.

I can do it either by doing you know $x_{k+1} = \Phi x_k + \Gamma u_k$ and $y_k = Cx_k$ this will be my linear plant simulation if I solve either alternative is I can actually solve $\frac{dx}{dt} = f(x, u)$ I am going to f of x u d so this I can and $y = \text{some } h \text{ of } x$ so this I can solve, solve using only solver so non linear differential equations and I directly simulate the non linear differential equations of the reactor okay if do that it is non linear plant simulation if I simulate this linearize model here as a plant it is a linear plant simulation that what I need okay.

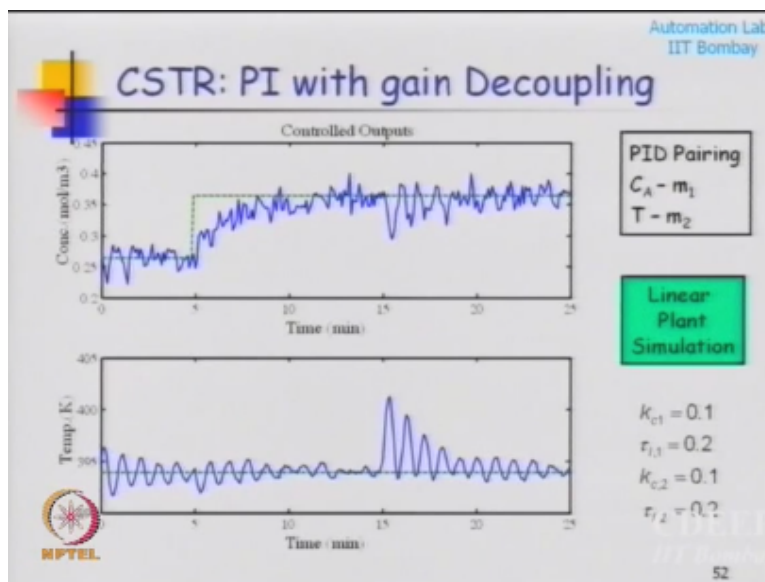
Everyone with me on this okay so look at this if I do linear plant simulation okay linear quadratic optimal control gives me near perfect control nothing can be better than this okay, there is settling time is almost you know 120^{th} of what you would get really okay so actually what I you can achieve through advance control multi variable control is far superior than what you can do with multiple variables you just cannot go there with multiple theoretical okay.

Of course to implement advance control you need much more knowledge about modeling estimation control and so on, so even though now those tools are valuable they are not being

used because you know lack of wariness and lack of you know background required to implement those advance tools, how it is being used now so what done you know what we do is we retune the PI controllers when you actually go to the so you have to keep changing the PID tuning parameters.

Till even after decoupling thus kind of satisfied now how do you do that because you know there is miss match between the point and the model and then it is not what you see in simulation is not working in reality, so that is you know that is why the operator experiment will come into picture or engineers experience we will just go and say okay I know we will use this gain by a 10 times and no we will put some concentrative value it will work yeah I mean people who do directly so that is what is a great way of going about okay even.

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Here may be you know if I done some trial error I could have made it settle you demonstrate I have just you know not try to do any tuning I have just said that take two loops do and see what happen so okay so if I blindly apply the method which I have in the literature it is not work oaky so maybe I can do something over you know had some more masala and then try to make it little what you say more cautious controller.

And then it might some over work but it is very difficult to get this what do I do yeah for this one PI controller are I have used the standard design methods given in the so this ach one of them have been designed using some one of the standard PI controller using methods I took the

transfer function then you know you look at pole placement then whatever there are methods of tuning PI controllers.

I have use them faithfully and then okay so this is what I want to get this where I want to get I want to get a near perfect control I want to get very good between the loops so you know so that is so even if I go to see when I go from linear you know there is one just this one shot you know step here and then comes back.

But then here when you go to the some linear plans simulation there is more oscillation little more oscillation but still it is able to control it is not the controller is has good so now how do we go here.

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Conclusions

- Multi-variable processes are more difficult to control because of loop interactions
- RGA and SVD analysis provides a systematic approach to choose pairing of input-output variables for multi-loop control
- When such simple measured do not work, it is advisable to use multi-variable controller (such as LQOC)

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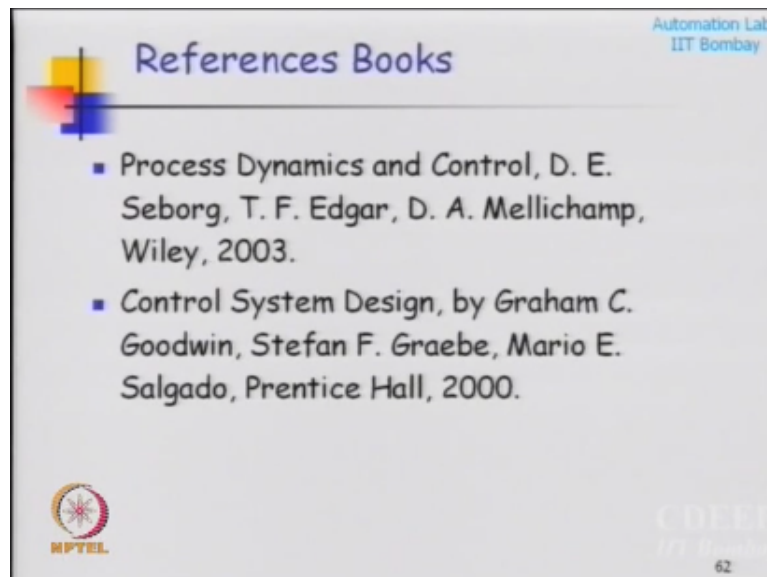
Starting from multi variable PI controllers as I said I do not know want to spend too much time in this multi loop strategy I just want due to make aware you should be aware of such things

exist and in case you do not have access to you know implementing model base controllers you should you can think doing of this.

