Advanced Chemical Reaction Engineering
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Lecture - 33
Population Balance Modelling –III

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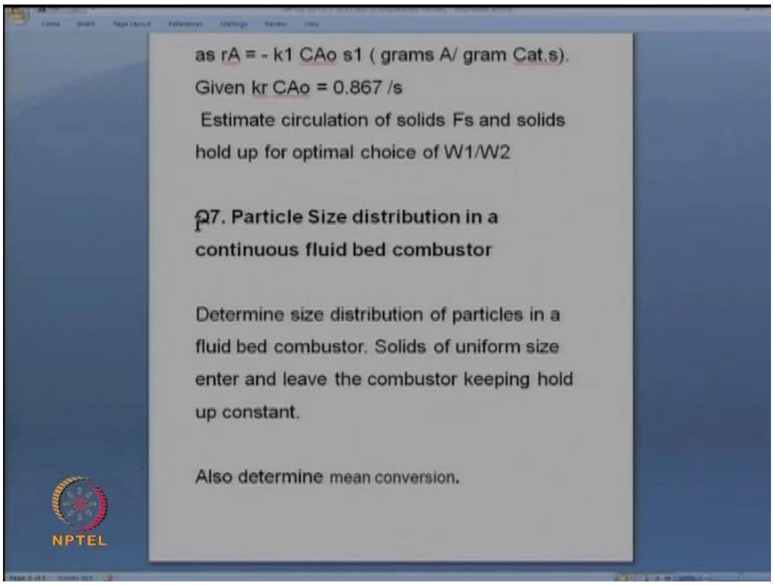

We want to look at next problem now that is particle size distribution in a fluid bed combustor.

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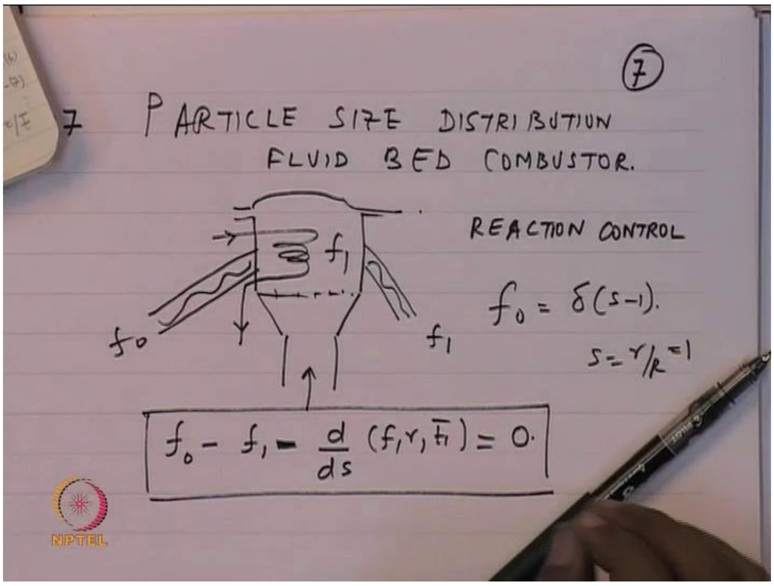

Particle size distribution in a fluid bed combustor; the context is the following you have coal going through ash coming you have gases; then you have all these hit transit tubes. Now, this is a very crude a crude way of trying to understand coal combustor. What we want to find out is; what is a size distribution of solids that come out to the fluid bed? How do we do this? The example I have taken here is reaction control. Now, just put the context of coal combustion in particularly in India; coals combustions are not in large scale fluid bed combustion are not combination.

But BHEL as has good number of fluid bed combustors now in working; it has many advantages has a pointed out to you to heat transfer efficiency are very large; it as disadvantages is that also we have seen the power consumption very large. So, there are some advantages there are some disadvantages. Now, our population balance look something like this; d by ds of f 1 r 1 t one by this how we have written our population balance. What is f naught? f naught is what is goes in f 1 it what come out, which is the same as what is they equipment this what we have been writing for quite some time.

But the context here is that we said whenever you write the population balance; we like to see whether these the distribution functions whether they are differentiable if it is not a differentiable function we said we should write it slightly differently. Now let us try to understand what is a f naught what is a f naught; it is the size distribution inlets solids if that size distribution is uniform of radius r then it is sum s minus 1, where s is r by R equal to 1. If all the particles at uniformly distributed at one particle size then it is a delta function at s equal to 1, where the entire particle are radius r.

