NPTEL

NATIONAL PROGRAMME ON TECHNOLOGY ENHANCHED LEARNING

IIT BOMBAY

ADVANCE PROCESS CONTROL

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Lecture No-14

Topic1: Stability Analysis of Discrete Time Systems –Lyapunov Functions

Topic 2: Interaction Analysis and Multi –loop Control

So we have been looking at stability of dynamical systems and yesterday I talked about a very simple method based on analysis of calculations or analysis of coefficients of characteristic equation and we can say whether the roots of the polynomial whether they are inside the unit circle or outside the unit circle using a very simple.

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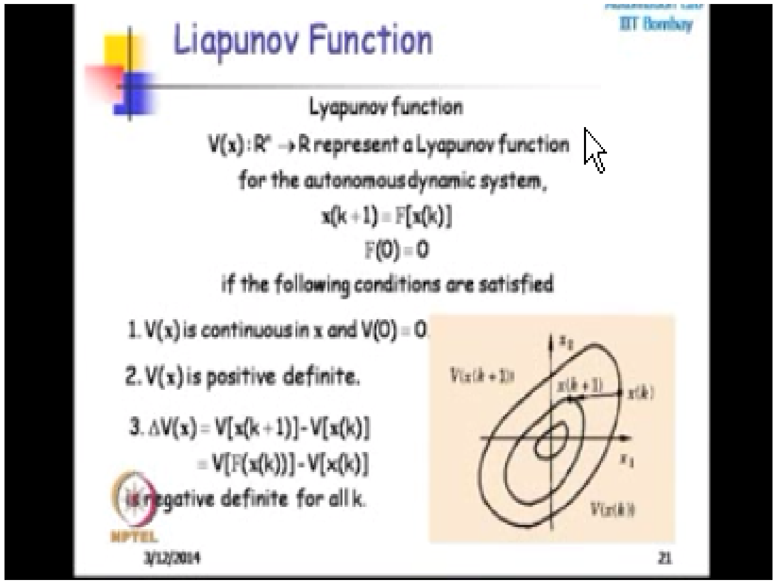

Hand calculation now this method has limitations first of all it is applicable only when you can do analysis using linear models ok transfer function models or linear state-space models it cannot be used when you have non-linear differential equations and real systems are nonlinear ok even though we are not going to look at nonlinear difference or differential equations as part of this course I am going to briefly introduce this idea of the Apollo stability.

Which is actually applicable to linear as well as nonlinear systems non linear systems can be very elegantly handled through this approach and actually it forms the foundation of modern theory of behavior of differential equations so this was founded by Russian mathematician and physicist who lived between 1857 and1918 his doctoral work on general problem of the stability of motion.

Which forms the foundation of this entire theory stability of motion has always been you know at the center of curiosity of scientific ability for example scientists IVs or physicists have been bothered about the stability of orbits around the Sun it should not happen that it is a it should be a center you see.

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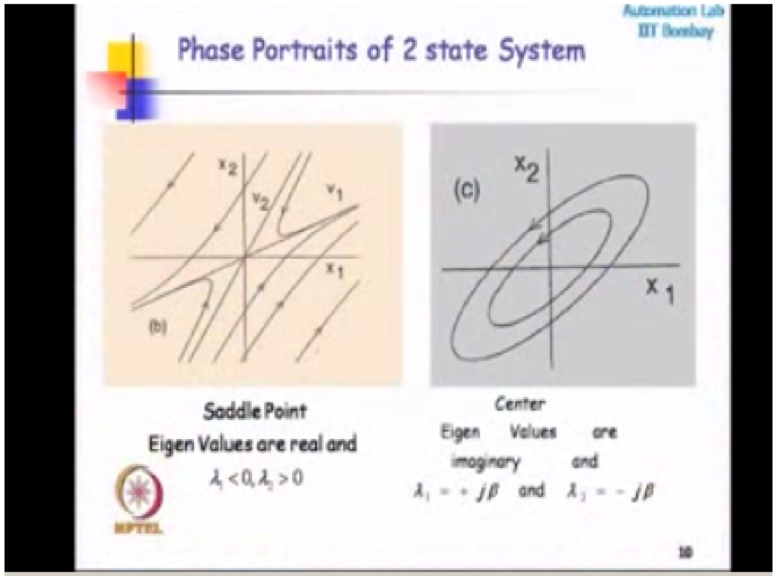

We talked about this face plane portrait you better I mean the solar system better behave like Center it should not behave it should not behave like this system.

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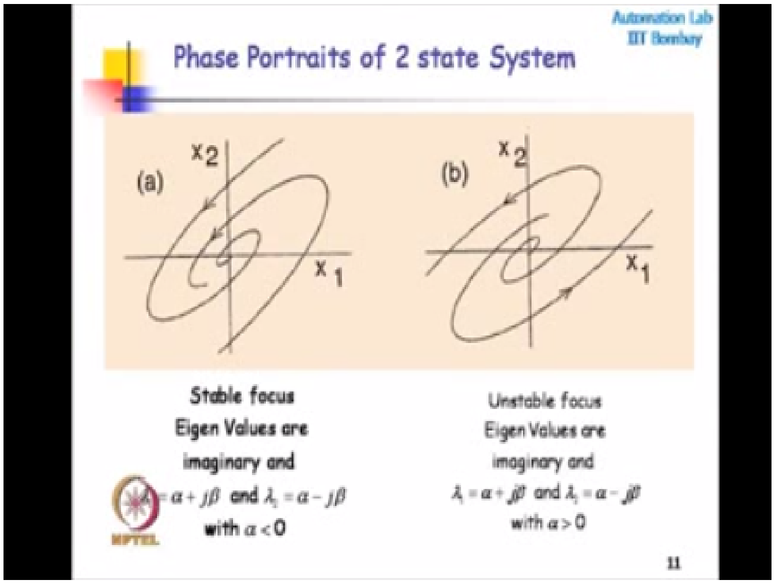

This one you know if the trajectory asymptotically dies and you know finally you merge into the center it will be disastrous for us okay so the stability of this stability of motions stability of orbits of planetary orbits different planetary orbits could be asteroids or it could be planets or comets it has always been a matter of curiosity and you want to and what is the problem the problem is that you want to predict.

how it will behave okay, now methods based on Eigen values or analysis of the roots of the characteristic polynomial can tell you whether the system is stable or unstable but as I said the linear systems is not a real-world problem linear system is a toy is an approximation that we have created in which we are very comfortable than doing things okay. The real world is nonlinear and you need so after looking at bibo stability.

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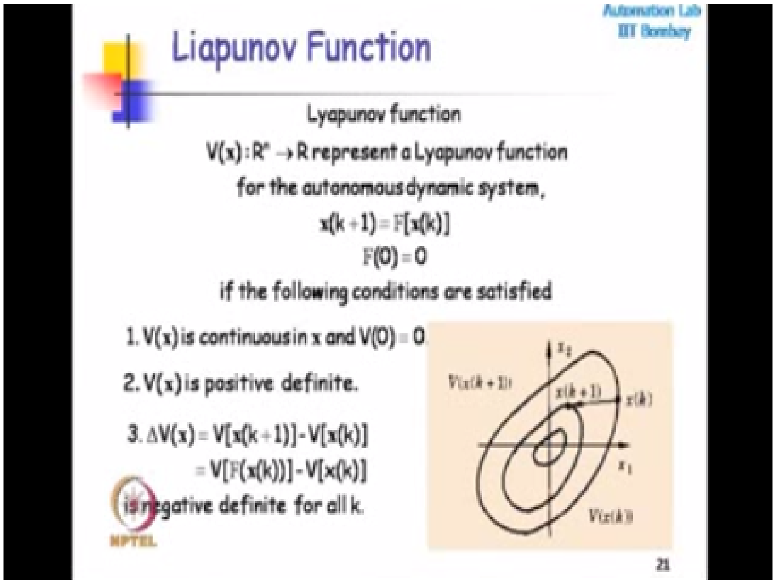

Some final concepts we want to now move on to they have no stability I am just going to give you half an hour introduction to this not really getting deep I am going to talk about things that are relevant to linear system analysis which we will be using subsequently in our in our controller design nevertheless I will show one example which actually shows the power of this method that so what is the Deon function the M&O functions are actually you know in some sense very crudely speaking they represent generalization of energy function okay now from physics.

We know that a system tries to take the path of minimum energy minimum energy principle so this is something which is a generalization of that basic idea okay, but not for a system which is governed by laws of mechanics okay but general dynamical system okay general dynamical system would be represented either through differential equations or through difference equations okay would be differential equations for convenience of computer-based calculations.

You can represent them as difference equations because you normally solve them using some kind of numerical method in computer and you advanced only in finite time finite time steps you can convert it into difference equations so for us the stability of motion would mean stability of different set of difference equations or set of differential equations which are coupled not a single differential equation okay so in general.

I am worried about a system which is of this form XK+ 1=F of XK and F of 0 is 0 so what it means is that if there is a steady state X bar we have done a transformation we have done a transformation and then we are you know dealing with a differential equation which is whose or difference equation whose steady state is nothing but the origin so if I have if I have difference equation.

