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ADVANCE PROCESS CONTROL

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Lecture No. 13

Stability Analysis of Discrete Time Systems

Okay, so far we have been looking at modeling loss dynamical systems. And by the end of these lectures for about 6 to 7 weeks, we know how to develop models starting from data. So we know how to develop dynamical models starting from physics, linearizing then coming up with a model which is controlled relevant. Alternatively we know how to start from data completely, come up with a model which is controlled relevant.

So now we are at a point where we have a model which can be used for controller synthesis, for controller design. And then, now we want to embark upon design of the controller. As I said controller design given that we have a good model, controller design is relatively easy task, it is not that difficult, it is not.

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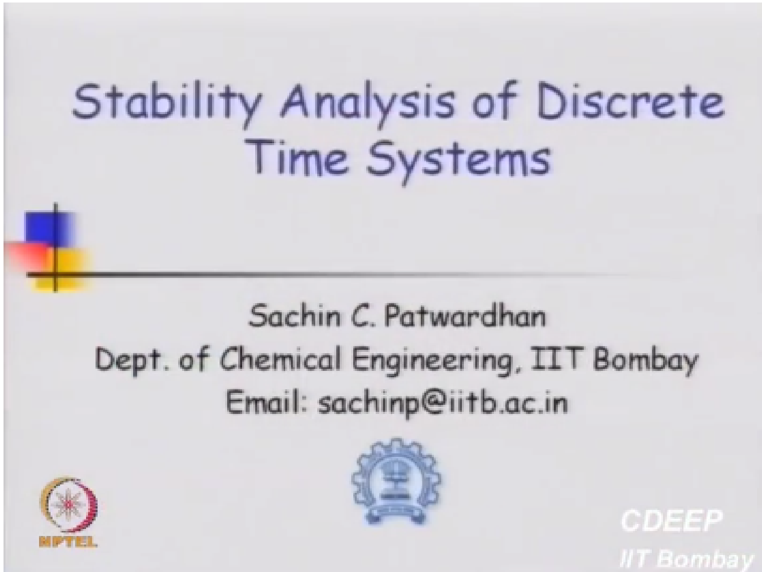

So to begin with I am going to view say one lecture on the concept of stability of discrete dynamical systems, because this is at the heart of desire okay. This is like a heart of desire, this concepts are relatively to easy to understand when it comes to linear dynamical systems. So we are going to restrict ourselves mostly to the linear dynamical systems. So I will touch upon little bit upon stability.

But that is not going to be my nonlinear system is not going to be my focus. We want to restrict ourselves into the, so stability analysis of discrete time linear dynamical systems, so that is the focus.

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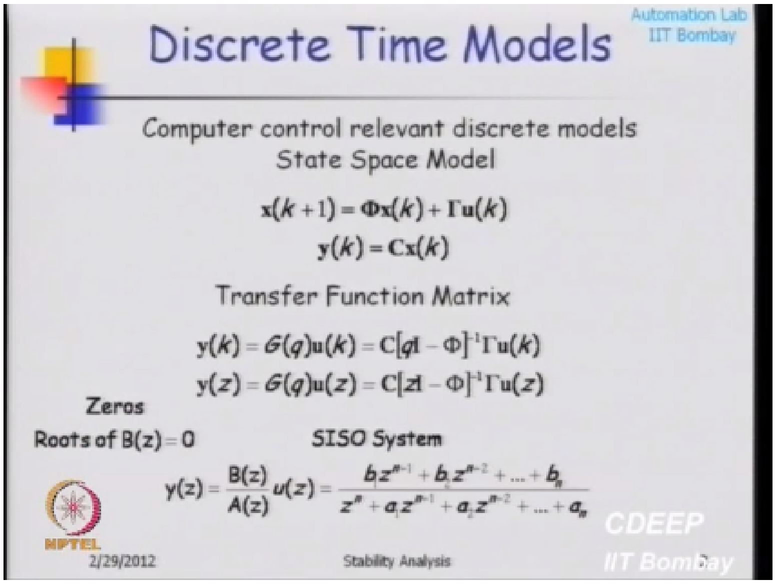

So at the end of what we have after modeling whether we are modeled using first principles or whether we are modeled using data, we have this model okay. Right now you will see that the stochastic component in the model is missing, I have just listed here modeled with respect to U inputs, known inputs okay. Right now do not bother about known, unknown, I restrict the model with respect to inputs okay.

X are the states, Y are measured outputs okay. Now X, if this model is coming from discretization of a dynamic OD model which is for a physical system, then X has physical meaning okay. If it is constructed as a realization of a time series model, then X may not have any physical meaning, may or may not have meaning okay. So we will not be able to associate some physics with X.

Nevertheless this generic form gives us, you know captures the dynamics in the neighborhood of a state remember that, this is the perturbation model, this is not a global model. This is a model which is controlled relevant only valid for a small range, what is that small region, very difficult to quantify okay. When you do perturbation studies, when you inject perturbations you can say that whatever is the range covered by a perturbations that is the – when you do linearization through teller series it is difficult to come up with that range okay.

When you do perturbation studies it is easier, because you have introduced, you know what is the sign of perturbations. And you can say at least for those perturbations this model is okay, not bad, good enough. But for linearization business we do not know okay. So we know how to get transfer functions, for this, this transfer function could be, those are in two transforms, or could be using Z-transforms.

So both of them well the form looks very, very similar you already know about this and tell this I am going to look at single input, single output systems okay. I would some of the results would send, talk about a value for multiple input and multiple output. But something's are resulted to single input, single output. So we are at this point right now, we know how to come to this point starting from physics, starting from data, that is what we know okay.

There are two things that I should both about, one is zeros of the transfer function. So I want to worry about the roots of this denominator polynomial or I want to worry about the roots of the denominator polynomial. In fact roots of the denominator polynomial is the main thing for design controller design. Of course that does not mean the roots of the denominator polynomial are not important.

They are also very, very important, but time concern when the time sees the roots of the denominator polynomial. So in control terminology these roots are called as zeros and poles, and we are going to be more worried in this lecture about the poles, we are not worried about the zeros so much.

Okay, now stability is a very, very deep conceptance not of dynamical systems. And again as I have been saying that, you know the stochastic processes which I introduced in last few lectures has matured over two centuries same is true about stability analysis. Stability analysis of dynamical systems has been studied since Newton. And what we study now in classroom probably took 100s of years to develop.

So it is not a easy concept to digest. Well, I am going to introduce two kinds of stability concepts, one is unforced stability, and other is the forced stability okay. What is unforced stability? Okay, let me just go back and view some concepts again. Is this a forced system or a unforced system? It is a forced system, the forcing function is U okay, so this is a forced system. Unless at certain situations this forced system particularly when you are designing a controller, the forced system can get transformed into an unforced system okay.

The forced system can get transformed into an unforced system. So unforced system what will be equivalent of unforced system here, if UK is 0 for all the time, if UK is 0 then you will get

$XK+1=\phi(XK)$ okay, this is unforced system. There is no forcing function, in that case okay. And so, you want to look at two stability concept, one is forced stability, another is unforced stability. Unforced stability talks about a system in which there is no forcing function okay.

Or by some manipulation forcing function has disappear, maybe you have put a controller, and your forcing function no longer exist. So it becomes unforced system or the other thing that we are worried about is what is called as input, output stability okay, this is forced stability. So these two are different concept, they are related, but not equivalent okay, there are certain differences.

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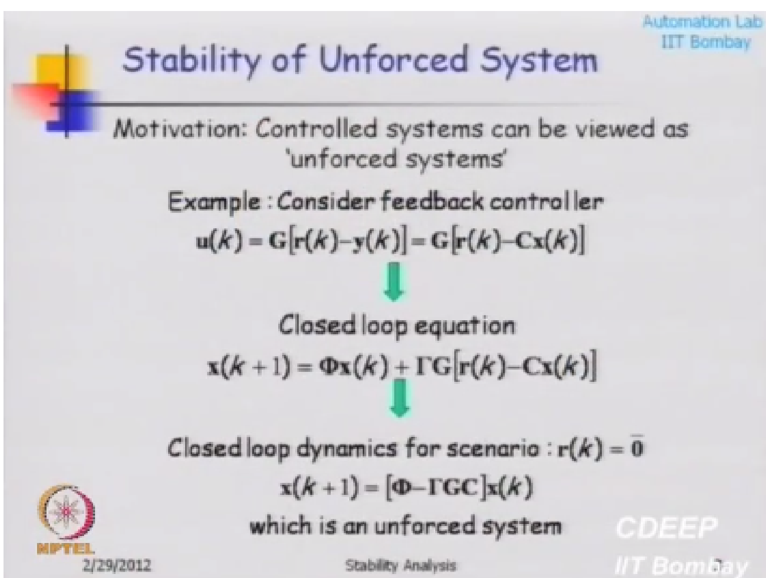

And as a control engineer we have to be aware of these okay. So why do I need to look at unforced stability, you might ask this question. Why do you look at unforced stability, we always have – you can control a system if there is a forcing function. If there is no forcing function there is no way of controlling a system right. If we cannot inject any input to the system okay, you cannot manipulate right, it exist on its own okay, like solar system, as a dynamical system okay.

We at least we as human beings cannot, at least still now inject any input from outside to perturbate right. So it works on its own, there is nothing that you can do about it, you can just observe, you can write a model, but not that you can manipulate something okay. So in a typical feedback control system, we want to put a feedback control law, let us look at a very simple feedback control law okay.

I am trying to come up with a very simple feedback control law, UK are the inputs, they are proportional to, let us take simple proportional controller. What is proportional controller? All of you know what is proportional controller from the first course in control, set point minus measurement okay, times some constant for a scalar case, for a vector case if I generalize be times some matrix.

So what I have done here is I have written this control law UK =G, G is a matrix, matrix times RK-YK okay, RK is my set point okay, YK is my measurement coming from the plant, and my control law is well. Actually let me put this internal form here, actually YK =C(XK) is some function of XK. So I am putting that here, and I am transforming this system now, I am just going to rewrite the system.