So multi variable process are difficult to control that is the first message from these two lectures because of the group interactions okay so life becomes very, very difficult when you move from your first course in control only worry about one loop to multiple loops okay it is not, it is not so easy to design a controller things like RGI SVD or yeah they will give you some help okay they provide some systematic way of paring and you know handling reducing the interactions and so on, but of course they have their own limitations.

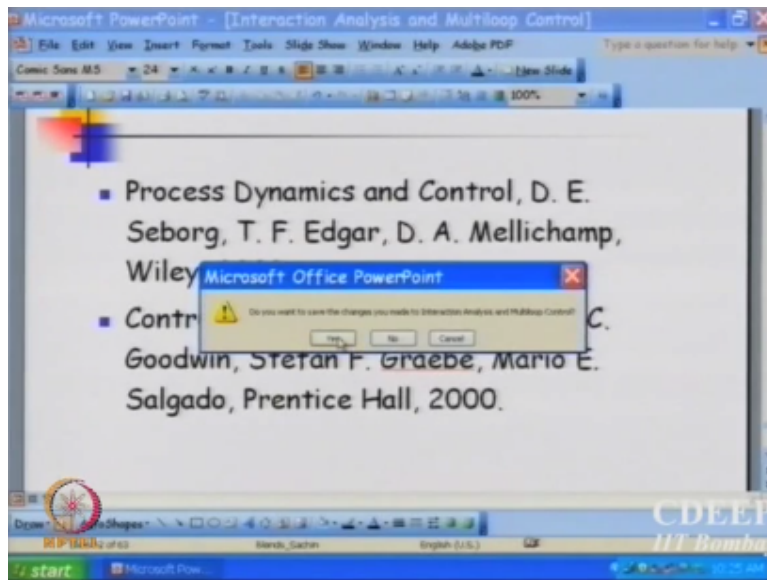
It is not many of these tools are based on gain they do not take into concession time constant and so on, and then a simple measures may not all this work like decoupling we know this work, so there is a motivation to go for you know something much better something much superior, okay so I want to get a controller which gives me the desired perfect behavior.

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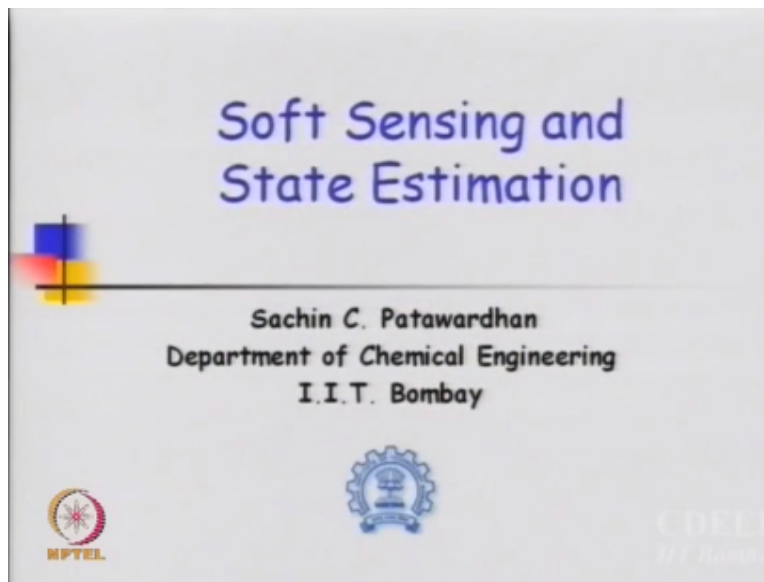
And that is what okay so they are two references which I have used for this part of lectures and now with this very brief introduction multi variable control mainly as the motivation for what we want to next and also want to keep you aware of.

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Other tools what is just we do not have but gain decoupling is to do and the study state.

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But there are dimension that during dynamics some has to write we are not the dimension during the dynamics we are only verifying the dynamic long term dynamics okay, so that is help it is not I am not saying it does not help so as I said if you do not have we cannot implement some very advance can you this is this will improve over existing PID controllers because decoupling together with multiple PID controllers it is like a multi variable control okay it is a intermediate between a full flex multi variable control and decoupled PIDs which do not talk to each other here when you put those.

Gain cross gains there is a attend to communication information between two loops right and somehow try to compensate for action of one loop on the other okay that is the basic idea that is what we should that a message is that when there are decoupler is implemented this attempt to develop a dialog between two loops okay which is not where when you have just those which w talk each other, so introducing decoupling whether it is study state or dynamic or whether it is study state what we about just using these.