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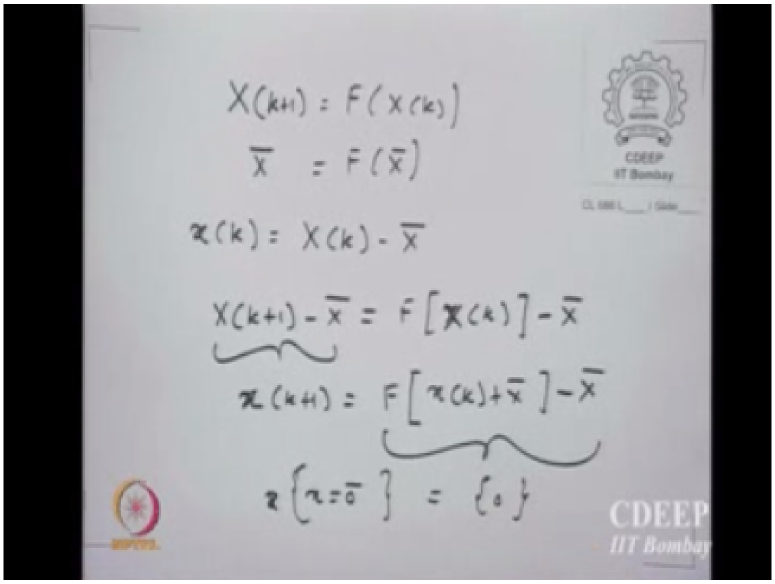

Let us say in the absolute variables XK +1 = X K and let us say X bar =F X bar if this is the steady state okay. I can define a new variable which is XK that is XK - X bar and then I can transform this equation as so this I can write as small X K + 1 and this I can write as F X K so this is capital X XK + X bar - X bar and then this new one this new function I can define as redefine as f so essentially if I if I do this transformation if I do this transformation.

I will get I will get a function when X when small X =0 the right hand side will be also = 0 so what I am expecting here is not something which is not possible when X = 0 the right hand side is also identically = 0 so let us say we have done this transformation they have actually done transformation of your wholly original difference equation such that the steady state is 00 this is very simple you just take then on-zero steady state and subtract.

And then you can do a translation origin translation so for the simplicity I am assuming that 0 ,0 is the steady state okay, now I am going to define a function which is energy like function energy function is a scalar function remember energy function the scalar function so this is I want this function to be a continuous function okay I want this function V of X I am going to call this function as V of X this new function V of X is called a Liapunov function.

The first quad first characteristic is that it should be continuous function of X well simplest function of simpler snap other function would X 1 square +X 2 square X 3 square and so on norm of X you know simplicity upon our function the scalar function norm of zero is zero right now a vector X zero is zero and it is a the second thing with it should be a positive function which means for any value of x for any value of x V of X should be positive okay. V of X should be positive simplest way of constructing such function which of course we are going to explore.

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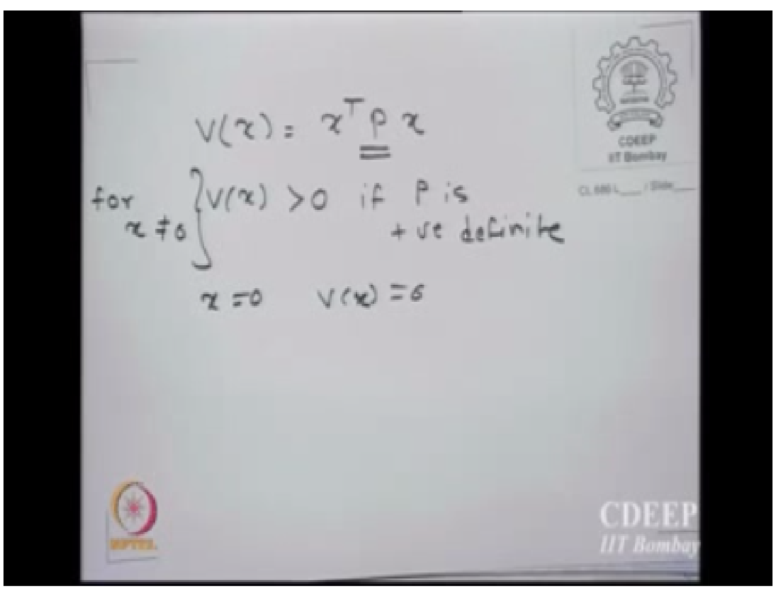

It in this is V of X is = X transpose p x where P is a positive leg matrix if P is a positive definite matrix for any X for any X V of X will be greater than 0 if P is positive definite right this is 4, 4

X ,X not = 0 will have way of X greater than 0 okay and X equal to 0 V of X will be = 0 ok this simple way of constructing a Liapunov function this is a candidate layer upon a function this is not the only layer for the function there are any function.

Which is a positive function okay and satisfies these two things and there is one more thing that it needs rectify to be qualified as early upon the function it is not sufficient that it is a it is not sufficient it is a it is a function which is positive function we also want one more thing we want Δ V that is derivative or difference of V pastime progresses should be negative it should be negative definite or this should be negative.

So in some sense there should be a decreasing function as time evolves this is not a real function coming from any physics this is a function which we are fabricating okay this is the energy like function that we are fabricating okay so it should have three characteristics one is that it should be at origin V of X V of 0 should be 0 second characteristic is that V of X should be a positive function okay and our positive definite function for any X okay.

We should get only a positive number and rate of change of B in time rate of change of V in time okay that should be negative okay so the energy function energy like function should be decreasing as time progresses that is the now whether it decreases or increases okay that will be governed by the system dynamics so whether a particular function qualifies to be a Liapunov function for a given system will be decided by the system dynamics.

Itself how does the system dynamics come into picture we will see that okay so basically what I am saying here is that when XK let us say these are the counters of V of X equal to constant C these are different counters of V of X equal to constant okay I can plot I can plot counters of V of X equal to constant in X 1 X 2 plane okay so as X K goes from XK to XK + 1 through dynamic equation okay on this counters.

I should move from outer counter to the inner counter that is what I mean okay it should continuously decrease.

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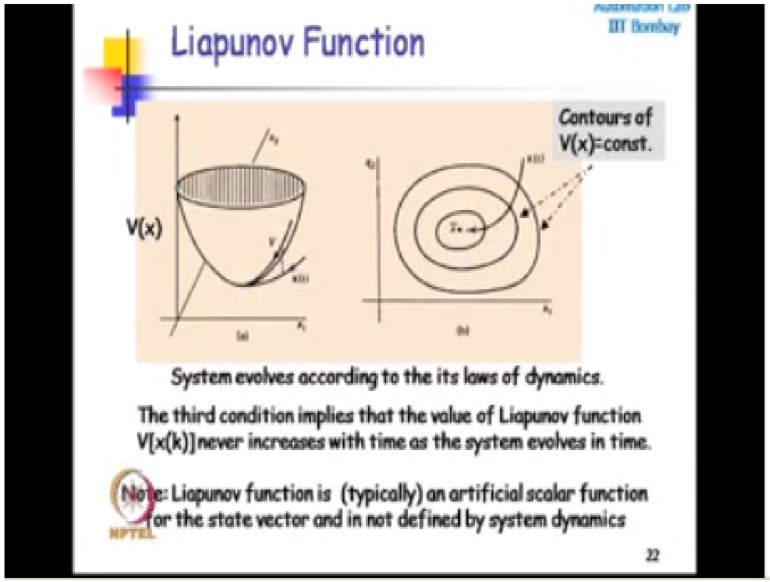

That is what I expect to happen okay so these are the counters if you plot a 3d plot it will look something like this two dimensional system and a upon a function for it so we have X is plotted on z axis and this is X 1 X 2 axis so a Liapunov function for a two dimensional system this is a simplified visualization of a D upon a function it should look like a valley okay with ashram minimum okay with a sharp minimum that is what it should look like and if you if you cut if you cut here different slices for V X V of X equal to constant see if you take any height here.

On this scale you know you will get V of X equal to constant and if you project that if you project that onto X 1 X 2 plane then it should look like concentric circles and what I want to happen is that as system evolves okay on the system on this surface of this function I should move inside and inside that is what I want to happen okay the system he was according to its laws of dynamics and the third condition actually tells you that the function never increases with time.

It should be less than or equal to zero so it should it should either stagnate it should never grow as time progresses that's what I want if I can find a function for a given system which obeys these three conditions then now whether this will happen or not will depend upon the system matrix you can just go back and see here see my system evolves according to X k + 1is = f of f of x k my system evolves according to this okay.

Now let me define this V of X so V of X V of X K let us say I have defined as I have defined as XK transpose P X K where P is a positive definite matrix what I want to happen is that V of XK plus 1 which is X K + 1 transpose P X K + 1which is same as F XK transpose p F XK right okay so this I compute weak XK +1 like this and then I want ΔV to be negative definite so you see where the dynamic centers here the dynamic centers here through this F of XK transpose okay.

So that is where the dynamic centers into the system so this Liapunov function does not naturally come out of the system dynamics we are defining an artificial function okay and if you can find inartificial function which obeys this characteristic then we can talk about the stability there is so many numerical methods for example you could use simplistic Euler method though you can have more complicated algorithms like we may get our methods.

So you can convert into a valid difference equation from differential equation there are particular corrector methods one could represent that into a all of them will be approximate they will not be exact but then so be it we can we have when we deal with nonlinear differential equations in computers we have to deal with this approximate distillation okay so now so this I want to stress that it is an artificially constructed scalar function okay.

If I am able to construct the scalar function which has certain properties then I am able to talk about the stability of the dynamical system if I am not able to construct that does not mean anything if I am able to construct then I can prove some characteristics of the dynamical system if I am not able to construct then that does not mean the system is unstable or anything of that sort okay.

