

NPTEL
NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING

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ADVANCE
PROCESS CONTROL

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Lecture No – 15

Interaction Analysis and
Multi – loop Control (contd.)

So we have been looking at multivariable interactions and in particular we are looking at interactions between single loop PID controllers by single loop PID controllers because most of the systems today seem to use multiple PID controllers for controlling achieving desired control now what is the problem with multiple PID controller so I said it is like having multiple drivers in your car multiple drivers who do not talk to each other who do not coordinate.

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Loop Interactions

- **Large loop interactions :**
can lead to poor quality of control due to lack of coordination among PID controllers
- **Solution strategy:**
 - Choose controller pairing with minimal interactions
 - De-tune the controllers to minimize loop interactions
 - Design multi-variable controllers, which simultaneously change all inputs by considering errors in all the outputs

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Now this can lead to poor control because of lack coordination between different controllers so what is the strategy well most of the large scale systems like chemical refineries or power plants nowhere companies have already invested into single loop PID controllers they obviously what to continue with that it is a historical package that we carry that we also there are other issues like even though they might have brought a computer control system a DCS.

Still lack of awareness of how to implement advanced control multivariable control would mean that they continue to use multiple PID controllers which interact and then the way this problem is handled in practices to have programmable logic controllers PLC's together with PID controllers this PLC's are something like you know they at like a boss if there is some problem they take some a dock actions whjkch are based on experience on some logic derived from the experience.

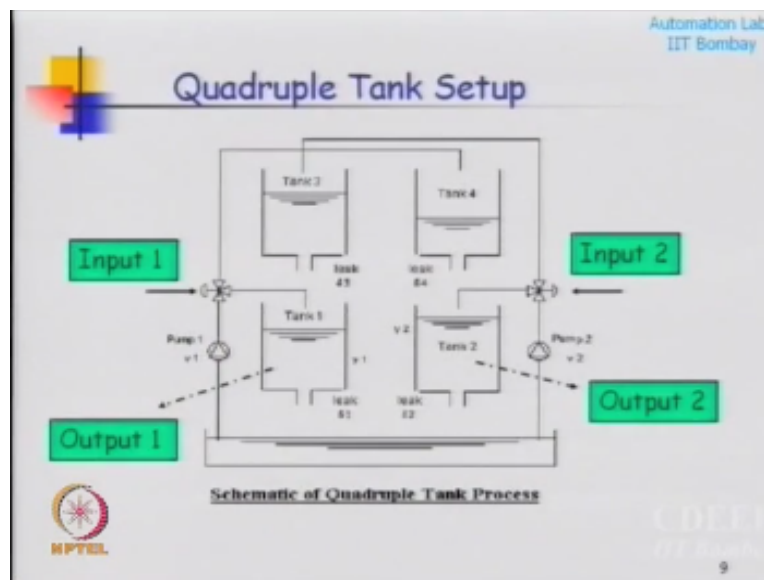
If this variable is high and if that variable goes low then shutdown the steam or whatever so there will be logical you know elaborate logic statements which will try to handle safety constrains by some kind of if the nulls XOR, OR, AND so on those kind of blocks and actually designing a system which is interaction between these multi loop PID controllers and this logic is a very complex business.

It is not so easy and that is because that is because the plant is multivariable everything you know many things effect many outputs many inputs effects many outputs and the controllers are actually trying to solve this problem using simplistic multiple single loop controllers which is

causing problem there are 2 solutions to this problem one is try to choose PID controller pairing in such a way what is pairing we will come to that.

Try to choose pairing in such a way that there is minimal fight okay after having done that okay tune PID controllers see we know methods of tuning PID controllers for a single loop that is single input single output systems kind of you know back off from tuning which is there for single input single output designed a detuned controller okay I am not going to go detail into the detuning of the controller I will just mention it and the 3rd option is of course discard them and go for advanced control go for multivariable control okay.

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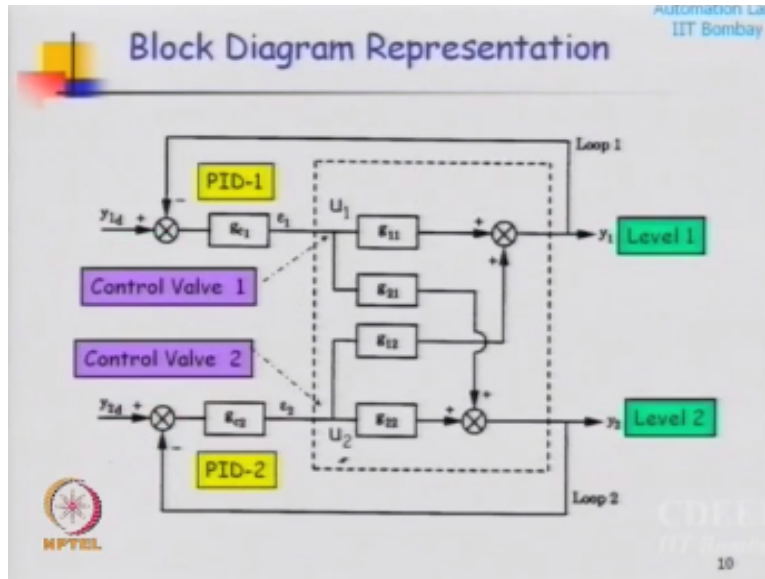


So those these example I was discussing in my last class this is the 4 tank system quadruple tank system we have actually a I think semi link model for this was Krishna you had shared the semi link model for this right and then you can actually if you have access to a mat lab you can run the semi link model there are 2 PID controllers or there is a PID controller design given in the paper which I have put on the net okay model.

Okay just impalement those to PID controllers you will see what happens in semi link it is just take a PID block attach it to this and see what happens when you try to control the system using 2 PID controllers we can actually simulate and see what happens now here this a multivariable system both the inputs affect both the outputs there are 2 outputs level in tank 1 and level in tank 2 there are 2 inputs.

And we know that both the inputs affects both the outputs so how to pair if I do I measure level 1 and manipulate wall 2 or wall 1 okay that is the question.

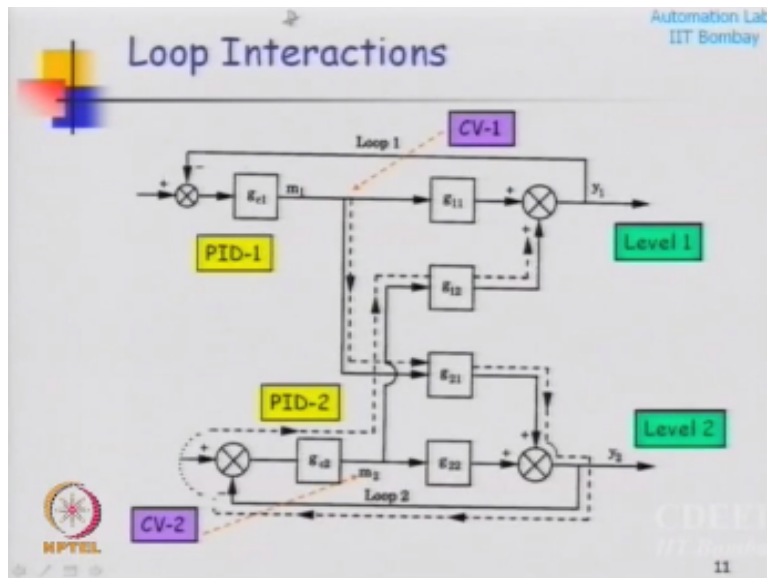
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Now there are 2 control walls u_1 and u_2 okay here I have shown you one possible scheme okay there is one there are 2 possible schemes here 1 possible scheme here is y_1 and u_1 now y_1 and u_1 right now I am just explaining some concept so numbering is arbitrary what is y_1 and what is u_1 is so right now so do not think that I have paired 1 and 1 that is what I want to convey okay so it did not be that 1 should be paired with 1 and 2 should be paired with 2 it is not like that.

So right now I am showing you 1 scheme other possibilities of course measure y_1 and manipulate u_2 and measure y_2 and manipulate u_1 so that is another scheme which is to choose and why okay so that is the question.

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Now as I was explaining to you in the last lecture when there is another loop okay see the difficulty with this kind of a configuration when there are independent controllers which do not coordinate between each other okay is that they can end up you know working against other they can fight each other see that is because if there is some action planned by the first controller its effect is transmitted through the second loop okay.

So the second loop gets disturbed as a consequence manipulate variable action of the first loop and you know it comes back, so you brought up this loop you unknowingly this PID controller see let us assume that everything is perfect right now both the levels are steady state somehow level 1 diverges from the set point what the reaction of the proportional controller PID controller here proportional action will immediately get into action.

It will start changing the manipulate variable so now there may not be an immediate effect felt because it has to go through this dynamics so initial action is what it is direction action is there okay well after some time the effect of this m_1 which is transferred to y_2 because you change m_1 level 2 gets this term controller 2 will come into action now action of controller 2 will come back through G_{12} to y_1 and this is a loop okay.

So and you can imagine if you have multiple such PID controllers which are not talking to each other you know this retaliatory actions of other controllers can be quite drastic okay so we have to get some kind of an idea okay if I have what is the change in the behavior what is the change

in the behavior of y_1 when this loop is open that means y_2 is not controlled and when y_2 is controlled okay that is a logical way of going about analyzing this system okay.

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Interaction Analysis

Assume a loop pairing say y_1 - u_1 and perform the following experiments

- With all loops open, make a step change in u_1 to $u_1 + \Delta u$ and measure the change in output Δy_1 . We will term this as a direct effect.
- With all loops except the u_1 - y_1 loop closed, repeat the change in u_1 . There will be change in y_1 because of the direct effect but also there will be a retaliatory effect because u_2 changes to keep y_2 constant.

We will term this change as $\Delta y_1 + \Delta y_{1r}$.

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That is what we are going to do in the interaction analysis okay so first thing which I am going to assume that the system is an open loop okay system is an open loop and I am going to do this an experiment I am going to give a step change in u_1 okay I am going to give a step change in u_1 by giving some magnitude change of Δu okay and then I will record output Δy_1 okay.

The experiment right now think of this other 3 experiment we are not actually going to perform it in the plant okay we will find some way doing calculations using just the gain matrix and looking at right now steady state interactions steady state defects I am kind of ignoring the dynamic the dynamic component and looking at large time component steady state behavior okay.

So if I give a step change in the input the output will go and saturate somewhere in open loop this all of you know okay the what am I going to do is that I am going to close the other loop see I have been looking at interaction between y_1 , u_1 okay looking at interaction between y_1 , u_1 what happens to y_1 , u_1 loop when y_2 , u_2 loop is closed okay and I want to come up with a major of not just y_2 loop, u_2 loop is closed all other loops are closed.

See suppose you have a system in which there are 5 PID controllers okay I am going to look at y_1, u_1 I am going to look at 1 pair okay and then look at the open loop gain and then look at so called closed loop gain what is this closed loop gain here is that all the other loops closed expect y_1, u_1 okay all the other loops closed expect y_1, u_1 so the effect that comes okay when the other loops are closed I am going to call it as a retaliatory effect of the other loops okay.

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Interaction Analysis

Ratio of these two terms can be defined as λ_{11} (for the y_1 - u_1 pairing) as

$$\lambda_{11} = \frac{\Delta y_1}{(\Delta y_1 + \Delta y_{1r})}$$

This is called **relative gain**

- Compute relative gain for each assumed input-output pairing
- Depending on the values of this index for various assumed loop pairings (step 1), decision can be taken on the final loop pairing.

