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NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING

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ADVANCE
PROCESS CONTROL

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Lecture No. 11

Development of Control Relevant Linear
Perturbation Models (Part 2)

Sub-topics

Model Structure Selection and
Issues in Model Development (contd.)

So we have been developing one step predictor for ARMAX model, and in my last class I showed you one way of developing this predictor okay.

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ARMAX: One Step Predictor

Alternatively, using residuals at previous instances

$$\varepsilon(k-1) = [y(k-1) - \hat{y}(k-1|k-2)]$$

$$\varepsilon(k-2) = [y(k-2) - \hat{y}(k-2|k-3)]$$

we can rearrange one step predictor as

$$\hat{y}(k|k-1) = -a_1 y(k-1) - a_2 y(k-2) + b_1 u(k-2) + b_2 u(k-3) + c_1 \varepsilon(k-1) + c_2 \varepsilon(k-2)$$

$$\varepsilon(k) = y(k) - \hat{y}(k|k-1)$$

We can start prediction with initial guesses

$$\varepsilon(0) = \varepsilon(1) = \varepsilon(2) = 0$$

and, given model parameters $(a_1, a_2, b_1, b_2, c_1, c_2)$, we can generate sequence $\{\varepsilon(k)\}$ using sequences $\{y(k)\}$ and $\{u(k)\}$.

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We distinguish between prediction of Y and Y measure, we should clearly understand the difference, prediction of Y based on the past information is different from Y measure, Y measured actually is the value of measured variable collected from data exist in system. Prediction is an estimate, it is not, there might be different ways of prediction, so there is no unique prediction.

But the measurement is, once it is obtained it is unique, it will not or it is in some sense invariance you cannot change the measurement okay. So I showed you one possible way of doing predictions, the idea was to bring everything that is Y and U into my prediction equation. This is an alternate way of doing predictions, we just go over it once more. So now other way is to use predicted residuals.

We used predicted Y, you can use predicted residuals and then do predictions okay. So here to begin with I am showing you two predictions suppose I have this, I have concept estimate $\hat{y}(k-1)$ given $k-2$, this is prediction of Y at instant $k-1$ using information up to $k-2$ okay. So same thing is here, this is prediction of Y at instant $k-2$ using information up to $k-3$. Let us go back to our equation, we were looking at second order ARMA model.

Second order ARMA model we had rearranged like this $\hat{y}(k)$, even $k-1$ was weighted sum of past measurements, past inputs, manipulated inputs or known inputs in the system. And now I am retaining these past two errors, now you just notice here earlier I had this term $E(k)$, $E(k)$ is the white noise which we hypothesize the dry is the model okay.

I do not have measurement of $E(K)$, I am going to use an estimate of this error, this estimate of the error between Y measured and Y predicted this difference, I am going to use them in poster okay, somebody you imitates the true innovations so called true innovations or so called true error. So I am concepting an estimate of E , the estimate of E is concepted using difference between Y measured and Y predicted at each instant.

And that is used in my model okay. So now I want to start doing predictions okay, I can make a simplified assumption that in the beginning of the, in the beginning of modeling is the size at time equal to 0, the two errors are equal to 0, expected value of $E(0)$, $E(1)$ and $E(2) = 0$ this is the simplifying assumption, this is the simplifying assumption. I want to do predictions using Y U and error okay.

If I have first three errors, then subsequent errors can be generated using this equation. I only have problem when I start doing using this equation from time 0 to time N okay, my only problem is first three instances. If I somehow have some guess for the first three assumptions, so I am going to make an assumption well, what error E is the 0 mean signal best estimate for this signal is 0 okay.

So you justify to use 0, 0, 0 in the middle way, so now what I am going to do is given this model parameter. Now in optimization what you do, you guess unknown parameters okay. Then using the guessed value is you construct the prediction errors and then you minimize some of this get a prediction errors that is the idea. So first three prediction errors we are arbitrarily choosing them equal to 0 okay, first three only.

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ARMAX: One Step Predictor

Given : $\{u(k) : k = 0, 1, 2, \dots\}$ and guess for model parameters
 Assume : $\varepsilon(0) = \varepsilon(1) = \varepsilon(2) = 0$

↓

$$\hat{y}(3|2) = -a_1 y(2) - a_2 y(1) + b_1 u(1) + b_2 u(0) + c_1 \varepsilon(2) + c_2 \varepsilon(1)$$

$$\varepsilon(3) = y(3) - \hat{y}(3|2)$$

↓

$$\hat{y}(4|3) = -a_1 y(3) - a_2 y(2) + b_1 u(2) + b_2 u(1) + c_1 \varepsilon(3) + c_2 \varepsilon(2)$$

$$\varepsilon(4) = y(4) - \hat{y}(4|3)$$

↓

$$\hat{y}(5|4) = -a_1 y(4) - a_2 y(3) + b_1 u(3) + b_2 u(2) + c_1 \varepsilon(4) + c_2 \varepsilon(3)$$

$$\varepsilon(5) = y(5) - \hat{y}(5|4)$$

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After that see what happens if you make that simplifying assumption, what happens is that, if I make this assumption then what is $Y(3)$, see $Y(2)$ is the measurement at instant 2, $Y(1)$ is measurement of instant 1, they are available, $U(1)$, $U(0)$ are available and $E(2)$, $E(1)$ we have assumed them to be 0, that is our simplifying assumption okay. So I get $\hat{y}(3|2)$ given 2, with this I can estimate $E(3)$ okay.

Then you know, I go to $Y(4)$, $Y(4)$ needs $E(3)$ and $E(2)$, $E(2)$ where assumed to be 0 okay, $E(3)$ can calculate it here okay, and then I can estimate for $E(4)$. Is everyone meet me on this, just check what I am doing okay. Once you give me a guess for parameters a_1 , a_2 , b_1 , b_2 , c_1 , c_2 and then make this simplifying assumption of $E(0)=0$, $E(1)=0$, $E(2)=0$. Then if you have a large data set, this first three elements, they assume to be 0 that will make a difference okay.

Typically, we will have large data set, and this does not really cause much problem, particularly if we make an assumption back to disturbance model, unknown disturbance model instable or inversely stable this is perfectly valued assumption making 3 to be equal to 0. So that is one could cause a problem in the identification. Well I can just go on doing this now, see once I have, you know $E(4)$ and $E(5)$ I can go on using them routinely in my recursively in my condition.

This is just writing for loop when I am writing a program, I am just writing a for loop going from time say time 3 to time n I just go on doing this calculations. For every guess of parameters I can estimate this okay, so what is there in optimization, in optimization you give a guess okay for a_1 ,

a_2, b_1, b_2, c_1, c_2 , for every guess we have knowledge of Y and U , we construct these predictions. Then we estimate $E(3), E(4), E(5)$ up to $E(n)$ okay.

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Parameter Estimation

Parameter Estimation by Optimization
Find θ that minimizes objective function

$$(\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2, \hat{c}_1, \hat{c}_2) = \underset{(a_1, a_2, b_1, b_2, c_1, c_2)}{\text{Min}} \frac{1}{N} \sum_{k=3}^N \varepsilon(k, \theta)^2$$

Subject To

$$\hat{y}(k | k-1) = -a_1 y(k-1) - a_2 y(k-2) + b_1 u(k-2) + b_2 u(k-3) + c_1 \varepsilon(k-1) + c_2 \varepsilon(k-2)$$

$$\varepsilon(k) = y(k) - \hat{y}(k | k-1)$$

Nonlinear optimization problem
Important to give good initial guess

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This just goes on, and then we minimize some of the square of errors with respect to these parameters okay. The θ here is nothing but this parameter set a_1, a_2, b_1, b_2 . And what is parameter resting optimization algorithm do, it will estimate the objective function based on the idea of the objective function, it will give you or some method galient or reason of the objective function.

It will concept a new guess, and you keep iteratively trying different values of algorithm of the parameters till you reach some local optimum, minimum okay, some optimization, some minimization that we are satisfied. So that is this problem has to be solved iteratively okay. And you can see for some simple systems, I showed you demonstration of MATLAB skill was last time.

This can be done very, very good, you know it takes problems in seconds now, doing these kind of calculations, optimization calculations. So this is the constrained optimization problem, so \mathcal{E} is actually computed using this particular equation. Of course you do not have to use constrained equation here, you can eliminate and use the simple unconstraint methods, unless you want to do constraint this a_1, a_2, b_1, b_2 and so on.

Well that is a little advanced thing, whether constrained name or not to constraint does not to – ideally we should constraint that, because we want the noise order to be stable and inversely stable, that if you try to pour of the constraint problem there are some difficulties. And there have been methods of finding them that this kind of part of there in research not in.

So it is very important to view in good initial cases it is an optimization problem. It is very, very important to design the inputs correctly, it is very, very important to choose scripture correctly the all kinds of things. So let me just summarize the prediction error method okay. Let us just go back and see what we have done, we have give for every days of parameter that I have been estimated.

We actually compute this $\hat{y}(k)$ and error okay, and then we minimize some of this square of errors respect to this parameters to be estimated that is the – what is, there is a time delay or dead time in the system that is typically estimated apriori based on some other method. And that is just used while concepting the model, time delay estimation using this becomes difficult, because time delay is the integer variable, because you have time delay of three samples or four samples.

And if you notice here these are all continuous values $a_1, a_2, b_1, b_2, c_1, c_2$ I take continuous values, they can bear the numbers. So we do not really get into solving a integer, mixed integer logging a programming problem that becomes very difficult.

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Prediction Error Method

Given data set $Z_N = \{y(k), u(k) : k = 1, 2, \dots, N\}$

Model : $y(k) = G(q, \theta)u(k) + H(q, \theta)e(k)$

Optimal one - step predictor

$$\hat{y}(k | k-1) = H^{-1}(q, \theta)G(q, \theta)u(k) + [1 - H^{-1}(q, \theta)]y(k)$$

One step prediction error is defined as

$$e(k, \theta) = y(k) - \hat{y}(k | k-1, \theta)$$

Parameter Estimation by Prediction Error Method

Find θ that minimizes objective function

$$V(\theta, Z_N) = \frac{1}{N} \sum_{k=1}^N e(k, \theta)^2$$

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Okay, so let me summarize this. So I have this data, I have done some experiments and I have collected this data for some N , N should be typically large, how large you hands, practicing engineer, knowing about the plan, knowing about the domain, the dynamics of the system you have to decide okay. You can probably come up with some health from statistics and linear algebra, but then it is a joint cause, that is not completely automatic.

Your expertise has a domain expertise very, very important here. Then you know we choose some model structure, how do you parameterize G , how do you parameterize H , whether you choose a pharma model, AR model, ARX model all that is up to you, you have to worry and come up with what is feasible, what is data length, you know very, very complex sessions, how it looks very simple here.

Then I do this optimal predictor, I will show you how to concept optimal predictor for two some simple cases. Now the nice thing about is optimal predictor is that it right hand side only involves known signals U and Y okay. It does not involve unknown signal E innovations. Innovations are actually constructed, E is a signal which is actually of fabricated signal okay. And you can see that, that signal is nothing but here $Y - \hat{Y}$.

So it is, we expect this to be a white noise assuming white noise signal. Typically it is 0 mean Gaussian white noise each one of them, stationary process white noise stationary process. And when do we stop, how do you know whether the model identify these good or bad, this ϵ sequence which you get after estimation of the model okay, should be a white noise very, very critical okay.