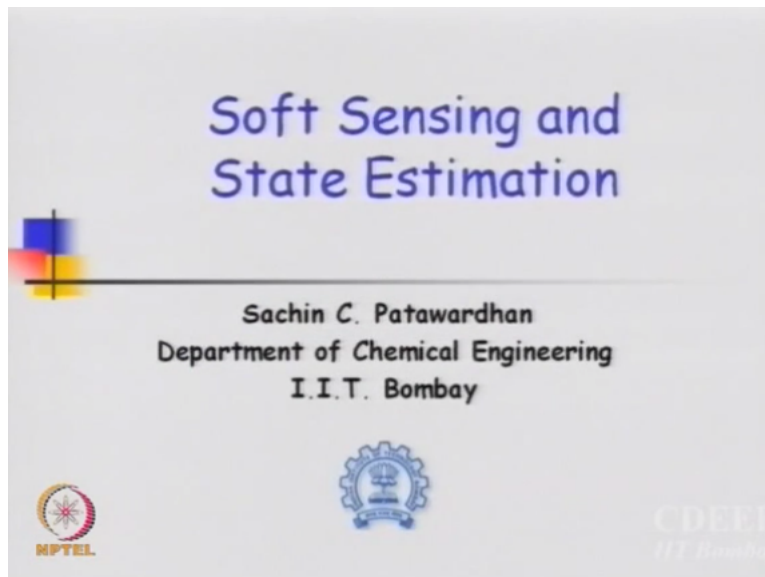
Distill better than not doing it all okay as I said you do not require any special hardware for that if you go to plant now and you will get a standard hardware which can actually implement is study state decoupling it is not reduce what is the advantage of decoupling study state decoupling I am not describing it totally computations are very simple you just multiply by gain and add to other value you know when we got to this multi variable controllers w realize that the every very

computational intensive I am going to solve an optimization problem online okay and it is going to be very, very computational intensive it is not going to be easy okay.

So implementing those multi variable controllers getting to what I showed you it is not going to be easy task it is take me at least you know till mid by the time we reach all the things that but so it has so im place can be implemented and particularly if you have a system which has very, very fast dynamics okay for example you are looking at automobile control problems right.

It might be worth doing because you do not have time to compute online you know you need computations in fractions of second if you need to decouple to loops just by putting some gain which you know create a dialog between two loops nothing like it okay it is very, very simple so it has its power it is simple at the same time it has the limitations you cannot do you know best decoupling that is possible okay.

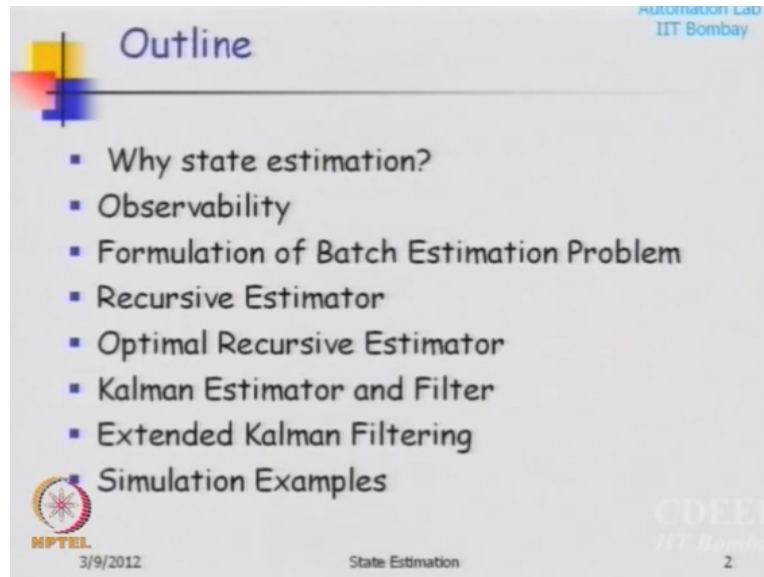
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I am going to start on this new topic it can take me at least five more lectures and this is the integral part of model control that is soft sensing is I would say a nice word to understand the that is why I put it in soft sensing but actually technical term for this state estimation okay so what is state estimation? Let us start looking at this and this is one of the main building blocks of my state feedback controller okay.

So I have a state space model I have a state space model somehow I have state space model somehow I have got this state space model I want to develop the controller based on the state space mode but this state are measurable okay the states are measurable I want to estimate the states using whatever is measurable that is my first takes okay so well of course I talk about why you want to do state estimation.

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I will move about fundamental properties of a system called observability I will talk about when fundamentally you can estimate or you can reconstruct a state just given them a measurements can you do that, that is the fundamental problem and that, that relates to some properties of the matrices that involved will form a problem of estimating state using batch of data and will find soon that is not useful.

In online context if you collect the batch of data and estimate the states in online the batch size will keep growing and then you cannot manage it so you need a what a called recursive estimators so will spend time on development of recursive estimators will move on to optimal recursive estimators what is the best way of designing the recursive estimators.

So here what I will come to what are called as Kalman's filters Kalman's estimators these are algorithm developed in earlier 60's actually this is Kalman of research over many, many years started with Gauss so the batch estimation problem was formulated by Gauss and then know later developments which were more computers have we appeared obviously when the

computers become available and suddenly used earlier 60's so Kalman's filtering is nothing but the original least square estimation problem recast for doing online estimation.

So elegant solution because the solution I am going to talk about extended Kalman filtering I will show you some simulation examples about so what is the motivation.

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Motivation

- Quality variables : product concentration, average molecular weight, melt viscosity etc.
 - Costly to measure on-line
 - Measured through lab assays: sampled at irregular intervals
- Measurements available from wireless sensors are at irregular intervals due to packet losses
- For satisfactory control of such processes: Quality variable / efficiency parameters should be estimated at a higher frequency

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Okay so just now I was talking about this reactor and in this reactor and in this reactor we said that concentrations are not measurable many times online even if they are measurable they could be very, very costly okay, they are very, very costly and I do not have money to just by you know there are so many concentrations which are required in a plant I do not have money to buy you know concentration sensor everywhere and then there are variables which are very difficult to you know measure directly.