I am going to write U, I am going to substitute this formula for U into my original equation okay, my dynamic equation. If I substitute this formula for U in my dynamic equation I get this equation, I am going to call this as a close loop equation, because now U has disappeared, U is function of R-Y, but Y is function of X okay. So if I substitute I get this, now let me look at a scenario where R is 0 okay.

Set point is 0 perfectly fine, I want to control the system at the origin, at the steady-state where they are linearized okay. My set point is 0, my X will be 0, if X is 0, CX is 0 okay, I want to control my system at the origin. This is the typical problem, regulation problem in any systems, say process system. You have a reactor or you have some label in the tank, you want to control it at 0, 0 and 0 perturbation.

You have designed some operating perturbation, you have defined some operating point okay, you want to maintain the 0 perturbation, the level should be at the desired point okay, so that is what I mean here. What is happened here, is moment I introduce this control law and I set R=0, set point is equal to 0, I get an unforced system, I think you might remember this little painfully, if you are not be able to do the last problem in the mid sem. This is the same problem right, little bit of manipulation just gives the same problem okay.

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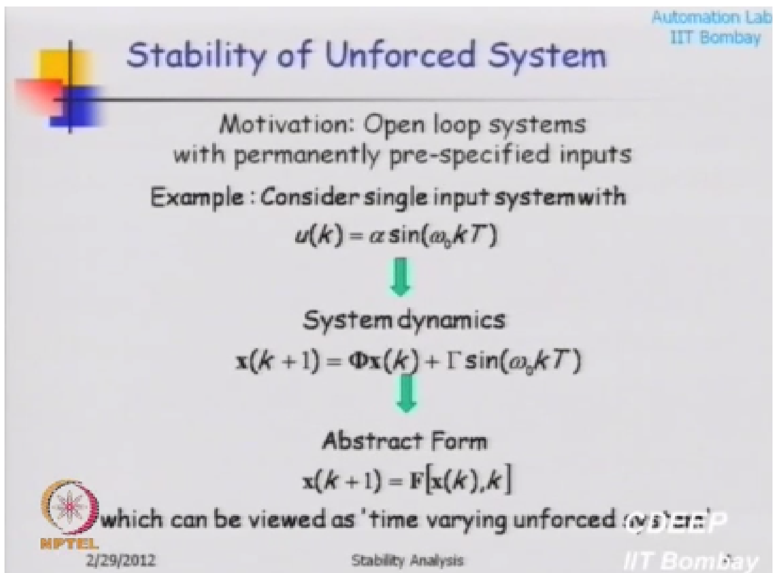

And we have already talked about stability of these kind of systems. Where are these stabled? Eigenvalue of ϕ-γGC are inside the unit circle, then they are asymptotically stable okay. So this is the unforced system and we know about the stability, I just want to – okay. So there is one more example why do we want to look at unforced stability. Let us ay, I have an open loop system, see one is close loop system.

So in the close loop systems, I get a situation where I have, you know a dynamic system which is unforced okay. And I am worried about stability of this unforced system. Well not only stability of the unforced system, I am also worried about the performance of the unforced system right. So if you just go back here, this matrix, eigenvalues of this matrix govern the dynamics okay. And here what is it that I have introduced as a control engineer?

Matrix G, matrix G is my choice okay. So I can move around eigenvalues of this matrix by appropriately choosing G, if that is a controller design problem. I choose G in such a way that poles of this matrix are at desired location right that is my controller designed problem. The other situation is that I have a system, let us look at a signal input system. And let us say the input is a sinusoidal function, it is al though a sinusoidal function, input to this particular system is always a sinusoidal function.

Let us say a circuit which is subjected to some, you know AC current permanently subjected to the AC current. I do not have, I cannot manipulate the input is always this AC current, and I want to worry about a dynamics of this particular system okay. In this case my dynamical system becomes like this right. This part here is fixed okay, again this is an unforced system, because this component is not in my control, this input is entering the system okay.

And I have to worry about – so I might generally say that I am really worried about unforced dynamical systems okay, in which the right hand side would be function of XK and in general of time K, K is the time here. So it could be in general function of time K, the previous example K was not explicitly appearing in this particular example K is explicitly appearing in my equation okay. So this is a time varying unforced system.

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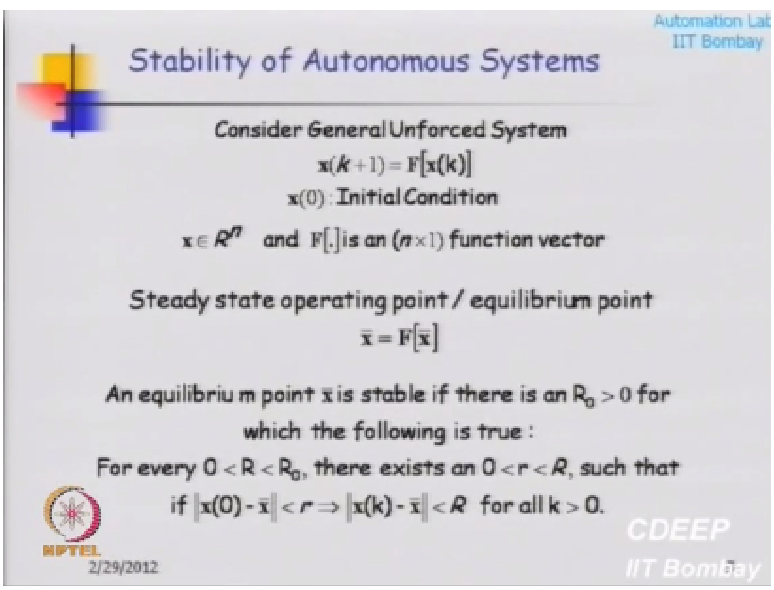

So general problem which we would be worried about is okay, but I am not going to look at least for this course, I am not going to look at the time varying unforced system that is more complex okay. I am going to study simple things, introduce concepts regarding unforced system okay, in which time does not explicitly appear okay. Now these things of course, we need to study in the real problems, but we are doing the first course on advanced control.

So we can excuse ourselves and just restrict to the simple ideas okay. Now this is a general problem, I have this dynamical system XA+1, it evolves according to this nonlinear difference equation okay. Where F is some nonlinear function, and X0 is the initial condition which is away from the equilibrium point, equilibrium point for the system, let us assume X00, perturbation system X is a perturbation model, some it could be linear or nonlinear okay.

Where I want to introduce this nonzero equilibrium point. What is the steady-state of this point system? When do you say a system at a steady-state? XK+1 is same as XK, I have written it by saying that Xbar is equal to F(Xbar) okay. So Xbar is my steady-state operating condition okay, and Xbar is equal to F(Xbar) is my – okay. So now this is where you are going to blink and then you are not going to be comfortable okay. Let us see what this is.

When do you say a particular system to be, let us before we look at the theorem statement, this definition sorry, definition of stability. Let us take a famous example which is often used to teach the stability. I will also take a chemical engineering example later, but a simplest example is a pendulum okay. Now a pendulum, see a pendulum can be, you know working like this okay, there is one more configuration for the pendulum which is inverted pendulum okay.

I could have a pendulum which is like this okay. So this example of inverted pendulum can be looked upon as a idealization of a scenario where, you know you have a rocket, space launch rocket taking off, and before it is taken off, you know it is just balanced on a platform right, it is balanced on the platform, it should not fall by taking off, it should remain vertical. And then go vertical, and then go into.

So it is actually, if you see, if you have seen those clips, we will see that before the take off it is clamped using some supports okay, you do not allow it to move. But you cannot do that when the rocket launches. So you have to remove the support okay, and then balancing that rocket on the platform below is same thing as balancing a stick on your hand, it is not different. There is a an equilibrium point where the rocket exactly stands vertical okay, provided the vector from a center of mass goes to the base and all that.

So if that does not happen the rocket might fall and then it might hit, you know some other country other than going into space. So that is the difficult problem okay, but so this is a valid configuration, we need it, we need this configuration and we want to control that configuration. So this configuration is the other configuration where, you know pendulum is moving like this, what will happen is if I put up the pendulum, the pendulum will come back after some time okay.

If you have a perfect pendulum no, no dissipation of energy okay, what will happen, it will keep oscillating okay. If you have a real pendulum where energy dissipates what will happen, you know you move the pendulum and leave it after sometime, you know it will, the oscillations will die down and it will come to the steady-state. What is the steady-state, you know it is vertically down that is a stead-state, it comes back to that steady-state.

So if I look at Xbar = F(X) for the pendulum system, there are two possible solutions, actually physically two possible solutions. Theoretically there are infinite solutions, because, you know this is a steady-state and so if you rotate once, this is also a steady-state, and then rotate twice this is also a steady-state. So mathematically, there are many steady-states, physically there are two steady-states. One is up, one is down okay.

Now how do I mathematically quantify this idea that if I perturb a little bit, the system comes back, the system comes back to its original steady-state okay. I want to mathematically express this idea, what will happen here in this case, there is no control. Suppose, you know, you are see, if you are 5 this pen is there on my hand, and if I am allowed to move my hand, I am using a degree of freedom, I am using manipulation to control this system okay.

But if I am not allowed, there is one perfect steady-state for which, if the system is at that steady-state it will remain at that steady-state. But what will happen if there is a slight perturbation? It will solve okay. I want to quantify this mathematically, mathematically the examples from chemical engineering, there are examples from other domains, like examples from other domains, like examples from chemical engineers is reactors, where you have multiple steady-states.

Typically what happens is that the reactor would have multiple operating conditions okay. So there are two operating conditions which are stable, one is low temperature where you do not get a product. One is high temperature, where you get good amount of product, but the temperature is very high, you do not want to control at that point. And in the middle temperature where you get less amount of product, but the middle temperature is, you know not steady. It is unsteady state point.