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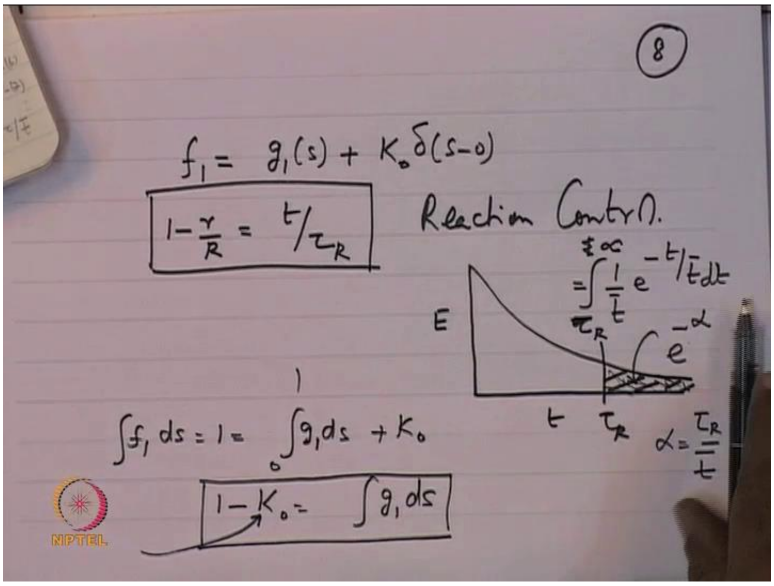

Now, second thing is what is our understanding of f 1 to context is we know that in reaction control this is the reaction control we know this. We know that our particle sizes are determine by where this is the time of complete consumption of the particle. So, what is our understanding a f 1? Viewed expect that there is since our fluid beds are like stirred tank; stirred tank has got exponential RTD. Therefore, if this is the time for complete consumption of the particle; this material would be completely consumed. Completely consumed means, what it is size is zero its size is zero, which means your f 1 should have a continuous part and then there is a discontinuous part, which is at zero.

Therefore, our size distribution of particles that emerge from the fluid bed this f 1 will have a continuous part which is g 1. It will have a discontinuous part corresponding to particle size corresponding that is equal to zero. So, keeping this will mind keeping this will mind we can say that therefore, integral f ds is 1 therefore, that is equal to integral g ds plus k 0. Therefore, 1 minus of k 0 is integral is 0 to 1 g 1 ds. What is this area?

Integral 0 to 1 1 by t bar e to the power of t by t bar tau 1; how much is this he said what is equal to give integrated many times is e to the power of minus of alpha. So, this area is e to the minus of alpha, where alpha is tau R by t bar.

You have done this. You can integrate in check for yourself, which means you are this fraction is completely consumed. Therefore, what is k 0 tau R to tau R to infinity I hope you understand this because at time goes from up to infinity that is why. So, what we have saying is that this value of this k 0 is such that this actually this value of this k 0 is known; because the fraction that is completely consumed is e to the power of minus of alpha. We know this on other words even before you solve the population balance you have some understanding what these solutions are going to look like you are not totally in the dark.

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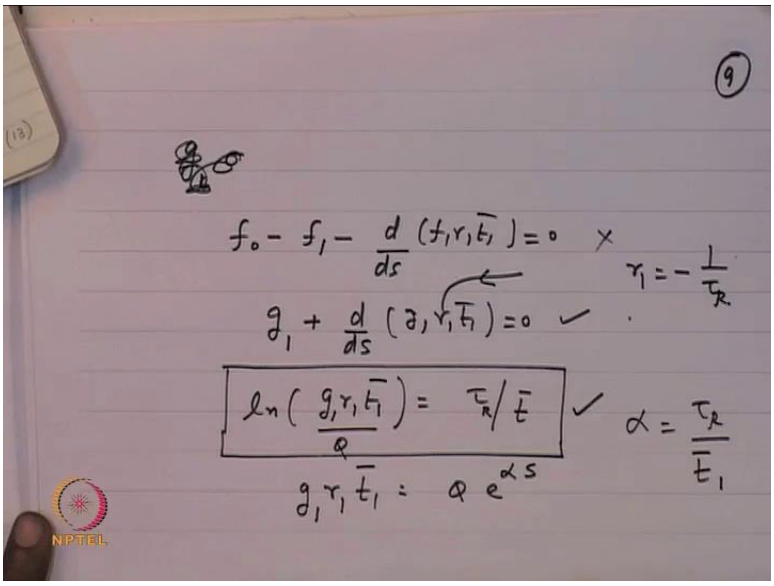