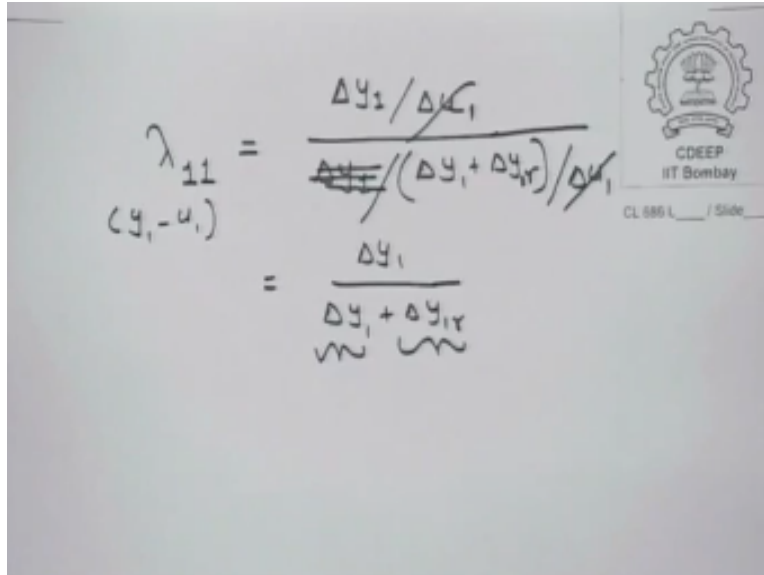
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So what is my 1st thing and then I am going to take ratio of these 2 I want to find out a ratio of open loop change see this is open loop change when all the loops where open this is change in y_1 total change in y_1 okay when all the loops except y_1, u_1 where closed okay every other loop is closed except the one the loop which is under consideration except that loop everything else is closed.

Now I look at a ratio of these 2 effects okay this ratio is called as relative gain okay so what we are going to do is we are going to find out relative gain for each input and output pairing okay and make use of this relative gain make use of this relative gain see this is a $\Delta y_1 / \Delta u$ I have given same change in both the cases in Δu $\Delta y_1 / \Delta u$ and this is $\Delta y_1 + \Delta y_{1r} / \Delta u$, Δy , Δy gets canceled okay.

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Handwritten derivation of the relative gain λ_{11} for the (y_1, u_1) pairing. The equation is:

$$\lambda_{11} = \frac{\Delta y_2 / \Delta u_1}{\Delta y_1 / (\Delta y_1 + \Delta y_{1r}) / \Delta u_1}$$

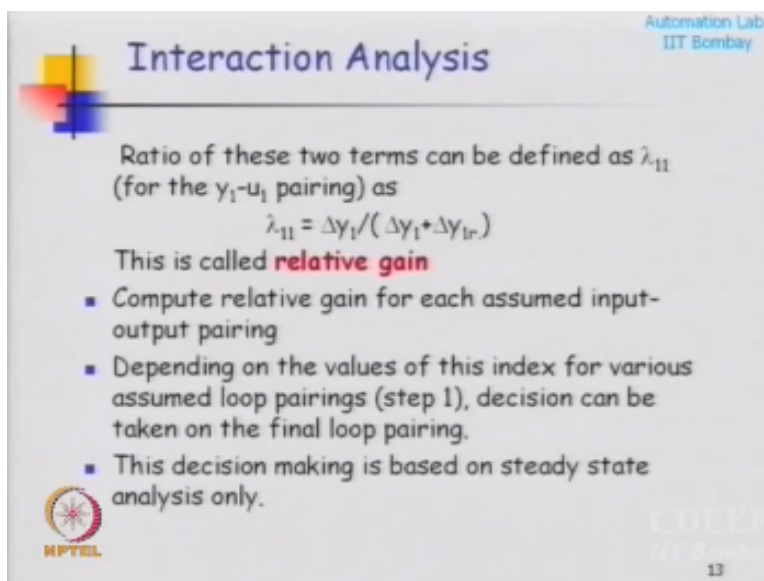
$$= \frac{\Delta y_1}{\Delta y_1 + \Delta y_{1r}}$$

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We have those pens well I am calling this 11 because we have done pairing of y_1 and u_1 if we have done pairing of y_1 u_2 I would have called this 1 to I will come to this little more details now see basically this equation you should look at as $\Delta y_1 / \Delta u_1 / \Delta y_1$ or let us say Δu_1 and / oh sorry $\Delta y_1 + \Delta y_{1r} / \Delta u_1$ now $\Delta u_1 / \Delta u_1$ gets cancels okay so this and this will cancel and you will get this to be $\Delta y_1 / \Delta y_1 + \Delta y_{1r}$.

So this is retaliatory effect this is the open loop effect linear system they we can just show that the 2 can be just added okay so.

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Interaction Analysis

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Ratio of these two terms can be defined as λ_{11} (for the y_1 - u_1 pairing) as

$$\lambda_{11} = \Delta y_1 / (\Delta y_1 + \Delta y_{1r})$$

This is called **relative gain**

- Compute relative gain for each assumed input-output pairing
- Depending on the values of this index for various assumed loop pairings (step 1), decision can be taken on the final loop pairing.
- This decision making is based on steady state analysis only.

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And then this index we are going to use for deciding the pairing.

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Relative Gain Array (RGA)

RGA: A measure of loop interaction used for deciding loop pairing of SISO controllers

$$\lambda_{ij} = \frac{[\Delta y_i / \Delta u_j]_{\text{open_loop}}}{[\Delta y_i / \Delta u_j]_{\text{all but } y_i - u_j \text{ loop closed}}}$$

y : Measured outputs
 u : Manipulated Inputs
 $[\Delta y_i / \Delta u_j]$: Steady state gain / sensitivity between i^{th} op and j^{th} i/p

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Okay so it is a measure of loop interaction and it can be very useful in pairing it is defined as λ_{ij} okay relative gain λ_{ij} is defined as if you do a pairing y_i u_j okay i^{th} output j^{th} input in general you will have a system which have multiple inputs multiple outputs okay and you want to do a pairing which is let us say you choose a pairing there are multiple possible pairings right if you have 5 inputs, 5 outputs how many pairings you can think of 5 to the power of 5! Barings some of them might be some of them you can eliminate just by a logical thinking that from engineering view point some of the things are meaningless okay.

But even if you do all that 5! Is a huge number if you want screen 5! Even for 5 x 5 system it is not an easy thing okay to screen so you need some systematic way of screening these multiple options okay so that is why we are coming up with this measure so what we are saying is that this is a relative gain this is between output y and input y_i and input u_j the gain in the open loop

divided by the gain between these two this input output pair with all other loops closed we are not worried how those are closed what is the pairing all the other loops are closed perfectly working nicely other loops are closed and only this loop is open okay so when other loops are closed okay.

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Calculations of RGA

Let us assume pairing y_1-u_1

Steady state model

$$\Delta y_1 = k_{11}\Delta u_1 + k_{12}\Delta u_2$$

$$\Delta y_2 = k_{21}\Delta u_1 + k_{22}\Delta u_2$$

k_{ij} : Steady state gain / sensitivity between output i and input j

$$\left[\frac{\Delta y_1}{\Delta u_1} \right]_{Open-Loop} = k_{11}$$

Step in Δu_1 →

$\Delta u_2 = 0$ →

Process

→ Δy_1

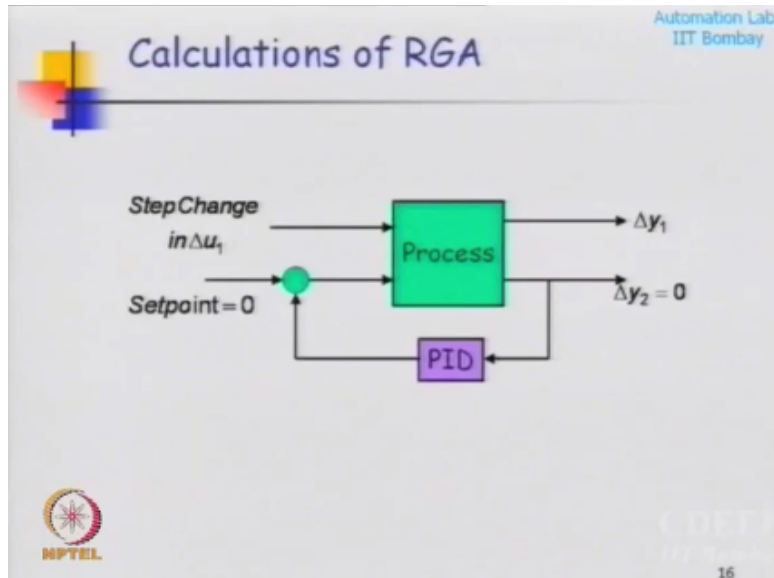
→ $\Delta y_2 \neq 0$

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So okay let us see calculation of RGF or a 2 x 2 system very easy task what is the if I give a step change here in Δu_1 okay open loop system this is the steady state equation okay if Δu_2 with 0 and if Δu_1 ids change what will happen see Δy_1 will change because of this equation Δy_1 will be k_{11} okay and Δy_2 will change which is not equal to 0 because you know you have given change in Δu_1 but Δu_2 with 0 okay but right now we are worried about gain between y_1 and u_1 okay. So open loop Δy_1 to Δu_1 is k_{11} okay.

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Now my 2nd situation is this how will you do gain calculations can you do it I have give you this model.

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Calculations of RGA

Let us assume pairing y_1-u_1

Steady state model

$$\Delta y_1 = k_{11}\Delta u_1 + k_{12}\Delta u_2$$
$$\Delta y_2 = k_{21}\Delta u_1 + k_{22}\Delta u_2$$

k_{ij} : Steady state gain / sensitivity between output i and input j

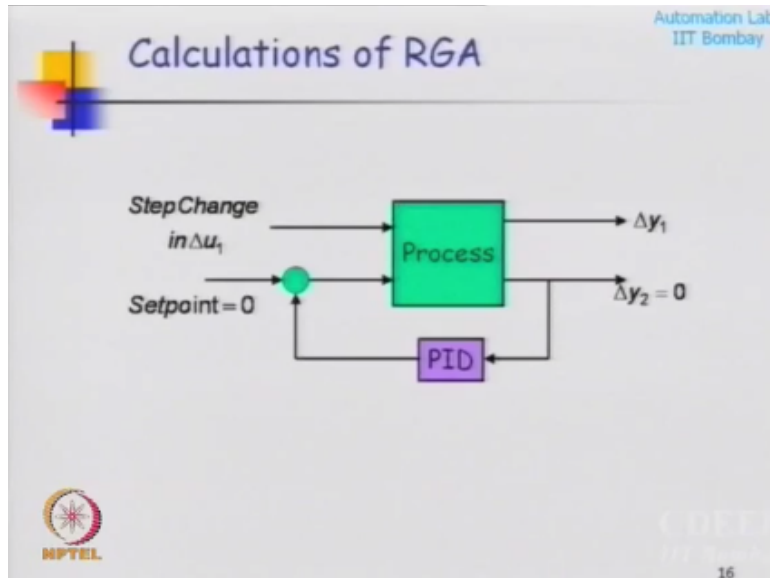
$$\left[\frac{\Delta y_1}{\Delta u_1} \right]_{Open-Loop} = k_{11}$$

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I have give in this model okay now.

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What if there is a PID controller what will happen if there is a PID controller here if I give step change in ΔU_1 what will happen see controller will make sure if it is a PI controller or a PID controller there is of set okay so the PID controller will act and it will make sure that Δy_2 becomes 0 okay can you calculate the gain for can you calculate the gain between Δy_1 and Δu_1 try it know. You have this equation.

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Calculations of RGA

Let us assume pairing y_1-u_1

Steady state model

$$\Delta y_1 = k_{11}\Delta u_1 + k_{12}\Delta u_2$$

$$\Delta y_2 = k_{21}\Delta u_1 + k_{22}\Delta u_2$$

k_{ij} : Steady state gain / sensitivity between output i and input j

$$\left[\frac{\Delta y_1}{\Delta u_1} \right]_{Open-Loop} = k_{11}$$

Δu_1 (Step in) \rightarrow Process \rightarrow Δy_1
 $\Delta u_2 = 0$ \rightarrow Process \rightarrow $\Delta y_2 \neq 0$

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Now because of PID controller is there Δu_2 will not be 0 Δu_2 will be non 0 in this case.