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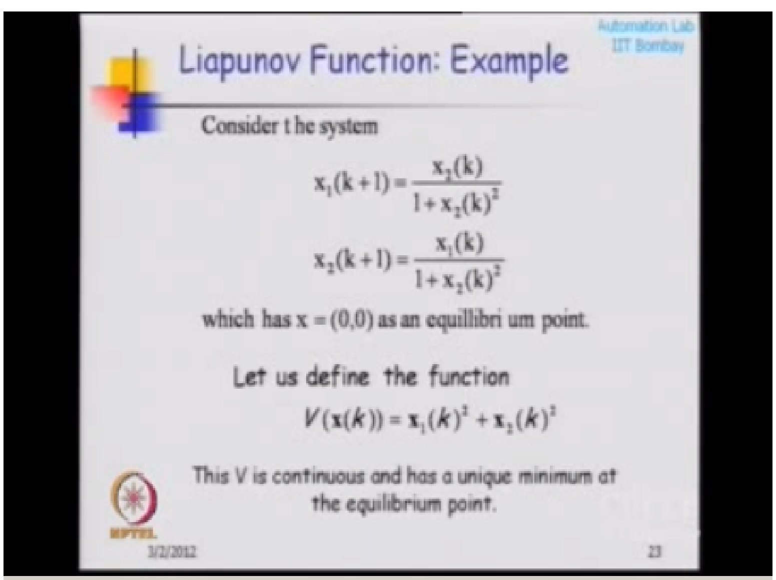

So do not stretch it the other way up now let us look at this particular system okay this is a very simple system. What will be the equilibrium point is 0 ,0 okay the linearization of this system if you do linearization of this system at 0, 0 okay you will find that the system has two poles on the you need circle for this particular system if you do linearization you find that there are two poles on the unit circle okay and based on our linear system theory you would say that well.

This is you know a center this is marginally stable okay actually using leer so what I said is that for a system when you do linearization if you get eigenvalues on the unit circle you cannot do analysis using linear approximation that is that is ruled out you can linearization based analysis holds for the nonlinear system only for two cases I similarly to this stable or unstable margin instability cannot be established using linearization.

So this is the classic example you cannot do that here okay let us define this simple layup on the function simplest that you can think of X 1 square + X 2 square okay the simplest Liapunov function one can think of V is a continuous function let us just check that is it a continuous function it is a continuous function now can you find out just do the calculations what will be V of XK + 1 and tell me whether the pit third condition is satisfied true condition is satisfied.

What is the first condition X 0 + 0 what is the second condition let us go back it should be a continuous function of X we X should be a continuous function of X which is happening right what is the second criteria it is a positive function X 1square plus X 2 square for any X 1 X 2is a positive function no problem what about the third one just try what do you think upon not positive value which is greater than 1 so if you if you substitute X 1 if you find out the value for V of XK + 1 okay.

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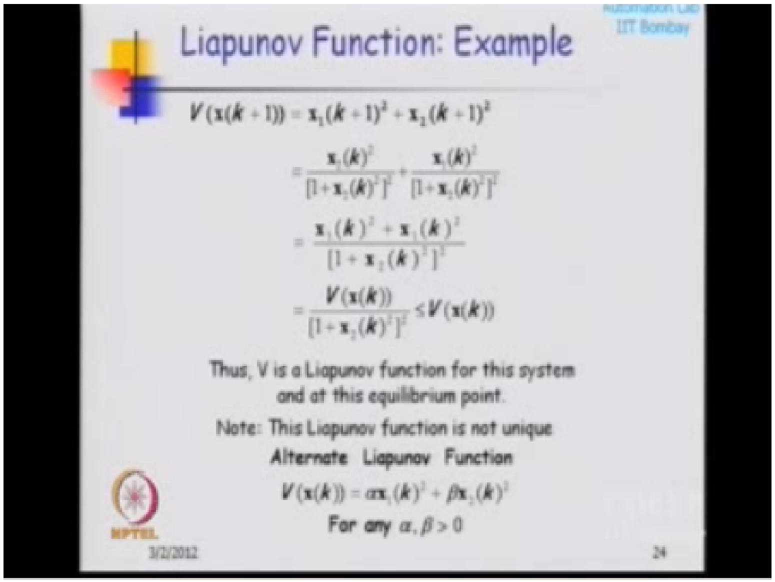

You will see that you will get this x2 square upon 1 + X 2 square whole square okay and this quantity this quantity is nothing but VX upon 1 + X2 square whole square the denominator is always greater than 1 the denominator is always greater than 1 so which means V of XK + 1 is always less then V of XK okay we have XK +1 sorry it always less then I made a mistake let me correct.

We have XK + 1 is always less then V of X K okay so as time progresses this Liapunov function value reduces okay time progresses the lay up on the function value will reduce and so this is a Liapunov function for the system now what is the significance of having a D upon a function well is the Liapunov function unique just try just try what happens if you take α α and ß α x1 square + ß X 2 square will it be a Liapunov function if you take any α and any ß which are positive.

Okay even they will turn out to be on the Liapunov function you will get the same expression okay so this will also be a Liapunov function so there is no unique way upon a function for this particular system fortunately you are able to find out infinite number of the Liapunov function.

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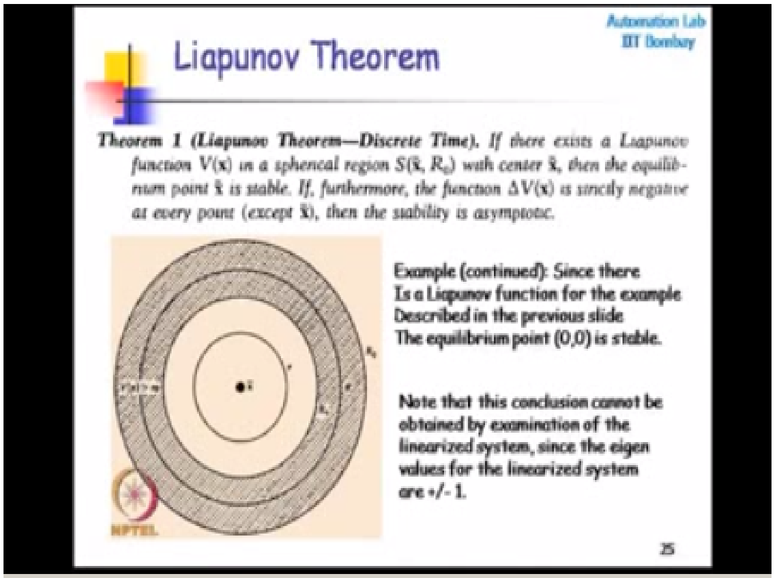

This may not happen for most of the real systems it is difficult to find another function but that's a different story well this is a very fundamental theorem of the epitome well let us not get into the theorem statement you can read what I want to say here or what the message here is that in the neighborhood of the steady state okay the particular case zero right in this particular case zero in the neighborhood of the steady-state if you can find a region.

Okay in which you cannot define a Liapunov function okay if you can find a region in the neighborhood of the steady state okay on which you cannot define a yep on the function what is the lay up on the function it should be positive okay it should be decreasing as time progresses okay a positive function which is decreasing and type progresses then okay it says that the equilibrium point is stable then it guarantees that it guarantees.

That the system trajectory will stay in the neighborhood of the stained and neighborhood of the steady state that is being investigated now there are two things there are two possibilities okay one possibility is C.

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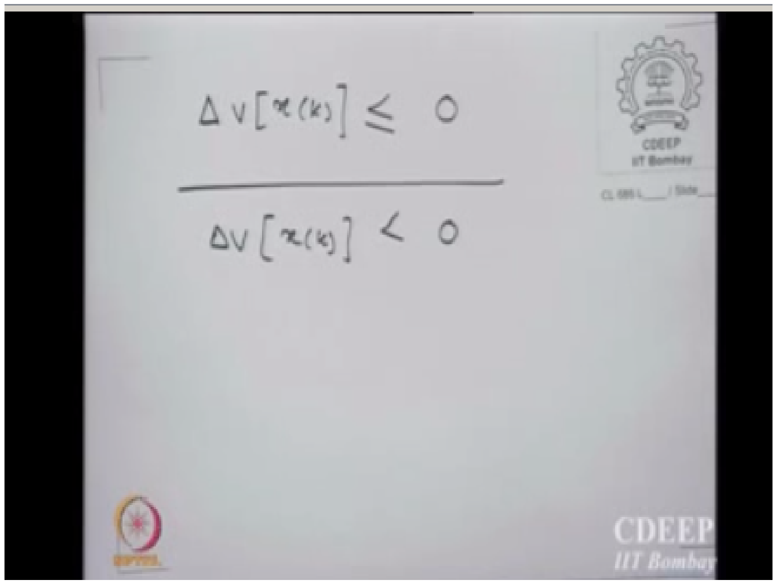

One possibility is that ΔV XK is less than or =0 and other possibility is Δ V X K is less than 0 strictly less than zero okay and these two are different possibilities in one case in one case we can be allow possibility that Δ V is zero okay. So there is no change there is no change in the value of so it means if I go here see suppose it happens that my you know suppose it happens that XK moves to XK + 1 O suppose.

It happens that XK moves to XK +1 and you reach this trajectory here once you reach the target ready the next point is trajectory in a sense this is the this is the constant V value this is the counter for V equal to constant the next move occurs such that it is again on the same counter okay third move occurs such that it is in the same counter what will happen is ΔV this is a this is a counter for V is equal to constant okay suppose the system gets tracked.

You know in a region where the value of the V is not decreasing okay if value of the V is not decreasing then all that you can say is that this that your dynamics will keep hovering in the neighborhood of the steady state the second situation is that this difference is strictly less than zero every time so what will hap pen every time every time it will move inside and inside okay and finally as time goes to∞ XK will collapse into the origin.

So these are two different situation in one case all that you are saying is that Δ V is non decreasing okay it is remain it is remaining it is becoming zero so you allow either decreasing or non-decreasing you know 0, 0 or less than 0 in other case you are staying strictly less than 0 these two are different two situations ok and these situations are captured through okay so if Δ V is strictly negative for every you know for every K then.

We know that the system will eventually go to origin okay if the for the given Δ V we just find that it becomes zero at some point okay and remains zero for example then you cannot say about asymptotic stability the this the first thing that I have talked about is asymptotic stability because you know every time every time system moves from a counter outside to a counter inside to a computer inside.

We should collapse if every time it is negative it has to move inside all the time so you know it will it will collapse into if sometimes it may happen that it is neither increasing non-decreasing the system stays you know in some trajectory in some bounded region in the example that we considered okay since there exist a Liapunov function so this theorem powerful theorem says that if you can find out the Apollo function for a given dynamical system okay.