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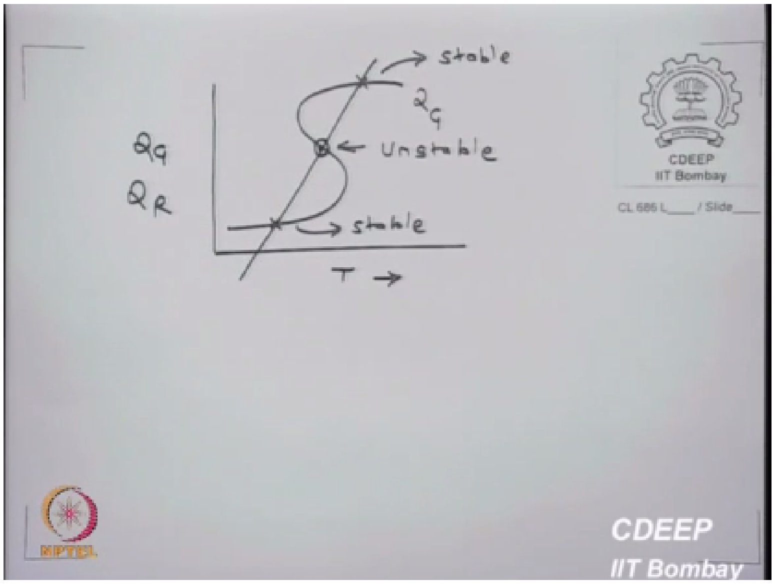

So actually for a reactor if you take this you generate it, and it released okay sorry, heat removed. Then heat generated is actually a S shaped curve okay. This is Q generated, and heat removed okay, there is a cooling jacket, and you remove the heat through the cooling jacket, and heat removal is proportional to cooling water temperature minus the temperature inside the reactor. So heat removed is proportional to the T versus QG, then you get three steady-state one is at a lower concentration, other is at the higher concentration.

And this one is a unstable steady-state, plus steady-state, and this is stable steady-state, and this is stable steady-state okay. And you want to control the system at this point okay, controlling the system, controlling the reactor at this point is same thing as controlling the stick, balancing the stick, inverted pendulum, no difference okay a slide movement will take the either to this point or to this point okay where will the system go if it is uncontrolled you do not know so to make the system stay at this point you have to apply a controller but this point is an unstable point okay now how do we understand this concept of how do we quantifier this

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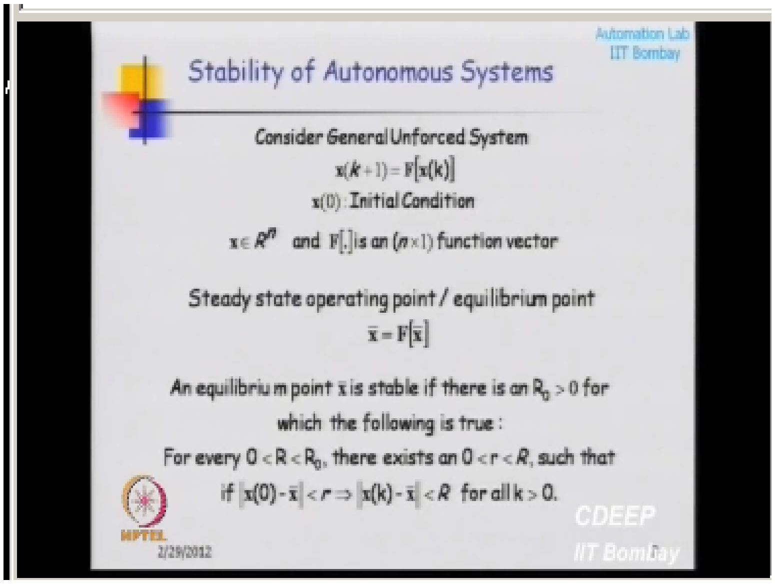

Okay so we are going to use a basic definition of continuity of a function if you see carefully this is a modified definition of conductivity okay so what I am saying here is that if I port up the system if I come up with the region suppose I am looking at a pendulum okay I look at some region okay in the neighborhood of the steady state okay which say I will give you that this region is where the movement motion of the pendulum should be restricted okay.

Now if I give you this region okay you have to tell me state of initial conditions for whi9ch the system dynamics will remain bounded in this region okay so think of it as some game you know I define the region okay I define this much region and then I will say that tell me all the initial conditions for which if I take my pendulum the system dynamics will remain within this box okay so suppose you come up with a set of initial condition I will say okay I am not okay I will string okay can you give me on the side on the initial condition for which the dynamics will remain inside.

This is okay you will come up with one more set of initial condition for the pendulum so dynamics inside this I will go on shaking this okay so whatever region i9 give if you are able to come up with a state of initial condition for which the dynamics will remain within this or within this okay then they say that the system is locally stable okay whatever region you come up with what are the region I come up with you have to give me set of initial conditions if that is possible for a particular system.

See what I am saying is for every hour greater than zero okay xk-x is my steady state xk-x is the evolution of dynamic evolution of the system okay if this remains less than this big R what I am giving you okay and for this system to be remain bounded in this region if you are able to find a state of initial conditions okay this is x0-x is a initial condition for this initial condition okay the system will let us go back to this okay let us see whether we can understand it from see.

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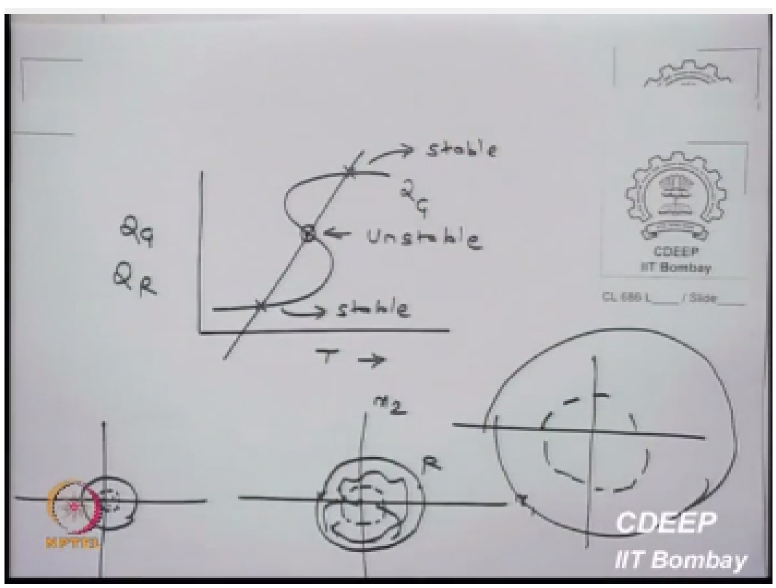

See suppose this is the point this is x1, x2 and initially you know I will give you that this is the region in which all the motion of the system x1 and x2 should remain bounded. So I wanted to be remain bounded here okay, I wanted to be remain contained in this region okay, so I have given some R okay, can you find out a set of initial conditions let us say this is the set of initial conditions for which if the motions starts from here from any point till here okay, the motion starts from any point in here it will remain bounded in this regions okay, it will remain bounded in this region.

If I happen to change this you know if I say that my R is this much still you should be able to come with some set of initial conditions you know if I shrink this if I say that my R is not big my R is very small you should be able to come up with set of initial conditions. For which if motion starts within this points it will remain bounded in this region okay, if motion starts within this point thus region it will remain bounded in this larger region, okay that is what I am asking.

Is this possible for every R whatever R I am saying it should happen okay, now why this every R is so important okay, if it happens for just one R is it good it is not good see for example, if I define my R which pans this region entire region okay, see for this system what will happen for this system the reactor is here it will either go here or it will either go here, okay. So if I give my R which pans both of this then you will say the system remains bounded and it is stable, right system remains bounded because if my region is we large okay.

Then there are set of initial conditions for which so shrink the limit bounded in this okay, but if I start shrinking this if I start shrinking gets there is a problem then you cannot find a set of initial conditions or which system will stay inside that boundary, okay so this is what is actually quantify by this definition.

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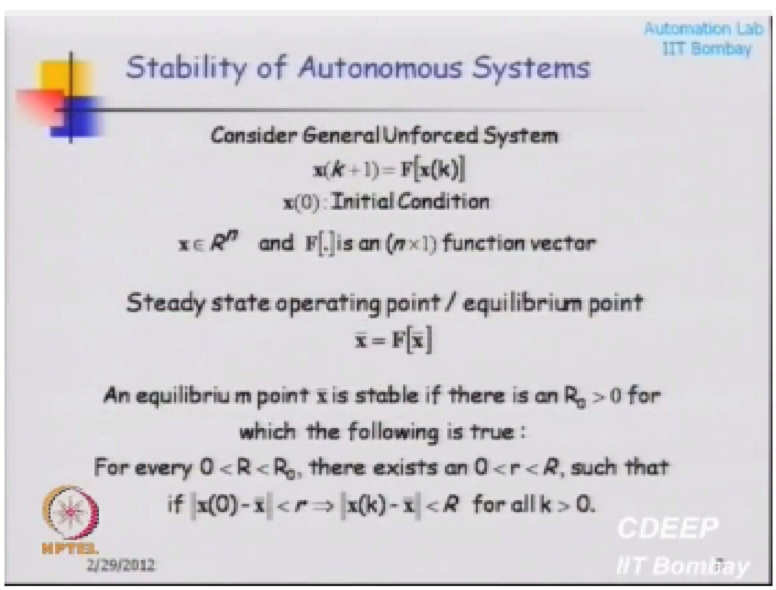

That given any R given any region can I find a set of initial condition for which the set dynamics will remain bounded in that region, if that can be done then we call the system to be stable unforced stable okay, also sometimes called as BIBO stable, okay.