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Calculations of RGA

Now consider the situation where y_2 - u_2 loop is closed and perfectly controlled

$$\Delta y_2 = k_{21}\Delta u_1 + k_{22}\Delta u_2 = 0$$

$$\Delta u_2 = -(k_{12}/k_{22})\Delta u_1$$

$$\Delta y_1 = k_{11}\Delta u_1 + k_{12}\Delta u_2 = \left[k_{11} - \frac{k_{12}k_{21}}{k_{22}} \right] \Delta u_1$$

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How will Δu_2 change where as to keep $\Delta y_2 = 0$ so find out and then you find out what should be the gain between Δy_1 and Δu_1 under this situation so first you have to solve for $\Delta y_2 = 0$ $\Delta y_2 = 0$ will give you Δu_2 substitute that in the first equation and then you will get gain between y_1 and u_1 you will get Δu_2 in terms of Δu_1 substitute that in the first equation what do you get what is the answer.

Yeah so if you do this calculations of $\Delta y_2 = 0$ and put it back okay you will get this particular equation right so you can see that the gain the steady state gain when the other loop is closed is different from the steady state gain when the loop other loop is not closed okay, so this part is coming because of the retaliatory action other loop is reacting and that is what is giving you know that is what is making Δy_1 different now this could be in any we do not how this is going to be okay.

Some times Δy_1 might be smaller sometimes Δy_1 might be larger it all depends upon all these terms k_{21} , k_{11} , k_{22} and so that so on.

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Calculations of RGA

$$\left[\begin{array}{c} \Delta y_1 \\ \Delta u_1 \end{array} \right]_{y_2=u_2 \text{ Loop Closed}} = \left[\begin{array}{c} k_{11} - \frac{k_{12}k_{21}}{k_{22}} \\ 1 \end{array} \right]$$

$$\Rightarrow \lambda_{11} = \frac{1}{1 - \frac{k_{12}k_{21}}{k_{11}k_{22}}}$$

Other relative gains can be easily computed as follows

$$\lambda_{12} = \lambda_{21} = 1 - \lambda_{11}$$

$$\lambda_{22} = \lambda_{11}$$

RGA Matrix (Λ) for 2 x 2 system

$$\Lambda = \begin{bmatrix} \lambda & 1 - \lambda \\ 1 - \lambda & \lambda \end{bmatrix}$$

RGA is independent of variable scaling

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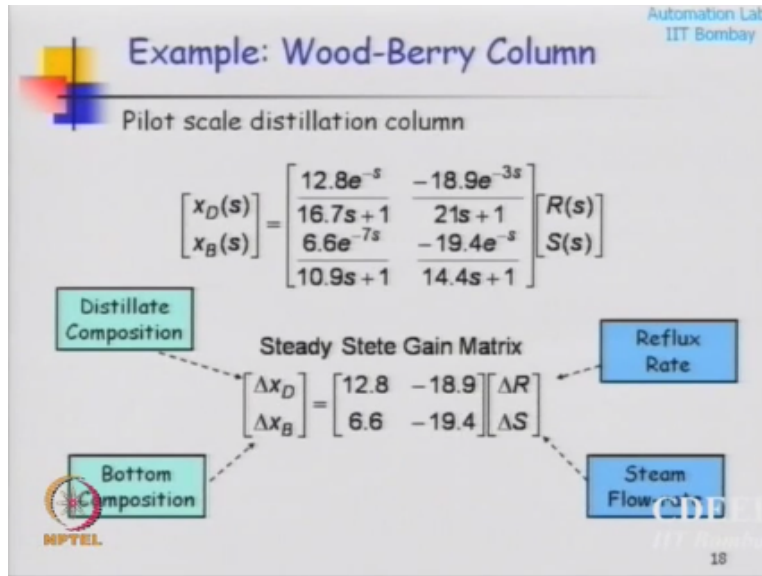
Okay so if I calculate the relative gain so what is $\Delta y_1 / \Delta u_1$ when the other loops is closed it is just given by thus value and if you divide k_{11} open loop by all other loops closed then this is what is the relative gain that you get okay so for a 2 x 2 system you can very easily show that it is enough to calculate 1 element all the other elements if this is Δ thus is will be $1 - \Delta$ this is will be $1 - \Delta$ and this is Δ .

This can be shown very easily for a 2 x 2 system okay this is relative gain for which pairing $y_1 u_1$ this is relative gain for $y_1 u_2$ this is $u_2 y_1$ okay and this is $y_2 u_2$ okay so we have calculated relative gain for each possible pairing okay and now we are going to take a call using values of this relative gain there is one very nice thing about relative gain see that gains themselves depend up on scaling or the units chosen to represent a variable right.

A, gain would depend up on but relative gain does not depend up on it is gain independent it is the it is unit independent it is scaling independent because it is ratio of 2 gains since it is ratio of gains okay or ratio of gains let us say because 2 is here well in this case yeah ratio of 2 gains in the same units actually weather it is multi variable or signal variable it is ratio of 2 gains and it is scaling independent that is very important.

This major which is not depending on any which you represent you are variables okay so this is scaling independent measure and that is why it is very popular well I do not know how much of it is still used in the industry but it is a historically this is a very important development.

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So let us looking at simple example gain now this is a one of the bench mark problems in process control literature this is called this is a transfer function matrix for a distillation column in which you know mixtures are separated into using relative volatility you separate mixtures of you know lower volatility and volatility mixtures are separated, so well from a control engineering view point you can look at it as 2 inputs, 2 output system the purity of the product at the top and purity of the product at a bottom okay.

Well simplest distillation would be you know you want we have mixture of water and alcohol and you want to get alcohol separated from water okay obviously you worried about the purity of alcohol which is product which as to be solid in the market so the top distillate composition let say it is alcohol and bottom product is water then you have what it as alcohol methane alcohol of course.

I do not about methane alcohol then alcohol is it will come as a top product right methane alcohol will be a top product and water will be a bottom product so you are worried about a compositions because you want to sell it to the market okay there are 2 inputs to the system one is reflex that is some part of the product is put back into the distillation column to get a good separation purity and you provide heat input to the system there is a re boiler at the bottom where you provide heat. So there are 2 inputs and 2 outputs what should be the pairing.


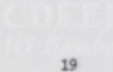
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Wood-Berry Column: RGA

$$\lambda = 1/0.498$$

$$RGA(\Lambda) = \begin{bmatrix} 2.0019 & -1.0019 \\ -1.0019 & 2.0019 \end{bmatrix}$$

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Okay so if I do if I take the steady state gains and find out RGA for this relative gain array it turns out to be it turns out to be 2 and -1 okay what is the meaning of -1 and what is the meaning of 2 how do you interpret let us go back to this definition.

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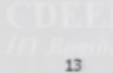
Interaction Analysis

Ratio of these two terms can be defined as λ_{11} (for the y_1 - u_1 pairing) as

$$\lambda_{11} = \Delta y_1 / (\Delta y_1 + \Delta y_{1r})$$

This is called **relative gain**

- Compute relative gain for each assumed input-output pairing
- Depending on the values of this index for various assumed loop pairings (step 1), decision can be taken on the final loop pairing.
- This decision making is based on steady state analysis only.

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Let us look at this if this ratio is negative what does it mean this try to interpret see let us take you change the input let us take the level case okay I change the wall position okay if I increase the flow what should happen to level it should increase okay but suppose RGA for the pairings are pairing that we are chosen it comes out to be negative means what if I increase the level wall then the level is actually decreasing it is going in the reverse direction okay.

So when the other loop is present it is completely changing the behavior you know from positive gain it is going to negative gain something which is quite dangerous you do not know what this kind of a pairing okay, what is the meaning of this going positive but higher value it means that retaliatory action is in the opposite direction but not too much opposite direction you know it is still smaller than the so Δy_1 is larger than $\Delta y_1 + \Delta y_1 r$ retaliatory action is reduced in Δy_1 .

Okay is reduced in Δy_1 but then you know it is not so harmful as negative pairing or negative RGA negative RGA means it is changing the direction completely here all that we are saying is that when it is higher than 1.

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Wood-Berry Column: RGA

$$\lambda = 1/0.498$$

$$RGA(\lambda) = \begin{bmatrix} 2.0019 & -1.0019 \\ -1.0019 & 2.0019 \end{bmatrix}$$

Any loop pairing will result in loop interactions

Pairing	k_c	τ_l (min)
$x_D - R$	0.604	16.37
$x_B - B$	-0.127	14.46

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Okay so definitely I do not want a pairing in which the gain is changing sign okay so I immediately reject this okay the only possible pairing for this is these 2 okay so only way I can pair this.

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Example: Wood-Berry Column

Pilot scale distillation column

$$\begin{bmatrix} x_D(s) \\ x_B(s) \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{2s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix}$$

Steady State Gain Matrix

$$\begin{bmatrix} \Delta x_D \\ \Delta x_B \end{bmatrix} = \begin{bmatrix} 12.8 & -18.9 \\ 6.6 & -19.4 \end{bmatrix} \begin{bmatrix} \Delta R \\ \Delta S \end{bmatrix}$$

Distillate Composition

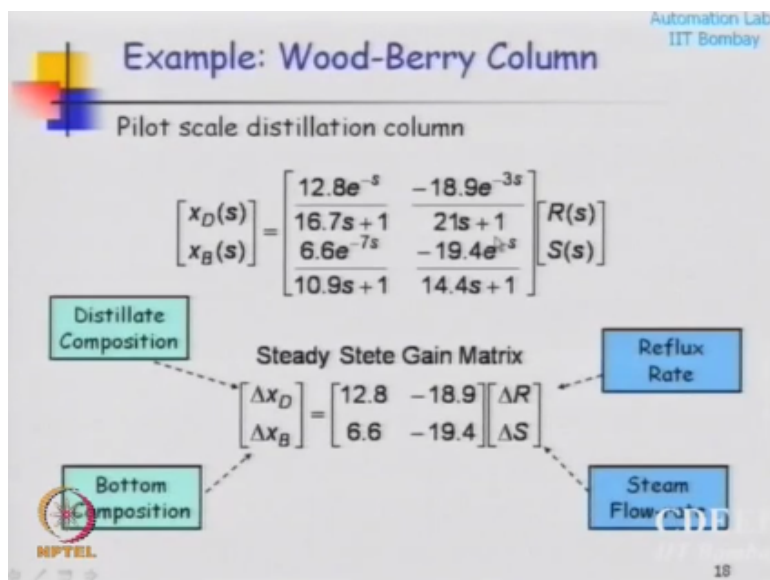
Bottom Composition

Reflux Rate

Steam Flow

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Is okay I have given the pairing here yeah.

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Wood-Berry Column: RGA

$$\lambda = 1/0.498$$
$$RGA(\Lambda) = \begin{bmatrix} 2.0019 & -1.0019 \\ -1.0019 & 2.0019 \end{bmatrix}$$

Any loop pairing will result in loop interactions

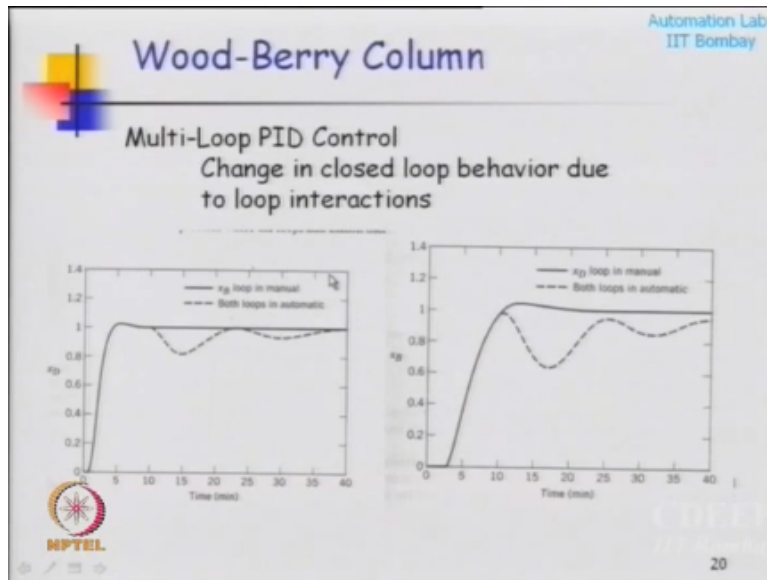
Pairing	k_c	τ_I (min)
$x_D - R$	0.604	16.37
$x_B - B$	-0.127	14.46

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Pairing here is the top composition is to be controlled using reflux ratio the bottom composition is to be controlled using or is to be controlled using the bottom flow rate as a manipulated variable okay bottom draw is a variable, so this is what comes out from this analysis okay here I have given some PID tuning parameters do not worry about them right now.