And then you minimize, typically you minimize some of the square of errors, somebody might ask you why some of the square why not some of the absolute errors, why not infinite norms. Now minimize maximum error, maximum of the absolute error, minimize maximum absolute error, you can do that perfectly fine okay. Why we use two norm is because, two norm is associated with mole of rich properties.

First of all those of you who have done this numerical methods course. Two norm comes with, you know in some invert space right. So you have lot of fixed areas associated with norm, and you can interpret it through projections, then in this case you can use some people pass those

theorems and then transform the norm, how to analysis using frequency domain, and there are lot of things which come into norm, it is very, very helpful to analysis.

It becomes difficult to do analysis with one norm or infinite norm, so if I use some of absolute errors, it is perfectly fine, but analysis becomes difficult okay. Analysis is very easy, particularly with two norm, that is the least square estimation so founded. Estimating covariance of parameters estimates, estimating some theoretical property. Well, I am just going to intend those properties right now, I do not have time to get into derivations or talk about it fully.

Because in this course, you know modeling is one aspect that we have to proceed and go to control and said estimation. So this lecture, the next lecture I am going to wind up the controlled error modeling. I am going to go over something's very briefly okay, which will probably cause some disconcert, because there will be some complex expressions here. But right now idea is to sensitize you that there are many more complex issues here okay.

I probably would need entire 40 lectures to really go into depth of these things. We many times run this course system identification and said estimation separately. This year it is not running, but if it runs next year maybe you can think of crediting or auditing that course. So that will give you much broader respective of what is happening.

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Two Tank System: ARMAX Model

Optimization formulation

Estimate $(a_1, a_2, b_1, b_2, c_1, c_2)$ such that objective function

$$\Psi = \sum_{k=3}^N [e(k)]^2 = \sum_{k=3}^N [y(k) - \hat{y}(k|k-1)]^2$$

is minimized with respect to $(a_1, a_2, b_1, b_2, c_1, c_2)$

Identified Model Parameters

$$A(q) = 1 - 1.651q^{-1} + 0.68q^{-2}$$
$$B(q) = 0.001748q^{-2} + 0.01154q^{-3}$$
$$C(q) = 1 - 0.8367q^{-1} + 0.2501q^{-2}$$

Residual $\{e(k)\}$ Statistics

Estimated Mean : $E\{e(k)\} = 4.3601 \times 10^{-3}$

Estimated Variance : $\hat{\lambda}^2 = 2.6813 \times 10^{-3}$

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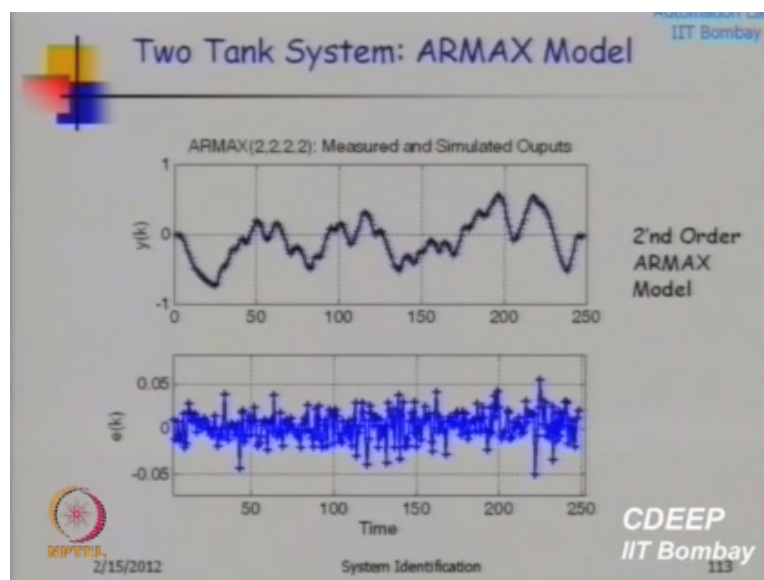
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Okay, so let us go back to our two time data that we have okay. If I actually do this parameter estimation using MATLAB tool box, this is the model that I get, and of course as I told you the model consist of these ABC polynomials, and it also consist of the statistical properties of noise E, which have been estimated from data. So the model has two components, deterministic component, which is coming by B and A.

Stochastic component which is coming from C and A okay, together with noise properties okay. We actually wanted $E(K)$ to be 0 mean it is almost 0 mean, it is not equal to 0 mean, it is almost 0 mean. And estimate of the variance, how is this way estimated of variance obtained using the sum of the square of errors divided by 1 upon and that will give you estimate.

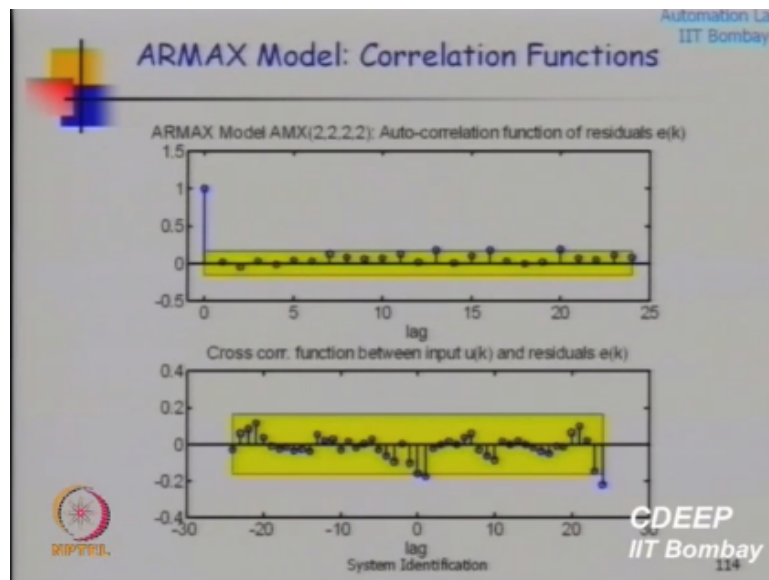
Actually we have given this formula, so this objective function itself divided by $1/n$ will give you estimate of the variables okay. So that is after you optimized for this particular optimum values, estimate of λ^2 , λ^2 is variance of E is given here okay. MATLAB till works will give you this information, you should know how to use it, that is very, very important okay. So even ψ lab which you are using in ψ lab, ψ lab involves or there are many statistical open force optimum and showed them, some of them have this times with modeling. So they will give you all this information, you should know what is the meaning of all this okay.

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So second order is ARMAX model is pretty good.

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I get, I show you these results earlier. The innovations are almost the white noise, then innovations like inputs are almost uncorrelated. It is a good model, we have instructed, and we have separated deterministic component and stochastic component okay. We have modeled for unknown disturbances, the model for unknown component, you can go ahead and use this for designers controllers.

Yeah, we have to go back and say that whether my model already is correct, should I go back and change from second order to third order. Both will change, if you want to change the deterministic order not change the noise model order, then you should note with ARMAX, you should be more genius, structure should be different.

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Parameterized Models

ARMAX: Auto Regressive Moving Average with exogenous input (ARMAX)

$$y(k) = b_0 u(k-d-1) + \dots + b_m u(k-d-m) - a_1 y(k-1) - \dots - a_n y(k-n) + e(k) + c_1 e(k-1) + \dots + c_r e(k-r)$$

Or
$$y(k) = \frac{B(q^{-1})}{A(q^{-1})} q^{-d} u(k) + \frac{C(q^{-1})}{A(q^{-1})} e(k)$$

Box-Jenkins (BJ) model: most general representation of time series models

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})} q^{-d} u(k) + \frac{C(q^{-1})}{D(q^{-1})} e(k)$$

$\{e(k)\}$ is white noise sequence in both the cases

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See we have this different structures, the trouble with ARMAX model, well why have we introduce ARMAX model is simple to teach and understand the concept. But the trouble with ARMAX model that it has common denominator polynomial for noise and for deterministic. This sometimes it is not a great idea, what you are saying is correct, this having same A and for noise and this thing, it will have sport for deterministic and stochastic component clubbed together.

You may have some difficulties because of that, if you want to separate them, you can separate the parameters. So this can be third order, this can be first order, you know you can do that okay. Only the algebraic terms is little more complex okay. Everything is same, you do $H^{-1}g + (1-H^{-1})/$ everything is same, nothing famous okay. So the prediction method can vary, but we used not an issue okay.

And you can separately parameterize the noise model, in fact a better idea is to separate the parameterize the noise model okay. So ARMAX model of course you can always convert this model into this model, suppose I do cross multiplication and I have a problem denominator okay, I can have a problem denominator which is $A(q^{-1}) d(q^{-1})$, you see that see this is you can do polynomial algebra just like fraction algebra know you can just multiple so I can define a model where this will be $d.b/ad+c.a/ad$ that converts it into this form okay by the way please understand

that these are just representative forms when I am writing this here it does not mean that this $b=b$ it is not that these I will just represent it we are representing okay.

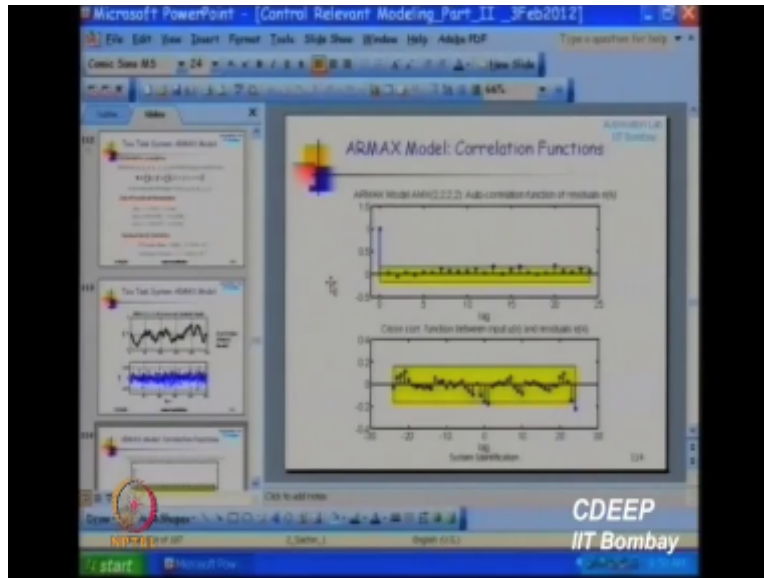
So when I am writing ARX model ARMAX model I still keep using ab you know actually for a same still ARMAX model probably should be a and you know v_j should be 8 cap or some other notation because they are different they will not be identical but if it now that will make that rotation so complex that these are this repetitions okay and you can see here you can convert this form into this form and so on okay a better understanding of this income off course we are solving those problems and we have those we have two more one or two more sections for before of the problem solving.

But the real understanding of this will come and when we start doing the project I been waiting for population this component so post start a project what you have done is we have collected the 5 different systems and then we are going to give good projects okay so you similarity the system you actually introduce noise to get data then you put it into I will get different models then you will get a feel of these numbers because how these statically things if you do use and had using some three or four values other make sense if you do it in met lap.