Which obeys these two these three criteria then okay you can guarantee stability in fact if it is if the difference is negative then you can guarantee asymptotic stability okay and this idea holds for any dynamical system you can see the sweeping generalization this reservation it is a different question whether you can construct a Liapunov function for a complex system but if you can construct it guarantees.

That you know it guarantees to analyze the stability of the dynamical system in the neighborhood of a given point okay it tells you whether it is linear or non-linear does not matter okay so very general result and now just going back to the system that we are we looked at just now okay if you do linearization you get two Eigen values which are on the unit circle and you have problem and you cannot analyze stability using linearization but you can talk about that particular system happens to be asymptotically stable.

And this you can never establish using a linearization based it is asymptotically stable in the neighborhood of 0 origin that you can never establish using Liapunov stability analysis because you get this difficulty of poles on the so linear world is ideal world and everything that all the results.

That you create there do not one-to-one transfer to the norm being you have you something else well we are not going to get into this yep ah no stability too much except we need some simple results for linear system theory which we will be using and you probably have to attend a separate course if you want to understand this thing in detail I am just going to derive some useful results for linear systems because we are going to use them later. When we talk about design.

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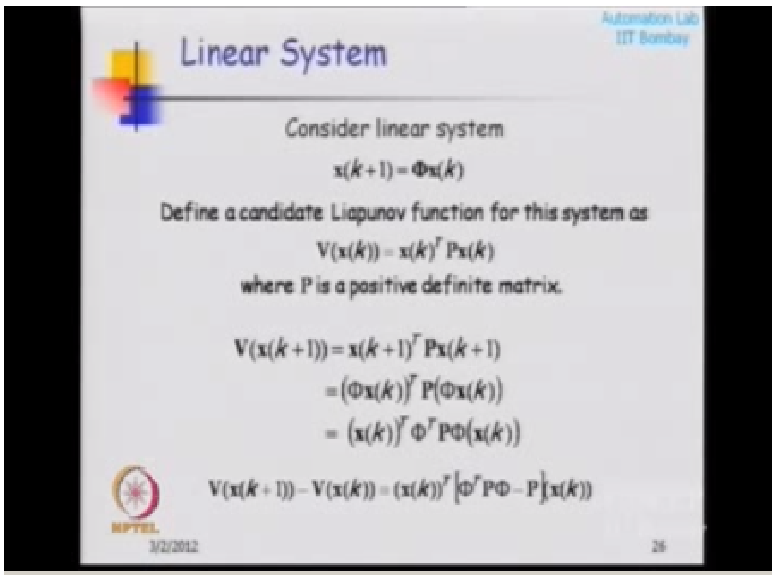

Or analyzing some stability of certain systems now let us look at this simple system well label no stability as I have defined right now it talks about on for stability okay you can of course talk about or you can analyze stability in presence of some inputs okay and those concepts are much more advanced and I am not going to touch upon them in this course so there is something called input to state stability and input.

To state practical stability there are many more notions for interstate the book by Khalil on nonlinear systems is a very good book I listed the book at the end of my lecture notes so also I think there are courses in the Institute siscon system fan control and electrical engineering offers courses on new systems analysis given we offer a course whenever it is possible elective one so now for the linear system the task of constructing a upon the functions is very simple okay for linear systems.

I need to construct the upper level function I can do it using any positive definite matrix so if you if you actually substitute V of XK + 1 this is a very simple calculation that you will get here V of this you will get this xk transpose π transpose p π into x k this is just by substituting the dynamic equation this is very straight forward and if I take a difference I will get this ΔV that is V XK+ one _ V XK will give me.

This matrix all that I need is this matrix this matrix should be negative definite if this is negative definite what is the problem -P should not I so there yeah so this matrix that is π transpose P π - P this matrix should be negative definite if this matrix is negative definite then we have constructed in the Apollo function for this particular system okay.

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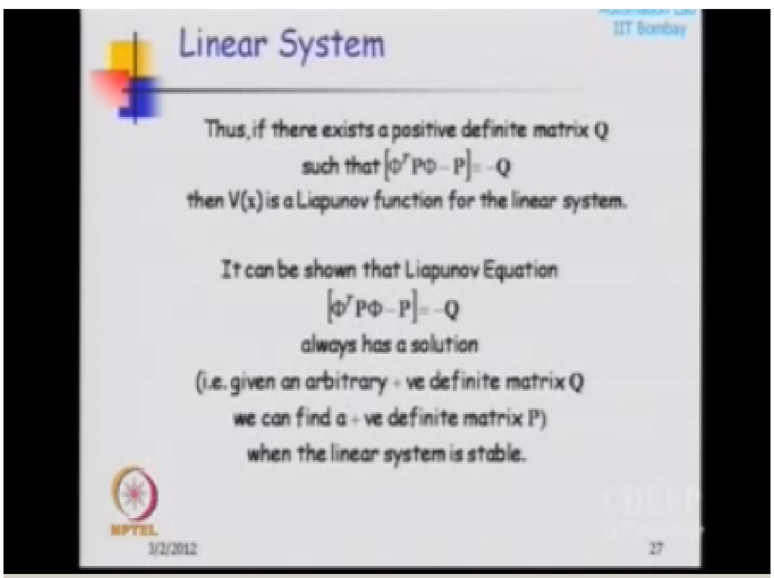

Now the question is when such a matrix exists okay you can show that such a max is such a matrix exist? Then of course the system is asymptotically stable so actually you can show that if I can values of π are inside the unit circle okay I can values of fire are inside the unit circle then you can always construct a matrix P such that π transpose P π- P this is negative that is always possible we won't but wonder why when you can analyze stability of a linear system using Eigen values and Eigen values can be computed very elegantly very easily now why do.

I need this particular method you will see as we progress we still need this classic equation this particular equation is called as the Apollo equation and it can be shown that this equation will always have a solution provided Eigen values of π are inside the unit circle if π is assumed would if I have Eigen values which are inside the unit circle which means the system is asymptotically stable you can always construct Liapunov function such that π transpose P π -P is negative definite okay.

So this is this is guaranteed and this is used this will be used in an analysis at some point later okay so there are times when you analyze some control their behavior closed loop behavior it becomes difficult to analyze using eigenvalues okay it is easier to analyze using the apana theory so that is why I am that is why I am developing this right now for linear systems it will have some other implications when it comes to design okay.

So in mat lab you have a function called the app okay so if you give π if you give five matrix and if you specify q matrix if π has Eigen values inside unit circle and if you specify Q matrix okay it will give you back P matrix okay it will give you back P matrix.

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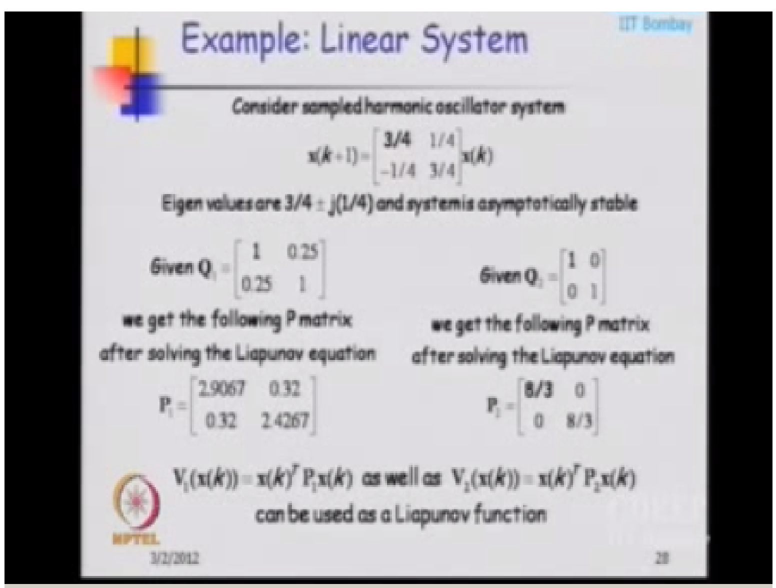

So I am just taking a simple example this is a simple harmonic oscillator system this has eigenvalues which are three by four and +or - one by four J. So the system is asymptotically stable so if I specify Q I can solve this equation I can specify Q C what is this equation this equation says that π is system matrix Q let us say I specify I want to design I want to design an Liapunov function I want to find early upon the function for this particular system so given what I will do is I will choose a matrix Q which is positive definite so that - Q is negative definite okay.

And then I can back calculate P okay and this Liapunov equation appears in many other contexts oh it is so if I give this P if I do Q one to be C I have chosen Q to be one point two five point two five one this the positive definite matrix so negative this is a Nega definite matrix okay and then f or this particular choice of Q I will get a Liapunov function which is which is P one matrix is given here if I choose Q to be one.

Then I will get this the apana function which is 8 by 38 by 3 so there is no unique way of constructing air for the function both this P 1 and P 2 both these matrices will give you a layup on the function for this particular linear system okay so other of them are validly apana functions that is X 1 transpose p 1 X 1 P 1 XK and XK transpose P 2 XK both of them.