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Okay this is just a demonstration of what happens if what is the effect of occurrence of the presence of the other loop see this is the control loop see let us look at these two control loops what are the loop see let us look at these 2 control loops.

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Wood-Berry Column: RGA

$\lambda = 1/0.498$

$$RGA(\Lambda) = \begin{bmatrix} 2.0019 & -1.0019 \\ -1.0019 & 2.0019 \end{bmatrix}$$

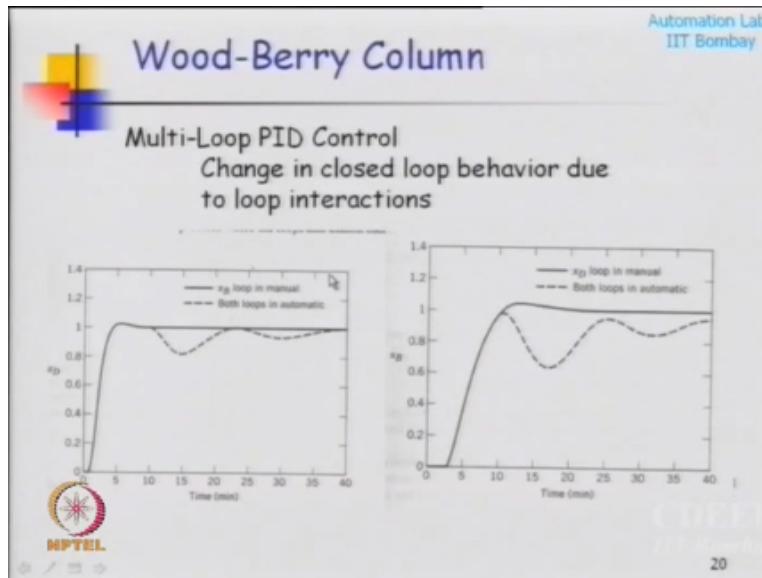
Any loop pairing will result in loop interactions

Pairing	k_c	τ_l (min)
$x_D - R$	0.604	16.37
$x_B - B$	-0.127	14.46

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What are the 2 control loops that we have it is X_D and R X_B and R okay.

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Now if you do an experiment in which X_D and R the loop for the top composition is closed other loop is open if I give a step change in the set point okay if I give a step change in the set point what happens here okay the you know it settles to the new set point within you know 7 minutes okay if I repeat my experiment with the other loop closed see what happens if my other loop is closed the settling goes to 35, 40 minutes okay.

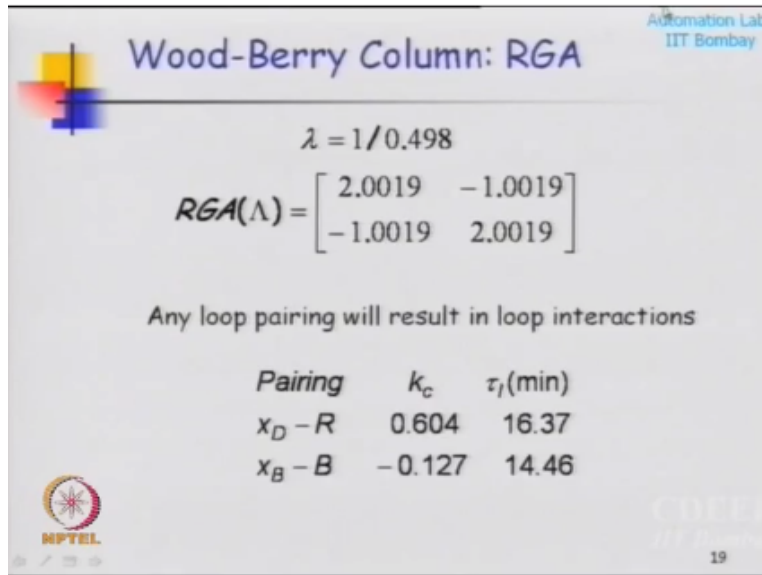
So if I do this experiment one loop you know 1st loop is closed the 2nd loop is open I give a change in the 1st loop okay the 1st loop seems to behave very nicely but when this tuning is not going to work okay this tuning is not going to work it is not a good tuning why because when the other loop is present you know the settling time of 7 minutes becomes 35 minutes 5 times increase.

Not a good idea okay so what I want to stress here is that I may have obtained this tuning just looking at one loop see what we do in your 1st course in control you look at tuning methods for single input single output okay we do not worry about what is there in the other loop if I just look at one loop and tuning it without worrying and the other loops okay then when all the loops start working together this will happen okay.

The same thing happens about the 2nd variable the second variable also very well tuned you know it is a very nicely tuned when it when the other loops are opened okay but movement I close the 2nd loop okay then you start getting this seivour oscillatory behavior it is no longer settling in 5 minutes or 19 minutes or whatever it is okay so 2 nicely tuned controllers tuned individually

when they are put together in this particular case because of heavy interactions. They start fighting what tells you there is a heavy interaction.

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Wood-Berry Column: RGA

$$\lambda = 1/0.498$$
$$RGA(\lambda) = \begin{bmatrix} 2.0019 & -1.0019 \\ -1.0019 & 2.0019 \end{bmatrix}$$

Any loop pairing will result in loop interactions

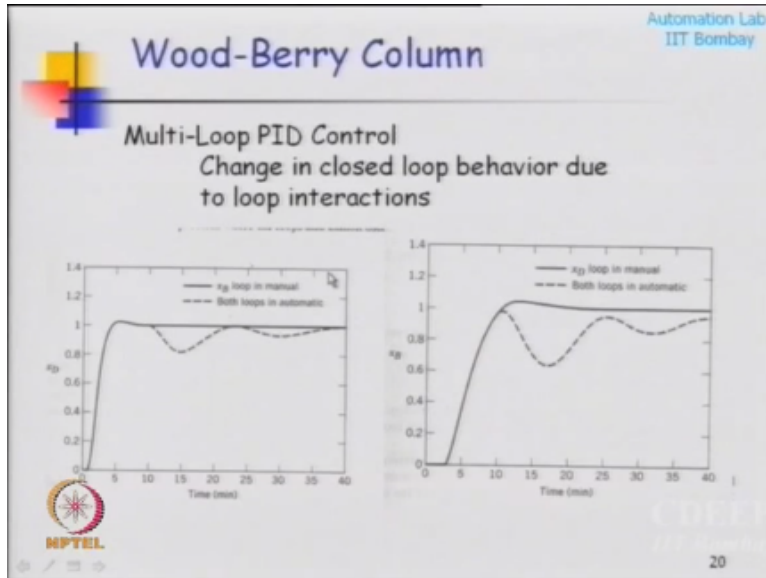
Pairing	k_c	τ_l (min)
$x_D - R$	0.604	16.37
$x_B - B$	-0.127	14.46

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This RGA this value is 2 this value is -1 there is lot of interaction between the loops so now how do you take a call based on RGA values okay.

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So there are some rules to be decided about that so just RGA of 2 is causing so much of change in the behavior from you know individually tuned controllers to all controllers working together okay.

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RGA for MIMO Process


$\frac{\partial y_i}{\partial u_j} = K_{ij}$ (steady state gain between input i and output j)

Steady state model: $y = Ku \rightarrow u = K^{-1}y = \bar{K}y$

When all but $y_i - u_j$ loop are closed and perfectly controlled

$y = [0 \dots y_i \dots 0]^T$

$$\left(\frac{\partial u}{\partial u_i} \right)_c = \begin{bmatrix} \frac{\partial u_1}{\partial u_i} \\ \frac{\partial u_2}{\partial u_i} \\ \dots \\ 1 \\ \dots \\ \frac{\partial u_n}{\partial u_i} \end{bmatrix}$$


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So you can imagine what will happen in a MIMO process well this particular part this particular slide as lot of little bit of complex looking algebra and you can go back and look at it more carefully I am going to go over it very quickly it is a simple derivation how do I calculate RGA for a multiple input multiple output system okay this derivation is for a square system okay it is for a square system there are number of inputs equal to number of outputs.

So 5 x 5 sectors is 7 x 7 whatever okay open loop gain of course is given by if you a steady state gain matrix the open loop gain is given by i zth element of that matrix okay now if a k is the gain matrix okay then k inverse is the gain inverse matrix I am going to call it as $k \sim$ here okay instead of calling it k inverse I am going to call $k \sim$ is k inverse okay now we want to find out a scenario where y_i u_j this loop is open and all the other loops are perfectly controlled.

Am I correct how do you find out relative gain you find out the gain between y_i u_j in open loop, all the other loops are perfectly closed so y should be 0 except y_i right input no here I have taken i , i output and j th no k_{ij} is $\partial y_i / \partial u_j$ no y is the output yeah output I so I want output I to be all output I to be perfectly controlled so i th output will be non zero all other will be 0 because we are assuming that there are PI controllers controlling all the other outputs. Oh this first one oh yeah this one is yes yeah, yeah this is j and i .

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RGA for MIMO Process

$\frac{\partial y_i}{\partial u_j} = K_{ij}$ (steady state gain between input j and output i)

Steady state model: $y = Ku \Rightarrow u = K^{-1}y = \bar{K}y$

When all but $y_i - u_j$ loop are closed and perfectly controlled

$y = [0 \dots y_i \dots 0]^T$

$$\begin{pmatrix} \frac{\partial u_1}{\partial u_j} \\ \frac{\partial u_2}{\partial u_j} \\ \dots \\ 1 \\ \dots \\ \frac{\partial u_n}{\partial u_j} \end{pmatrix}_c$$

$$= \bar{K} \begin{pmatrix} 0 \\ \dots \\ \frac{\partial y_i}{\partial u_j} \\ \dots \\ 0 \end{pmatrix}_c$$

$$= \begin{pmatrix} \bar{K}_{1i} \\ \dots \\ \bar{K}_{ji} \\ \dots \\ \bar{K}_{ni} \end{pmatrix} \begin{pmatrix} \frac{\partial y_i}{\partial u_j} \\ \dots \\ \frac{\partial y_i}{\partial u_j} \\ \dots \\ \frac{\partial y_i}{\partial u_j} \end{pmatrix}_c$$

i'th column of gain Matrix inverse

$\Rightarrow \bar{K}_{ji} \left(\frac{\partial y_i}{\partial u_j} \right)_c = 1 \Rightarrow \left(\frac{\partial y_i}{\partial u_j} \right)_c = \frac{1}{\bar{K}_{ji}}$

Relative gain $(\lambda_j) = K_{ij} \bar{K}_{ji} = K_{ij} [K^{-1}]_{ji}$

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Yeah this is steady state between the output input j and output i yeah okay so now I want to find out this okay now to find out this see to find out what is the effect of u when all the other loops are closed I am going to use this inverse equation I am going to use this inverse equation together with this particular vector okay.

So the way I am going to do this is I am going to find out $\frac{\partial u}{\partial u_i}$, $\frac{\partial u_j}{\partial u_j}$, $\frac{\partial u}{\partial u_j}$ why $\frac{\partial u}{\partial u_j}$ why I want to find out this matrix vector because when the other loops are closed see when other loops are closed u_1, u_2, u_3 , or whatever $u_{j+1}, u_{j+2}, u_{j+3}$ all of them are going to change in response to change made in u_j I want to find that out okay so essentially I want to find out $\frac{\partial u}{\partial u_j}$ okay yeah at steady state I am looking at only steady state why no we have PID controllers no open loop they will be 0.

Not in the close loop we are looking at situation where all the other loops are closed yeah except $y_i - u_j$ other loops somehow are been paired and have been closed we do not ask how they have been closed they are perfectly working all of them are PI or PID controllers so of set okay see this is a 3rd experiment this is not we are never going to perform this okay we can just we just want to find out and index using gain matrix okay.

So this my so do you agree with this just look at the deviation all that I have done is $\frac{\partial u}{\partial u_j}$ I have used this equation here say I will use this equation and then finally you know you have to little bit of manipulation and then you get this column which is nothing but a appropriate column

see in this y all or 0 except y_i okay so you will get the column of this matrix corresponding to the i^{th} output okay.