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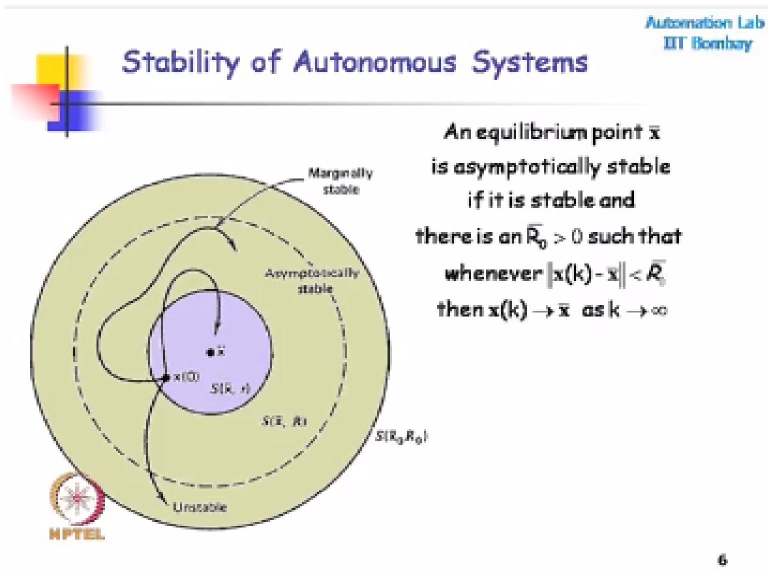

This is picturization so given any initial condition or given any bounded region can I find a set of initial conditions or which the motion will remain bounded within the given region okay, in fact when you start looking at stability you get tempted to think the other where out okay, you start saying that given a initial condition can I found the region that is not a way correct way of thinking in fact that was mistake which was made historically when people started looking at suddenly they first said well, given a initial condition can I found the region in which it will remain okay, that is not there might be one region in limit might remain you know, what is important is so you know this the question is if I perturb a little bit will it go outside see if I define a region which covers even this steady state.

If I define a region which also covers this steady state then you will say that system is stable but actually you know the system is not stable, so this definition will not up the other way out okay, only way it will work is saying that for every small region so you know I am not interested in the region which covers this steady state and this steady state I am here right now I give a small region around this. Now can you find out a steady state, can you find out a initial condition for which system will stay here except at the steady state there you cannot even move a little bit even if you move a little bit this will fall.

This will relieve the region okay, so that is very, very important now think about this look at more examples this is not a thing which you will go down so easily. Well, we have another concept which is asymptotically stable, asymptotical stability comes if not only if the dynamic remains bounded within a region but the trajectory goes to 0 as all trajectory goes to the steady state as k goes to infinity.

See if it happens that this trajectory is starting from x(0) and not that is just remains bounded but it also converges finally to the steady state that will happen for a pendulum right, it will you perturb it okay it will oscillate and come back to the steady state wherever you perturb it okay it will oscillate and come back to the steady state so pendulum which is not inverted the regular pendulum is asymptotically stable system it will come back to the but a pendulum regular pendulum which has no damping is not asymptotically stable which has no damping what is meaning of no damping it will keep postulating it will oscillations will not grow oscillations will not died on okay they will not grow they will neither grow not ideal.

So those systems are marginary stable systems neither oscillation is grow norm they write down if the oscillations eventually died on as k goes to infinity okay then those are called as asymptotically stable systems okay, or the equilibrium point is called as let me device we cannot say system is asymptotically stable or system v misused this terminology for linear systems because linear systems stability concepts are global you know everything either it is globally asymptotic stable or globally marginally stable or globally unstable okay.

In real non linear systems you have to talk about local stability in neighborhood of a point x bar okay, that is what make sense the real non linear system okay, so one thing that because we are introduce to stability in the control course where you start looking at linear dynamical systems one thing that we start thinking about is that unstable system means something going to infinity okay the trajectory should go to infinity that is not what it means it only means that the locally that point is see the pendulum dynamics it does not go to infinity because you have a unstable point okay.

So that is only ideal world of linear systems it happens that trajectory goes to infinity in reality all that it means is that is the point where local any trajectory does not tend to stay it is point where system or the tendency in to go away from that point that is all unique, so stability is a local concept in the context of non linear systems or kinds of real world systems and we can talk about an equilibrium point to be stable or a equilibrium point would be unstable okay that is that makes more sense okay.

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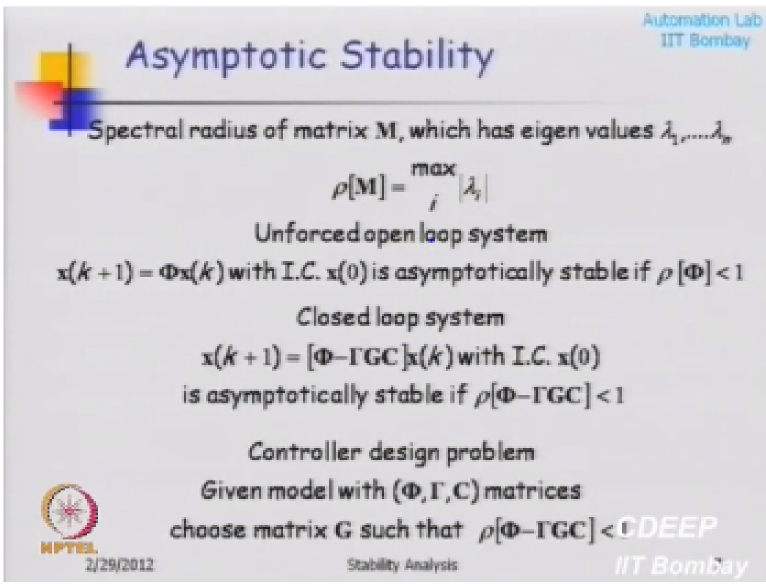

Well asymptotic stability of equilibrium well we know something already we have done this when is linear dynamic systems stable spectral radius is less than one strictly less than one it is asymptotically stable if it is less than or equal to one it is marginary stable and it could be you know one of the Eigen values is equal to one when or pare of Eigen value is on the imaginary access it is marginally stable it will okay.

So we know this result that spectral radius if it is less than one so we can applied it in the different context and both the context now you are aware of right you could applied to open loop systems if you with 0 you could apply to the close loop systems if u is function of y intent function of x you know I could applied to, so of course my controller design problem is to chose g in such a way that you know controller design problem is to chose g in such a way that spectral radius is always less than one okay.

In fact I might decide and say I want it to be 0.1 okay, so that rate of change of error will also or the rate of change or the rate of auto measure system written to the steady state also we will get defined if you specify the poles now there is a problem you know this result which is very powerful for the linear systems cannot be taken to non linear systems as it is okay, is a difficulty.

You can use this result for non linear systems through local linearization we have done local linearization we started from delusory is non linear model locally linearized now the question is well a golden question analysis of the local linear models does it also hold for the global non linear model it is a relevant question, because ultimately you have going to base your design decisions and the local linear model but the true system is not local linear you know local true system is non linear okay.

Now trouble is you can carry over these results of linear, now this results which are listed here okay these are for ideal linear system ideal linear world which we have created as you know applied mathematicians not it does not exist anywhere it exist only in our imagination and text books okay. There is no perfect linear system anywhere in the world always in approximation which.

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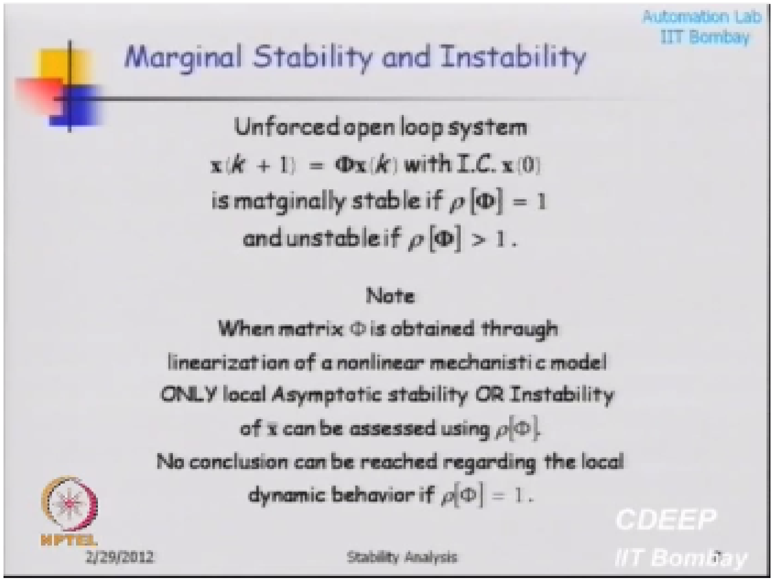

Now we also know this that open loop system is marginary stable if ρ = 1 it is unstable ρ > 1 okay now the trouble is this results or analysis based on local linearization can be used only for two cases if locally matrix or locally system is asymptotically stable or if locally system is unstable okay if locally system is asymptotically stable then behavior of the local linearization and non linear system is you know you can make some common based on the local linearization you can make a commend upon non linear system behavior okay.

If locally system linearize system is unstable then the non linear system is also locally unstable okay, but if local linearization is marginary stable I cannot say anything about the non linear system this is a trouble okay. I cannot translate results okay so marginal stability you cannot establish using local linearization you can establish asymptotic stability we can establish instability but not marginal study okay.

And there are marginary stable system which we keep using all the time is not that marginal civil systems going to exist you must have seen old pendulum clocks, pendulum there is a marginary stable system we neither dk is okay and nor it grows right, so oscillators u oscillators you know at so stability of such marginary stable systems which are actually non linear cannot be establish to linearization asymptotic stability yes you can okay in stability you can.

That is why you are still able to use linear control theory and design controller for real world non linear problem okay but it has a limitation because marginal stability of linearized model cannot say anything about the stability characteristics of the non linear system okay. I will give an example where linearization says that the system marginary stable but actually the system is asymptotic to the state okay.

So if linearize system says marginal stability you cannot draw any controller linearize system says asymptotic stability yes the non linear system will also locally asymptotically stable okay linearize system says unstable non linear system is locally unstable okay.