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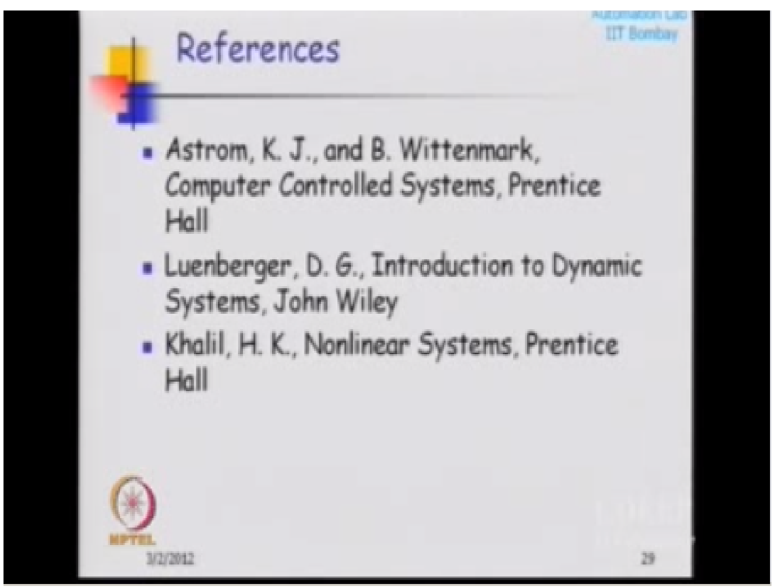

Are can be used as Liapunov functions now. When I am stopping it here I am not getting into more details of Liapunov function I have introduced this knee upon of stability because it will be useful later when we analyze state feedback control this that is where it is going to be useful okay, it is a very brief introduction they cannot be shorter or something shorter than this couple of stability theory.

So as I said if you are interested you should actually read this book very well-written book for those who have flavor formats it is a very nice book Colleen nonlinear systems and nonlinear systems and this book is of course available in our library so even the second the second book is a very nice book Khalil's book is a advanced text it is useful if you are going to do research in control theory this is probably the primer this book is will give you brief introduction the kind of thing which I have given you in 1 or 1/2like ours but it will give you a practical knowledge about what it is learn Berger's book is intermediate it is not at a level of Colleen it also introductory book very nicely deals with dynamic systems and a third book of course today's book is specialize book where so why I have done this that is because I need this little later.

So I am going to use two tools for analyzing stability one is roots of the characteristic polynomial I there is Liapunov function okay so this is just a brief background to the stability analysis and we are going to move on now we are going to move on to something different now so now let us move on to the controller design what I want to do eventually is controller design using state feedback or using the state space models that we have developed okay.

But before I move to those controller design problems or code controller design methods I need to give some motivation as to why do I need to-do all that why is there any benefit because in your first course in control you study about PID controllers and if you go to industry if you are done any industrial training or if you are spend some time in industry before coming for your post graduation you would realize that PID controllers are everywhere.

And you go to any company of any power plant any chemical plant it is just full of hundreds and hundreds of PID controllers okay and Here I am talking about state feedback controller why what is so great is there a motivation to go for something more complex than simple PID controllers also PID controllers which were taught to you in your first course are they do they remain simple when you have hundreds of them together okay is the question okay.

Yeah it is possible to define for continuous time systems continuity is a different notion from continuous time and continue T what is continuity of a function that is a different notion altogether you know you do not consider is between continuous time system and a continuous function these two are the word continuous is common does not mean that they are referring to the same things what is the continuous function for every epsilon greater than zero.

What something what we should be able to find out Δ greater than zero huh what is it tell me who remembers left limit and right limit that is a simple way of looking at it yeah so for every epsilon greater than zero so if you advance f of X more of f of X -f of X + epsilon X -epsilon difference of this okay so you should be able to find X - X tilde which is less than Δ such that yeah for any value of x in this in this interval you know f of X will be less than Epsilon f of X minus f of X tilde more of f of X tilde so that is the definition.

You can of course go for multi-label function then talk with norms okay continuity of a continuity of nonlinear function is different from continuous time systems and discrete time systems look don't confuse it to two things I can get into those things but we have a limited time so and then II think you are attending those mixtures right on nonlinear systems where I Δ about continuity different notions of continuity yeah we talked about Lipchitz continuity.

And you know uniform continuity so there are okay so I just want to move on for a very short period to another topic.

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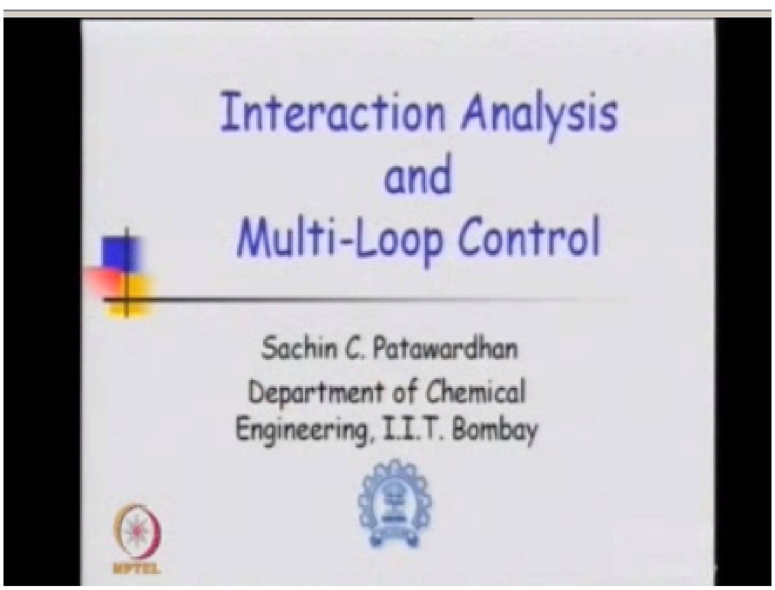

So now finally we start doing the main business of control okay now that we have models we have tools of analysis we are two tools of analysis one is Eigen values and then we have Liapunov functions okay and now let us get into the business of doing control okay, the nice thing now is that we are in our imaginary world.

Where we have a model and we believe that the process dynamics is represented by this model now this model is something which we can play with we can turn it around we can you know introduce some new terms we can do all kinds of things later we have to worry about how to translate that into reality if you design a controller in the space of models which are linear models okay we will have to then take it back.

To the reality through some computer implementation or something but now once we, once we have develop the model we have, we have you know description of the dynamics and now the idea is can I alter the dynamics the way I want okay that is the main idea we believe that this model is a good representation of the real system so if I do manipulations with this model the similar manipulations will hold for the real system under that belief we carry out a design okay.

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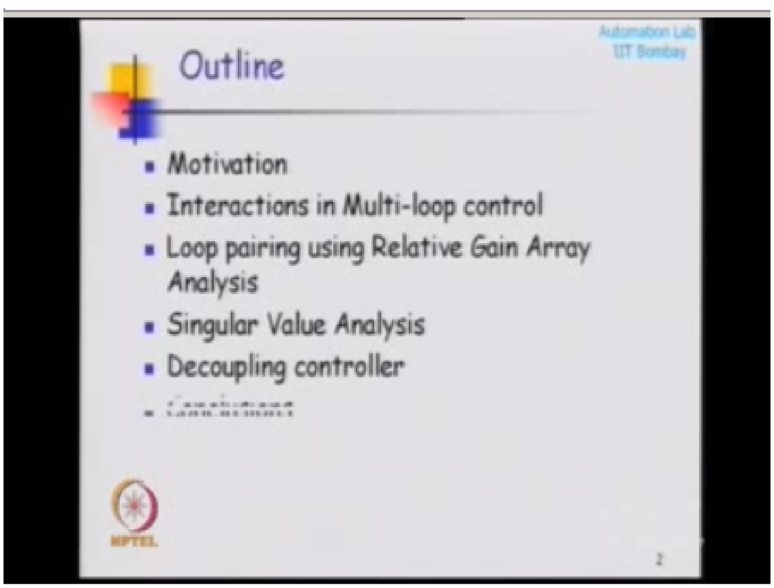

So let me see what we have here I will talk about something called multi loop control I want to go before you go to advanced control I want to talk a little bit about these PID controllers which we have looked at in your first course and what happens when you have multiple PID controllers what is what is the problem then is there a way of intelligently choosing PID controllers that is what we look at through something called relative gain array analysis there is a single event analysis.

Which will help us in finding out how to go about doing you know choosing PID controller pairing what is this pairing we will come to that then there is a concept called decoupling controller self introduced this is coupling controllers and then conclude well this part.

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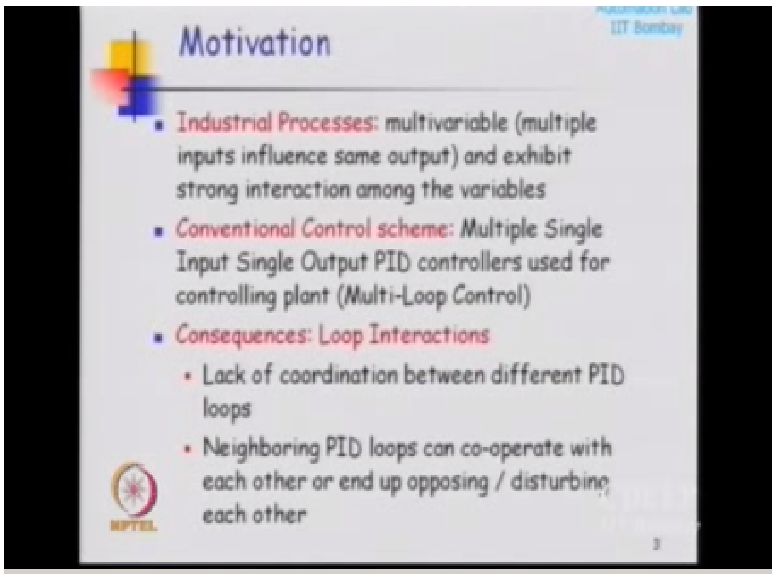

I am going to do very quickly because yes it is important it is a important link between what you were studied in your undergraduate as a control course. And what we are going to study next now unless I do not want to emphasize it too much I will go through it somewhat quickly there is lot to it if you a lot of work has been done on this area but what I am going to do for next maybe one or two lectures is not completely representative it just gives you a flavor okay.

So real systems the other industrial systems are multivariable, multivariable systems there are multiple inputs and there are multiple outputs. To be controlled you have multiple things at your manipulation the example that I keep using in many lectures on advanced control is your car the car has you know you want to control speed direction and you have three inputs at your disposal you can accelerate you can brake and you have steering okay there is real industrial systems there are hundreds of measurements and hundreds of multiple inputs that are available.