We will get i^{th} output column corresponding in the inverse matrix and what you can show just go over the slide again you can derive it yourself it is not so difficult just one slide derivation that is all I will give you the final formula which is given in the next page.

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RGA Calculations

For general $(n \times n)$ system
Given Steady State Gain Matrix K

$$\text{RGA}(\Lambda) = K \otimes (K^{-1})^T$$

where \otimes denotes Schur Product
(element by element multiplication of two matrices)

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Okay so how do you find out relative gain array of a particular system if you have given a matrix I will tell you the algebraic formula okay algebraic formula is take the gain matrix okay inverse transpose okay and then do what is called hadamard product is Schur product sorry Schur product is you know element multiplication do element by element multiplication this is a algebraic formula okay so what I am saying here is something like this see you have this gain matrix.

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$$\lambda_{11} = \frac{\Delta y_2 / \Delta u_1}{\Delta y_1 / (\Delta y_1 + \Delta y_{1r})} = \frac{\Delta y_1}{\Delta y_1 + \Delta y_{1r}}$$

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} \tilde{k}_{11} & \tilde{k}_{12} \\ \tilde{k}_{21} & \tilde{k}_{22} \end{bmatrix}^T = \begin{bmatrix} \tilde{k}_{11} & \tilde{k}_{12} \\ \tilde{k}_{21} & \tilde{k}_{22} \end{bmatrix} = k^{-1}$$

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You know k_{11} , k_{12} , k_{21} , k_{22} okay then find out let us define $k \sim = k$ inverse see this is my k and this my $k_{11} \sim$, $k_{12} \sim$ so this is $k \sim$ which is same as k inverse okay and then take element by element product which means k_{11} , $k_{11} \sim$, k_{12} , $k_{12} \sim$ you need to transpose this here sorry and you need to transpose this and then take oh yeah so.

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$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} \tilde{k}_{11} & \tilde{k}_{21} \\ \tilde{k}_{12} & \tilde{k}_{22} \end{bmatrix} = \begin{bmatrix} k_{11} & \tilde{k}_{11} & & \\ k_{21} & \tilde{k}_{12} & & \\ & & & \\ & & & \end{bmatrix}$$

So we write k_{11} , k_{12} , k_{21} , k_{22} k_{11}^{\sim} , k_{12}^{\sim} and then RGA is simply k_{11} , k_{11}^{\sim} , this is element by element by product okay very funny product we do not use this in matrix multiplications normally this is element by element this is called Schur product or also I think it is called Hadamard product there is a subroutine in mat lab which you just give matrix and ask you to do element by element multiplication it will just do it for you.

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RGA Calculations

For general ($n \times n$) system
Given Steady State Gain Matrix \mathbf{K}

$$\text{RGA}(\Lambda) = \mathbf{K} \otimes (\mathbf{K}^{-1})^T$$

where \otimes denotes Schur Product
(element by element multiplication of two matrices)

Properties

1. Summation of all elements in any row = 1
2. Summation of all elements in any column = 1
3. RGA is independent of scaling used for input of output variables

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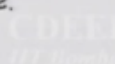
Okay there is one very nice property of this matrix is that all the elements sum to one okay all the columns sum to one so this is very nice property of RGA matrix and the nice things is RGA is independent of scaling whichever way you have used to compute the gain say does not matter okay you get a scaling independent measure of interactions okay.

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Analysis of RGA

- If $\lambda_{11} = 1 \Rightarrow$ retaliatory action is not present. So assumed loop pairing is correct because there is no interaction from the other loop.
- If $0 < \lambda_{11} < 1 \Rightarrow$ retaliatory action is comparable to the direct action but is in the same direction. The assumed loop pairing may be chosen only if the index is closer to 1 (say 0.8).
- If $\lambda_{11} = 0 \Rightarrow$ Retaliatory action is much greater than the direct action. The assumed loop pairing is incorrect. The loop pairing u_1-y_2 is preferable.

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I am just giving you some rules of how to use this RGA to do pairing okay if what if RGA is let us take a 2 x 2 system let us go back to 2 x 2 system if λ_{11} is 1 what does it mean that means the other loop is not making any difference okay it is not interacting okay so ideal situation is λ_{11} is 1 other loop weather it is present or absent is making no difference okay ideally it should be =1 but that may not happen it should be close to 1 okay so what we should do is we should look for that pairing okay.

In which RGA element is closed to 1 because if RGA element is close to 1 other loops have not bothering this loop okay that is what I means what if it is less than 1, but greater than 0 if it is less than 1 but greater than 0 it means that the other loop is acting in the same direction okay it is increasing so retaliatory action is suppose the you know the original action if you give a change in the flow rate the level increases the retaliatory action further increases the level and that is why you know the RGA is smaller is less than 1 there is interaction but we roughly say that interaction is strong if it is between 0 and 0.8 okay.

Between 0.8 and 1 we say that the interaction is low it is no making too much effect on the so this is little bit there is a heuristics coming in here if it is 0 what is the meaning of 0 interaction is very strong other loop is nullifying the effect of I do not want to choose this okay if other loops are presents it is all most nullifying the effect of the action of you know u_1 action is getting nullified by the presence of other loops I do not want this pairing.

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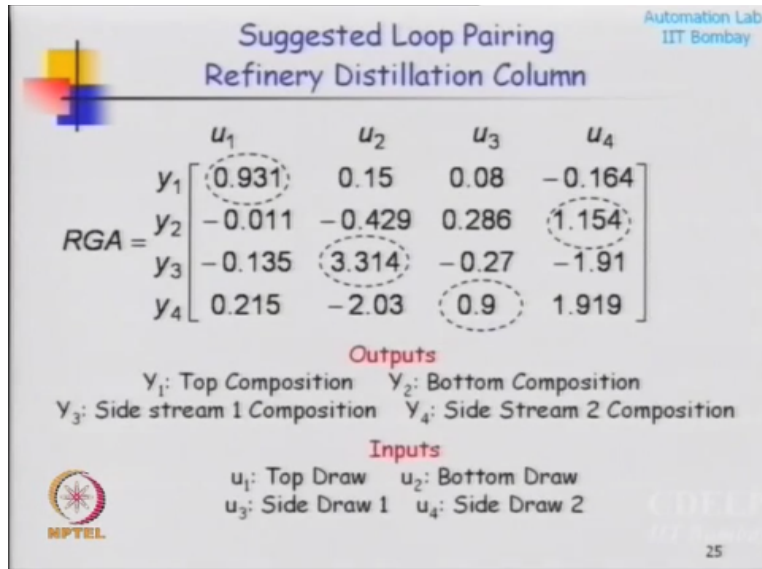
Analysis of RGA

- If $\lambda_{11} > 1 \Rightarrow$ Retaliatory action is in opposite direction to the direct action but is smaller in magnitude than the direct. The assumed loop pairing may be chosen only if the index is close to 1.
- If $\lambda_{11} < 0$, Retaliatory effect is larger and opposite in direction to the main effect.
Do not chose this loop pairing.

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What if it is > 1 there retaliatory action is in the opposite direction okay yeah it is reducing the effect of open loop okay but still it is not bad as having negative okay it is not so strong, so if it is < 0 we do not want that pairing okay we just reject that particular pairing we do not want that and so λ is < 0 we do not use this.

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So I am just going to show you this for a little more complex case this is there are a refinery distillation column there are 4 measurements top composition of the top product there are in a refining system when you let say you have a cured oil coming and you are refining it into different products the top product could be you know light hydrocarbons a little below that in the huge column little below that will be petroleum then you will get kerosene the you will get aviation turbo fuel then you will get heavy oils and so on.

From the same column you draw a different, products of different volatility so these are the draws they are called as side draw side product, so there are 4 composition which are important an you have 4 manipulated variables you have top, flow rate bottom flow rate and two side flow rates there are 4 flow rates which are manipulated 4 compositions which are controlled look at this as a control engineer it is a system with 4 inputs 4 outputs you want to put 4 PID controllers.

Okay you have let us say you are given steady gain matrix okay now if I do RGA calculations for this particular system it turns out to be this now can you tell me how choose pairing what about y_1 I should choose $y_1 u_1$ okay because this is very close to 0 okay this is very close to 0 I do not like this there will be lot of retaliatory effects if I choose $y_1 u_2$ pairing or $y_1 u_3$ pairing okay there will strong interactions $y_1 u_4$ of course is completely ruled out it is negative okay.

So I should choose $y_1 u_1$ pairing what about y_2 should be u_4 okay y_3 we have no choose but to go for only positive pairing there is lot of interaction but we cannot help it all others are not expectable okay and here this is closest to 1 okay now how many possible pairings you can think

of for this system 4! That is how much 24 possible pairings okay from that you have arrived at 1 possible pairing okay just using this simple analysis look at the power of this simple method okay.

It can tell you what is that choice of pairing which will give minimum fighting between the loops okay this does not mean the fighting does not exist because look here this is going to cause problem for you this is going to cause problem for you never the less this is best possible pairing this is the least fighting or least interacting loop pairing that you can think of so this method is quite powerful very simple method based on just gains and then you can choose pairing.

So question is should we go for multiple multivariable controller or multi loop controller in plant if you do not have choice if you have to go for 4 PID controllers your employer says well I do not believe about model based control and observer and all that you talking about want to go to the market buy 4 PID controllers and put them you know then you should choose that pairing which is giving you least interaction.

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Automation Lab
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RGA: Non-square Systems

Consider process with 2 measurements and 3 inputs

$$\begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.07 & 0.04 \\ 0.004 & -0.003 & -0.001 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

Best
Combination

$$RGA_1 = \begin{matrix} & \Delta u_1 & \Delta u_2 \\ \Delta y_1 & 0.84 & 0.16 \\ \Delta y_2 & 0.16 & 0.84 \end{matrix}$$

$$RGA_2 = \begin{matrix} & \Delta u_1 & \Delta u_3 \\ \Delta y_1 & 0.76 & 0.24 \\ \Delta y_2 & 0.24 & 0.76 \end{matrix}$$

$$RGA_3 = \begin{matrix} & \Delta u_2 & \Delta u_3 \\ \Delta y_1 & -1.4 & 2.4 \\ \Delta y_2 & 2.4 & -1.4 \end{matrix}$$

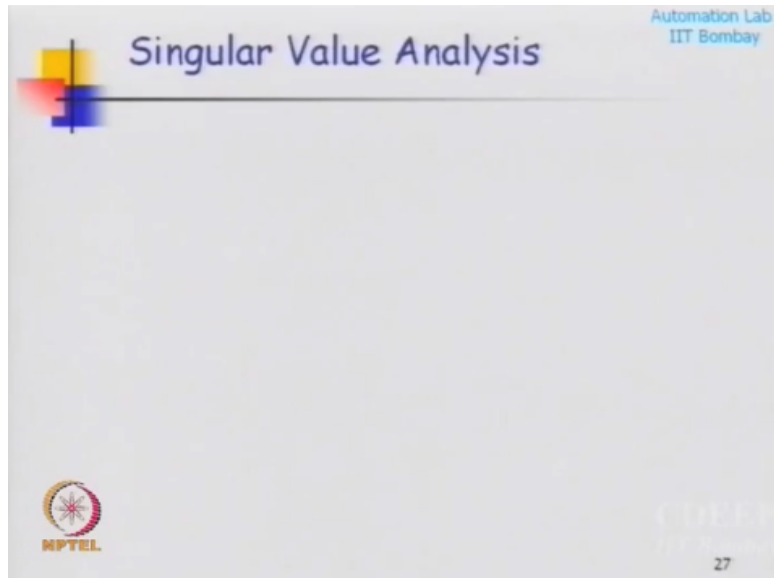
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It can be also used for screening options see suppose I have 3 inputs and 2 outputs okay I have 3 inputs and 3 outputs y_1, y_2 are the 2 outputs and u_1, u_2, u_3 are 3 inputs can I use RGA to screen out I can use 2 PID controllers okay I can use 2 PID controllers because PID controllers you know 1 input 1 output so which one to use so in general for a bigger plant this problem is much bigger.