Let us now look at the population balance; we said whenever we have all these kinds of problems; f naught minus of f 1 minus of d by ds of f 1 r 1 t 1 bar equal to zero. What we have said is that we will only look at the continuous part and then our f we will not look at f we will look at the continuous part of the distribution. Therefore, we are only look at the g function g 1 plus d by ds of g 1 r 1 bar t 1 bar equal to 0. So, we will only solve this this equation and not this equation why did we say that we said whenever we have this kind of discontinuities.

Simply, look at the problem minus this entire problem all these difficulties and create appropriate boundary conditions to account for wherever these kind of unbounded discontinuities exist. So, let us integrate this. So, please integrate and tell me I am writing the final answers I will integrate please tell me whether what I done this correct. What I get tell me whether this is right. What is r 1 here? 1 minus of s equal to t by tau R therefore, ds dt which is r 1 equal to minus of 1 by tau R. For reaction control 1 minus r by r is t by tau R we have derive this r 1 minus of s is t by tau R.

We are looking at the property how did determine the rate function is simply look the variation of that property with respect to time. So, we have done that. So, you get 1 minus r. So, r 1 here is minus of tau R recognizes that. So, here the r 1 here is r 1 equal to minus of r 1 tau R what we are saying. For reaction control, we have derived this this we have derived in our class r by r is s. So, 1 minus s is t by tau R d by dt of minus of 1 by tau R for different rate for controlling the things we get different kinds and rate functions this is tau R. So, now, tell me whether the solution is correct; r 1 is minus of 1 by tau R put it here integrate and tell me.

So, it is being quite means what if not is it or not it should be an in fatty guess. So, therefore, g 1 r 1 t 1 bar equals Q times exponential ill call it as alpha s, where alphas is tau R by t bar. So, it is only a single so the single fluid bed sees our system is this. It does not there is only one system here. So, therefore, I did not bother see I should put even I could not put even everywhere and I will put even here. How do you find Q? So, to find Q off course there is many ways, which you can began.

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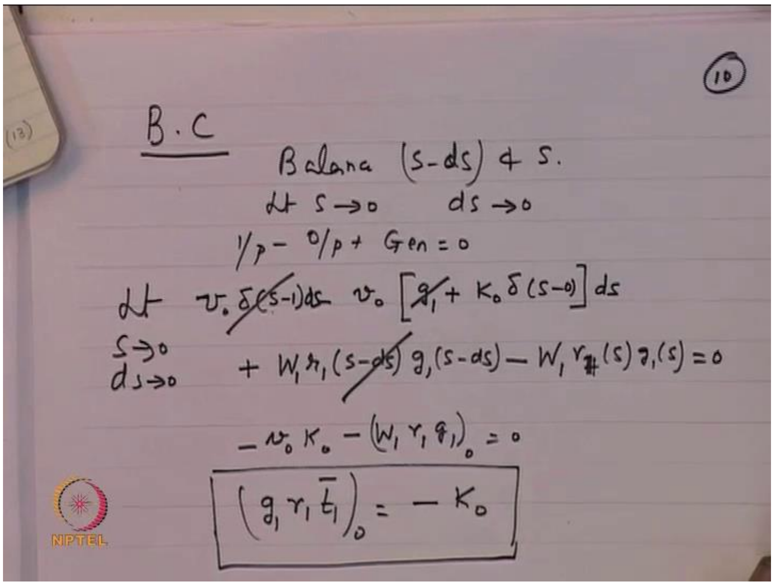

But let us just formulate the boundary conditions just to get a boundary or B C s; let me do a balance I will do a balance between s minus of ds and s; limit as s tends to zero ds tends to zero. I will write all the inputs I will write all the outputs and the generation equals to zero. What are the inputs? u naught delta of s minus 1 this is the input. What is the output? v naught that is going out g 1 plus k naught delta of s minus 0 then ds. Inputs and outputs are correctly written. Now our rate functions W 1 r 1 at s minus ds g 1 at s minus ds minus W 1 r 1 of s g 1 of s equals to 0. So, limit s tends to 0 ds tends to 0 I put ds ds ds here.