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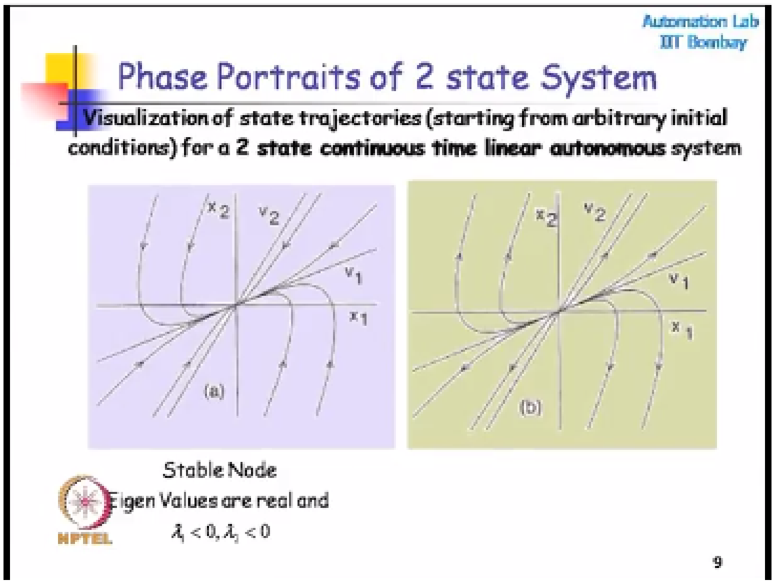

I will just give you some visualization of a two dimensional system this phase pain pirates is something which is very often used to visualize dynamical behavior of two states establish okay, so I am right now talking about continuous time systems so if you have a system a non linear system or a linear system for a the time mean where is a linear continuous time system two differential equations okay.

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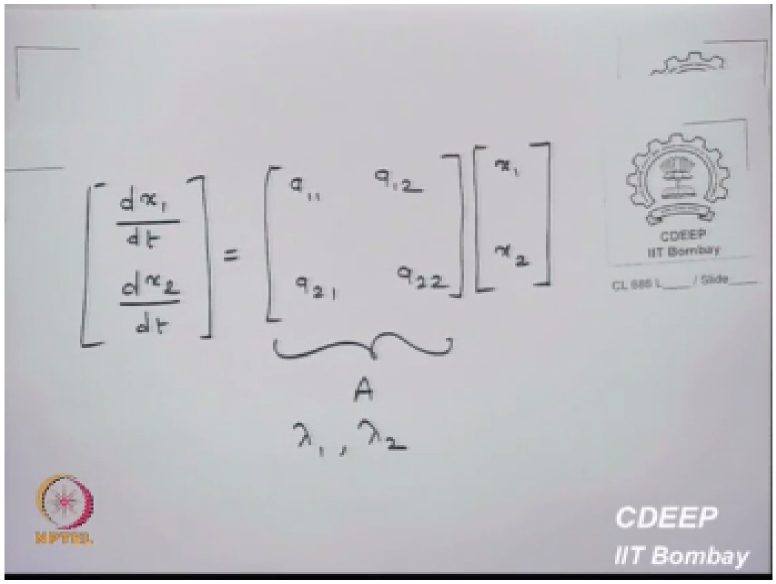

So I am actually worried about for the time being what I am going to show you is dynamic behavior of this is my a matrix okay for the time being do not worry where this estimate come from you have this system or you analyze the system okay what you know is that the dynamic behavior of this system is given by Eigen values of a matrix right so a will have two Eigen values λ 1 and λ 2 right two will have I have two Eigen values λ1 λ 2 of course if you want you can list this and convert in to a discrete form but there is a mapping between continuous and discrete and results of dynamics will not change okay.

So this is just a visualization of what is happening it is important in the view point of you know I am just putting this figure to enforce the concept that these are local behaviors okay so one thing that we look at is a system which has two Eigen values which are you know negative real part.

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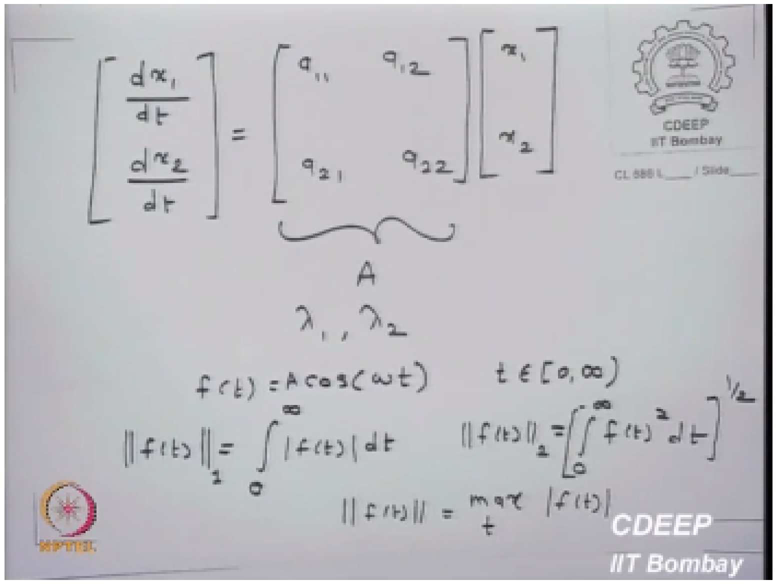

I am talking of continues time system right now okay for a continuous time system where both the Eigen values have negative real part what happens? Mathematically we say that the solution dk is 200 asymptotically stable pendulum okay the motion comes back to the steady state okay, perturbation state is steady state so you know any trajectory that starts in the state space these are different trajectories in x1 x2 plain I just plotting the time trajectory is an x1 x2 plain you start from some point the trajectory will eventually come back to the original okay you start some other point it will come back to the original.

So locally because the Eigen values have negative real part all the trajectories will come back to the original so this is called as stable node other possibility is unstable node, what is unstable node? All the trajectories diverge from this point okay, nothing stays here okay that is why I wanted to visualize when so if you are perfectly here see pendulum if I am perfectly here at 00 it will remain at 00 but any small perturbation will take the system away okay in reality for the pendulum it is not going to go to infinity.

So do not associate in stability means something going to infinity that is not on the thing okay it only means that this point is such that nothing stays at this point you know a system tried to run away from this particular point, no this is for there are slight differences for second order system.

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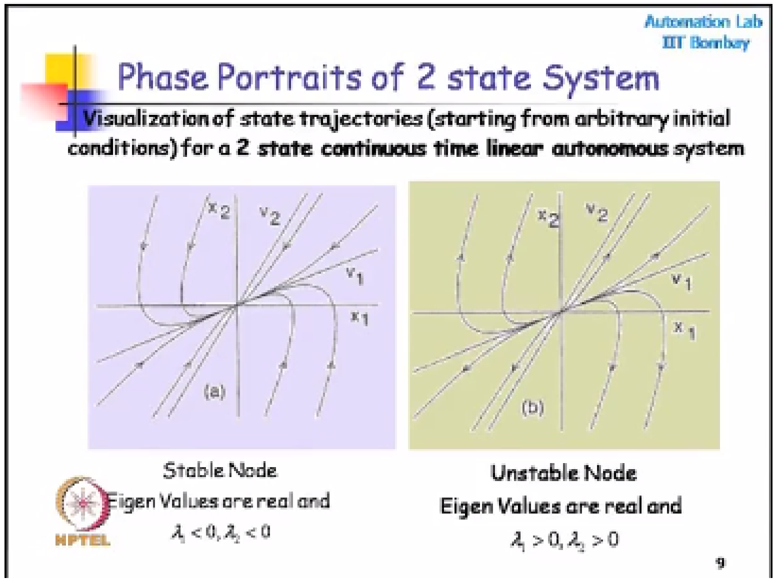

You have something call saddle point so if one is so it is unstable but saddle point there might be certain directions in which system will not become unstable so if you have this situation that one Eigen value is less than 0 and one Eigen value is greater than 0 okay and the system is you know somewhere in between actually it is unstable system technically because okay but there are some trajectories okay if you happen to latch on to those trajectories you will not leave those trajectories so you will not go to infinity.

So that see here what happens is that there is only one point where you know if you are there you stay there otherwise any perturbation from there you will go away okay.

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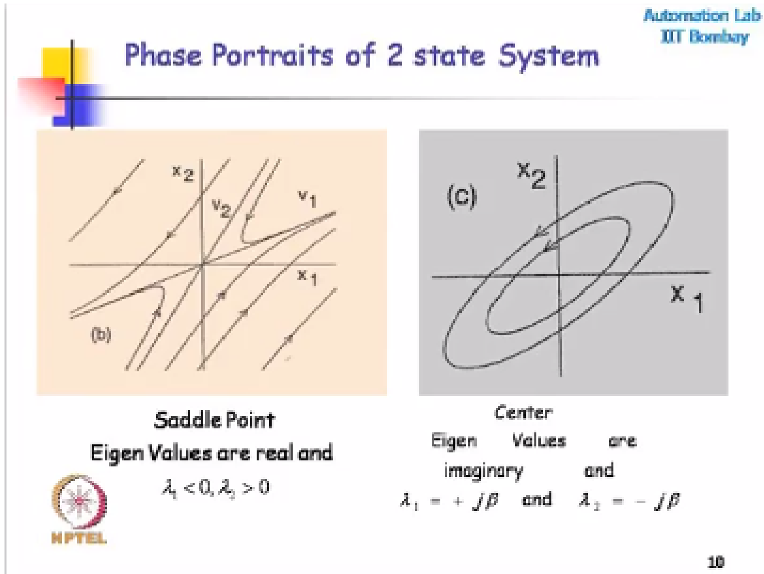

Now for a saddle point what happens, if you happen to stay I think on this b1 sorry b2 if you happen to stay on b2 then you will remain on b3 okay but anywhere else it must tend to go away so little funny situation okay so this is only mathematical procedure not there the real system which are one Eigen value which is outside unique circle or one Eigen value which is right of plain system is unstable of practical purpose.

But this is a final point which this is classified is saddle point well the other thing is you know center, center is where both the Eigen values are complex okay, what is the equivalent thing in terms of discrete time here one Eigen value is inside unit circle one Eigen value is outside unit circle here both the Eigen values are on the unit circle okay.