For example, if you have controlling inner in a you are producing a polymer of certain quality you would want to control its molecular weight estimating molecular weight or measuring molecular weight online is not a trivial task, okay. You can measure viscosity you can measure density you can measure temperature but you cannot measure molecular weight online you know that is difficult, but nevertheless your property that you want to control these molecular weight okay, so that will decide by the property of your polymer which is coming out.

So what is beside is something different but you can measure is something different, now can I estimate what is decide that is a thing, that is the main. So sometimes you may have measurements but that irregular offline and develop you know found once in a while, okay so in such cases I want a continuous measurement, I want a measurement which is coming at every sampling instance so that my controller can take action, okay. Now there are other problems which are modern problems which were did it is 20 years back, now you have wireless sensors right, and a wireless sensor has packet losses and I am sure every day you experience packet losses and you try do use your mobile phone okay, the packet losses signal weaker instruction is bad and you do not hear about the other fellow is saying at the other end.

So packet losses in a wireless confirmation is a inherent problem same thing is too when if you try to do when many people are going for wireless plants okay, sounds good but there are dangers the data packet losses are there okay, we have one such beta side in our lab it estimate wireless control for and even about three four years back, so all the temperature sensors all the walls everything where fitted into the device which would transmit and receive data and you have a wireless controller settings somewhere I have a computer which is far of and then we are just communicating through wireless in the room and then trying to control.

But there are packet losses, if there are packet losses then information loss and then there is a trouble because you know my all control theory works with regular interval sampling this is possible when you are dedicated lines, data line okay, but movement you have wireless you have uncertainty keeping in, so there is a gain because you know you are cutting on the wiring cost, huge cost detection because of wiring but uncertainty increases there are price to pay.

Now that is where this state estimation come into picture, if you have irregular measurements and if you have a model then you can reconstruct missing measurements that is the idea of soft sensing, that okay I do not have a measurement right now but I have a model I can reconstruct, I can predict and estimate and I can use the estimated value for controller other than the measure value, so that is what I am going to do in soft sensor.

Yeah, to get a wireless transmitter yes, wireless transmitters probably can be done cheaply they are not so I mean if you can have a mobile phone took a lot of piece, wireless transmission may not be that costly. Just look at the mouse when you have wireless mouse or keyboards now, it is a wireless mouse must be some 300 rupees so they are able to have a wireless transmitter receiver

in the mouse okay, which is must be less than 20 rupee otherwise you cannot afford to sell that temper.

No, no but wired also you know you have to maintain all those wires and then wires have all kinds of problems you know are real mice can come and eat the wires right, so maintain those wires is a big problem it is not so easy, okay. So I mean each one had it is own pros and cons some cases it is good to have see in some cases you have very hazards environments okay. So where wiring can be difficult so wireless transmission can help it surfaces so, no I do not know I do not think so, I do not know this to give you tell me your opinion.

So there are always parameters that are not directly estimate able but you would like to measure them okay, so if you have a model you can actually construct so what is the remedy, what is the cost effect remedy, you have a model use the model in a computer I estimate the missing measurements from use of model. The main thing is some measurements are available so I should make use of those measurements and use the model use them together to construct the soft sensor, okay. How to merge the model and data that is what I am going to do next, that is the soft sensing all above.

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State Feedback Controller Design

Aim: Design state feedback controller

- Step 1: Assume the states are measurable and design a stable control law / controller
- Step 2: Design a stable state estimator using measurements
- Step 3: Implement the controller using estimated states

Separation principle ensures nominal closed loop stability with state estimator-controller pair

Advantage: Multi-variable systems can be controlled relatively easily

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Okay, so what is the state feedback controller let me just go over it again, the aim is to develop a state feedback controller, so the way we design this, way we do this we assume first when we design the controller the controller design I able to do little later, but at that time we are going to

say somehow the states are going to be available they are measurable I will design the controller okay, so that is controller design step.

Then I will design a stable observer what is the stable observer I will come to that, state estimator so only certain measurements are available can I reconstruct the state from this measurements and the model that is observer design, so actually step 1, step 2, step 3 which order we should come as no meaning. Probably one should design the observer first probably to design the controller first.

So I am choosing to design I am choosing to do step 2 first and I will do step 1 later, okay. so that 1 and 2 has no meaning, what is what of course has to happen is the step 3 has to happen after step 1 and 2 so I can after I design the observer and controller then only I can merge them and then you know. See observer is mathematical even you try to put it, it looks very, very formidable but observer or estimation of unmeasured state is something which happens you know we also keep doing every time all the time.

For example, doctor you know he is trying to estimate what is the, you know what is the faulty condition inside your body he will ask you to do some measurements right, go and take some blood sample and the he will take blood pressure and in the put stethoscope and take ask you to breathe so these are the measurements which are coming and from this he has to back calculate the state of the body, okay.

Now it is not a easy task because same measurements, same faulty measurements can be obtained for different diseases and so you need more and more measurements of course to separate the thing but state estimation is basically this you have measurements and it is what you what to know is whether it is malaria or typhoid or whether it is you know bone kindles and then you take some measurements and then from that you estimate the.

Now what guarantees that you can design a estimator separately and controller separately and then merge the two and work. There is something got separation means we will look at that later, so which guarantees that at least in the nominal case in a perfect model you can design controller separately, overseer separately merge them together still the close look will be stable.