Tendency is one problem which is showed you there are 12 inputs and there are 54 outputs okay which subset of which 12 among the 54 see how many combinations are there and that 1 thing which is showed you as a tendency is plant is a small section of a chemical plant there are many such units each one will have you know many inputs many outputs and the pairing problem is a huge problem it is not a simple problem.

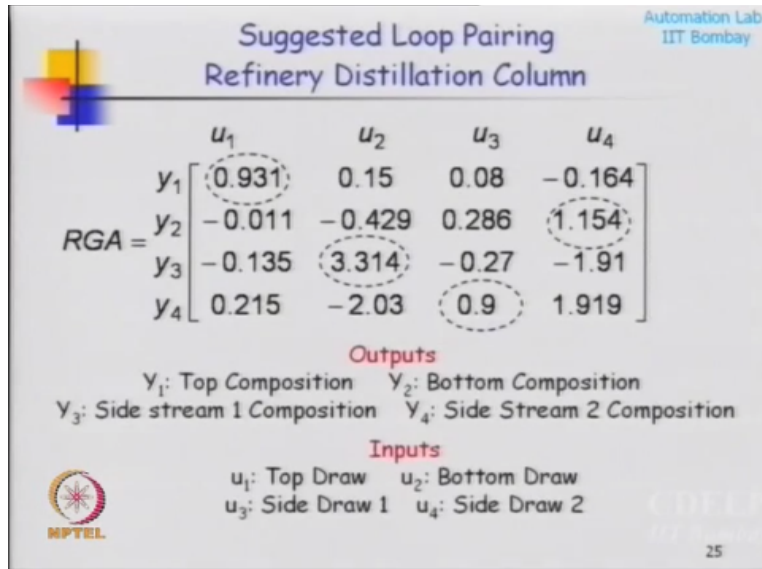
So what I will do is I will finds out RGA with respect to each possible pairing okay so I can take $y_1, y_2, u_1, u_2, y_1, y_2, u_1, u_3$ and y_1, y_2, u_2, u_3 okay which one is which one you will go for you will go for this pairing see because here this 0.76 is higher interaction that 0.84 okay in this case this is negative and this to high okay so you do not except this okay so it is very in this particular case is easy to say that this is the best pairing okay. You can notice one thing here if you add the columns they will add to 1.

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If you add the rows they will add to 1 okay.

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Same things here just check here if you add the columns will add to 1 if you add the rows they will add to 1 this is a nice property tells me to give an problem in the exam I can give you an RGA with missing elements you have to fill in just know that summation of ∂ or summation so suppose you the but this is not just the thing of giving a problem in a exam in a plant suppose you know some gains partially okay.

Some gain elements are no partially and if you are able to compute RGA elements for some of the if you have just partial RGA information you can fill in the matrix using this property.

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Singular Value Analysis

- Powerful analytical tool for
 - Selection of controlled, measured and manipulated variables
 - Determination of best multi-loop configuration
 - Evaluation of robustness (insensitivity to changes in plant behavior) of a control scheme

$\Delta Y = K \Delta U$ (K : Steady State Gain Matrix)

Singular Values $(\sigma_1, \sigma_2, \dots, \sigma_r) =$
 Square Root of Eigen - values of $K^T K$
 Roots of polynomial
 $\det(sI - K^T K) = 0$

Condition Number (CN) = $\|K\|_2 \|K^{-1}\|_2 = \sqrt{\frac{\max(\sigma_i)}{\min(\sigma_i)}}$

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It is very okay now this is one measure 1 big problem I want RGA is that you know you are using steady state information you are not using dynamic information there have been attempts to extend this to dynamics in frequency domain I do not want to get into that right now but it becomes very complex when you go to frequency domain analysis using RGA kind of frame work.

It is not so straight forward well when you actually do a this pairing it is not when you have a multivariable plant and when you are trying to put multiple PID controllers it is a difficult problem you cannot just really on measure that is RGA we use one more measure which is singular decomposition okay it is a powerful analytical tool that can be also used for robustness analysis I am just giving some idea about this what is singular value of a matrix do you remember.

A transpose A or AA transpose Eigen values of A transpose A or AA transpose these are called as so here what we do is we find out Eigen values of k transpose k okay you take the gain matrix and for the gain matrix we can find out singular values okay now singular value you know what is a condition number the ratio of the maximum magnitude singular value divided by minimum magnitude single value okay square root of that.

That is called as condition number a system which is as high condition number form linear algebra what do you know it is difficult to inverse right it is difficult solve does a control inverse control controller design does it involve inversion somewhere let us look at steady state forget

about the dynamics look at the steady state okay what is Δy is the output and what is Δu is the input okay.

When you design a controller you get a set point what do you give a set point on output you give a set point on the output and what do you want the controller to do you want the controller to find that input which will take the system to the desired output, so if you forget about a dynamics look at the problem only as a steady state problem actually you want to solve this problem given Δy find Δu right very simple way of looking at the you know control problem that given Δy I want to go to certain set points okay find the inputs that will take me to the set points okay.

Which means well if k is square what does it mean k^{-2} what is the u that will take you to that Δy k inverse into Δy that is the u that will take it to that now k inverse okay very practical application of you know condition number while you have seen in the numerical methods it as lot of you know meaning in terms of stability of numerical systems here difficulty in control can be quantified using condition number okay.

If the gain matrix as high condition number which means you know your system is condition okay a system is in condition difficult to control okay well there is only one trouble with this analysis it is condition number is depend up on the scaling of or choose of scaling that is used for gain calculations so this is not a gain this is not a scaling independent measure never the less this gives you a good idea about how good is your so for example I yeah no it is not it is ratio of Eigen values Eigen value is depend up on scaling it is ratio of the Eigen values but the Eigen value is themselves depend up on the scaling.

See you have different gain elements in the matrix each yeah so each one you can different scaling and if you change the scaling this ratio will change that is not independent that is not the case about RGA see RGA your doing point to point calculations here you're not doing point okay.

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Automation Lab
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RGA: Non-square Systems

Consider process with 2 measurements and 3 inputs

$$\begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.07 & 0.04 \\ 0.004 & -0.003 & -0.001 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

Best
Combination

$$RGA_1 = \begin{matrix} \Delta u_1 & \Delta u_2 \\ \Delta y_1 & \begin{bmatrix} 0.84 & 0.16 \\ 0.16 & 0.84 \end{bmatrix} \\ \Delta y_2 \end{matrix}$$

$$RGA_2 = \begin{matrix} \Delta u_1 & \Delta u_3 \\ \Delta y_1 & \begin{bmatrix} 0.76 & 0.24 \\ 0.24 & 0.76 \end{bmatrix} \\ \Delta y_2 \end{matrix}$$

$$RGA_3 = \begin{matrix} \Delta u_2 & \Delta u_3 \\ \Delta y_1 & \begin{bmatrix} -1.4 & 2.4 \\ 2.4 & -1.4 \end{bmatrix} \\ \Delta y_2 \end{matrix}$$

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So for example in this particular case I could have compared this just like I compare them using RGA I could have compared them using RGA I could have compared them using condition number okay which with sub system is well condition in terms of inverting okay I can check that so condition number is a useful measure.

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Singular Value Analysis

- Non-square systems (number of inputs not equal to number of outputs)
 - SVA can be used to find a square subset with least difficulties in control
- Larger singular value implies difficulties in controlling a system
 - Among multiple possibilities, choose subset with minimum singular value
- **Limitation: Singular values are dependent on scaling of input and output variables**

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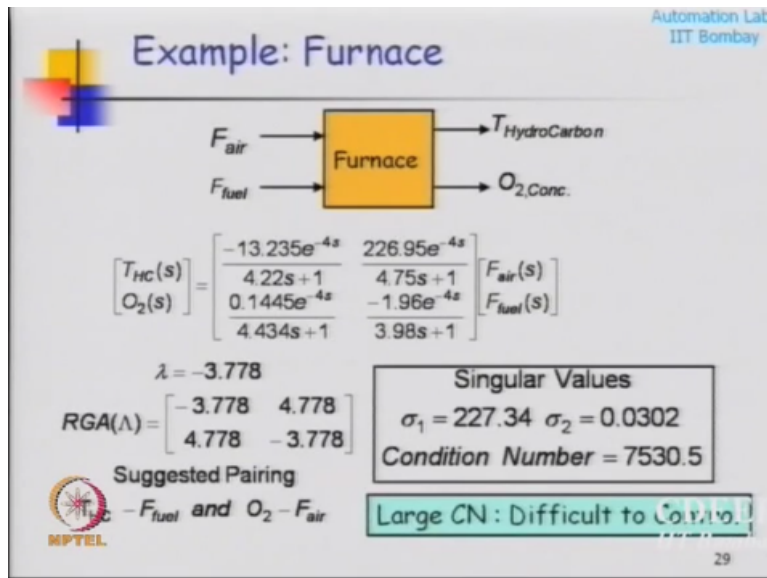
So you can actually find out that sub set of variables which is well conditioned in terms of control using the condition number that is a larger condition number employs you know you have trouble difficulty in the sense that good determinate well not determinant is singular it means that inversion is difficult okay inversion is unreliable okay.

In computationally it means that when you try to invert the answers that you get so see it is to be taken little bit of qualitative interpretation okay you think of it that design a controller is like inverting the gain matrix okay now when you try to realize gain matrix inversions numerically okay because of gain matrix is ill condition okay computation of it inverse can get into numerical trouble.

You can get spurious answers okay I can show you some example maybe I can put those notice on the model where you take simple 3 x 3 matrix or 4 x 4 matrix 3 x 3 matrix okay and if it is ill conditioned if you find out inverse using mat lab mat lab will give something with a warning saying that this inverse is not reliable if multiply the 2 you will not get identity matrix you will get some obituary things okay.

So trying to invert a ill conditioned gain is inherently difficult is what one has to realize okay so one can phase problem in realizing a controller that is what it indicates okay.

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See for example I am just giving you simple example here this is a upper Furnace look at it as a black box as a control engineer there are 2 inputs air flow rate and fuel flow rate okay and what is important is the temperature of the hydrocarbon which is measured okay inside a temperature of the furnace or hydrocarbon is being heated here you know have a furnace in which you are burning fuel and your heating some hydrocarbon to some temperature.

And you monitor the exit see you are burning fuel you monitor the exit concentration of oxygen okay in the flue gas you monitor the exit oxygen concentration okay so we have the 2 control outputs okay exit concentration of oxygen in a flue gas and you are heating some fluid some hydrocarbon in the furnace so the outlet output temperature of the fluid the transfer function for this particular system is given by this simple system there are time delays and four gains are give to you okay.

With reference to 2 flows you can appreciate here the gain values you know are quite magnitudes are quite different that is because see this is temperature this is concentration okay these are 2 flow rates one is air flow rate other is liquid flow rate let us say you have liquid flow rate for fuel other is liquid flow rate so the values can be completely different and a gain values will depend up on the choose of units that you choose okay.

So if you look at the RGA here see you should use only one measure you have to use multiple measures to do scaling of variables RGA shows that there is huge interaction okay these 2 loops interact a lot but RGA of course gives you pairing what is pairing it tells you that you should

never pair a the temperature with air, air flow rate you should pair temperature with a you know only with the fuel flow rate that is the other pairing is not allowed okay.

And in this case singular value analysis also indicates difficulty in control because you know you get this condition number which is very high 7000 a condition number above 100 is considered high okay 7000 is very high which means you will have difficulties in realizing the controller here yeah what can be up to you condition number can be it is up to you but you know it is like a at least some symmetric way of thinking about the problem.

Yeah so what you try to do normally is that you try to find out each element in the gain matrix something like a dimension less you know you have to do lot of a what I would say pretreatment of your gain matrix before you can you know so what I would do in such a case to find the gain matrix instead of using I would say percentage change by percentage change okay so you in two depending up on whatever is your measuring device okay let us take it is maximum and minimum.

And then define a percentage with respect reference to that so try to make is as unit independent as possible and the you know but what he say is right it can be this singular value analysis can be desuetude and it depends on the gain.