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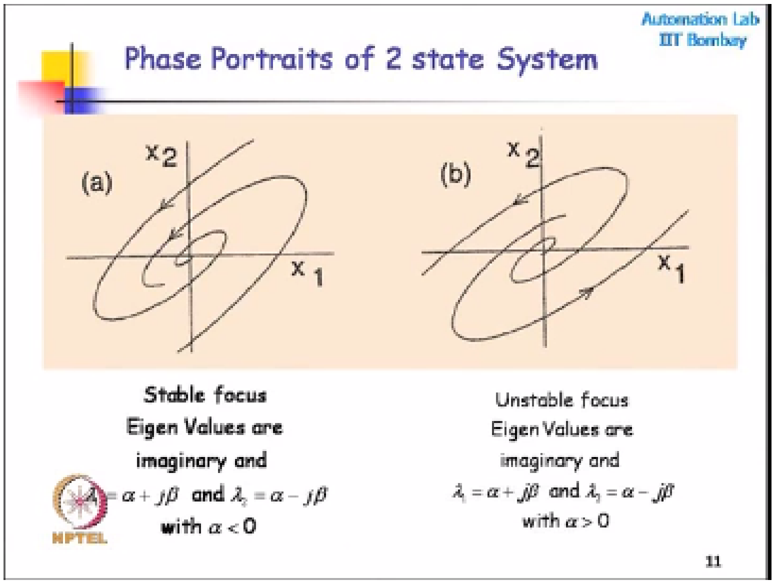

Then you again differentiate between unstable systems so you have a stable focus a stable focus is where your complex Eigen values but the real part is negative okay pendulum you know it will oscillatory behavior but oscillation is die oscillation is don not grow okay. So unstable focus is one from which locally you know the system diverges but in a oscillatory manner okay so that will be an unstable focus so these are local behavior that is what I go to emphasize through this graphs nothing else okay.

We are looking at locally at some point then neighborhood of that point we are trying to asses everything of these cases we can analyze through local linearization of non linear system expect this case if you take a nonlinear set of system two differential equations to non linear differential equations come up with linearization and then linearization tells you that actually it is a center you cannot believe that result you cannot extrapolate that to the non linear system. So whatever you come up with analysis with linearization only holds for all cases except center okay.

Is one more concept which is very crucial in control theory, so till now I talked about un - force stability okay, so un – force stability can arise because you are lost degree of freedom how did you lose degree of freedom because you are put a controller okay, the second thing is you know in out and output stability when I want to talk about stability concept when I have some degrees of freedom okay and I still going to know about behavior of the output with respect to inputs okay mind you there was one difference earlier we were worried about the evolution of states okay.

I was only talking about x whether x is a physical meaning or does not have physical meaning is a different story I was talking about behavior of x okay in input output stability I am going to be worried about y and u, u is my input y is my output I am only worried about observed states or observed behavior okay, and not just say I have a boiler okay there are so many states inside the boiler okay we should write a differential equation and then come up with the model you will get many states for temperature and level.

And what not if I am just measuring level and temperature okay and if I am giving two inputs one is fuel other is pole water flow I am only worried about behavior of you know level and temperature with respect to pole water flow and fuel flow input output I am not worried about what is happening inside I am not saying that I am not worried about I am not talking about it when I talk about BIBO stability.

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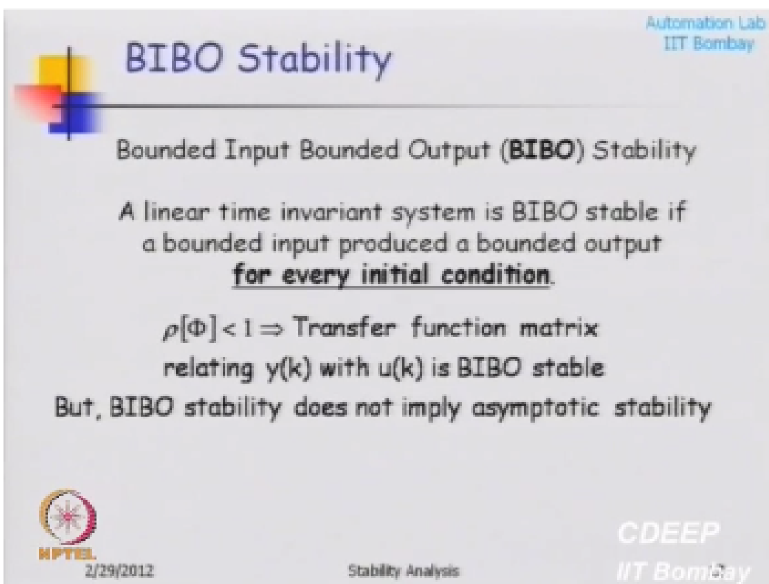

Well you might say that why I am introducing all these mathematically find motion we cannot help it control theory you are dealing with dynamical systems and you have to understand this final points otherwise okay, so BIBO stability is a linear in variant system is BIBO stable if a bounded input produced a bounded output okay if I give a bounded input now what is bounded good question okay because when you say when you call a function bounded we are dealing with functions input will be a function say cos function okay or you know input is ut is step function so is step function a bounded function it depends on what?

Tell me improve upon it, Sumitha, something else we talked about in the last course what about norm? I could talk about a norm of a function okay; if norm is bounded then the function is bounded okay. So bounded input bounded outputs are actually gets tide up with the norm of a function, so question comes what is a bounded function okay now is actually magnitude is not a bad idea but is but you have to put it in the right context now each step function are bounded function how to define roundedness?

Suppose I give you norm of a function so now I am talking about a function ft okay which say which is equal to a cos t a cos ω t okay now t belongs to 0 infinity okay. What is norm of this function, how do you define a norm, how do you say this function is bounded or unbounded? Integral well you could define different ways you could define one norm you could define two norm you could define infinite norm okay this function might turn out to be bounded according to one norm but not according to some other norm that is possible okay.

See for example this function I can define a norm of ft as integral 0 to infinity node ft so norm of ft you know I am calling one norm is equal to 0 to infinity dt, what is true norm right what is ∞ nom max over a t mod of ft okay so these are three different nom okay now let me whether step function is bounded according to one now you need step function no it is not bounded according to 1 it is bounded according to ∞ now yes okay so this bounded input bounded output actually also depends upon the way you define the nom typically you would use ∞ nom will say if you decide to ∞ nom then according to ∞ nom step function is bounded function cosaωt is the bounded function.

Okay you can have very simple definition it give a region the system input function remains bounded in that region then it is bounded according to infinite nom not according to 1 nom or 2 nom okay it can but when I have to choose mathematically use have to be the reason for that so yeah but what I wanted to point out was that it crucially depends upon how you define what is bounded.

Okay it is not that so exponential function is bounded according to 1 nom e^{-t} as t goes to ∞ or e-18 is bounded function according to I nom according to 2 nom according to infinite that does not mean step function is bounded according to all these so this is convenient to use the infinite nom when you are doing this divorce ability okay now this part is very important here okay this last thing which I have underlined very important.

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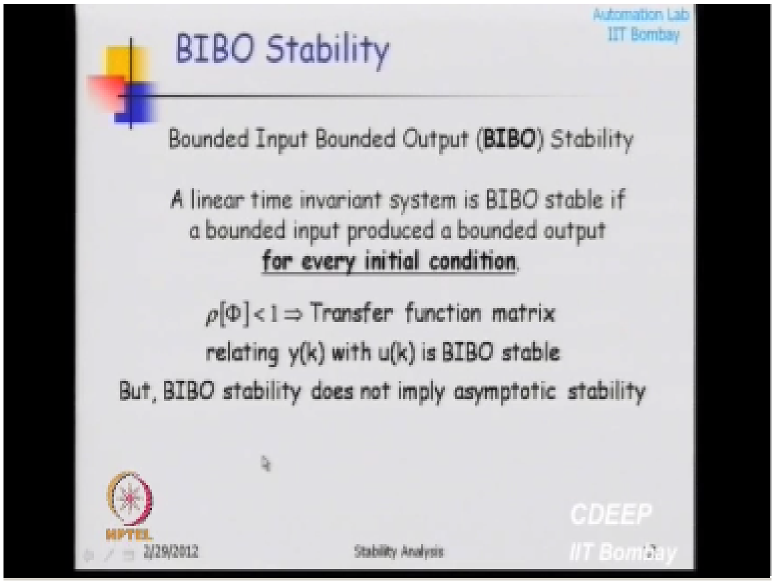

It should happen for every initial condition okay so one way to find system is unstable to find regionally condition for which this will not be done now there are final things here if you have a system which is a you are talking about a state place model and overcall spectral radius is less than 1 then you know then the system relating transfer function relating y and u in table why is that.

Because what is the relationship between the denominator of the transfer function and poles of 5 Eigen values of 5 they are same Eigen values are 5 is nothing but the characteristics equation okay so if one of if all the Eigen values are inside the unit circle then but I am what I want to say here is that a system which is asymptotic stable in the sense of enforce system is asymptotic stable okay if you take this.

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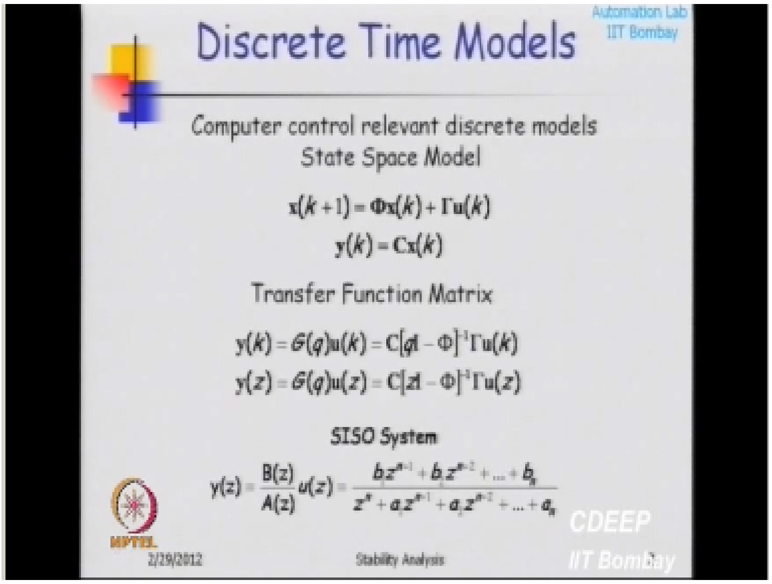

Let me go back to this so when I am talking about stability I was talking about only xk+1=ϕst okay when I am talking about BIBO stability actually I am worried about this stability of this transfer function okay are the systems for which the transfer function is stable but the state dynamics is unstable possible that is possible okay unstable or marginally stable if possible that there are four 0 cancellations.