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State Feedback Controller Design

Discrete time State Space Model


$$\mathbf{x}(k+1) = \Phi\mathbf{x}(k) + \Gamma\mathbf{u}(k)$$
$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k)$$

State feedback multivariable control law

$$\mathbf{u}(k) = \mathbf{G}(\mathbf{x}_r(k) - \mathbf{x}(k))$$

- Step 1: Assume the states are measurable and design a stable control law / controller
- Step 2: Design a **state estimator** which constructs estimates of states by fusing measurements with model predictions
- Step 3: Implement the controller using the estimated states

Separation principle ensures nominal closed loop stability with state estimator-controller pair

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So let us start with this as I said of these two components one of them I have to do first I choose to do observer first, then I will move to controller design okay. So let us begin with this model this model we have okay, and as I said right now I am not worried about how this model has come from it could have come from linearization of the kinetic model it could have come from data completely and tool box some time series modeling tool box and state realization.

So these are the three steps which are, is repeat on the slide and then I want to do this state feedback controller design which is of this form this is where I want to reach, okay finally.

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Inferential Measurement: Basic Idea

Since fast sampled (primary) variables (temperatures, pressures, levels, pH) are correlated with the quality variable, can we infer values of quality variables from measurements of primary variables?

On line state estimation:
Feasible after availability of fast Computers

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So the major question is like this, I am going to separate measurements coming from a plant into primary measurements and secondary measurements okay, primary measurements are ones which can be done at a first rate okay. The secondary measurement would be one which is done intermediated, see for example let us take the doctor and patient analogy if you admit a patient in say ICU you can keep track of pulse rate continuously BP you can you know have some monitoring system that measures BP, but if you want to measure blood glucose you know you may not have an online blood glucose measurement I mean now people are moving towards that but it is right now so cost effective that we have blood glucose measurement.

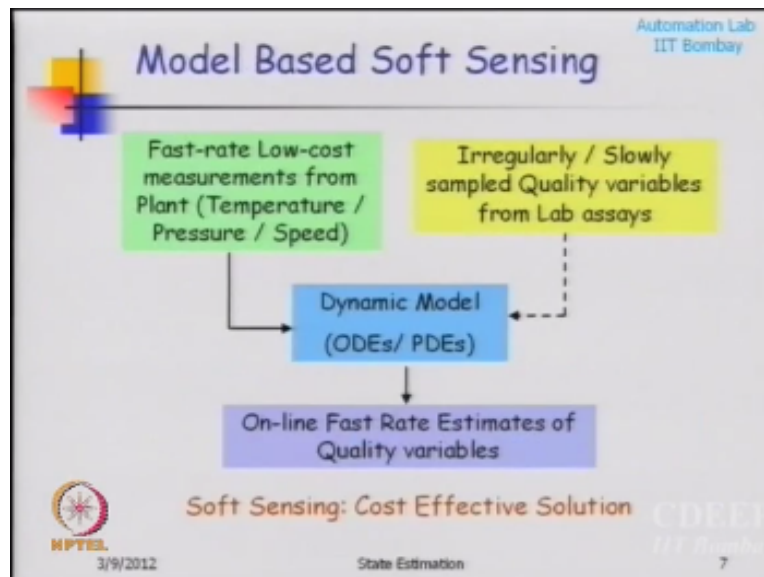
So you will take samples in between so the samples which you take for blood glucose will be secondary measurements but pulse rate or BP which you are measuring suppose by putting some device continuously you will get online measurement of Bp okay, for your plant, plant is now operation how is sleeping out there. Yeah, so we will have to worry about can we increase samples, can we so there are many, many issues ideally you should but even if you are not you can develop models which are which can take into account variable type delays.

See they are not getting into all those multi rate systems where able time delays in this first course I mean I probably have to teach one more course if I want to start off may goes to issues. In practice see what actually I am doing here is one-o-one of advance control, when you actually go to client that is much more complex you will have to build up on what I am teaching you okay, so many of these things are two simplistic easy to understand mathematics.

But then for example, you may not have a situation where the time gap between two samples is constant they might be variable, then you can develop discrete time models with time varying it is possible to do that okay, I am not talking about it because you know those are too much of details you can once you know this you can you know graduate to something that is more complex. Okay, so what we know is that let us take a disillusion columns chemical engineer or let us take this reactor which we are talking about.

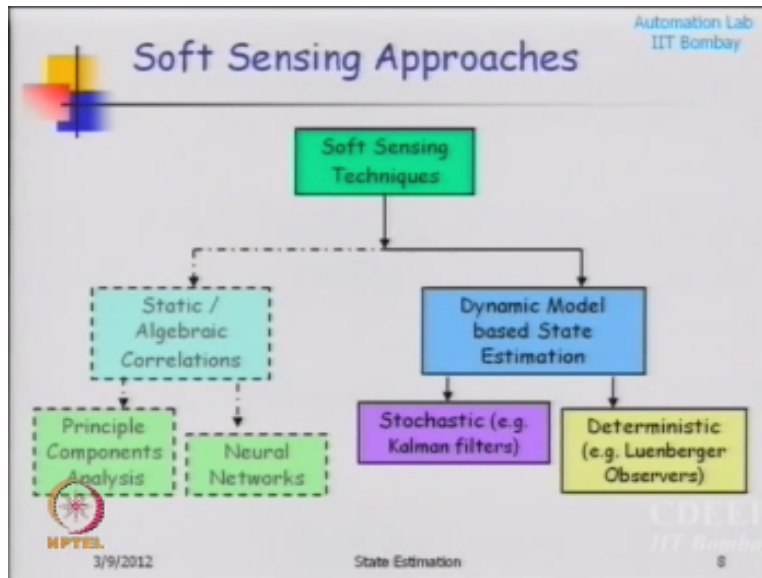
We know that the temperature and concentrations are relegate okay, so if I just measure concentration can I exploit the fact that temperature and concentration are relegate through a model and reconstruct my concentration measurement that is what I want to do.