So, s tends to 0 this term goes away this goes away what else what else goes away this goes away. Why these terms disappear? So, these terms disappear because it is s equal to 1 we talking about s tending to 0. So, this disappears. This is the continuous function ds tend to 0. So, therefore, this goes away and but this is k naught. So, this becomes minus u naught k naught minus W 1 r 1 g 1 at 0 equal to 0 or r 1 t 1 bar g 1 at 0 equal to k minus 0 and we can learn to do this at other boundary also, but it is not necessary.

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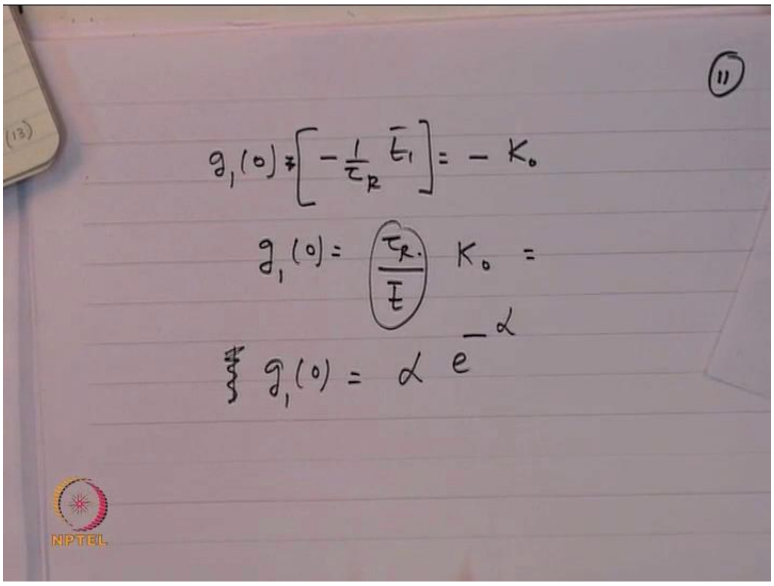

Now, tell me what is g 1 0? Now from this what do we get g 1 0; what is r 1 minus 1 by tau R t 1 bar equal to minus of k 0. So, g 1 at 0 equal to tau R by t bar k 0. So, this is alpha k 0 we already said that it is minus of alpha. So, where our solution is let us look at our solution now so, we said g 1 0 from here can we find Q; this is our solution here I want Q. Now, let us find the value of Q and tell me what the value of Q is.

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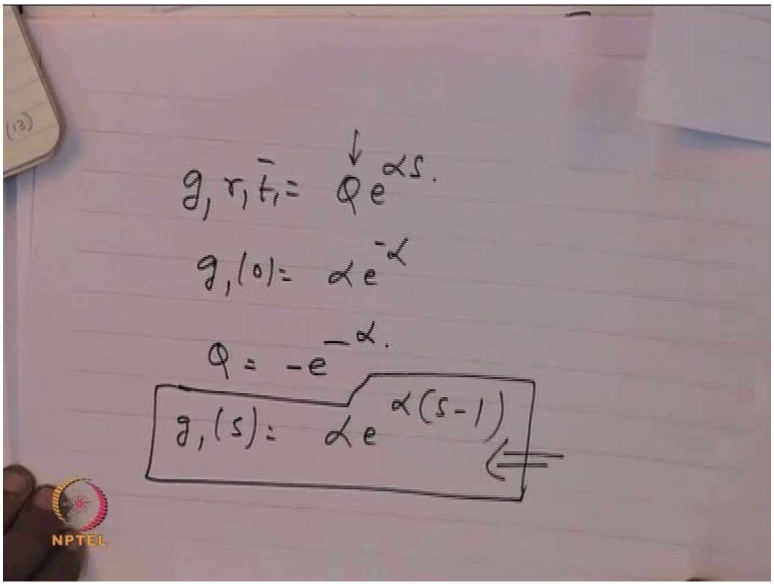

This is our solution g 1 t 1 bar equal to Q times e to the power of alpha s; we know that this is equal to the alpha times e to the power of minus alpha. So, what is our solution? What is Q? I get Q to the power of minus alpha and solution as this. This is my solution all of you please see if you get this result alphas time s minus 1. See, you look at this equation from here you will be able to find Q is Q equal to minus e to the minus alpha is right g 1 s equal to this; the solution fine some more Q equal to now we have got our solutions let us look back at our solutions.