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4 Component Distillation Column

Table 18.2 Condition Numbers for the Gain Matrices Relating Column-Controlled Variables to Various Sets of Manipulated Variables (Rost et al., 1986)

Controlled Variables			
x_{11}	Mole fraction of propane in distillate D		
x_{14}	Mole fraction of isobutane in tray 04 sidestream		
x_{15}	Mole fraction of <i>n</i> -butane in tray 15 sidestream		
x_{21}	Mole fraction of isopentane in bottoms B		
Possible Manipulated Variables			
L	Reflux flow rate	B	Bottoms flow rate
D	Distillate flow rate	S_{04}	Sidestream flow rate at tray 04
V	Steam flow rate	S_{15}	Sidestream flow rate at tray 15

Strategy Number ^a	Manipulated Variables	Condition Number
1	$L/D, S_{04}, S_{15}, V$	9.030
2	$V/L, S_{04}, S_{15}, V$	60.100
3	$D/V, S_{04}, S_{15}, V$	116.000
4	D, S_{04}, S_{15}, V	51.5
5	L, S_{04}, S_{15}, B	57.4
6	L, S_{04}, S_{15}, V	53.8

^a In each control strategy, the first controlled variable is paired with the first manipulated variable, and so on. Thus, for Strategy 1, x_{11} is paired with L/D , and x_{21} is paired with V .

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So another example where you have 6 you have 4 outputs and 6 inputs okay and in this particular case if you see there are different subsets of the different subsets of manipulate variables taken okay and then for each subset you find out the condition number outputs are same there are 4 outputs I have to choose 4 inputs which 4 input to choose okay so this can be used as per screaming in this case these 2 are see this one definitely not a good combination because this is 60000.

This is you know 1160000 this is 9000 so these 3 are ruled out okay among these 3 are competing okay now I can use RGA on these 3 okay see I have use it is like you know it is like some bags of linear algebra tools that you have and then you are trying somehow come up with some simple measures yeah why should I so we are not going to use this only thing now we are going to use multiple so there are for example you know doctor ask you to do some when there is a malaria or this thing he ask you to do a blood test.

Somehow those blood test are disputants still you they are asked to do it right because they give you some indication okay see they give you a relative measure once you have chosen set of way of choosing gains they give you a relative measure so even if you what is important to note here is that even if you rescale your gains these relative things are not going to change the values of singular values might change okay.

But the fact that this particular combination has higher singular value that will not change to much okay so it is a toll which you have to use in a complex senior with no as it communication numbers it will not happen that you know see this condition number actually depends up on some orientations of the you know Eigen vectors of and it will not fundamentally you know it will not change so much that so as a relative measure to you know check between multiple rings it is it is a good indicator I am not saying you should rely completely on this okay.

So just because blood test is dispute at time particularly when you want to check for malaria does not mean you do not do blood test you also do blood test you also do something else okay so not that you completely rule out because it is dispute.

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Neiderlinski Index


Consider MIMO system whose inputs and outputs are paired as
 $y_1 - u_1, y_2 - u_2, \dots, y_n - u_n,$
 i.e. T.F. matrix $G(s)$ is arranged such that transfer functions relating paired inputs and outputs are arranged along the diagonal.

Further, let each element of $G(s)$ be (a) rational and (b) open loop stable.
 Also, let n SISO feedback controllers with integral action be designed such that each SISO loop is stable when all the rest $(n-1)$ loops are open.

Under these assumptions, the multi-loop control system will be unstable for all possible values of controller parameters if the Neiderlinski index defined as

$$N = \frac{\det[G(0)]}{\prod_{i=1}^n G_{ii}(0)} < 0$$

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All this it will proportionally shrink yeah it will proportionally shrink so comparative it is many times useful yes comparative it is useful okay there are many such measures which were developed in the literature because people wanted to deal with to now loops which are interacting and this is one of them neiderlinski index you know you have multiple input and multiple outputs and then you want to distinguish between you know you want to find out whether something there is something called integral control ability of a system okay.

So well these measures have served probably because they are simple okay on simple information just gain matrix you know you can come up with some pre screening okay you have to use this prescreening together with your intuition as a or together with your undemanding of the system as a engineer physics of the system this is just an tool to help you okay it is not the solution.

Okay and it is good because it is the solution so that I keep saying as very course that if it was the solution mat lab would solve it for you then you me are not required right so good these tools are fuzzy and then you know you need somebody human being to say take the final call what is the right thing to do so now what you do in this neiderlinski index another index that tell you whether a system is whether it is possible to control the given system using multiple controllers that have integral action okay.

So this is a ruling out tests whether it is possible or not possible are you doing something fundamentally wrong by putting up multiple PID controllers that you can find out using this

hidden index okay so you rearrange the matrix in such a way the transfer function matrix so is that your chosen pairing appears on the diagonal see you have already done pairing let us say using RGA okay RGA did not consider any anything about stability it just told you know pairing which way to do and it rejected pairings which can potentially lead to instability negative element pairings can lead to instability.

So those where rejected it does not say anything about whether it is possible to control okay so what you do here is this is gain the dependent index which you can find and you know it shows that if this particular index okay this index is first thing that you have to do is to rearrange the system the transfer function matrix in such a way that the paired variables appear on the diagonal okay and then you compute this index okay I am giving you this without a proof if this index is negative system cannot be controlled using multiple PID controllers okay that is what it tells you so it is a screening test these are screening test and you have to use them as you know bunch of tools to analysis a multivariable system.

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Example: Furnace

$$\begin{bmatrix} T_{TC}(s) \\ O_1(s) \end{bmatrix} = \begin{bmatrix} -13.235e^{-4s} & 226.95e^{-4s} \\ 4.22s+1 & 4.75s+1 \\ 0.1445e^{-4s} & -1.98e^{-4s} \\ 4.434s+1 & 3.98s+1 \end{bmatrix} \begin{bmatrix} F_{TC}(s) \\ F_{TC}(s) \end{bmatrix}$$

$$RGA(\Lambda) = \begin{bmatrix} -3.778 & 4.778 \\ 4.778 & -3.778 \end{bmatrix}$$

Suggested Pairing: $T_{TC} - F_{TC}$ and $O_1 - F_{TC}$

Rearrange transfer function matrix such that paired variables are on main diagonal

$$\begin{bmatrix} T_{TC}(s) \\ O_1(s) \end{bmatrix} = \begin{bmatrix} 226.95e^{-4s} & -13.235e^{-4s} \\ 4.75s+1 & 4.22s+1 \\ -1.96e^{-4s} & 0.1445e^{-4s} \\ 3.98s+1 & 4.434s+1 \end{bmatrix} \begin{bmatrix} F_{TC}(s) \\ F_{TC}(s) \end{bmatrix}$$

$$Ni = \det \begin{bmatrix} 226.95 & -13.235 \\ -1.96 & 0.1445 \end{bmatrix} \times \frac{1}{226.96 \times 0.1445} = 0.209 > 0$$

\Rightarrow Process is integral controllable

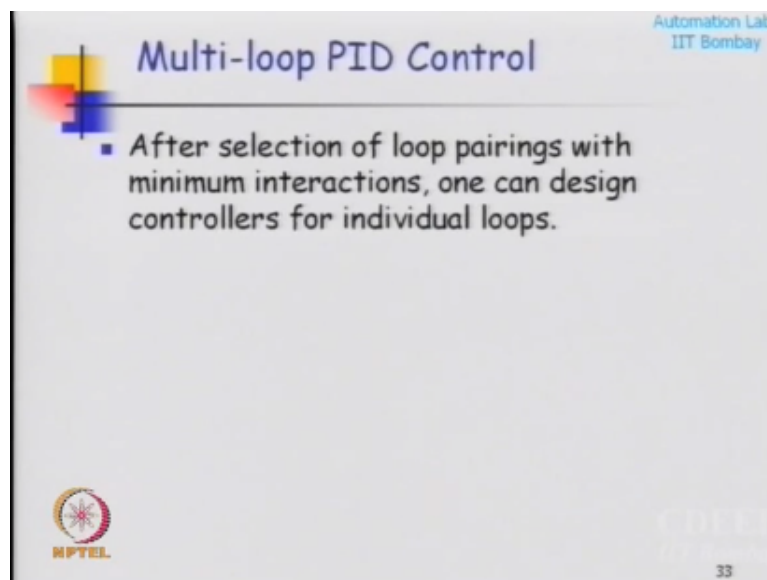
MPTEL

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I will just go back to this furnace example in this furnace example RGA showed as this particular pairing okay and I am now reorganizing see what is required once you have chosen the pairing you reorganize this transfer function matrix such that the paired variables appear in the diagonal okay so I have what I have done is see here air and fear was there I have change the matrix solvent air okay so that paired variables appear in the diagonal, so T hydro carbon and fuel this is

the one element and O2 and air this is 2, 2 element okay this is the paired so I have reorganized this matrix and then I just applied this we will scheme index.

To this particular index is positive which means it tells you that you can control the system using two controllers that I have integral actually, okay if this index had come out to be negative then we can where we use all these things, see suppose you have a very complex plant and you come up with 2 or 3 possible ways of pairing okay now how do I skill those further I checked integral control everything for each one of those possibilities when I can screen further you know so the systematic variables reducing a options okay to and come up with the.
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So what you do is multi loop PID controllers after selection of the pairing one can design individual PID loops individual PID loops okay and in the presence of interactions there are different ways of dealing with the interactions okay one of them is I do not want to get into this details I am just going to touch very, very easily there are different methods given in the literature that you know you can tune one loop then close that loop tune then close the other loop and tune the second loop.

With one loop closed then you know loop 1 loop 2 closed tune the loop 3 and so on, so there are different approaches given the literature okay to do this tuning this details of.

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Biggest Log Modulus Tuning

SISO Loop Design Review

Closed loop characteristic equation

$$1 + g_c(s)g_p(s) = 1 + G(s) = 0$$

$g_p(s)$: Process Transfer Function ; $g_c(s)$: Controller transfer function

Nyquist plot:

depicts real part of $G(j\omega)$ on x-axis and imaginary part of $G(j\omega)$ on y-axis as $\omega \rightarrow \infty$

Nyquist Stability Criteria:

A feedback control system will be unstable if the Nyquist plot of $G(j\omega)$ encircles point $(-1, 0)$ on the -ve real axis as $\omega \rightarrow \infty$.

The number of encirclements correspond to the number of roots of the characteristic equation that lie in R.H.P. of s-plane, assuming that the process is open loop stable.

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This biggest log modulus tuning you can see in the notes and well my intension in this particular course is to go to the multi variable controllers you see just that the some exponential to the multi loop controllers that you need that is because when you go out in a field you will multiple PID controllers and you have to deal with them it will at least know something about that situation okay, so you can have and look at the notes for this there is one method called biggest log modulus tuning.

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BLT Method

A major of distance of $G(j\omega)$ contour from point $(-1,0)$ is given as

$$L_c(s) = 20 \log \left| \frac{G(s)}{1+G(s)} \right|$$

Suggested design specification for log modulus: $L_c^{\text{min}} \leq 2\text{dB}$

Log modulus design: iteratively choose PI controller parameters (k_p, τ_i) such that design specification is met.


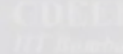
Multi-loop PI control of MIMO Processes

$$Y(s) = \left\{ \left[I + G_p(s)G_c(s) \right]^{-1} G_p(s)G_c(s) \right\} R(s)$$

Closed Loop Characteristic Equation

$$\det \left[I + G_p(s)G_c(s) \right] = 0$$

If Nyquist plot of $\det \left[I + G_p(j\omega)G_c(j\omega) \right]$ encircles origin, the closed loop system is unstable.

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And we can have details some of important and the defaults you can see of this.

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BLT Method

If we define a new function $f(s)$ as

$$f(s) = -1 + \det[I + G_c(s)G_p(s)]$$
 then encirclement of $(-1,0)$ by $f(j\omega)$ would indicate instability.

Defining a multi-variable closed loop log modulus

$$L_{cl} = 20 \log \left| \frac{f(s)}{1 + f(s)} \right|$$

Suggested design specification for log modulus: $L_{cl}^{min} \leq (2n) \text{dB}$
 where n is dimension of MIMO system.