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Okay, so basic idea here is to merge online data with model and create and estimate of unmeasured variables quality variables.

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And well there are different ways of doing this okay, just an overview you can do it using there is a way handling this through you know neural networks and algebraic maps like what you are looking at PCA and PLS and those kind of things, the way is of course using dynamic models what you can do through dynamic model is much more rich and complex these are simplistic solutions like correlations you know, you develop a correlation between temperature and concentration by some doing some offline experiments.

And in online you just put that correlation and say that if temperature is this concentration is this, so these are those simplistic approaches which are clubbed into one side okay, but those things have limitations you cannot do it beyond. Well we are going to look at two classes of dynamic model based soft sensing one is using stochastic approaches okay, and the other one you using deterministic approaches.

The stochastic approaches one is so called celebrated Kalman filter which was developed by Kalman, I think 62 or 63 he was seminal work and at the same time almost at the time this work by Luenberger which appeared in the literature completely deterministic approach completely different way of looking at a problem okay, two people are look at a same problem the two different tools okay, and by now these two are major research areas in control okay, stochastic observers and deterministic observers.

People of course are not still looking at linear system they are looking at non linear systems so these actually here this represents a branch of modern control and this represents another branch

of model control. I am going to talk about the first two developments that occurred right in the beginning okay. So as I said we are just we are going to look at one-o-one okay, so going back 50 years.

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Example: Continuously Stirred Tank Reactor

Consider non-isothermal CSTR dynamics

$$\frac{dC_A}{dt} = f_1(C_A, T, F, F_c, C_{A0}, T_{in})$$
$$\frac{dT}{dt} = f_2(C_A, T, F, F_c, C_{A0}, T_{in})$$

States (X) = $[C_A \ T]^T$ Measured Output (Y) = $[T]$

Manipulated Inputs (U) = $[F \ F_c]^T$

Measured Disturbances (D_m) = $[T_{in}]$

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So let us go back to our old CSTR system we will have there are two measurements there is only one measurement there are two states concentration and temperature I only want to measure temperature I cannot afford to measure concentration okay. There are two manipulate variables but manipulate variables values are known to me I am giving them from a computer okay, so my controller is whether my controller is advance controller, whether my controller is PIT controller I know manipulate input values, okay.

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Example: Continuously Stirred Tank Reactor

Consider non-isothermal CSTR dynamics

$$\frac{dC_A}{dt} = f_1(C_A, T, F, F_c, C_{A0}, T_{cw})$$

$$\frac{dT}{dt} = f_2(C_A, T, F, F_c, C_{A0}, T_{cw})$$


States (X) = $[C_A \ T]^T$ Measured Output (Y) = $[T]$

Manipulated Inputs (U) = $[F \ F_c]^T$

Unmeasured Disturbances (D_u) = $[C_{A0}]$

Measured Disturbances (D_m) = $[T_{cw}]$

If model parameters are known accurately,
can we estimate C_A from measurements of T alone?


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So these two feed flow rate and coolant flow rate are known to me okay, there are two disturbances inlet concentration fluctuation this one is an or feed concentration that is a unmeasured disturbance I cannot afford to measure this, I cannot afford to measure the product concentration forget about measuring the inlet concentration so concentrations I am not able to measure. Well the temperature of the coolant which is coming in that is also fluctuating, this is also a disturbance.

But I can afford to measure it okay, temperature measurement is cheap I can measure inlet coolant temperature I can put it tank in my overhear reservoir and measure the difference not an issue, okay. So my question is if I have a perfect model okay, can I reconstruct or can I estimate concentration from temperature alone so these are the parameters and then what I am going to do is of course I am going to do linearization of this model and then.

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CSTR: Continuous Perturbation Model

Continuous time linear state space model

$$x(\tau) = \begin{bmatrix} C_A(\tau) - \bar{C}_A \\ T(\tau) - \bar{T} \end{bmatrix}; u(\tau) = \begin{bmatrix} F(\tau) - \bar{F} \\ F_c(\tau) - \bar{F}_c \end{bmatrix}; d(\tau) = C_A(\tau) - \bar{C}_A$$


$$\frac{dx}{d\tau} = \begin{bmatrix} -7.56 & -0.09 \\ 852.72 & 5.77 \end{bmatrix} x(\tau) + \begin{bmatrix} 0 & 1.735 \\ -6.07 & -70.95 \end{bmatrix} u(\tau) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} d(\tau)$$

$$y(\tau) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(\tau)$$

Discrete time linear state space model
Sampling Time (T) = 0.1 min

$$x(k+1) = \begin{bmatrix} 0.185 & -0.01 \\ 73.49 & 1.33 \end{bmatrix} x(k) + \begin{bmatrix} 0.005 & 0.13 \\ -0.73 & -1.8 \end{bmatrix} u(k) + \begin{bmatrix} 0.06 \\ 3.9 \end{bmatrix} d(k)$$

$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(k)$$


State Estimation
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You know how to get this model okay, I at some operating point I got this the operating point is given in the previous slide and then you get this linear perturbation model where the two states are perturbation in the concentration perturbation at a picture two inputs are perturbation in the flows and so on, and then I will discretized this model I will get this discrete state space model, okay.

Now I have this discrete state space model with me let us assume that it is very big model, it matches very well with the plant behavior in this small region in which I am operating there is not too much of plant model mismatch the model is good, okay and my y here is only temperature measurement see it is 01 only temperature is being measured, okay. And now here I want to do a state estimator that will reconstruct concentration from measurements of temperature alone.