To you look at a power plant will have many manipulated variables available to you many temperatures pressures flows all data coming in and you want to simultaneously control everything you want to control the state of the system okay and the conventional approach to doing this is using multiple PID controllers okay this is something which is being done last for 40 50 years 60 years very complex thing when you do it using multiple PID controllers.

Using multiple PID controllers to control it is multivariable plant is like having three different drivers or two different drivers in your car one who only manages accelerator one who only when he is the brake the third who only manages steering okay not a funny thing to have in your car when you are driving if you have three people.

Who do not talk to each other we do not know about existence of each other you can have chaos and that is why controlling industrial plants is quite difficult when you have multiple PID controllers typically the various PID controllers are implemented today they are implemented in such a way that they do not talk to each other they do not know about existence of each other okay and so it is like 100 drivers simultaneously driving a plant.

Okay and that is why you need you know a team of plant engineers and operators who are continuously on the watch what is happening 24 hour there has to be a watch so you will start wondering how it still it works so this is what I am going to call is a multi loop control study multi loop is multiple drivers okay and multi variable controller is you driving your car single person making decisions about all three things simultaneously by taking into consideration.

Both speed and direction this is the ideal situation one driver for your car not or at least if there are you know if there are thousand variables to be controlled and manipulated you do not want thousand drivers you can probably reduce it to twenty drivers its sector than having thousand drivers okay so that is so multi variable controllers is what we want to eventually go to so what is the problem with loop interactions.

So typically there is a lack of interaction there is lack of coordination between the loops now the neighboring loops okay can collaborate and help each other or they can destroy it either they can fight okay, if they fight you know you have trouble how do you find out whether the loops are fighting or not fighting how do you pair which controlled output should be controlled using which manipulated variable if I am controlling speed should I use accelerator.

To control or brake to control what is what my strategy is if I have multiple ways of controlling a system what is the pairing because PID controller means I have a single input and single output right I have a single measurement single manipulation okay.

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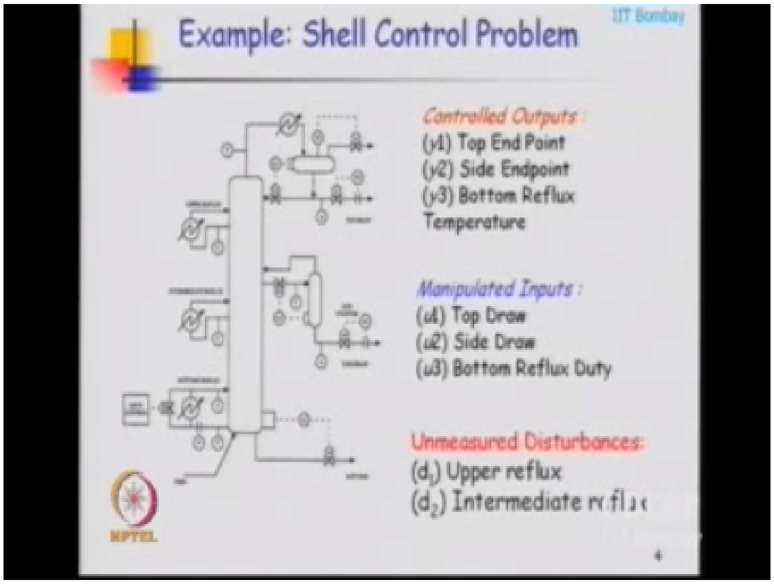

So even if let us go to this plant this is a distillation column plant do not worry about if you do not understand distillation there are things that are to be controlled that you can definitely appreciate top endpoint this is a distillation column in which you separate chemicals mixture of chemicals using you know difference between their relative other dies it is so this is the industrial column of shell you want to get top end point means top product quality.

Okay side endpoint means side product quality and temperature here at the bottom these three things have to be controlled or have to be are the controlled outputs from the viewpoint of operating the system I have three manipulated variables I have a top drop amount of liquid that I draw from here this is called top drop okay then there is aside draw I can draw some liquid from the side and I can input heat here this is called bottom reflux duty so there is a boiler here you put in some heat here say through steam.

So I can heat this system look at this as input output system okay there are three inputs there are three outputs there are multiple states okay what is happening inside is very complex but as a control engineer I have developed a model using you know might relapse toolbox or whatever tool box that you have scarab toolbox you have a model which is probably a great idea both model and you know how the dynamics of this system is you know how the dynamics behaves.

In the middle of some operating point so you have a model point now question is if I want to put PID controllers how do I put them should y1 be controlled or y1 betide up with a PID controller that manipulates u1 or you two or you three you know there is a combinatorial problem here okay what should be my control output what should be my manipulatevariable okay I am allowed to put three PID controllers.

Okay and which one to go which one goes with which one and what is the basis for choosing that okay well the life is not so simple there are also other inputs which you cannot manipulate there are true to heat there is a heat exchange between some other stream in the chemical plant and some liquid inside here on some trays so there are two disturbances which keep influencing the plant this is one of the standard problems.

Which the shell group has floated in the control literature okay so there are two disturbances there are three inputs and three outputs I am defining a simplified problem they have gypped the problem which is more complex they are given a problem with seven out puts five inputs and three disturbances I have created a simplified problem here so now the question is which one which one should I pair with which one you know how do. I couple these inputs and outputs into one PID controller every time so difficult problem.

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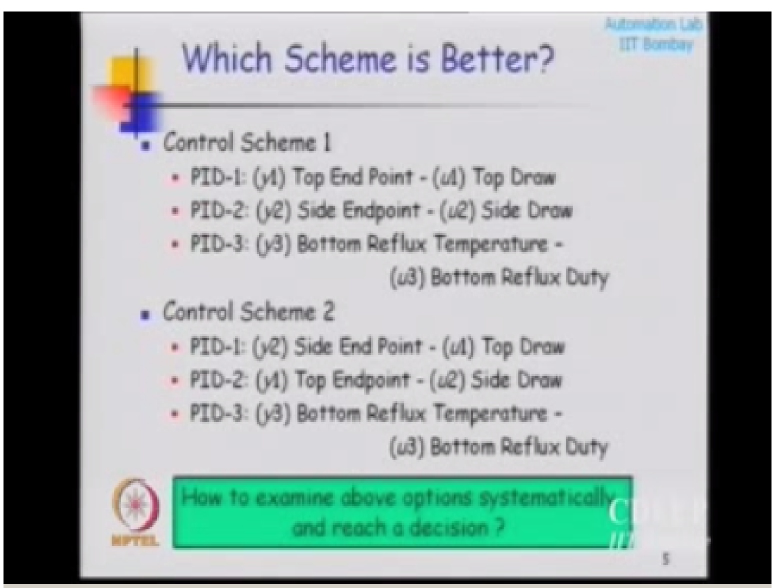

So which scheme is better I can come up with many schemes right I can say that top end point the you know I can say that physical proximity is important so if the manipulate variable is here and I want to control this concentration I should tie them up together okay but somebody might say no this heating here has a very high influence on the concentration here so this concentration should be tied up with this heating well that is also logical okay so there are possibilities.

Which are similar okay and then one needs a filter to screen out these possibilities okay I need some filter to screen out this possibilities so fundamental question is which is the better scheme which is this scheme better that is y 1 u 1 Y 2 u 2 y3 u 3 now this numbering u 1 u 2 u 3 is arbitrary okay so I have written some scheme here 1 ,1, 2, 2, 3, 3 okay I can come up with some other scheme Y 2 u and y 1u 2 and so on okay.

So typically way people or practicing engineers have been dealing with this is through so-called experience okay people have a printed plants for years and they know that if you actually do the second combination it may not be a great idea and this knowledge is transferred from generation to generation and then you know you keep doing those things so is there a way of systematically reaching a decision how do. I do pairing in a complex plant.

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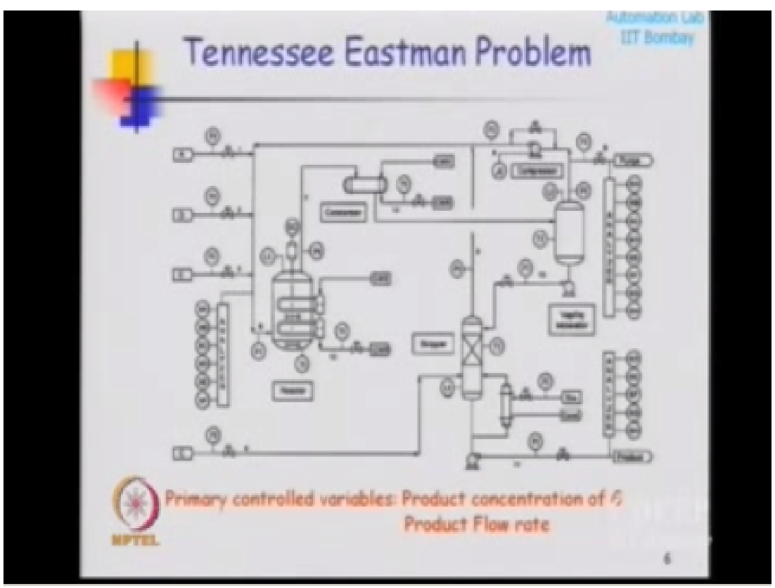

I just show you one simple example of a complex block so this is a Tennessee Eastman problem which is floated by Dennis Eastman company now well we can appreciate this plant as a control engineer without knowing what is physics this is a reactor here and in this reactor there is some reaction being carried out the reaction gives rise to product which are gaseous so these products are coming out here they go to condenser.

So you condense those projects products what happens unfortunately is that what comes out is not just the products reactants and products together come out okay so you have here a mixture of reactants and products so you need to separate them because you want the product okay so this is done in this vapor liquid separator okay and part of what is recovered is feed back through compressor to this particular reactor so I do not want to throw out you know good stuff.