And you know you know you have system which is the state space dynamics is partially stable but this is BIBO stable so this kind of things are possible now the question is if spectral radius of ϕ>1 that means all the roots of this equation are inside the unit circle okay and then we can talk about you know you can say that if this system is asymptotic stable in the sense of unforced system they are asymptotically stable then this transfer function is BIBO stable okay if this system if this ϕ as all the Eigen values inside the unit circle.

Then this system is bounded input bounded output same vice versa is not true suppose I do not say anything about this I just give you the transfer function in a transfer function that you found that the transfer function is bounded input bounded output stable input can you say that you know all the Eigen values of ϕ will be inside the unit circle that is trigger thing you cannot go other way out you can say that if this is asymptotically stable this is BIBO stable you know the original system can be marginally stable or something.

So there is a problem so that one way thing works in other way it does not work observable do not confuse with this observability will talk later on this right now there is no question about observably Bilbo can be so what I want to say is that symbiotic stability is the stronger concept then BIBO stability so a symbiotic stability implies BIBO stability but BIBO stable implies stable okay so this is an example where this is an example of an harmonic oscillator okay

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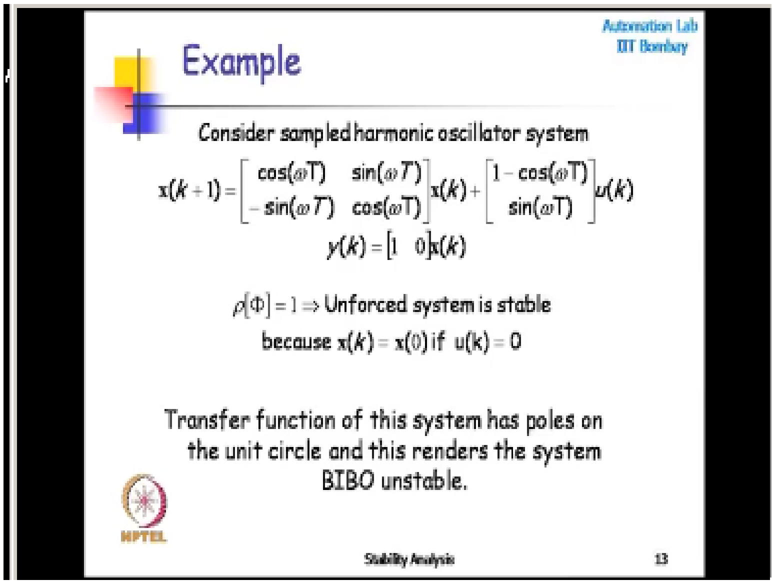

This unforced system has stable okay which means its spectral radius is equal to 1 okay it is stable in the sense of the unforced system is stable if you take this system okay do not worry about the U put u=0 you take this system okay for this system Eigen values are on the unit circle this stable is okay now this particular system has poles on the unit circle and actually that makes it BIBO unstable

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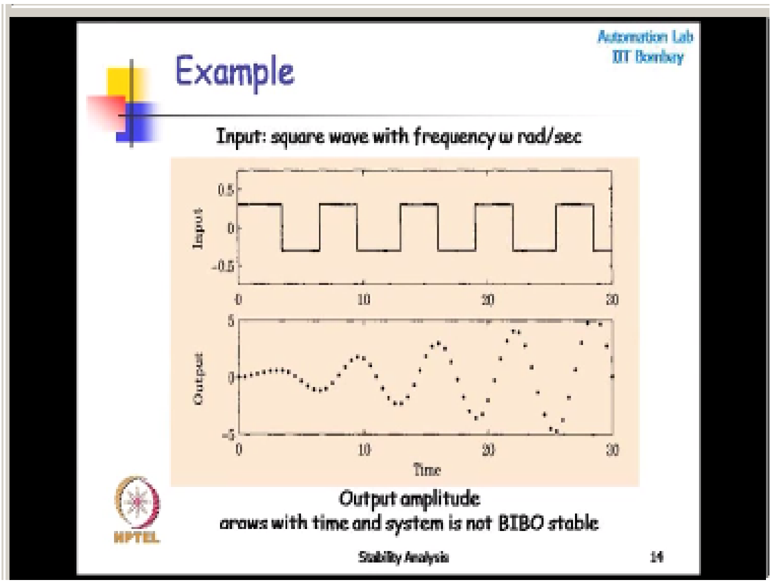

So actually if you introduce bounded input into this particular system okay then the output actually grows with time okay so the conclusions that you reach by putting u=0 okay about the stability are different from the conclusion that you reach by putting a forcing function u on this particular system if you introduce cyclic you then the output will grow okay but, if you say u=0 then spectral radius is equal to 1 and then you will say this is stable to the certain differences between these two you have to go back.

And think about it I cannot explain everything we will look at more examples you will get some cases it is stable system okay so the marginal stable so stability of ϕ as BIBO stability okay so here I am showing a situation where original system is marginally stable okay but it go on stable okay so these two are different already I want to point out all that I want to point out that is these two are different notions.

And only when you have state space model to be symbiotically stable then you can guarantee BIBO stable okay you can say the same thing unstable also but there is a trouble when it comes to Martian stable systems even for linear cases okay why are we worried about BIBO stability off course when set pint is not equal to 0 and I have a close loop control this is my close loop equation I derived this some time back and then I am worried about behaviors between r and y okay

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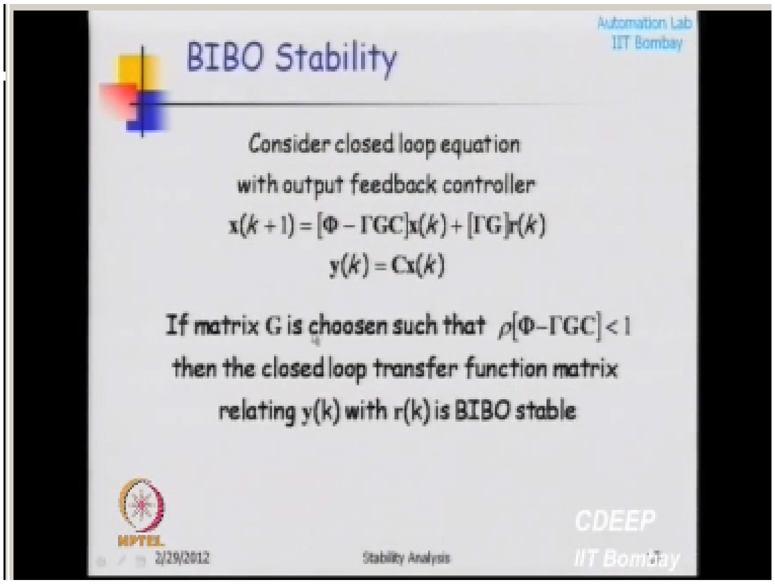

So in this case I am worried about behaviors between r and y okay so the question is the system BIBO stable that means is y with reference to u sorry y with respect to r, r is now input to the close view okay where the system bounded input so if I give bounded in the set point will the output remain the bounded okay.

And then can you guarantee that if you make sure that 5-γgc is symbiotically stable which means all the Eigen values are strictly inside unit circle okay then you can say that this system be bounded input bounded output stable which means if I give bounded in the set point output will be bounded okay so that is why we worry about in the design.

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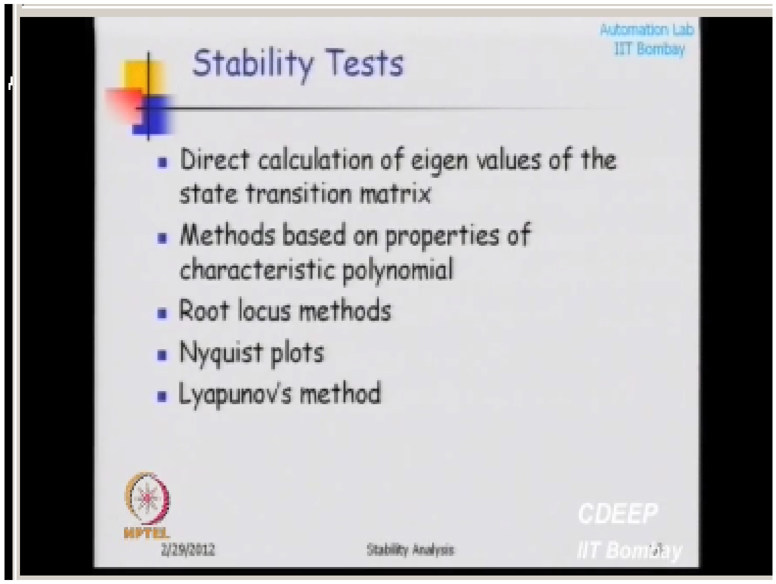

Okay now stability tests I am going to do one simple stability test in the context of stabilize context only this is most simple method that is direct calculations of Eigen values you have five matrix or you determined polynomial you find out the Eigen value okay and simple tricks like prove its criteria or roots through this criteria in today's context does not make too much sense because we have very powerful computers.

And you would download some program and say roots and you will get or you calculator itself you know might have some matrix Eigen value calculations so there are some nice methods of anal sizing roots of characteristics equation you are aware of criteria right so off course other methods is root locus method so I am not going to get it to the root locus method I am going to take one method which is based on properties of characteristics polynomial plots take analysis in Nyquist plots.

I am not going to get the Nyquist plots analysis either I am going to talk about lyapunov's method very deeply because I need this in my subsequent development this method of properties now I need more for depending upon problem in my exam and also nice tool where you can find out whether a poles of a particular system because they are inside the unit cycle or outside the unit cycles this is by doing some simple hand calculations you do not require a computer so I will talk about that particular I think this is very elegant method for timing out.