And have some sections where you start doing these projects so there are two or three models from chemical engineering then two model from chemical engineering to the reactant models one two models from bio domain which I think anyone can understand there is some module for monitoring insulin a monitoring less blood glucose concentration using insulin in future some three equation model simple model very, very simple model.

So those models will cut cross any blank system is going to be human body very close to you so and then you can appreciate what happens if glucose goes higher you do not need to need any special engineering domain or that official what will also tell you.

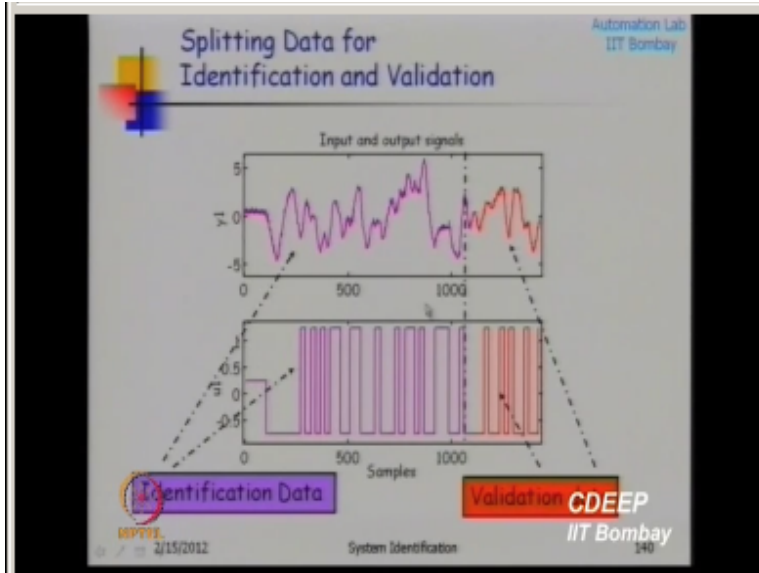
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Is that these methods that we are talking about modeling and control I just cut cross I mean the boundaries of disciplines that belong to this so that does not matter they the modeling 2z can be used for any system okay so let us go to this model we get I showed you various range of accessing the model quality okay we actually what we do is we compare model predictions see you pick data you divide it into two parts one part is called model identification.

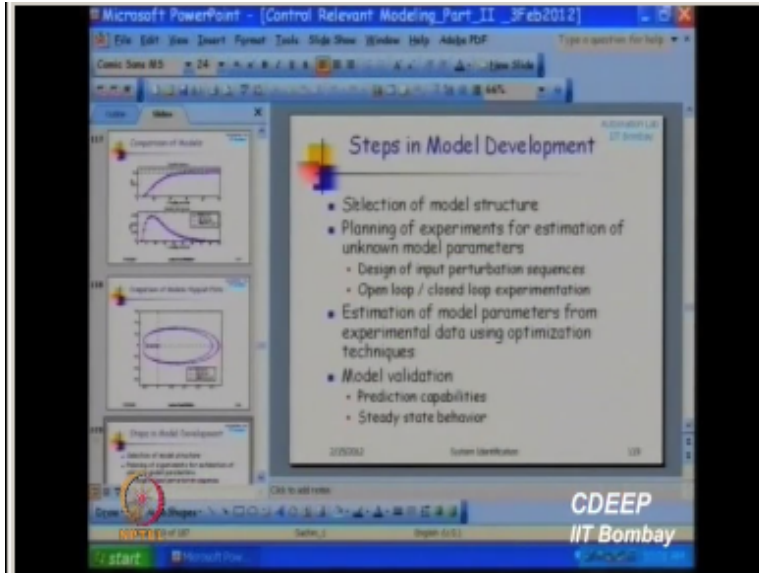
Other part is for model violation a model identified using part a of the data should be able to predict part b data because see this data has been used I will show you that in a particular thing this data has been used one part of the data has been used for I will show you this case study so this is what is done.

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You conduct an experiment you take this part of the data for model identification and this sort of data you carry for model validation if I identify model using this part of the data it should predict this data then only the model is good how do you know see when you are minimizing the sum of the square of the errors on this the identification you know will show you perfect results it should predict the unknown situation then the model is good okay.

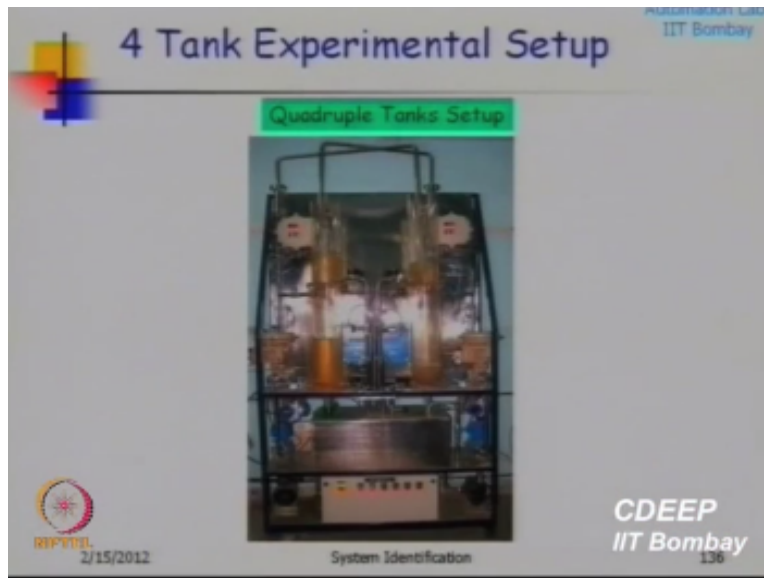
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Maybe I will present I will rearrange this I will present this case study first move to some of the issues that I want to talk about yeah actually nothing like that we can have both of them saying you can do two different experiments to the similar characteristics and then use one experiment said for model identification the other expensive no nothing like that we will use basically if I conduct an experiment of IR well I have taken larger data set for model identification.

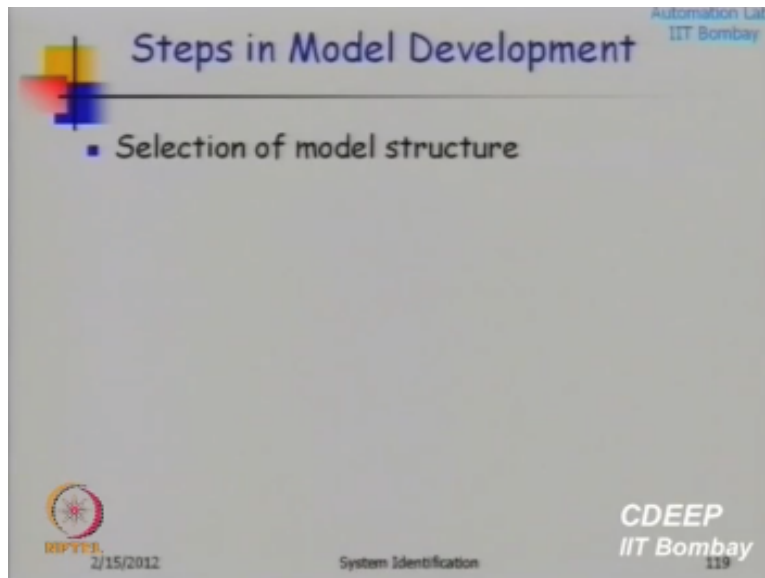
Because to keep the parameter estimation variances of parameter estimates low I need to n to be large okay that is why when you if you conduct only 1 experiment then you tend to take the larger data for identification smaller data for if you conduct multiple experiments if you are trying to do that then not an issue you can have two different experiments conducted 1 dedicated only for model identification the next for model validation absolutely fine. So this is what I move to some of the issues in identification.

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That we present at case study okay there are lot of okay let me present selectively some two slides before a new one and then we move on to case study.

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Then we come back to issues so what are the states of model one of the most important thing is model structures selection you have this apply first of you know what is available in your basket what are the different model structures available to you I have talked about a limited slide linear difference equations okay linear stochastic difference equation they are stochastic difference equations mind you.

Because you have they have been driven by known inputs as well as a innovation white noise okay 0 mean white noise signal so it is a set which is linear stochastic difference equation what we are using again within that you know ARX R max R mole OE all kinds of things and you have to make a judgment what you want may be in some situations you are not bothered about disturbance modeling you just want deterministic component correctly you should go for output error model structure.

Okay but as any issues that even though get good estimate of deterministic component you may have to model the noise okay so that part is I am not going to cover that aspect so much deep I will just show point as I said if I do justice to all those this probably I need to spent entire so this so which I do not wanted to do so model structures selection is very important it is about important before you start you know experiments.

Because if where going to do ARX models okay are finite impulses pluses models the number of coefficients is going to large you better conduct this experience for larger time so all these decisions are correlated they are not independent you have to understand this and you have to

understand this particularly with the back drop that you know all these modeling tools are available to you at a figure tip.

Okay so when I was graduate student say 20 years back we had to write our own program to make this and you know writing a program for this itself would be some m tech project to write a PL method now it is not even a course tutorial you know so it is reduced to that so but you should know what you are using you have hammers but you know you should know what you are doing.

I just talked about linear difference models there is a whole lot of things called non linear ARX models nor max models non linear you know you have all those u and x and wave makes and there are so many other options by which you can construct non linear difference equation what it is we have just talked about one simplest set why linear difference equation models because linear difference equations are mathematically capable very you know very easily treat in fact linear difference equation.

You can do design of controllers and all that very easily that is why we but in some cases in some situations linear difference equation models will not be useful and in those cases you have to take a call what model you want to use linear or non linear and then once you go for a non linear model you have to of course see this non linear controller design methods and so on.

But my prescription would be simple things first try linear difference equation models if they do not seemed to work or if you know from physics that linear linearity is not going to work in this particular situation then only you should go for the more complex structure next thing is that you have to plane satiable set of experiments that you design input perturbations sequence okay you have to worry about whether to do the experiment in open loop or close loop now I tell you why you are using linear models.

Okay now linear model is not valid if you develop a linear model for plane which is operating at 1 operating conditions in the operating conditions change tomorrow okay which can very well happen in a chemical plant let us say you have refinery which is working using gules rule and you decided to shift to Bombay high rule okay the characteristics of Bombay high rule are different operating conditions change completely.

The linear model which is valid for gules rule will not be valid for Bombay high rule okay so these are ultimately perturbations models okay or when we go to that model okay the model if the models is non linear if you linearism at one operating point the predictions will not match the behavior at some other operating point so if the operating condition changes the glucose level changes from say 100 to 30 the predictions at 30 will be different you cannot use the same linear model.

So you may have to you know inject the perturbations at 30 and now the questions is that when you attend this injecting perturbation should be open loop or should it be in closed loop open loop is that no insurant is being is given okay may be is the glucose level are to level and low insane is given no control is there okay and you have perturb in the plan you know you are on the dangers predatory okay you better control a system and inject the perturbations okay.

So in a reactor you may be in the region where it is you know high pressure and temperature it is dangerous operate if you inject perturbation without the controller being present so there is some controller designed earlier that is already active we found that the controller is not performing well you want to change the model retune the controller this is the scenario I am talking about okay.

Now in such a situation should I take the controller off or should the controller be on operator is more comfortable if controller is on okay he does not want to take of the controller and as a you know molder you will say no open loop is better you know open loop is not particle sometimes now if you decide to do perturbation in closed loop there is huge mathematical implications the noise in the measurement becomes correlated with the inputs u .