Which is the reactants are gaseous and liquid and then the product which comes out is gracious what is the product Tennessee spoon has not said everythingthey have published a paper as a challenge problem in for control engineers and they just say abcdefgh okay.

So there are six components abcdefgh and they have given some reactions which you do not understand and you write to them or you go to their webpage you can download a program which will simulate this plant okay so you can actually give inputs there are twelve inputs to this plant you can give twelve inputs there are different inputs here you can see the measurements given here these are symbols of measurements.

These are the concentration measurements available at the poles and these are the concentration measurements available at the product now what happens is the bottom of this particular vapor liquid separator comes to this unit called stripper and in this stripper at the bottom you get the product and again for stripping you use this one of the reactants the C is a reactant which is used to strip some components.

Which are remaining still in my bottom liquid and rose again recycled here okay, and a product is withdrawn here now you can see here that this is a coupled system okay, what modern systems modern plants are always very tightly coupled integrated plants we do not want to waste even a you know kg of a material so whatever is might you know in olden days you may not have such couple things you just create a product and then store it somewhere.

Then you know separate it and then afterwards might think of reusing it and so on whereas now you know you do not have time you want to do it online separate unreached material put it back into the reactor to take the products and then send them for packaging or whatever so any small perturbation in this reactor will have effect on this heat exchanger will have effect on this way political equilibrium will have effect on this stripper.

So these are coupled interacting systems okay it is stupid to put you know 12 drivers12 PID controllers I am allowed to put for this system it is as you can see this going to see this you know that if you put 12 drivers driving this plant who do not know about each other it can be chaos okay so how do you choose this 12PID controllers how do you select pairing okay should I should I measure pressure here.

And manipulate you know this cold water flow should I measure pressure here and manipulate the flow out what should I do everything affects everything okay so it is very hard to make that decision okay, and we want a systematic method for reaching that decision yeah that is a very good guess very good thinking so I want to systematize.

This which variable will have maximum effect so what is the primary when you develop models say transfer function model what will you look at gain Jane will give you sensitivity yeah steady state gain steady state gain I can look at a city state with is there a trouble with using looking at steady state gains uh it does depend upon tau -yes sensitivity does depend upon tau but it let us not let us right now look at a steady state model simple suppose.

I have a simple gain model like he is saying sensitivity in some sense can it be used see if you look here the variables of are of all kinds some of them are mole fractions some of them are precious some of them are temperatures okay some of them are flows okay so what all things I can manipulate I can manipulate this Inlet flow I can manipulate this load I can manipulate this flow I can manipulate this flow this for flows I can manipulate this compressor feedback line.

This feedback line I can manipulate this particular ball on the compressor feedback line this is called purge means you let out some bleed stream because you want to maintain balance of some things that are not reacted in a reaction always happens that there are some things which come in which are not useful for example you want oxygen but you have to put your pumping in air and nitrogen comes in.

And you have to keep pumping out night nitrogen and that is done through this I mean I am giving a very crude example but you manage through this purge you keep purging the nitrogen out through or some gases out which are not useful so this purge is you can change the purge flow rate you can change the flow rate from this you know vapor liquid separator to this stripper this flow rate can be changed.

This cooling water flow rate can be changed this cooling water flow rate can be changed this product drawer it can be changed so there are so many paired variables okay what do I want to control okay I want to control pressure level temperature here I want to control pressure level temperature here C L L I PIR pressure indicator and they will indicate a temperature indicator any reaction I want to control reaction pressure reaction temperature and amount of reactants inside the reactor they will okay I of course want to control the product purity that is why I am worried about putting the analyzer here okay.

I am worried about the product purity if I am producing you know let us say alcohol they did it better be of particular quality okay otherwise it is useless for me.

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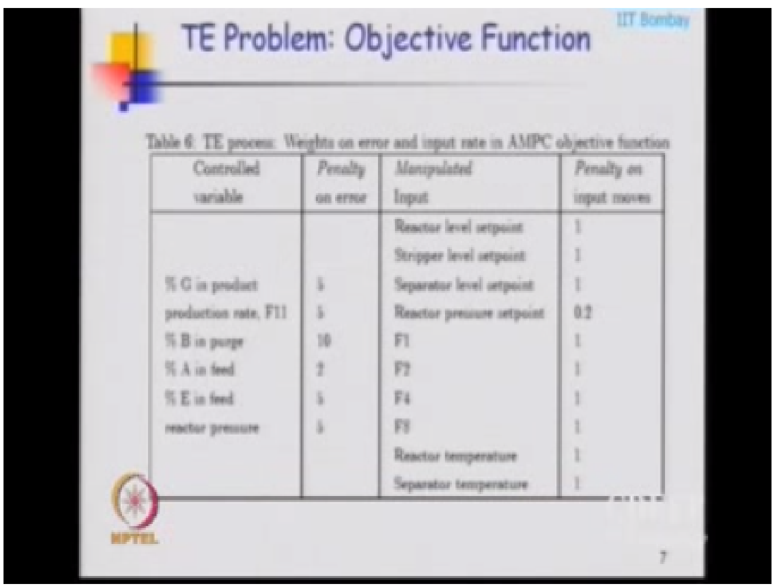

So the quality is very important so putting up how do you systematically come up with pairing of which controlled output and which money paired variable if I want to put12 PID controllers for this system it is not a trivial exercise it requires lot of okay.

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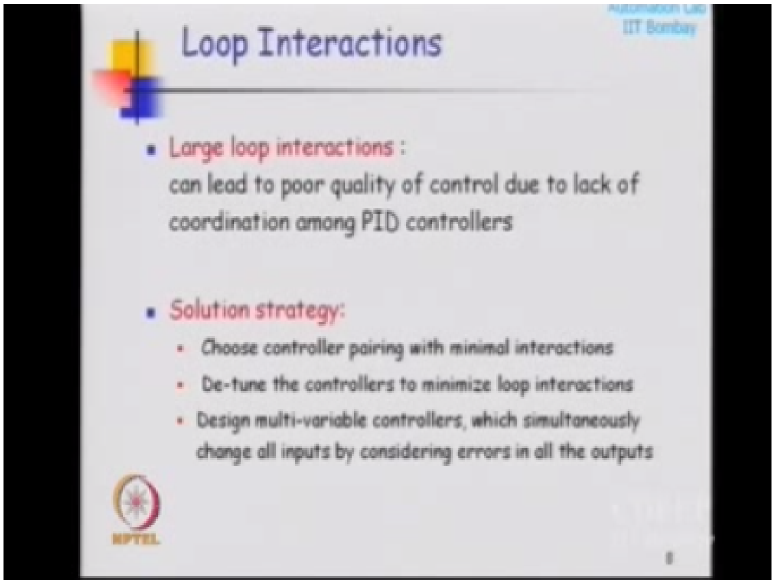

So there are large loop interactions the loop start fighting then you can have poor quality of control so is there a configuration is there a configuration of 12 PID controllers or whatever n PID controllers such that they fight least okay is what I want to find out I want to find this out mathematically okay I am just going to use simple information of gains now gains on their own are not useful that is because gain value depends upon.

The unit used for calculating the gate okay and units of each variables see pressure might be Newton per meter square it might be 10 to the power 5 okay temperature is 100 ,100 okay so if I find out change in pressure by change in flow that value might look very large the pressure is expressed but the same thing if I expressed in terms of atmospheres instead of Newton per meter square it might look very small.

So you know comparing gains of different variables becomes very difficult you cannot compare gain so easily so you need a method which is gain independent sensitivity you should look at sensitivity is no doubt but directly looking at sensitivity does not help because those values can be deceptive they can be unit dependent ok so you find a value which is per hour or a per second will give you different values and you know difficult to make a call on.

Whether this gain is high or this gain is low it is very difficult to say okay so what is done a practical way of dealing with this problem is to try to find a configuration of PID controllers such that they have you know in some sense fighting least okay and if you can come up with a least fighting configuration then you choose that configuration and hope for the best that is you try to tune them in such a way that okay.

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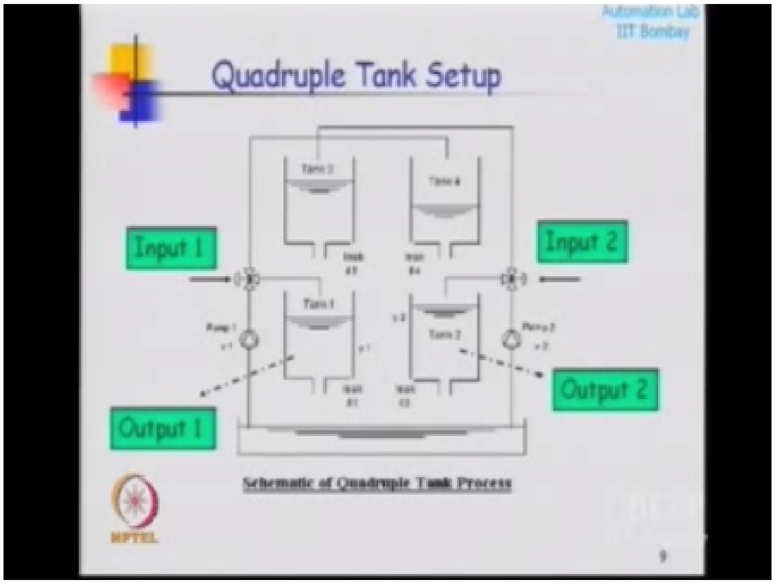

Let us take this good old example of quadruple tank ok I have two inputs and two outputs question is which input I want to put to P I control this or do PID controllers whether level one and input one or level one and input two okay that is the question.