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Example: Quadruple Tank System

$$\frac{dh_1}{dt} = -\frac{a_1}{A} \sqrt{2gh_1} + \frac{a_2}{A} \sqrt{2gh_2} + \frac{\gamma_1 k_1}{A} v_1$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A} \sqrt{2gh_2} + \frac{a_1}{A} \sqrt{2gh_1} + \frac{\gamma_2 k_2}{A} v_2$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A} \sqrt{2gh_3} + \frac{(1-\gamma_1)k_1}{A} v_1$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A} \sqrt{2gh_4} + \frac{(1-\gamma_2)k_2}{A} v_2$$

Manipulated Inputs: v_1 and v_2
Measured Outputs: h_1 and h_2

If model parameters are known accurately, can we estimate levels in Tanks 3 and 4 from measurements of levels in Tanks 1 and 2?

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State Estimation
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Let us move on to the problem which we are familiar with, as I promised I will take you to the lab and show you the system. In this particular system I have four tanks again you level measurement is costly each level measurement device cost now well in when I am aggregated this system in 2005 it was 30,000 so I am sure it is now 60,000 you know so good quality I think those are UcoGoga or some very good quality level transmitters each one is 60,000 I cannot afford to buy two transmitter four transmitters.

But I want to measure I want to have an estimate of I am measuring level here I am measuring level here and I want to have an estimates of level in tank 3 and tank 4 just using the model this is the cost effective solution. Turning a model in my computer does not take too much, no I just have to buy one laptop which is or some control computer which is anyway going to be there, so I will use that computer to the construct.

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State Estimation Problem

It is desired to implement a state feedback control law. However, all the states are not measured.

Thus, given

Computer control relevant discrete model

$$\mathbf{x}(k+1) = \Phi\mathbf{x}(k) + \Gamma\mathbf{u}(k) + \Psi\mathbf{d}(k)$$


$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{v}(k)$$

and input-output data

$$\{\mathbf{y}(0), \mathbf{y}(1), \dots, \mathbf{y}(N)\} \text{ and } \{\mathbf{u}(0), \mathbf{u}(1), \dots, \mathbf{u}(N)\}$$

Can we estimate state sequence

$$\{\hat{\mathbf{x}}(0), \hat{\mathbf{x}}(1), \dots, \hat{\mathbf{x}}(k)\} ?$$


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State Estimation

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Okay, so given this model I have this model now okay, there of course there are troubles we got them measurements can have error measurements can have error here okay, measurements are not perfect and then we have talked a lot about measurement errors sometime back when we talked about system identification okay, and colored noise and colored disturbances and all kinds of things. So even if you take a very simplified view that this \mathbf{v} here is a white noise even then it is there right.

So there is some error in the measurement you never get perfect measurements. This \mathbf{b} here is unmeasured disturbance okay, there are concentration fluctuations in the reactor at a inlet I am not measuring them but they are effecting the dynamics and then I need to estimate the state, so estimating the state in presence of these two trouble makers one is the measurement noise and other one inputs unmeasured inputs that are going into the state dynamics is not an easy task it is going to be quite tuff.

So what is the problem, the problem is I have a model now see earlier the model the problem that we are looking, what are the problem that we are looking at earlier, the problem that we are looking at earlier was given input and output data okay, we wanted to estimate Φ γ ψ and \mathbf{C} matrices that was the modeling problem identification problem okay. Now I am just turning it around I am saying that well I know Φ γ okay, I probably know ψ I know \mathbf{C} okay, and I know \mathbf{y} and \mathbf{u} , \mathbf{y} and \mathbf{u} are known input is known, output is known, okay.

Can I reconstruct x illiterate problem okay, in one case what was known and in once case what is known is just turning around now I am saying I have the model okay, I have data for input and output I want to reconstruct can I estimate the state this is the abstract way of saying can I develop a soft sensor for the system okay. As I said grand dual problem, being studied from the days of Gauss you know when even before that if Gauss came up with the solution people were looking at this problem much before.

And a classic example or classic system which human that days where bothered about was planetary positions you know exact knowledge of planetary positions by the time Gauss worked on this problem you of course turn the differential equation that given the system, okay and you have measurements coming from telescope can you estimate the state of each planet okay, that is.

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Simplified Problem Statement

Consider ideal situation where

- disturbances and measurement errors are absent
- model is perfect

Problem: Given measurements $y(0), y(1), \dots, y(N)$ and inputs $u(0), u(1), \dots, u(N)$ together with model

$$\mathbf{x}(k+1) = \Phi \mathbf{x}(k) + \Gamma \mathbf{u}(k)$$

$$\mathbf{y}(k) = \mathbf{C} \mathbf{x}(k)$$

Estimate state sequence $\hat{\mathbf{x}}(0), \hat{\mathbf{x}}(1), \dots$

$\hat{\mathbf{x}}(0)$

State Estimation

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So let us take a situation, let us go by very, very slowly I am going to take situation initially where there are no disturbances okay, I am not going to worry about measurement noise too much in the beginning I will systematically go towards that point how to deal with measurement noise, so let us take the most ideal case, most ideal case is there are no disturbances zero disturbances there are no measurement others, okay.


Now I have a perfect model okay, everything is perfect model is perfect no disturbances, no measurement error now the first question that I ask is given this model and then measurements under the scenario that there is perfect model okay, no measurement noise no state disturbances.

Can I reconstruct x_0 from measurements of y okay, so let us start answering this question? Okay, now let us call this initial see to use this model what is known here in this model ϕ is known, γ is known now we have done modeling u is known, now suppose I want to do a simple online estimator is use this model online you will say use this model online okay, you know what is u okay, keep giving u to this model this model will predict what is x , that is a soft sensor is not it a very, very and you would be wrong it is fine.