And then entire things becomes basic math's becomes very basic you cannot use the methods which I described in the straight foreword manner you have to lot of math's to clean the data of the noise so the input and output is some sense become correlated that causes problems in the estimation I have hinted at this in my tutorial to solve the tutorial I have given once small problem with proportional feedback I am sure that some matrixes this Ω matrix will become random definite and so on okay.

Well we do that exercise w will realize so identification weather you do it closed loop or open loop is a tricky question and when you are actually reidentifying the plant while the plant is

running or operator would prefer that you do it closed loop but closed loop identification is a tricky nascence there are very few commercial products right now people are working are actually developing products which can do model identification online while the controller is on okay.

It is a hot research topic and very much valued by industry yeah there are so many issues yes that is also one of the argument which is advanced that you know it might drift some where the other argument which is advanced is you might when the controller is on you actually excide the plant in the frequency range which is relevant to closed loop which is not possible when you loop and so many evaluations so there are people who are for return for against it.

So the debate which is still inconclusive debate you have to take call on depending up one the particular situation do not they have no there are no global answers to this what should be done yes in some sense yes what your saying is yes so how do you know our priory if the plant is not really so there are chicken and egg problems you know what tends to of strength.

So how do you design an input itself becomes a very critical thing so typically use optimization techniques and model value you have to perform by multiple ways you have to see whether the steady state properties on the model log good or not in a game is correct sort not you know you expect the gain with positive and tells out to be negative from estimation something wrong which I have molded.

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Model Structure Selection

Issues in Model Selection

- Process application (batch / continuous)
- Time scale of operation
- Type of application (scheduling/optimization/MPC/Fault Diagnosis)
- Availability of physical knowledge / historical data
- Development time and efforts

Model granularity decides how well we can make control / planning moves or diagnose / analyze process behavior

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So all kinds of things which you have to do so there are multiple things you know whether you see I do not know many of you know what I am saying here because a chemical student's batch and continuous is not difficult to understand a chemical plant as you know the crude oil is coming continuously from the sun source some oil may let us say or some that is continuously being processed and the products are continuously being pumped to different destinations.

So let us say you have crude oil then you separate it into any of fuel and diesel and gasoline and this question is happening continuously all the time the feed keeps coming and it gets separated and these are called continuous processes the other kind of process are batch processes there is something that starts then there is some operation and then it stops best example in our home for a batch process is cooking okay.

So you take rise in the cooker then you raise the temperature at a certain point then you operate it for look at from a control view point you operate it at a constant temperature for some you know 9 minutes or 11 minutes or whatever you wait for some wishes and then you switch off then a temperature should come down right before we can open it so it is a control problem which we will do by hand.

Real life situation in chemical engineering is specially be chemicals pharmaceuticals you produce a you know some special be chemicals are drugs in a not a cooker but a reactor which is called a batch reactor or some batch reactor in which you put this okay and let them cook let them react so you have some recipe okay there is a batch recipe you heat it up to a certain point and the reaction starts the reaction might be exothermic.

So you may look into and so there are all kinds of you know the other batch process is air craft flight view take it off to a certain level close it and then it stops right and now on the analog continuous time analog for this kind of system would be of chemical engineering would be a satellite a satellite never once it takes off it is there I mean for so it is if you take if that if never stops.

It is a continuous line as to much as once it is there it is there you know in the orbit and so it also depends up on what kind of application you have in mind I will be talking about this prediction so what kind of model you want to develop depends up on what is the availability of data you know how much time you want you have at they are discuss in so there are I mean

modeling if you ask me if the control project what is most important part is modeling and not the control once you have a good model you know doing control as relatively much easier.

Because you know once you put the system behavior into nice difference equation differential equations we know how to go about you know we are master at working in this ideal world of equations but translating the reality into something that is similar equations which we make the plant or the system is in it very well weather it is mechanistic model weather it is grey box model weather it is black model whatever it is.

Doing that is the most tricky part and in complex control schemes where people implement them to people who can do modeling are paid much higher probably and those who can operate because if you know how to model if you have done 70% of the work in a control project yeah you ask some question distillation can be done in both batch or continuous so the cure have discretion which I was talking to you is like a continuous process.

It probably this they do I mean if you take very large view point of 100 years it is a batch process but we call it continuous because once you start we will not stop for one year also once you start this process for 1 year okay at a larger times scale everything is batch process okay but because sometimes you have to do start up and some time you do a shutdown you call it continuous because once I start it for one year it is going to run okay.

But the other batches you start talking about see let us say I man repapering some specialty drug okay so once react it for some one day let us takes to react and get mixture where you get the formulation then you get a mixture which is as product and reacts both together now you need to separate the products okay.

Now that as be done in the distillation but that is called batch distillation because you are going to only process you know some 100 kg of material it will take some 4 days not 4 days it may take some 4 hours or five hours okay so that is I called a batch process okay so this batch and continuous is relative of course so if the crude oil supply the rise up well the flag is it is not so when you say continuous it is so what I would say is that the model granularity what kind of model of you want to develop first principle model, linear model, black box model, grey box model you know non linear grey box model whatever, it all depends up on what is the application, what is the system, what is the data available you know you have to take a call.

So please remember we are doing with data driven modeling your role as engineer the most physics is not less important it is equally important you cannot develop good models unless you know physics with the system well, okay so this is the tool which has to be use in conjunction which your know physics it cannot be.

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PEM: Parameter Estimation

$$\hat{\theta}_N = \underset{\theta}{\text{Min}} V(\theta, Z_N)$$

Typically, the resulting parameter estimation problem is solved numerically using

- (a) Nonlinear optimization
- (b) Gauss Newton Method

If it is desired to emphasize certain frequency of interest, then, we can minimize

$$V(\theta, Z_N) = \frac{1}{N} \sum_{k=k_0}^N [e_F(k, \theta)]^2$$

where $e_F(k) = F(q^{-1})e(k)$

$F(q^{-1})$ represents a band pass filter

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So that is why as a control engineer I am better so they could develop models in chemical industry because my back ground is in chemical engineer and a mechanical engineer is better so they could develop models for robotic problem because he knows those issues or for a car or so those things are important. Statics part anyone can pick up the domain knowledge you know it is important.

How do you solve this problem typically there are two methods one is non linear optimization then you also this whether called Gauss Newton method which is also quite popular which is used and those are few who are not done this should probably look at what is Gauss Newton method. There are number of issues you know do you want to emphasis certain frequency contained in your model okay, I know that the model the plant actually operates in this frequency range this knowledge is known to you from your physics have you understanding on the system.

And you can use it shape up the objective function, so instead of minimizing the sum of the square of errors okay, I can choose to minimize a filtered version of this, I can considerable fraction on pass it through a shaping filter okay, and then it is a band pass filter and does band pass filter will allow you to cut off some higher frequencies from this and focus on low frequencies how to choose this filter. Which one no, no we want a white noise but we are minimizing a filter value of it.

Objective function is different okay, we still want that is belong to be white noise okay, but is being minimized this yeah whether you always get a white noise or not is a good question I too have to, if you do this are you guaranteed to get a white noise is a question that is quite critical yes. I too have to go back and check whether you are grant to do at white noise.

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Model Order Selection

Model order determined by minimizing
Akaike Information Criterion (AIC)

$$AIC(\hat{\theta}_N) = N \ln \left[\frac{1}{N} \sum_{k=1}^N \varepsilon(k, \hat{\theta}_N)^2 \right] + 2n$$

n : Number of model parameters

AIC = {Prediction Term} + {Model Order term}

- ✓ Prediction Term: estimate of how well the model fits data
- ✓ Model Order Term: measure of model complexity required to obtain the fit

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So there are issues like model order selection okay, see one thing which you can which will find when you are doing any data effective okay, if you have a model with four parameters okay, and if have model with ten parameters, ten parameters model will always seem to fit better okay, as compared to four parameter model. Now which one I should use virtually it looks like you know ten parameters model is better.

So like sometimes they say in parameter estimation you know if you give me ten parameters I can fit in any element into data and if you give me 12 I can make the elephant walk so you can you know make seemingly better and better model if you have both parameters. But then there

are so many issues more parameters means larger data length it means longer experiment time means loss of production and so what is done is that when you are comparing different see actually when you are identifying a model for a system for which you are guessing the order okay, you will not know when you are given some data simple system of two times you can say okay, the second order or third order I you go to a real plant where you have power plant. No you have some complex boiler system what is the order very difficult to listen into make.

So you will start trying different orders second order, third order, fourth order smaller the better okay, and but I go to the call what order model I should fit, so we actually use this criteria called alike information criteria okay, to decide which model is good so it uses two things if you see here it uses some of the square of error see just because of a small sum of square of errors does not mean the model is better okay, we use two things one is information criteria base some of the square of errors it also wears the number of model parameters, okay.

So we compare a model with five parameters and with ten parameters using this criteria if you see here a ten parameter model we have this component more it may have this component the less but this component more okay, so in this criteria it has to balance between number model parameters and some of the square of errors which reduce because the number of model parameters okay, so you compare different models using so prediction so there is one term which is prediction term which is this term because this tells you how well the model fixes the data.

And the model N is some sense you know measure of a complexity of the model more the parameters, more complex model you have.

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Model Order Selection

Akaike Information Criterion (AIC)

Basic Idea:
Penalize model complexity (measured by n) and obtain a model, which is reasonable w.r.t. variance errors and model complexity

AIC strikes a balance between low residual variance and excessive number of model parameters, with smaller values indicating more desirable models

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So model of the selection you have to gain experience and doing this I mean go book field level tell you how do choose the model order, you have to see the other data you have to fit in data use that model of models and then you will get, so you penalize in this akaike information criteria you penalize the model complexity by adding the including those term or some penalty on N number of model parameters.

And then you know this is not the only thing that you would bothered what, you have to bother about variance errors you have to bother about some things are bias errors I am coming to that, so typically you use akaike information criteria I have given you where one information criteria there are more information criteria in the literature is the basin information criteria and some other methods of comparing models.

But is the curial question if you have data if you are developing models of different order how do you compare them, okay you might be developing simple correlation you know, simple correlation model. Whether to use a first order polynomial second order polynomial third order polynomial, okay which is better so you can use this akaike information criteria not just for time series modeling you can use it anywhere, okay.

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MIMO System Identification

- **ARX Model:**
Method for ARX parameter identification can be extended to deal directly with multivariate data
- **OE / ARMAX / BJ Models:**
Typically, an $n \times m$ MIMO system is modeled as n MISO (Multi Input Single Output) systems

$$y_i(k) = G_{i1}(q)u_1(k) + \dots + G_{im}(q)u_m(k) + H_i(q)e_i(k)$$

$i = 1, 2, \dots, n$

MISO models are combined to form a one MIMO model

Input excitation

Inputs can be perturbed sequentially or simultaneously

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So that is how will I use to compare models well real systems are multiple input multiple output low real system is you know truly single inputs single outputs, single inputs single outputs the study in a first course because everything is easy to understand the concepts you know digest the concept is easy and many cases you know some symbol systems approximating in the SESO system books, okay so not every time you require very complex since but real complex systems in today's world they are tightly integrated and they are difficult to use single loop controllers.