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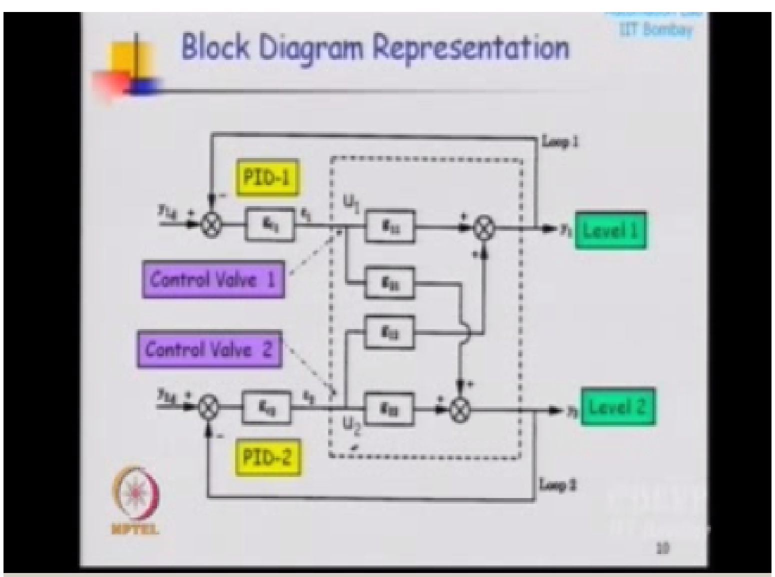

Yeah but that is multi variable controller so I am I will talk about multi level controller separate so can I have a ID controller which controls multiple output simultaneously that is a different class that is not right.

Now used in the industry it is not really the available of the shelf you can buy a single loop PID control then you can go to market and say I want a PID controller you will get one input one output at the most you can do cascade with it or some bit forward control but that is it okay though modern dcs allows you to implement multi variable PID controllers people do not know about it they still use you know a hammer to kill.

And so they have a very powerful tools in a dcs you can actually implement multiple PID controllers people just do not know about it so they just keep implementing multiple PID loops okay why because we are comfortable doing that over ears okay how will you normalize yeah but then your values also the way you normalize also will play a role now is not it how do I compare gain with respect to pressure and gain with respect to temperature.

I mean if I take temperature variable that pressure normalize with respect to what maximum but maximum of pressure means what which maximum which can occur under disaster or maximum which occurs under normal operation which maximum right see normal operation it could be that you know it + or _ PI degrees disaster maximum could be + or _ 5 degrees which one do you might use so your decision there it will influence the gain value calculations what you say is a good thing we actually do that we do dimensionless kind of gains but even then they do not help they help only to a limited point okay nowhere is the problem where is the trouble okay look at two PID controllers this is my process.

Whatever is drawn here this is my process this is the representation mathematically representation of the process okay we have seen we have seen that if I if I change input one tank one changes and tank for change level changes also that means tank to level changes okay same is true here if I change input to tank to level changes and through this leg even tank one level changes so if I actually find a transfer function matrix we have done this earlier.

If I want a transfer function matrix it was the full matrix okay and I have just represented this here graphically g11 is a transfer function between u 1 y 1 G 1 2 G 2 1 is between u 1 and y 2 2 1 is a transfer function between u 2 and y 1 and this is a transfer function between y 2 and you do okay and there are two PID controllers okay I will just try to show you what is happening what are the parts.

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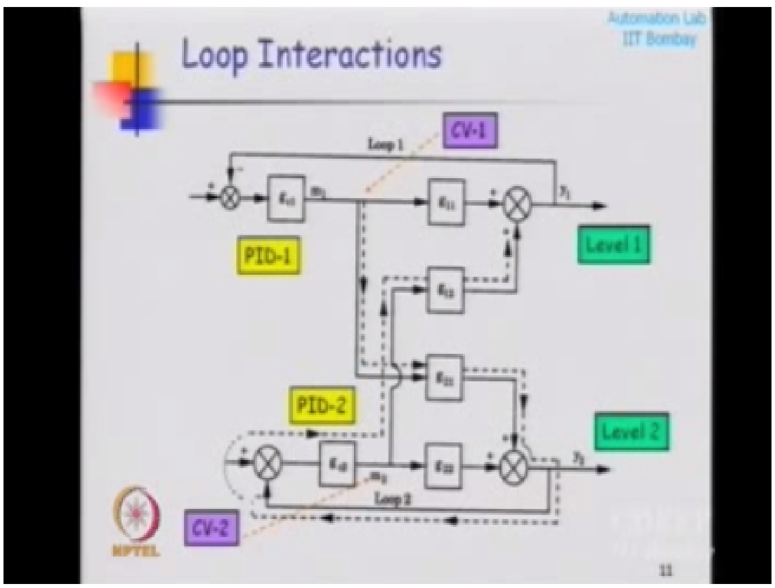

See this particular this particular look at this output of this particular. Controller m1 influences y1 through this path right direct path there is a direct path between M 1 and y1 through this G 11 and there is an indirect path which is this right to trace this path so when there is an another PID controller okay whatever this controller is doing is affecting y1 through 2 channels through two routes one is the direct route okay and other is this other is this route through the other loop and just imagine.

What would happen if there are multiple such controllers okay there will be interactions between different loops and a given loop okay and then I need to actually is everyone clear about this what is happening here is a simple very simple expression of when and typically this two controls are independent they do not know about existence of each other okay even though they are implemented through same hardware they might be two controllers running parallel.

In this computer but they do not talk to each other they do not exchange information okay if the no direction information you ask why do not exchange information that is because we started using PID controllers historically using analog hardware okay initially the PID controllers were implemented using you know pneumatic hardware yeah bellows and you know things like that Springs and by those and then we moved to electrical circuit sop-amp circuits okay.

Where gain was you know op-amp and integral and derivative was realized through capacitance entities and inductance okay so historically we have that baggage of using electronic controllers then you know what happened after that from in the pneumatic controllers to electronic controllers - we moved to microprocessor based controllers now microprocessor based controllers you do not have you do not.

What you do is you are solving differential equation or a difference equation you do not have to stick to you know those old forms why those old forms were thought of P I and D because those three fundamental forms can be realized through a physical hardware very easily okay RLC circuits you can actually fabricate and realize a differential equation you can come up with the equivalent of a differential equation you can have a value of op-amp gain.

Which is same as you are decide to gain you can have a value of L and R and C such that you know you get design integral time and derivative time and so on so those were done okay because of certain constraints that existed in those types and in 80s when we are in 90s when we moved to digital control okay so these were onboard computers which were used for doing control and they could do much more than you know.

Just implementing a PID controller PID controller is one differential equation solving it online you know is child's play for using oh my purposes are even the price preliminary ones which existed in 80s okay so that is you know highly under use of what is existing but that is because of the historical baggage we still continue to use multiple PID controllers in the plants because we have been using them for the last 50 years and we have a lot of experience okay.

And how to design controllers multivariable controllers is something which is not so well known yet yeah but it this PID controller will work only according to output y1 no see this PID controller will only look at y1 and this PID controller will look at why - it is not cross-links see this PID controller does not know when you when you do.

This it does not know that m2 will actually will have an effect on y1 which will through this loop will have effect on by - now it is just trying to control y1 it does not realize that you know if I make up change here so ideally what we should do see if I ask you if I if I ask you to control this plant.

If I give you two knobs you know they said this o actually we can we can now actually start this we have this set up in the lab we can go and do experiments so if you what will you do you will not if you if you are put in the job of doing it you will look at both the walls you will look at both the levels and look at both the walls and try to control both you will never try to you know say that okay will only look at this okay now just the - situation that you are on looking.

At one wall and he is looking at one wall and then you do not talk to each other okay you only look at you know your level which is strange okay you are reacting you know to only level one and he is reacting was it to level two and you are taking independent control actions okay so there is a problem yeah no so I came to track level two will create a disturbance in level one control okay same way attempt of attempt to travel track level one very precisely can create a problem in level two okay.

So what we need to do is that if they start fighting we call it detuning we need to sort of not tune each controller aggressively okay see you may have designed a single loop controller using some beautiful method for a single loop design it would be very tight control but you know to try to control this if they start fighting.

The trouble okay, so we need to sort of back off from two type designs to do you know there has to be some compromise which has to be struck between them first of all you should choose them properly pairing should be chosen properly okay so that is the first question.

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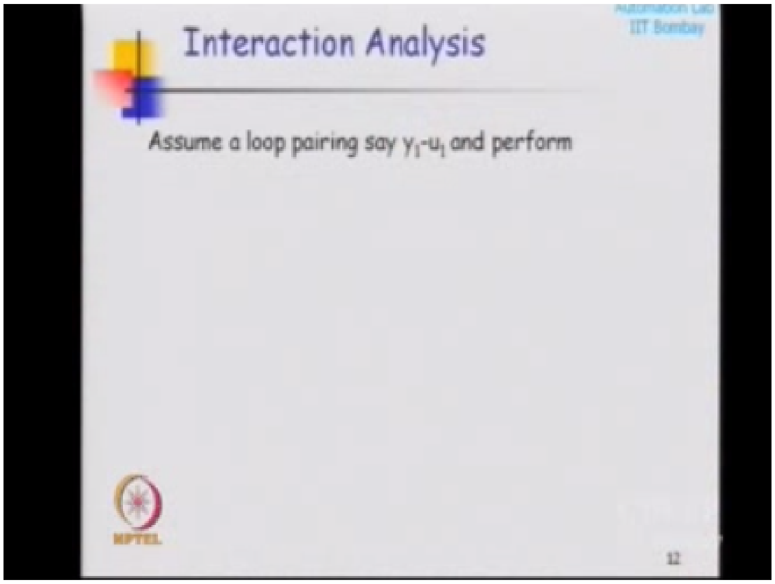

So we will actually do this using what is called as interaction analysis so we will continue looking at interaction is in the next lecture so I will talk about this interaction analysis how do you analyze. You know how do you analyze these loops behavior okay particularly in presence of other loops and in the absence of other loops can you compare and make some judgment what will happen you.

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