It will give you a very crude soft sensor yes, if you just use this model in my computer I just program this model in my computer. Now the trouble is when I start this model what should be x_0 , what is the initial state see I do not know what is the initial state of the plant if I give a wrong initial state this model will go somewhere the plant will go somewhere okay, I do not know I am not measuring x_0 I am only measuring y , okay.

So to kick of this model minimum information that I need is x_0 , if I have x_0 okay and if I know input sequence u I can just use model and then I will get x_0 is known u_0 is known ϕ and γ is known I will get x_1 okay, then if I know x_1 and u_1 I can get x_2 and I have a soft sensor is not it, this is fine I mean in ideal world computer world you will get this.

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Initial State

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Let $\hat{x}(0)$ denote initial state estimate
and given input sequence
 $\{u(0), u(1), u(2), \dots\}$
we can use model to estimate

$$\hat{x}(1) = \Phi \hat{x}(0) + \Gamma u(0)$$

$$\hat{x}(2) = \Phi \hat{x}(1) + \Gamma u(1)$$

$$= \Phi^2 \hat{x}(0) + \Phi \Gamma u(0) + \Gamma u(1)$$

$$\hat{x}(3) = \Phi^3 \hat{x}(0) + \Phi^2 \Gamma u(0) + \dots$$



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Okay, so I can get this x_1 if I happen to know x_0 then I can compute x_1 then I can compute x_2 okay, but you will realize is that x_2 actually depends on x_0 okay, it depends up on u_1 and u_0 , u_0 and u_1 okay, what about x_3 you will see that x_3 will also depends up on in a dynamic system, dynamic system has a memory it also admits new inputs you can modifies its behavior, you can give inputs u_1 , u_2 they will have insulation on what happens in future but the passed also will have its effect on the future, okay very, very important.

So now the question is how do I find out x_0 okay. Well if you guess then no, no in this case there is no measurement error there is so I do not know to guess I want to come up with an exact value and which is possible okay, so which is what I am going to say now that.

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Estimation of Initial State

Given measurements $y(0), y(1), \dots, y(n-1)$
 And inputs $\{u(0), u(1), u(2), \dots\}$
 we can write


$$C\mathbf{x}(0) = \mathbf{y}(0)$$

$$C\mathbf{x}(1) = \mathbf{y}(1) = C\Phi\mathbf{x}(0) + C\Gamma\mathbf{u}(0)$$

$$\Rightarrow C\Phi\mathbf{x}(0) = \mathbf{y}(1) - C\Gamma\mathbf{u}(0)$$

.....

$$C\Phi^{n-1}\mathbf{x}(0) = \mathbf{y}(n-1) - C\Phi^{n-2}\Gamma\mathbf{u}(0) - \dots - C\Gamma\mathbf{u}(n-2)$$


State Estimation
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See I can write this first equation $C\mathbf{x}(0) = \mathbf{y}(0)$, then my second equation will be $C\Phi\mathbf{x}(0) + C\Gamma\mathbf{u}(0) = \mathbf{y}(1)$ I am just using those previous equations okay, now see and this equation I am going to rearrange because what is not known to me $\mathbf{x}(0)$ is not known to me okay. Now $\mathbf{y}(1)$ is known to me $C\Gamma\mathbf{u}(0)$ is known to me okay, so that I have taken on the right hand side okay. I can just go on doing this okay, so I have collected all the terms on $\mathbf{x}(0)$ on the left hand side all the known quantities \mathbf{y} is

known u is known all the y and u terms are appeared in my equations and then taking them on the right hand side, okay.

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Observability

Observability: System is said to be observable if initial state can be uniquely estimated from output observations

Initial state can be estimated from measurements of inputs and outputs if following rank condition holds

$$\text{rank} \begin{bmatrix} C \\ C\Phi \\ \dots \\ C\Phi^{n-1} \end{bmatrix} = n = \text{state dimension}$$

Observability Matrix

Observability: (Local) system property

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So now okay, so what happens here is I think it is level let us do in the next class, let us stop here so we have these equations I can start them together.

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Estimation of Initial State

Given measurements $y(0), y(1), \dots, y(n-1)$
And inputs $\{u(0), u(1), u(2), \dots\}$
we can write

$$Cx(0) = y(0)$$
$$Cx(1) = y(1) = C\Phi x(0) + C\Gamma u(0)$$
$$\Rightarrow C\Phi x(0) = y(1) - C\Gamma u(0)$$

.....

$$C\Phi^{n-1}x(0) = y(n-1) - C\Phi^{n-2}\Gamma u(0) - \dots - C\Gamma u(n-2)$$

Can we uniquely estimate the initial state by
Solving above set of linear algebraic equations?

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Because see what is the coefficient here C , here it is $C\phi$ and it will be $C\phi^2$, $C\phi^3$ and finally to do $C\phi^{n-1}$ okay, so I can stack all the equations together what is not C is known ϕ is known only x_0 is known on the left hand side okay, and everything on the right hand side is known okay, so it is like solving n equations in an unknowns you know when does the solution exist okay, we will find a fundamental property of the model here called observability which comes down to the rank of this particular matrix.

This particular matrix will appear and you can show that is this matrix has rank n then alone you can reconstruct state from the measurements, is the rank is not equal to n then you cannot get all the states from the major bad challenge, you may have to add more measurements to make system observable so this is the fundamental property of the system call observability we will relook at it again starting from the next class. So you can design your observer for a system if this fundamental property is satisfied by the system equations otherwise you cannot, okay.

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