You have to use multi variable controllers, okay this is more and more so because of tight energy integration mass integration then all kinds of complex this I's are they coming up to know longer has system which are you know can we done this so you have to use multi variable, you have to develop multi variable models, multiple inputs affect multi outputs simultaneously and you have to model in a power plant or in a boiler and you can appreciate.

If I change the fuel flow it is going to change variety of things not just you know one parameter okay, it is going to change pressure it is going to change the level inside the ball where, it is going to change the oxygen contained in the you know fuel gas it is multiple things what happens and this is for everything you change one thing which has effect on multiple outputs. So in reality we have to develop MIMO models multiple input multiple output models problem is much more complex.

Now the issues when I have multiple inputs divide put up them simultaneously pr you have put up them one at a time okay, one at a time which nice you know you can so I get effect of one

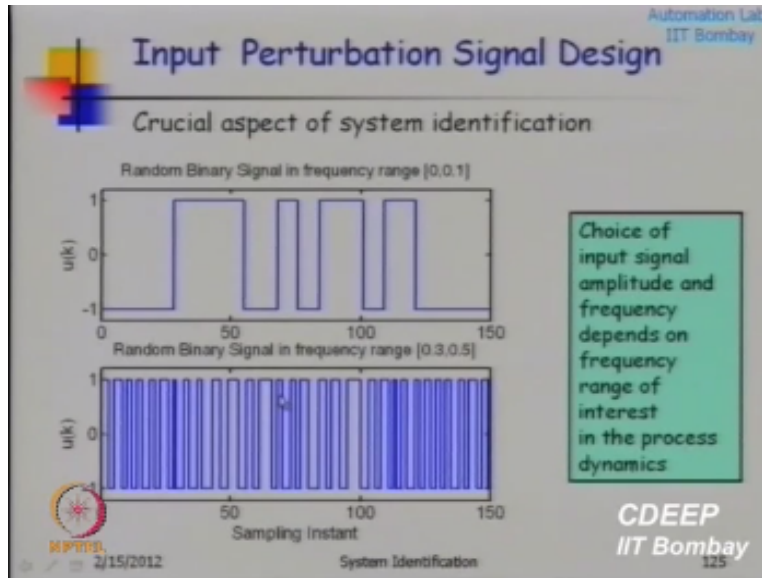
input but not that taking because if you have a boiler which has 5 inputs okay, you put a for one input for one day and 5 experiments for 5 days you are wasting you know the steam for 5 days not affordable you better put a all 5 simultaneously do experiment for one day okay.

Collect your data and do modeling nobody is going to allow you to do experiments of 5 days so these are the issues in real life that you have to worry about. Well ARX model can be very easily converted into MIMO model multiple input multiple output not an issue, okay the trouble is the number of parameters to do estimated just blows up okay, so some initial data levels trying to use ARX models you know we had 5 outputs 7 inputs with respect to each input output pair I required 40 parameters to get so huge number of parameters of course the data was there for 23 days so I could do lot of, I has used data so I could use a module which has lot number of parameters.

But you need lesson work parameters, if you have ARX model you can do simultaneous modeling of multiple input multiple output but and you can get solutions very easily that least square estimation just scales up does not matter whether you are working with scalars or vectors it just works very well. But there are issues in terms of number of data points these models output error model ARMAX model, Bosh Jason model what we do is, you developed what I call as MISO models multiple input single output models.

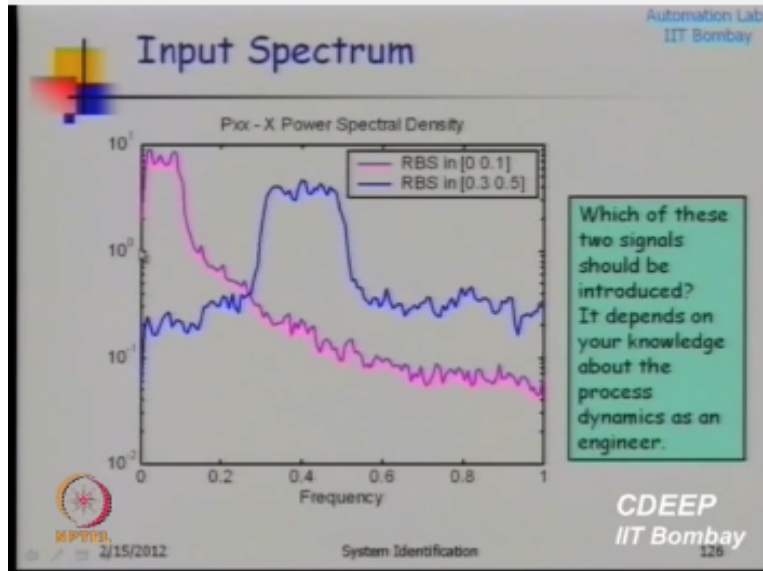
And then you fuse them to create a MIMO model that is typically what is done, okay so this MISO modeling is done. How you do excite the input well it is a tripping as this as I said, they will do it sequentially and simultaneously and you still find people writing papers what is the best way of doing it and not a certain issue.

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What should be my thus so many issues when you design an input signal what should be my choice able input signal should I use this signal or this signal both are silver random binary signals but they have different frequency content you can see that this has very low frequency depended thus a very high frequency content.

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How do I know this I look at the power spectrum okay, this signal here has this power spectrum it has lot of frequency content lot of power at low frequencies. But it has almost no power at high frequencies okay, whereas this signal and almost no power at low frequencies but significant power at middle frequencies okay, which excitation I should introduce golden question okay. you should know the system what is at what time scale are or what frequency scale the system operators.

Unless you know that you cannot make this call and this is very, very, very curial call if you do not introduce right you know frequency perturbations your data will not be useful for modeling because it may keep you a modeling a wrong frequency region why will you modeling around frequency. Well, I will talk about that a little later.

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Persistency of Excitation

A signal $\{u(k)\}$ is said to be persistently exciting of order 'n' if following limit exists

$$r_u(\tau) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N u(k+\tau)u(k)$$

and matrix

$$R_u(n) = \begin{bmatrix} r_u(0) & r_u(1) & \dots & r_u(n-1) \\ r_u(-1) & r_u(0) & \dots & \dots \\ \dots & \dots & \dots & \dots \\ r_u(1-n) & \dots & \dots & r_u(0) \end{bmatrix}$$

is positive definite

Persistency of excitation is essential for well conditioning of the parameter estimation problem

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One issue that is very, very important is what is called as persistency of excitation I have tried to teach this concept to a exercise problem of talking about estimating parameters of a two parameter model in which step input is given and I have tried to show that stable input will not be useful stable input will give you, you know this matrix to be prime definite and then you will have trouble estimating model parameters if you have.

So how do you put up the plan you actually becomes an art and guided by maths and the physics both you know, so the persistency of excitation is actually a technically defined as rank of this matrix correlation matrix of inputs if you want to identify Nth model with N parameters which is reference to the input this rank should be equal to N that is critical, if it is not your model will be you know the results that you get from your identification exercise even which we use the best package the results that you get is garbage that you have to understand.

See now you have good powerful packages when you are giving data and you are not making any mistake on numerical side okay, if you are getting a wrong model if you are getting and even a good package there can give you garbage because you have fading garbage that you have to understand. If you do not excite the plan properly if you do not make sure that this matrix rank is you know consistence with the choice of model order okay.

See if I want to identify model of third order then this matrix should have at least frankly probably I mean if I model of fifth order this matrix should have at least rank 5 and so on, okay. So it is not that given a data I can develop a model of any order okay if you are input is not you

know persistency exciting of order N you cannot develop a order 10 model very, very important. Where do this parameters, where do these things appear in the equation.

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Properties of Parameter Estimation

Similarly

$$\lim_{N \rightarrow \infty} \frac{1}{N} \mathcal{C}^T \mathcal{Y} = [-r_p(1) \quad -r_p(2) \quad r_{ps}(2) \quad r_{ps}(3)]^T$$

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See here this r_u , r_n , r_0 these are appearing here okay, this sub matrix here rank of this sub matrix actually is very, very critical rank of this sub matrix. It should be two if it is not two you have a trouble okay, and it will cost trouble given here if this sub matrix rank is not two you will have trouble even here so there are issues in which are deeply regressive matrix rank if you make a wrong choice of the input excitation okay, very, very important.

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Asymptotic Bias and Variance Errors


In reality, true model order is not exactly known, data length is finite and data contains unmeasured disturbances / noise. This results in two types of errors in estimation

$$G(q) - G(q, \hat{\theta}_N) = [G(q) - G(q, \theta^*)] + [G(q, \theta^*) - G(q, \hat{\theta}_N)]$$

$[G(q) - G(q, \theta^*)]$: structural or bias error induced by fact that model set is not rich enough to exactly characterize the plant

$[G(q, \theta^*) - G(q, \hat{\theta}_N)]$: noise induced or variance errors due to unmeasured disturbances / noise

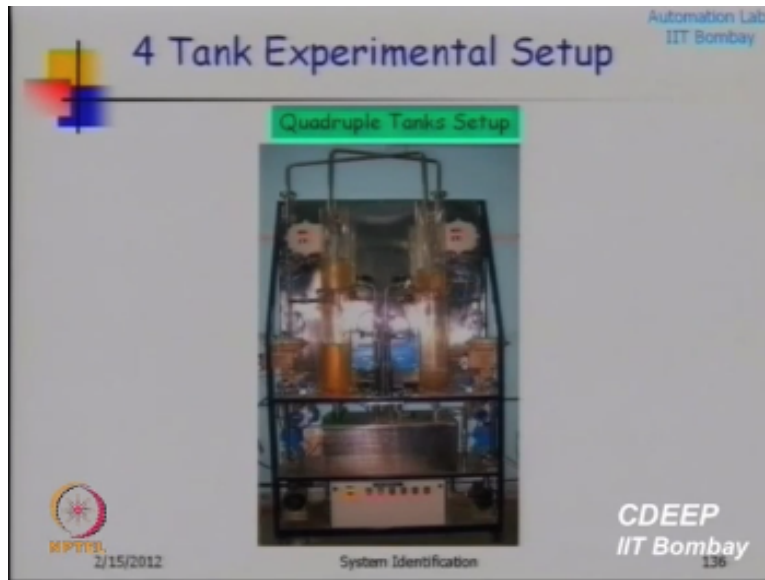
$$\left\{ \begin{array}{l} \text{Total Error} \\ \text{of Estimation} \end{array} \right\} = |\text{Bias Error}| + |\text{Variance Error}|$$


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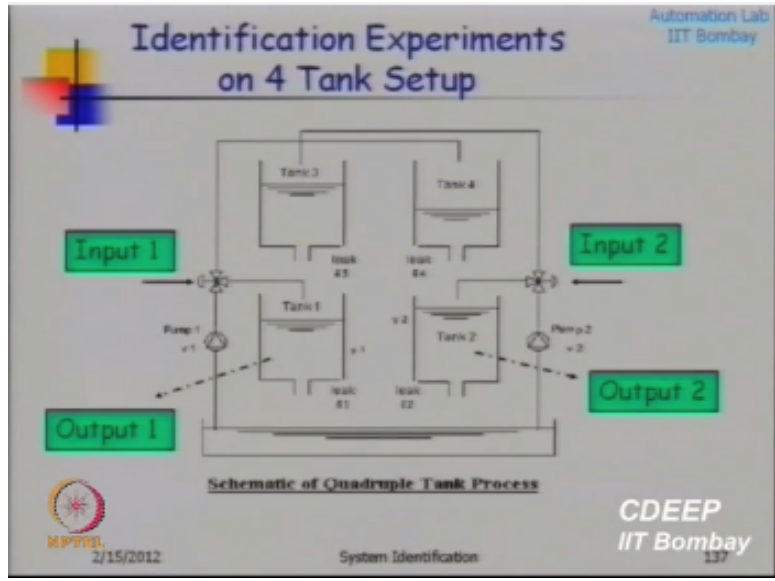
Now before sensitizing you know something which is visibly advanced I am not going to look over this apart though I have given some explosive to this part towards the end of my nodes after the last slide I have put some extra slides on this issue. There are two kinds of errors that can occur when you identify model one is called as the bias error other is called as the variance error. I will just give you sensitizing about this after I talk about the real example. So let us take, let us depart from here so to this problem, okay.