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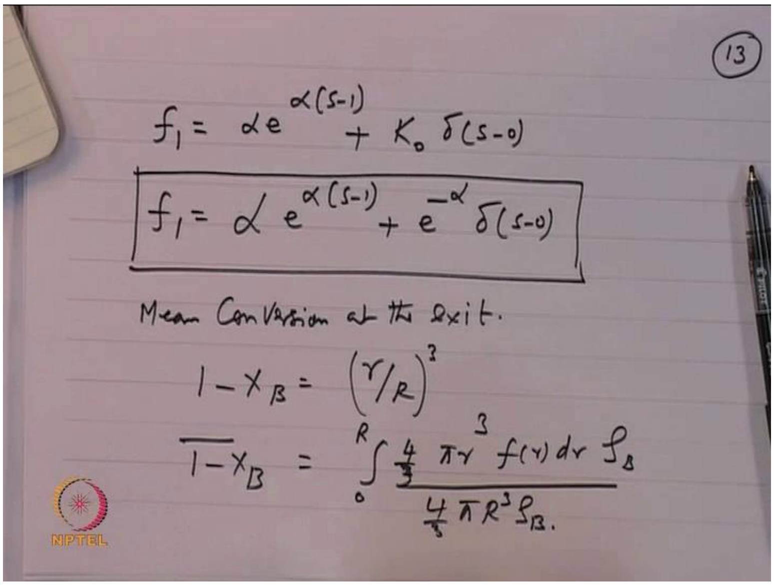

Our solution is f 1 equal to alpha e to the power of alpha s minus 1 plus k naught delta of s minus of 0; once again I write this as alpha times s minus of 1 plus e to the power minus of alpha delta of s minus 0. So, this is our solution also what is the distribution of the sizes of the exit from the fluid bed; it is given by this function. Now, we want to know what the mean conversion at the exit is. How do you find mean conversion? Let me write mean conversion like this. This is for each particle now we have to now there is all these problems f. So, the mean value is given by integral 0 to R 4 by 3 pi r cube f of r dr times density divide by 4 by 3 pi R cubed rho B. Do you all agree with this what I have written? This is what the numerator is.

This is see what is this f of r dr f of r what is f r this is the distribution we have found out.

What we have found out? We are going to put in terms of s shortly, but this is easier to understand that you know particle that enters the fluid bed; because of the exponential RTD you know all these distributions result. We have to find the means we have to find these integral to find out the mean value. The mean conversion is given by this integral is this with everybody, why is this given by this? Because the particles that enter because of this exponential distribution we have all these features therefore, we are doing this. Why are we having the size distribution in the fluid as bed? Now, what is f of r?

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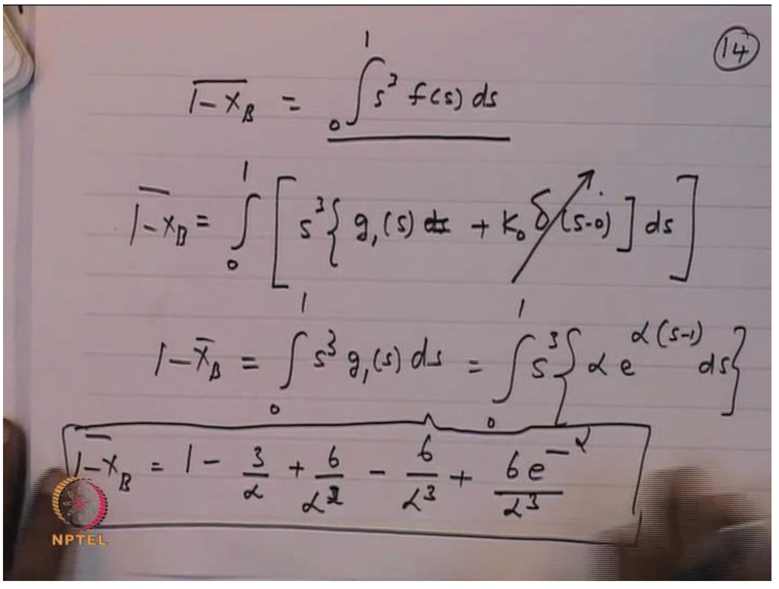

Suppose, I want to put it in terms of s; can I write this can I write this integral s cube f of s ds 0 to 1 what in saying is that this this same in writing it likes this? Now, s cube g 1 s ds can k naught delta s minus 0 ds. So, this is the mean. Now, what happens to this term? Why does it disappear? s cubed times k naught delta s minus of 0 that is why it is identically 0; s cube times k naught delta s minus 0 ds is 0 because s is 0. The area of c v the point is that the delta function the area it takes the value of the function. What is the function? Function is s cubed. So, therefore, the area takes the value of the function there is something you would have learnt in your control theory anyway. So, our answer is simply this. So, the mean conversion is actually this.