Tuning Procedure (Luyben, 1986)

1. Calculate Ziegler-Nichol's tuning for n individual PI controllers

2. Assume a factor F such that $2 \leq F \leq 5$

3. Re-tune PI controllers as follows

$$k_{c_j} = k_{c_j}^{ZN} / F \quad \text{and} \quad \tau_{i_j} = F \tau_{i_j}^{ZN} \quad \text{for } j = 1, 2, \dots, n$$



Notes so I want to just skip this.

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BLT: Examples

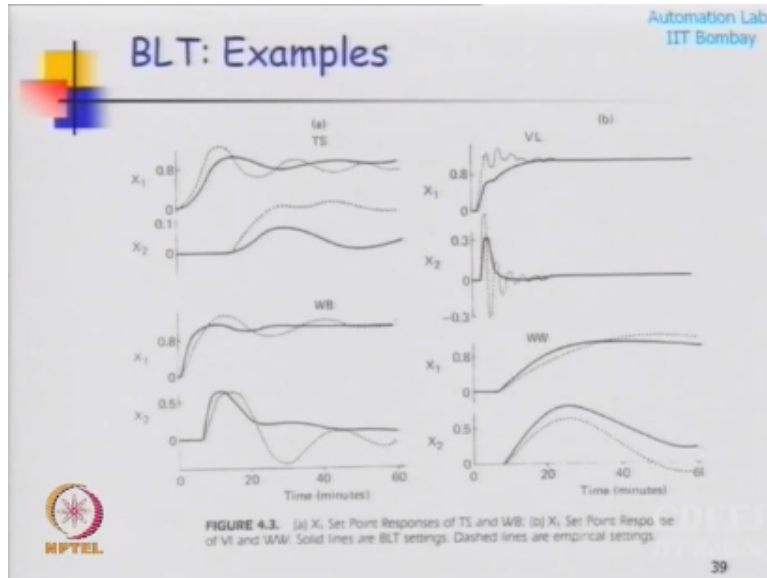
TABLE 4.2.
2 × 2 Systems

	TS <i>(Tyreus stabilizer)</i>	WB <i>(Wood and Berry)</i>	VL <i>(Vinante and Luyben)</i>	WW <i>(Wardle and Wood)</i>
RGA	4.35	2.01	1.63	2.69
NI	+0.229	+0.498	+0.615	+0.372
empirical				
K_c	-30, 30	0.2, -0.04	-2.38, 4.39	18, -24
τ_I	∞	4.44, 2.67	3.16, 1.15	19, 24
L_c	1.74	10.1	13.3	8.4
Z-N				
K_c	-166.2, 706	0.96, -0.19	-2.40, 4.45	59, -28.5
τ_I	2.06, 8.01	3.25, 9.20	3.16, 1.15	19.3, 24.6
L_c	unstable	unstable	13.3	18.5
BLT				
F	10	2.55	2.25	2.15
K_c	-16.6, 70.6	0.375, -0.075	-1.07, 1.97	27.4, -13.3
	20.6, 80.1	8.29, 23.6	7.1, 2.58	41.4, 52.9



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And then I have given some example where if you do this tuning procedure follow this de tuning procedure you can come up with reduction in the interaction between the loops.

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De-tuning of PID

BLT De-tuning Method


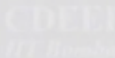
$$k_c = \frac{k_{2N}}{F} ; \tau_i = F \tau_{2N}$$

(k_{2N}, τ_{2N}): Ziegler Nichol's Tuning
 F : Detuning Factor ($2 \leq F \leq 5$)

Adjust F till Biggest Log Modulus (BLT) achieves a specified value
 BLT : measure of how far system is from being unstable

Sequential Tuning Method

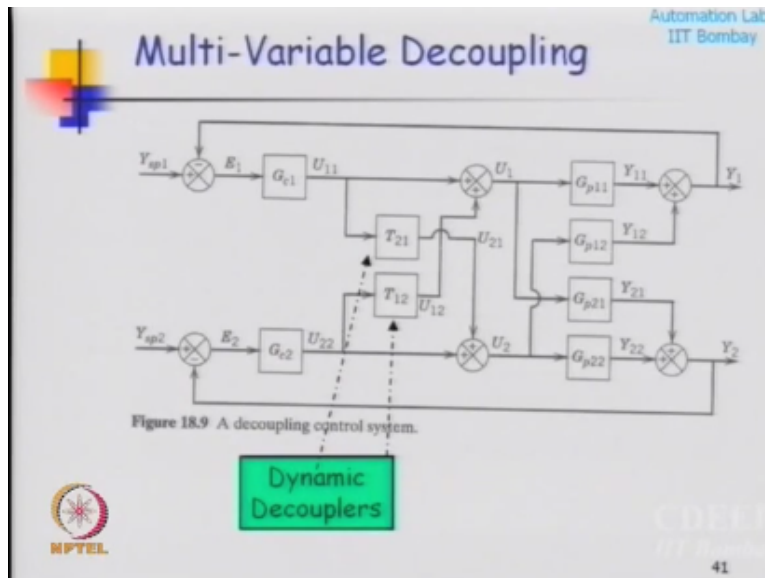
1. Tune one of the controllers in selected pairing and close the loop
2. Tune next controller with the first loop closed and so on

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If you want the summary of this it is something like this we individually design each controller okay and then put some factors so that you release you the gain and increase the integral time in such a way that the loop interactions become smaller okay, so this is the iterative design method which there are sequential tuning methods so you tune one loop keep the plant open tune 1 loop close it, with 1 loop closed tune the second loop close the second loop closed okay with valve into closed tune the 3rd loop go on doing sequentially and so on.

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Okay what I am interested in talking is this idea of decoupling and this decoupling is finally going to read us to multi variable controllers, so now I am getting into the area of multi variable controllers, so look at this diagram here see till now we were talking about two PID on controllers okay, and a process part had interactions okay now can you have a controller which is slightly different so this controller has one more led here which provide some kind of compensation here okay.

The same thing is here there is one more compensator element so this controller together is not this and this it is g_{c1} g_{c2} T_{21} and T_{22} and there are two elements cross links that I'm introducing now, so this is a multi variable controller it will try to simultaneously change if I change the set point it will change you one month so this root it is also try change simultaneously U_{12} if I give a set point change in 1 and do not change the set point 2 what should happen the set point the level 2 should remain constant.

Only level one should change okay this has to happen then has to be simultaneously you know we will take simultaneous action of into you to such that only U_1 changes a only y_1 changes and y_2 does not changes this two happen I need this cross links okay, so these cross links are called as dynamic decouples okay.

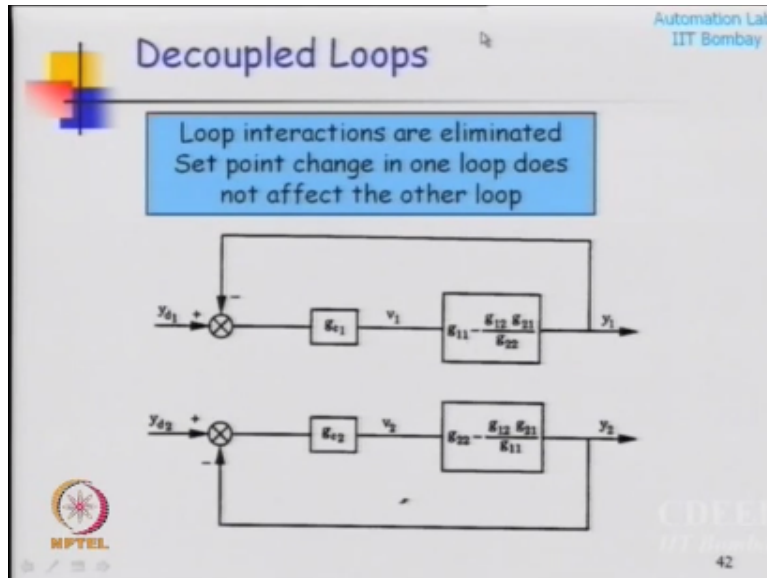
This cross links are called as dynamic decouplers and this decoupling elements help you to eliminate the interactions okay now this decoupling kind of controllers can be implemented through modern DCS or PLCs okay while that can be implemented through these modern

computer controller systems which are commercially available people just do not know about it, so they still continued to PID controllers while it is possible to do this because now when you implement.

It is piece of software right actually when you all implementing this PID controller I will be giving you this programs now I will be formulating an problems for you to solve in that w will realize the PID controller or any of these controller so just a piece of soft okay w are solving some different equations online okay, so if I am solving one difference equation I might have to solve 10 difference equation no matter, okay so now implementing a multi variable controller is not a bog level just.

Solving some more equations okay which is very easy with given the computing that power that we have today, so developing such complex controllers cross link controllers is not at difficult it is very, very easy it just that people do not about it that using existing hardware and software which is there in the DCS distributed digital control system or computer control systems or PLC it is possible to do this, so we will stop here, I can see lot of people get doubt in this point and what I want to do is can I do this okay I will just.

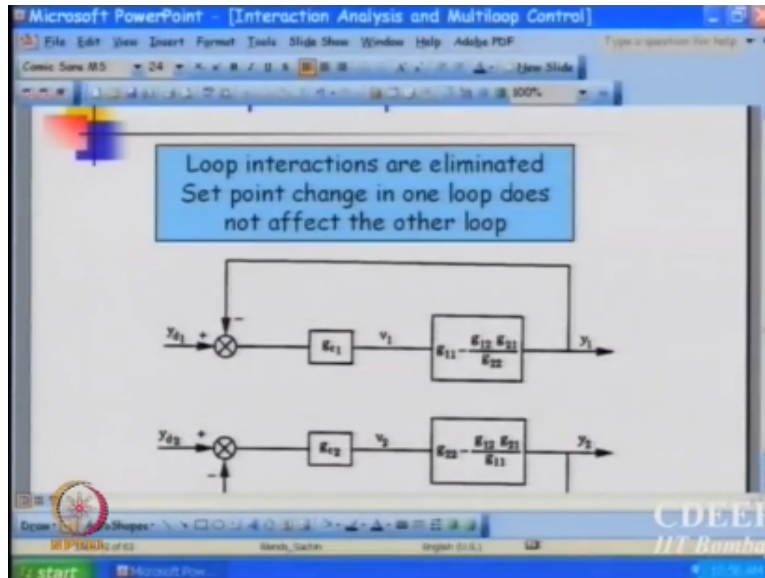
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So this lecture next lecture I want to use to do a transition from multi loop control to multi variable control okay, so I will start with a multi variable control but this is what I want to do can I introduce two extra elements okay can I introduce two extra differential equations for difference equations computer control system will b difference equations or continuous time system will differential equations.

Such that effectively system behave like this as if there are two decoupled loops okay is it possible to do that this is called decoupling okay using extra blocks I am going to separate the effects, I am going to view them as to separate things, so how do you do this if you have a transfer function matrix we will look at it very briefly okay and then you will move on we will move on to states place state feedback controllers I will go on talking about observers Kalmen filtering quadratic optimal control and so on.

(Refer slide time: 1:19:06)



Okay so now what I want to now which is the course is going to become more intense now because now we will start the course is going to become more intense now because now we will start doing the computing assignment okay, so we will set up this examples by Friday I want to form groups roughly 3 students in a group okay and tell me three main components of this assignment okay, so you can choose to solve it whichever were you want one person is leading one component other person takes leading to other components.

So three component are the three components of this of course on component is parameter identification modern identification whatever we need till made some, so you take this plant simulates it is dynamics collect data okay and use it to develop a model from system identification of that is part 1 part 2 is observer design what I am to cover next part 3 is the controller design so whatever we learn as a part of this course you implemented on this particular thing.

Better into project this is going to be a very, very important component about 25% of the marks okay, so you can choose your partners okay I am going to we are going to put 5 problems and if you think you can bring a problems on your domain like you want to do some automatic domain fine okay then bring a problem from your domain just a contact and show to us and we want to monitor this simply so we are in three evolutions of this and I will give you demo program okay let me a basis from which you can write your all programs okay, so and using this you should

modify to suite to your case and I should write those controls so complete case study okay of whatever we do in the this course.

Okay so system identification state estimation and control from mook they studied on the simulated system which as at least let see depending up on the situation it could be multiple input multiple output or single input single output so we try to set up will partially give you the cod beyond that part you have to write on your own course okay.

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