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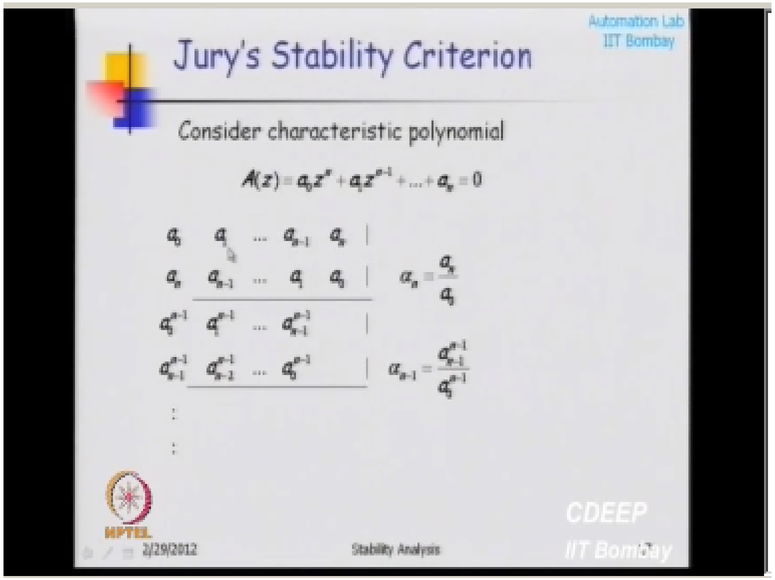

And this is called jury's stability criterion in the case of discrete time systems you have this jury's stability criterion in the stability something which we discuss later the stability is Russian scientist mathematician increased to a lived in 19th century and spread several contribution to the theory of stability of differential equation so let us look at jury's stability criteria postpone the stability to the next class it is fairly involved.

So this is simple polynomial test given a polynomial I want to find out whether the roots are inside the unit circle outside the unit circle okay and so this is my polynomial okay I have this convection of numbering a0, a1 so the coefficient of n is called a0 and the last coefficient is called an you construct this stable okay now how do you construct this stable well I will tell you a simple rule you have written this a0 to an here and if you notice the second row is an-1 to a0 okay.

Now all that I want to do is to I want to multiple this row by some factor and eliminate n okay I want to eliminate an so this things here becomes 0, 0 okay then see here I got this new row whi9ch has one element less okay then what is have done is the same thing see here we first started with a0.

And an then we wrote this row in the reverse order same thing a0 I got this row then I wrote this row here in the reverse order I want to eliminate this n-1 okay I go on doing this till I get finally you know only one number finally it somewhat similar to what you have done later here for eliminating once you do it for some simple system you will understand how to do this it is very.

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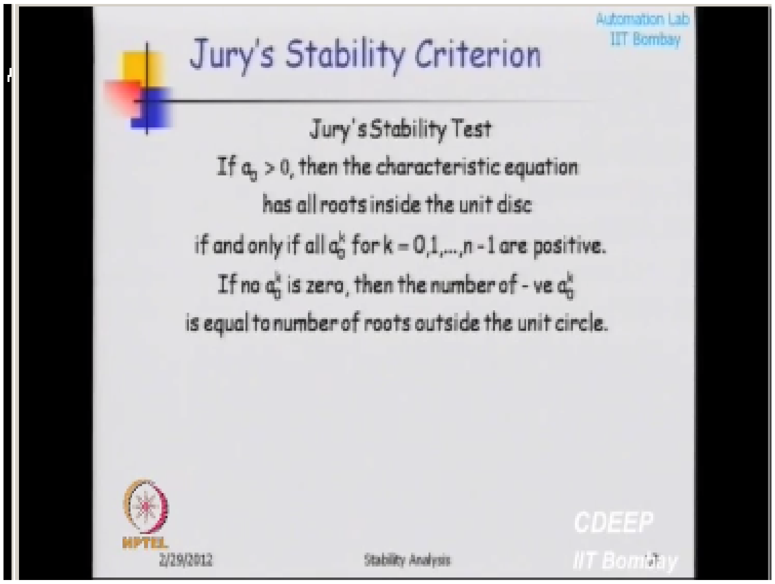

I mean on the next class in the beginning we will just do this and then there is a test you know if all if a0>0 and if all elements in this first see you have to look at elements here a0, a0n-1 you will get a0n-2 and so on so if all of them are positive okay all the roots are inside the unit circle let me check if all this a0 is positive and if a0 1 to k that is all the first elements in the first column are positive okay very simple test which you can do.

And calculations and without even or simple calculator calculations you do not need some you know matrix Eigen value calculation program you can just find out whether the roots are and you can find out you can write some necessary and sufficient conditions of stability based on analysis of this will look at this problem

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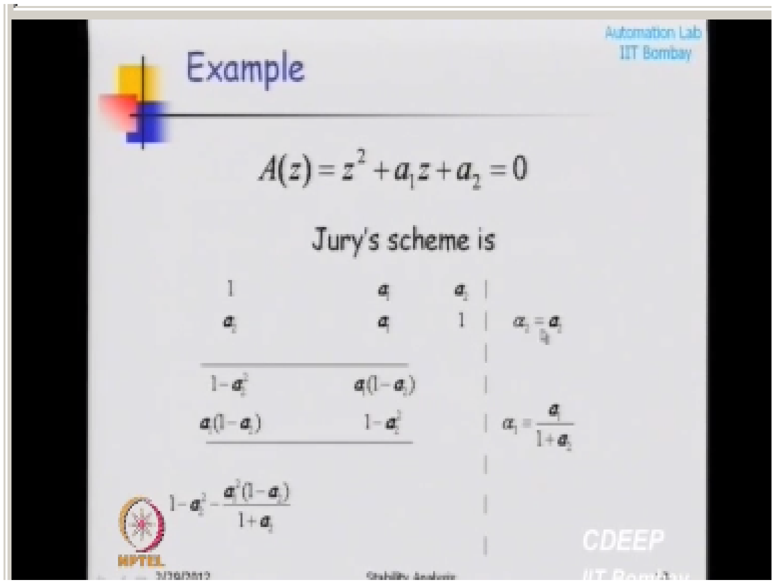

I have this problem and if I use jury's scheme see this is a1, a2 I have written a2 a11 okay I multiply this entire row by a2 and subtract if I subtract this last thing here will be 0 then I get this $1\text{-}a2^2$ and a1.1.a2 I write this in the reverse order so this element as come here this element as come here now you want to element this term okay multiplied by approximate factor and subtract you will get only one term so what is the jury's stability criteria so this should be greater than zero okay if all of them are greater than 0 and the system is stable.

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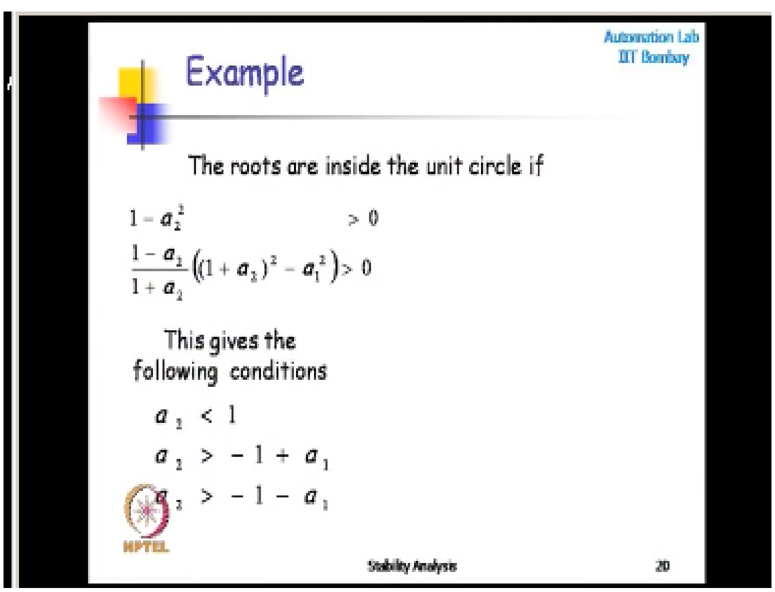

Well which wide is stable we are talking about roots of the characteristics polynomial all of them are positi9ve then we have semiotic stability is guaranteed so if you are tiger error if you are using the g matrix okay if you guess some value and if you want to do hand calculations and see whether you are inside.

The unit circle or outside the unit circle you could just apply this simple test on the paper and then find out whether you are controller designed is locally stable or locally unstable okay so in this case if you so this –a2>0 and this second thing should be greater than 0 it gives you these three conditions and then you can actually plot in a1 a2 plan you can plot so any value of a1, a2 in this region you will have a stable transfer function that is what so this business of

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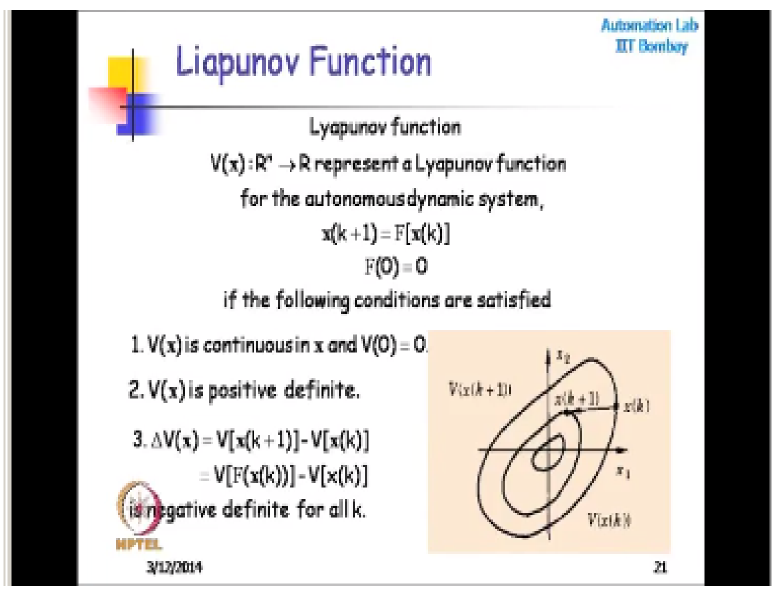

This liapunov stability will look at it later okay so this was a very, very deep concept which actually forms the entire foundation of stability analysis of differential equation difference equations entire model theory of stabilities holder on this one pieces was written by the liapunov as its doctoral work the seminal work which vary, few times it has happened that but doctoral would entirely form a new branch of analysis so you will have a very brief look at this because we need this theory when we as we go along.

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