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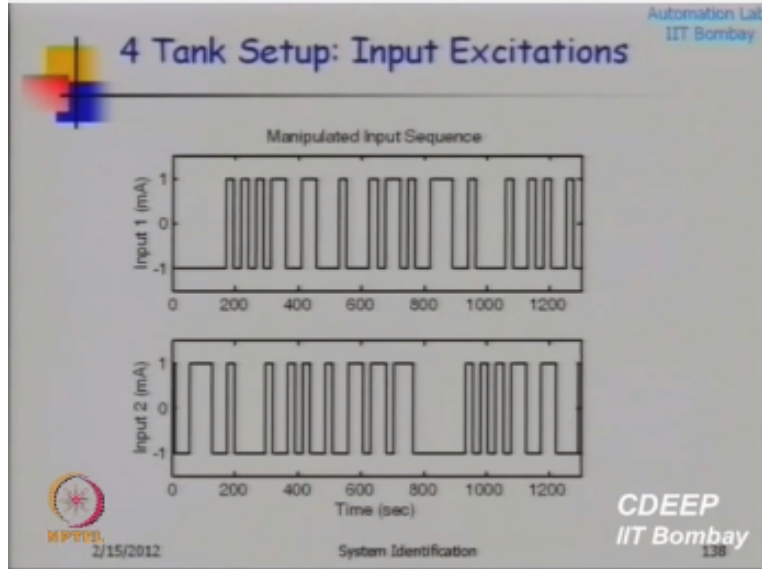
Now this is the system which I have in my lap okay, I want to develop a black box model okay, I will develop a black box model.

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If you want to apply with the data I can put in model and you can download the data and ply with the data put it into identity to works and then you can develop all kinds of model yourself. There are two inputs okay, and there are two measurements the four time problem which we are been looking at for the long time same problem.

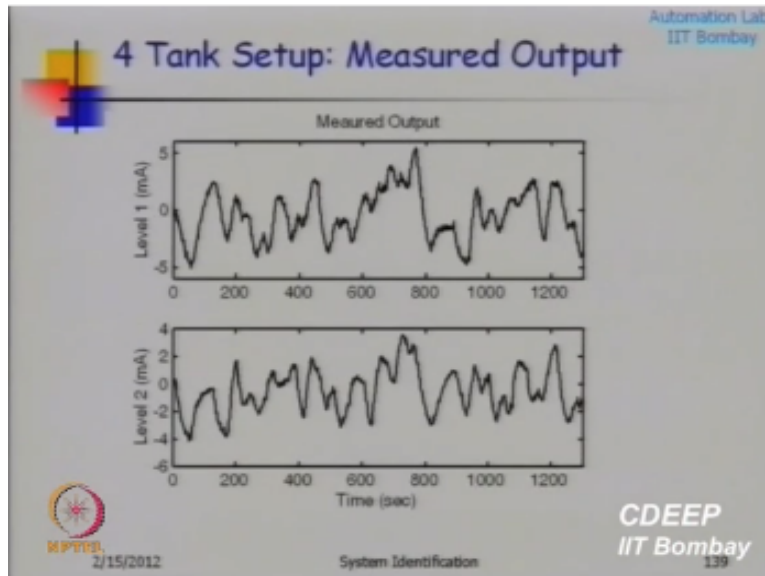
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Some time what do you afterward might send you will actually visit that then I will show this setup I have the setup you can actually and some of you are interested you can do a project on that you have freedom to do that. Instead of doing project on a toy simulation example you can choose to do a project on maybe I give a different vectors if you are going to do that, that is much more difficult.

But you will learn far more I should do the project on the real data then if you do it with you know tattoo, you have problems are much more complex so I simultaneously perturb both the inputs two walls simultaneously perturb okay. Frequency contain I could choose based on the knowledge of the time constants, rough idea that I am constants I know that this particular plan time constants are all over two minutes I am based on that I have made a call and how to do this.

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So these inputs let me generated is in mat-lap identification tool box there is a function called ID input okay, you can give what type of input you want to generate what should be the frequency contained with references to the MISO frequency π/t , what fraction of mat-lap frequency t is the sampling interval π is π/t gives you matrix frequency with references at mat lap frequency what fraction you want to so what is that fraction that you should decide doing a plan frequency, okay that is critical. So it is very, very important that when you are exciting two inputs they should not be identical if they are identical then that means of this matrix will not be N.

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Persistency of Excitation

A signal $\{u(k)\}$ is said to be persistently exciting of order 'n' if following limit exists

$$r_u(\tau) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N u(k+\tau)u(k)$$

and matrix

$$R_u(n) = \begin{bmatrix} r_u(0) & r_u(1) & \dots & r_u(n-1) \\ r_u(-1) & r_u(0) & \dots & \dots \\ \dots & \dots & \dots & \dots \\ r_u(1-n) & \dots & \dots & r_u(0) \end{bmatrix}$$

is positive definite

Persistency of excitation is essential for well conditioning of the parameter estimation problem

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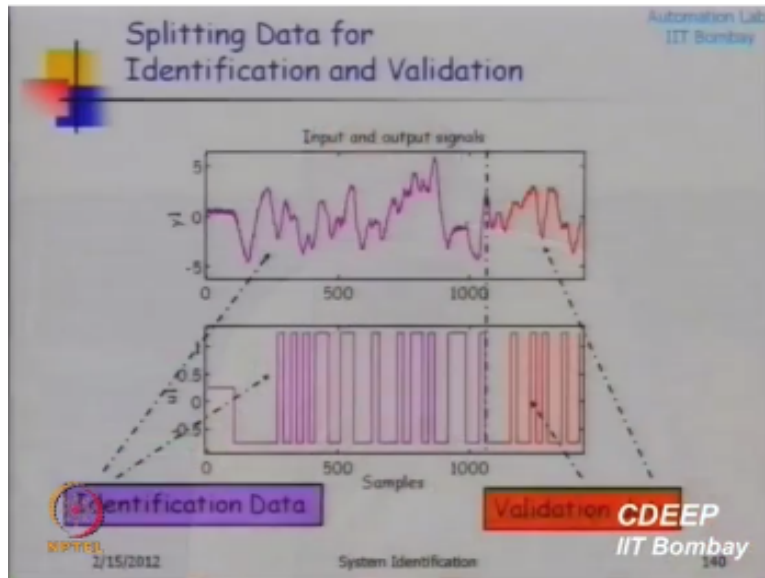
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Okay, they should not be scalar multiple of each other if you are simultaneously perturbing a plant they should be on core related and ID input to make sure if it is simultaneously generate to two inputs it will generate uncorrelated inputs, very, very critical that these two inputs should not have any correlation okay, very, very critical that is the meaning of actually persistency of excitation node.

So these are the two outputs I collected both the levels remember each level is changing because of both the inputs simultaneously it is not just function of 1 okay. Now I am going to do this.

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I am going to take part of the data for modeling and remaining for validation okay, the model which is developed using this part of the data should predict this behavior then this is model is good okay.

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MISO OE Model

MISO 2'nd Order Model

$$y_1(t) = [B_1(q)/A_1(q)]u_1(t) + [B_2(q)/A_2(q)]u_2(t) + e_1(t)$$
$$B1(q) = 0.1393 q^{-1} + 0.04704 q^{-2}$$
$$B2(q) = 0.002375 q^{-1} + 0.01105 q^{-2}$$
$$A1(q) = 1 - 0.2454 q^{-1} - 0.6571 q^{-2}$$
$$A2(q) = 1 - 1.887 q^{-1} + 0.8903 q^{-2}$$

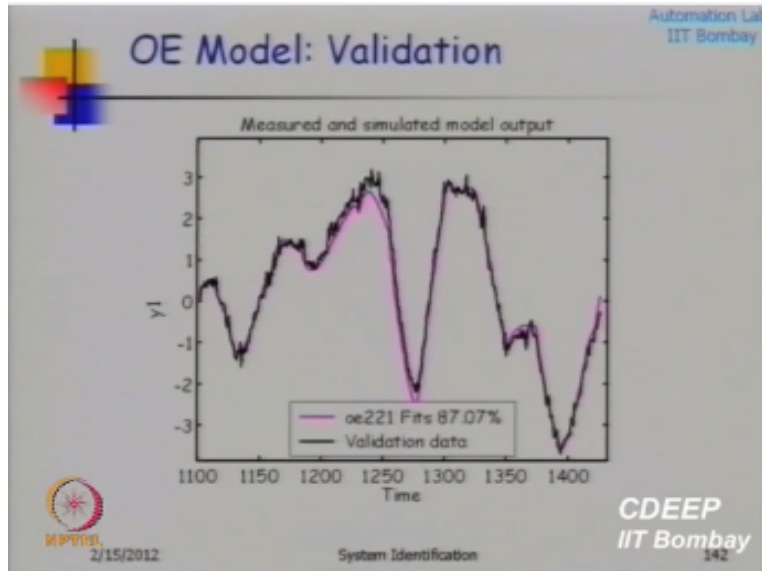
Estimated using Prediction Error Method

Loss function 0.114719 Sampling interval: 3

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How developed variety of models I have developed second order output error model then you know.

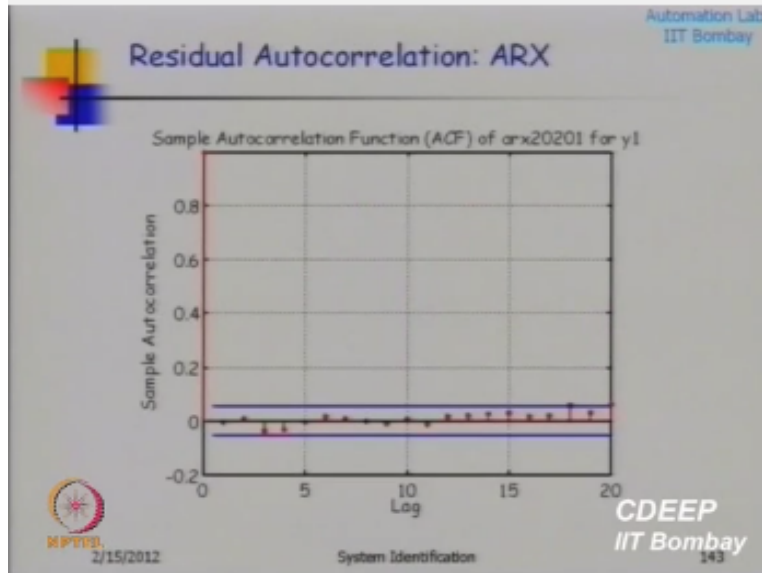
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The predictions seems to be okay 87% fit means it is explaining sum of the square of error there is a this system identify works defines something called percentage fit using sum of the square of errors, so it gives you a good measure of how good is the data and mind you this is not so this part of model validation has been carried out on this part of the data, this data. So these inputs have been given to the model identify I want to check how well the prediction so these means I have just shown here one graph but actually up to 1000 I think up to 1000.

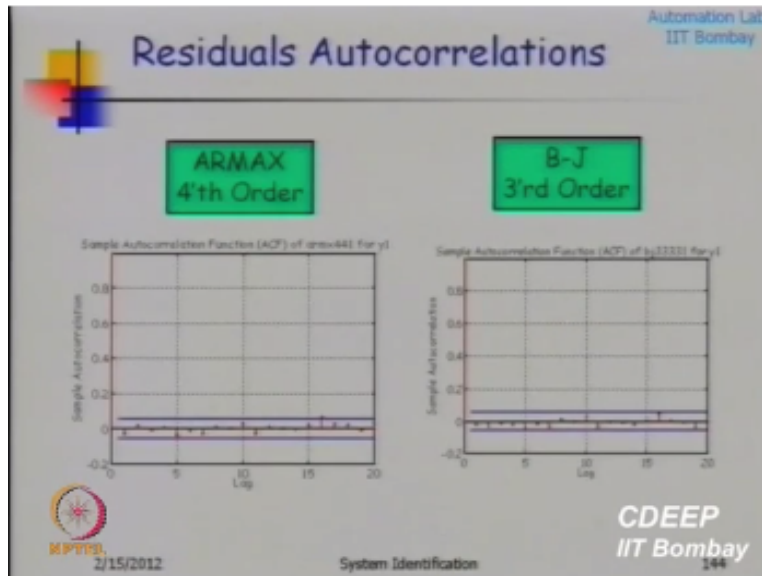
Yeah, upto 1000 okay, I will use for identification and 1000 to whatever 13,000 so this part both the inputs starting from 1000 to 13000 have been injected into the model and I am going to predict how well is level 1, so you can see that it is a good model. Unknown data is being predicted residual is in second order model.

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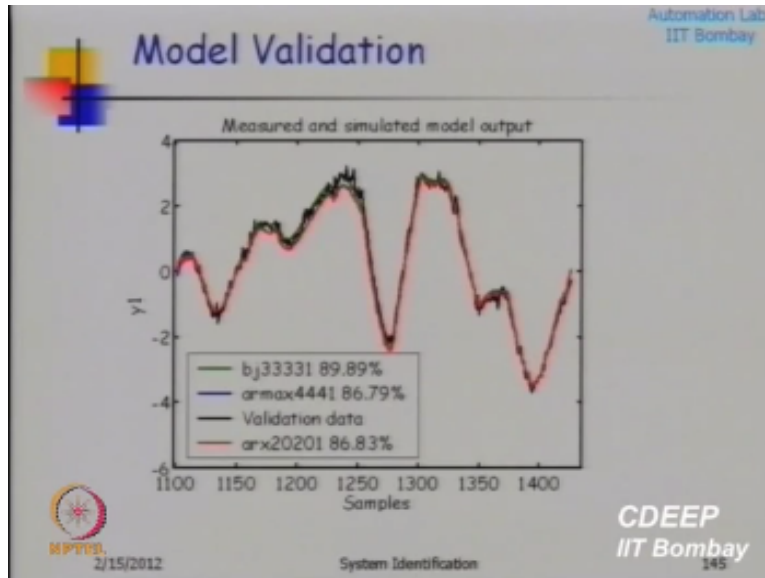
Well in this case the noise is uncorrelated great okay, no problem.

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You can go on developing different models you can compare them on ARX criteria I am just showing here auto correlation of residuals I had developed a third order Box Jenkins model I will looked a fourth order, I am comparing different models.

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Well, box zining use 89% little more little better than the other and so on, so you can actually compare the model output.

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ARMAX Model


ARMAX (4th Order)

$$A(q)y_1(t) = B_1(q)u_1(t) + B_2(q)u_2(t) + C(q)e_1(t)$$
$$A(q) = 1 - 0.6236 q^{-1} - 0.8596 q^{-2} - 0.0758 q^{-3} + 0.568 q^{-4}$$
$$B_1(q) = 0.08324 q^{-1} + 0.02757 q^{-2} + 0.02681 q^{-3} - 0.1214 q^{-4}$$
$$B_2(q) = 0.004045 q^{-1} + 0.03261 q^{-2} - 0.01841 q^{-3} + 0.0201 q^{-4}$$
$$C(q) = 1 - 0.4695 q^{-1} - 0.8017 q^{-2} - 0.1065 q^{-3} + 0.4855 q^{-4}$$

Loss function 0.0243707

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These are the model parameters which will be.

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Box-Jenkins Model

$$y(t) = [B(q)/F(q)]u(t) + [C(q)/D(q)]e(t)$$

$$B1(q) = 0.08196 q^{-1} + 0.1035 q^{-2} + 0.1323 q^{-3}$$

$$B2(q) = 0.01197 q^{-1} + 0.001306 q^{-2} + 0.01304 q^{-3}$$

$$C(q) = 1 - 1.976 q^{-1} + 1.126 q^{-2} - 0.1453 q^{-3}$$

$$D(q) = 1 - 2.096 q^{-1} + 1.209 q^{-2} - 0.1128 q^{-3}$$

$$F1(q) = 1 + 0.3058 q^{-1} - 0.5066 q^{-2} - 0.6204 q^{-3}$$

$$F2(q) = 1 - 0.897 q^{-1} - 0.9828 q^{-2} + 0.8861 q^{-3}$$

Loss function 0.0239039

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Obtained okay, so this case study you can actually do it yourself I will upload this data and you can take this into Mat lap tool box and then you can play with it, you can put all kind of models and see what model parameters you get then you can take it to you know LTI view you can compare the model frequency responses, state responses, emphasis responses like the sorts you will get better inside into what is happen.

So you can develop variety of models at your finger tips and it is not so, of course when you go for controller design when you data call which one of them you are going to use and how, so the next part logically is going to be you know using them for controller design and my next lecture is going to be converting this models. See I have this models into this transfer function form okay, my entire course is based on discrete time state space model.

Whether when should work using transfer function models or state space model, well it is a matter of choice. In control there are two communities one fair by transfer function others like only state space I belong to the state space category I will like to work with state space because you can use linear algebra okay, not that you cannot use linear algebra when you are using transfer functions but a masteries containing polynomials of q there manipulations become very missy if you go for higher order systems. For state space in my opinion whether you work with single input single output or multiple input multiple output max remains same everything is, so industry is thus now does the first to take space lot of controllers are return using state specific variations. So what I am going to in my next lecture is.

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State Space Realization


Consider 3rd order SISO T.F.

$$y(k) = G(q)u(k) = \frac{b_3q^3 + b_2q^2 + b_1q + b_0}{q^3 + a_2q^2 + a_1q + a_0} u(k)$$

Problem : Derive state space model of form

$$x(k+1) = \Phi x(k) + \Gamma u(k)$$
$$y(k) = Cx(k)$$

such that $C(zI - \Phi)^{-1}\Gamma = G(q)$

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What is called as state space realization of a time series model, I want to convert my model into a state space model so basically I will start with this okay, and systematically explain how do I come with a state realization okay, which will look like this in the standard form okay. In this particular case x may not have any physical meaning unlike when you start from physics and develop a state space model okay, the state x has a physical meaning temperature, pressure level whatever it comes from physics.

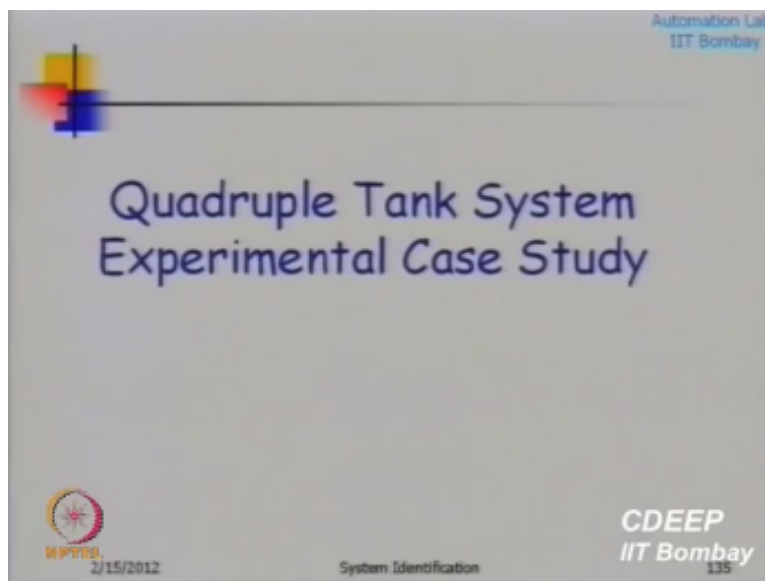
In this case when I start from this transfer function model and go to this then call state realization okay, x has no physical meaning what is physical meaningful here is U which is the input and Y , Y is the measurement and all that you are saying is that this difference equation okay as a same transfer function as this okay, so the next part of this is going to be connecting up with this standard form if you remember we develop this standard form starting from physics starting from first principle model we linear zed developed the standard form now I am going to develop the standard form starting from this data driven black box model okay.

Finally when I start working with the controller development I am just going to use this standard form I am not worried where does it come from whether it is come from physics whether it is come from data whether it is come from some merger of data and physics it is up to the model developers okay I am just going to work with this standard model form for control a design of the system whatever.

So that is how the game planning that you could start with physics if your physics model you could start with your data if your data do a model finally you come up with this one standard form and then do your Maths on the standard form see once you have translated the reality of the system in to a mathematical description okay then you are in the world of mathematical model you can do can do all kind of manipulations okay.

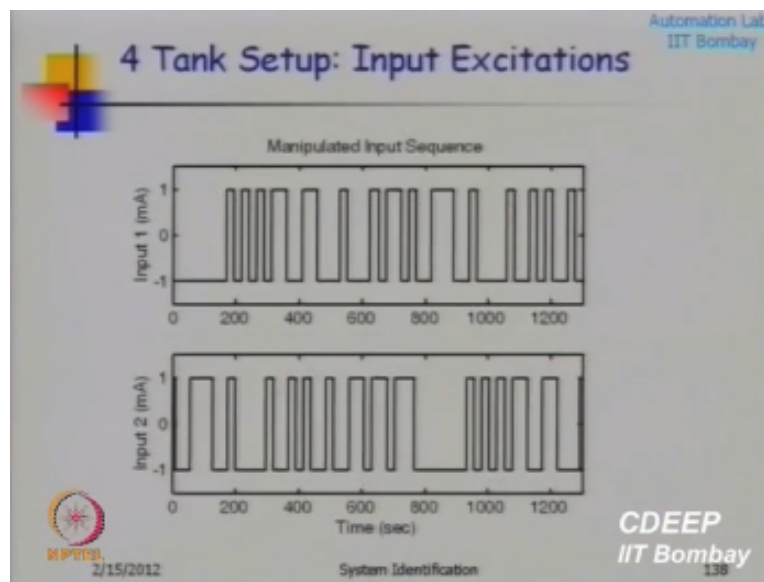
So I am going to stick with the standard form and do all the monopolizations that is my game plan okay so how to do this from starting from is there a unique way of doing this the all kind of questions will also discursion you know how do you go from here to here okay. that is the main way of doing it there is no unique way of doing it you can show that there are infinite way of doing it so we are going to prefer and do it by some 2 3 different way which are popular. But that let us reserve that for the next class.

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So that is where the story ends that you start with data you come up with this time series models you convert in to state space model and then I will use state space model for control okay yeah.

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They should not have co- relation they may or may not have different sequence it depends upon some inputs have very fast influence on the outputs some inputs have slow influence in the outputs you know the fuel generalize it depends upon the system okay. When you are validation in to that thing is not so critical in validation data can be correlated identification you better make sure that the inputs are uncorrelated particularly if you are simultaneous exiting okay.

So statistical properties of the input signal and frequency domain property of the input signal at equally important when you design is impossible for identification validation does not which

input what input is white noise, if you give white noise well on paper ideal input is white noise, okay then if you have white noise then you know then the persistency of excitation is as much as you want okay develop any model okay.

But white noise is not a great signal to inject okay so we need typically we do not inject white noise we inject a color noise so see you can see here know that if I inject this signal if I am injecting this signal this it is color noise it has only see what is white noise it has all the frequencies so that will be ideal signal and if you inject white noise then you know white noise is the perfectly presses in the exiting signal that thing better than that but white noise cannot be inject the reality in computer simulation you can inject white noise you will get great models.

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Persistency of Excitation

A signal $\{u(k)\}$ is said to be persistently exciting of order 'n' if following limit exists

$$r_u(z) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N u(k+z)u(k)$$

and matrix

$$R_u(n) = \begin{bmatrix} r_u(0) & r_u(1) & \dots & r_u(n-1) \\ r_u(-1) & r_u(0) & \dots & \dots \\ \dots & \dots & \dots & \dots \\ r_u(1-n) & \dots & \dots & r_u(0) \end{bmatrix}$$

is positive definite

Persistency of excitation is essential for well conditioning of the parameter estimation problem.

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But not practical okay so this matrix will be you know perfectly condition if you introduce by pass because you know this matrix are diagonal if you introduce white noise is matrix is diagonal nothing like it but you cannot do that white noise.

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Asymptotic Bias and Variance Errors

In reality, true model order is not exactly known, data length is finite and data contains unmeasured disturbances / noise. This results in two types of errors in estimation

$$G(q) - G(q, \hat{\beta}_n) = [G(q) - G(q, \theta^*)] + [G(q, \theta^*) - G(q, \hat{\beta}_n)]$$

$[G(q) - G(q, \theta^*)]$: structural or bias error induced by fact that model set is not rich enough to exactly characterize the plant

$[G(q, \theta^*) - G(q, \hat{\beta}_n)]$: noise induced or variance errors due to unmeasured disturbances / noise

Total Error of Estimation = |Bias Error| + |Variance Error|

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I will just deeply talk about this now maybe I will continue in the next lecture there are two fundamental issues and to understand this actually you have to change that gear and it become much more complex than whatever we have been doing till now if you consider whatever they have been till now is simple then this is relatively more complex okay.

The analysis has to be done frequency domain okay, because in time domain you can do identification in all those things but the incites come only from frequency process to go from time domain to frequency domain incites using two norm is very critical because there are certain relationships transformation from time domain to this come only because you define to now and you can best books for this system identification volume or sort of like books are have the set them in my they are too control engineers who have excellent books which gives lot of incites in to these aspects.

Now let me explain what is this animal see you have $g(q)$ let us assume that there exist for a given system they are exist something call a true transfer function okay do not ask me question whether for a real car or that does not exist it does not exist it is a hypothetical scenario suppose there it is exist suppose it is exist then the question is will you reach it when you identified from data this question you can definitely ask for a computer simulation if I am when I am simulating the plant inside the computer I am got I am creating the world inside my computer program.

So I know that there your perfect system g_q and I give to a modeler only to a data and ask you to identify question is will be able to reach the truth only from data okay now there are two problems see if I give you a data for a system let us say for a boiler okay.

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Asymptotic Bias and Variance Errors

In reality, true model order is not exactly known, data length is finite and data contains unmeasured disturbances / noise. This results in two types of errors in estimation

$$G(q) - G(q, \hat{\theta}_N) = [G(q) - G(q, \theta^*)] + [G(q, \theta^*) - G(q, \hat{\theta}_N)]$$

$[G(q) - G(q, \theta^*)]$: structural or bias error induced by fact that model set is not rich enough to exactly characterize the plant

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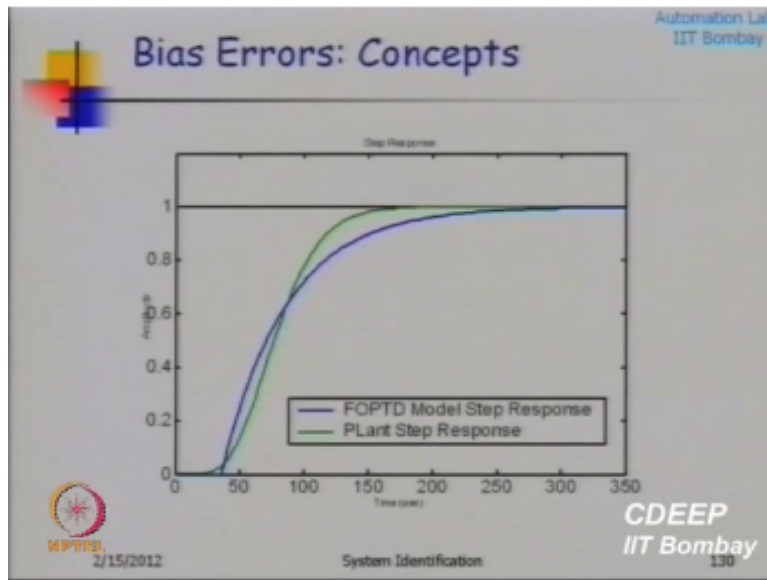
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You know that boiler is something which is the real boiler is inter connected a system of multiple system okay, so actually what should be the order of this system it should be high order it is typically high order system okay, but as modeler I do not want to use large number of differential equation for differential equations I keep them small so I tend to tend to chose a low order model okay, so quasic example is you know let us say in process control we use this kind of models these are called as time delay at day time plus time constant model.

For certain time constant plus a date time this is use to approximate a high order model why this because this is simple model one time constant and one time delay has compare to 8th order model okay, 8th order model means 8th differential equation or 8 first order differential equations okay, so instead of that I will like to work with one differential equation you know so but what is happen if I for a real plan which is like this if I approximate using this model okay there is structural error between the true plan and the model okay. And this gives raise to something call bias error okay.

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See if you look at this step wise concepts they look similar okay this blue is the true take this points this red responses of the first FOPTD is first order plus time delay this models are available in systematic tool box you can go and it will tell you which kind of model you want o identify you can say that you do not want ARX OE I want FOPTD model it will give you a model which is of this form okay.

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Bias Error: Concept


Real systems are of very high order and model is always chosen of lower order

Thus, bias errors are always present
in any identification exercise

Classic Example in Process Control

Process Dynamics: $G(s) = \frac{1}{(10s+1)^8}$

Identified FOPTD model: $\hat{G}(s) = \frac{1}{(50s+1)} e^{-36s}$



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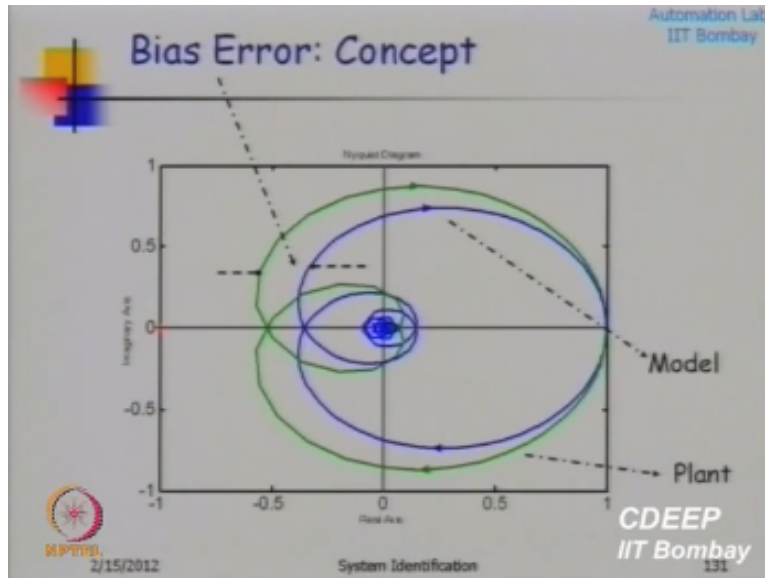
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If you just compare the state responses of these two they look very similar, you know it say good model and I am getting there you know the state response are similar if I compare the frequency responsible of these two models see if I say that this is truth map this is the estimated model of the truth map then you know and if I just show you this bio relation will say good model I will use this for control okay, look at the frequency responses.

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The comparison of frequency responses this is the model frequency response and this green one is the plant it this is my approximation which is my first order time delay constant and this green one is the plant 8th order model this is my approximation which is first order about time delay approximation it is very good in the low frequency Maths is very good in low frequency region okay.

But this high frequency region this match is not good suppose you are controller in close loop you are operating in this region in this frequency band this is the large mismatch between the model and the plant I will give you big problem when you control I will give you big problem when you control okay.

Now what you can do is okay this is called bias error and this is going to occur in most of the situation real situation real situations real problems are very high order okay we always approximate them using low order model so when you approximate a high order system is in a low order model this always a bias error okay, because your model is not able to capture the reality it has limitations okay.

What you can do of course is that you can shift this error you can say that well instead of good mat in the low frequency I want the good match in this repeats how do I do that I go back and say that I will use this signal for input exestuation then using this signal. So using this signal or instead of this signal will it ensure that you will get good model in that frequency range yes so that I will talk about in my next lecture I will just end that next.

My next lecture you can show that well the you can shake the difference between the true and the estimated using the power spectrum of inputs so how do you shake the power spectrum of inputs is lot of bearing in what region what people see band model is good what frequency band the model you band so all these analysis comes only with two norm that is why we use two norm yeah.

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Bias Error: Interpretations

$$\lim_{N \rightarrow \infty} \frac{1}{N} \int_{-\pi}^{\pi} \left[\hat{G}(e^{j\omega}) - \tilde{G}(e^{j\omega}) \right]^2 \Phi_s(e^{j\omega}) - \Phi_n(e^{j\omega}) \frac{1}{|\hat{H}(e^{j\omega})|^2} d\omega$$

- Bias distribution of $|\hat{G}(e^{j\omega}) - \tilde{G}(e^{j\omega})|^2$ in frequency domain is weighted by Signal To Noise Ratio
- Input spectrum can be chosen intelligently to reduce variance errors in certain frequency regions of interest

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So this I am going to sensitize you not really get in to deep because this might need 2 or three weeks of lecture really we get in two.

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