What is g 1 s? g 1 as this is integral 0 to 1 s cubed g 1 as you have found out it is alpha times e to the power of alpha s minus of 1 ds. This integral you have to do. Now, I have done this integration. So, therefore, we do not waste time. So, I have put down the final form you can check at home it comes it comes nicely. So, this is now this is the result, which is there in books also it comes nicely; there is no it is a little messy integration, but it comes very nicely. Now to put it in the context the context is that in a fluid bed. in a fluid bed; you have size distributions and what you get is the mean that is the only message you have to get to know.

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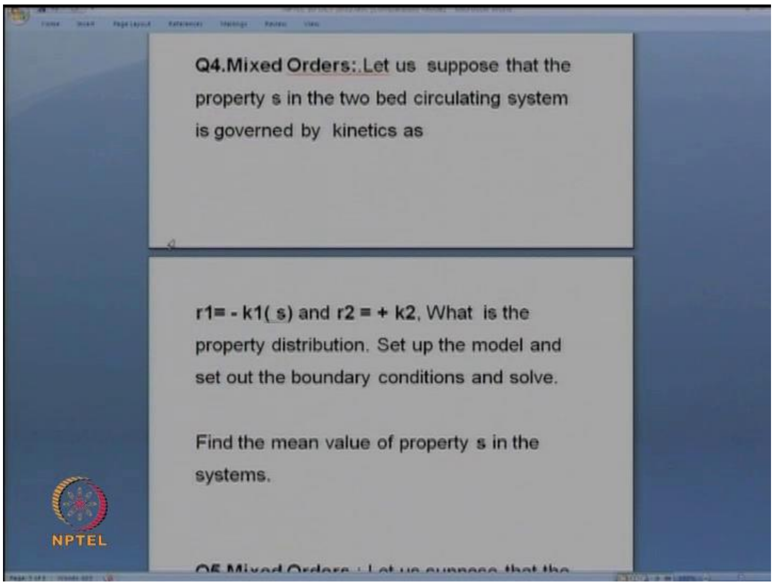

Let us solve one of these problems mixed orders.

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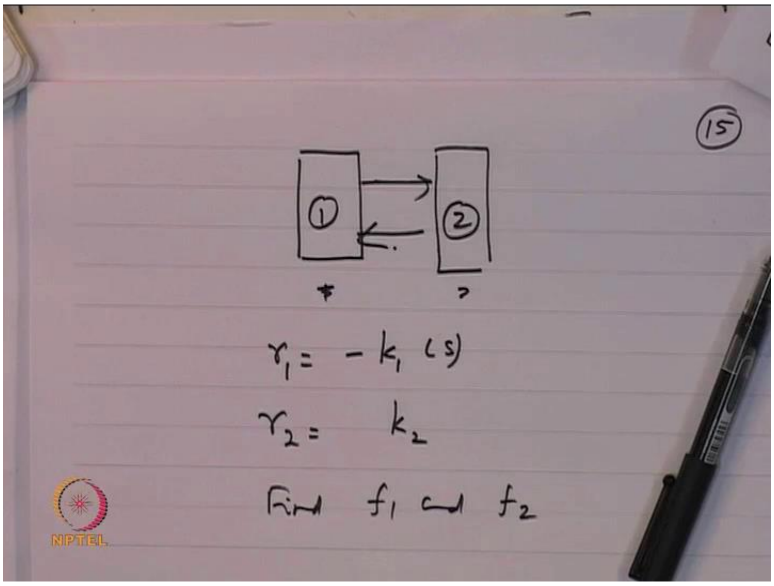

Now, this one is you have r 1 is minus of k 1 s; r 2 is k 2. Find f 1 and f 2. This is the problem what are the problems you have solved so, far. So, have solved for the case both are first order; you have solved the case where both are zero order that also we have done when it is mixed how do we handle; this is what I have done is the following.

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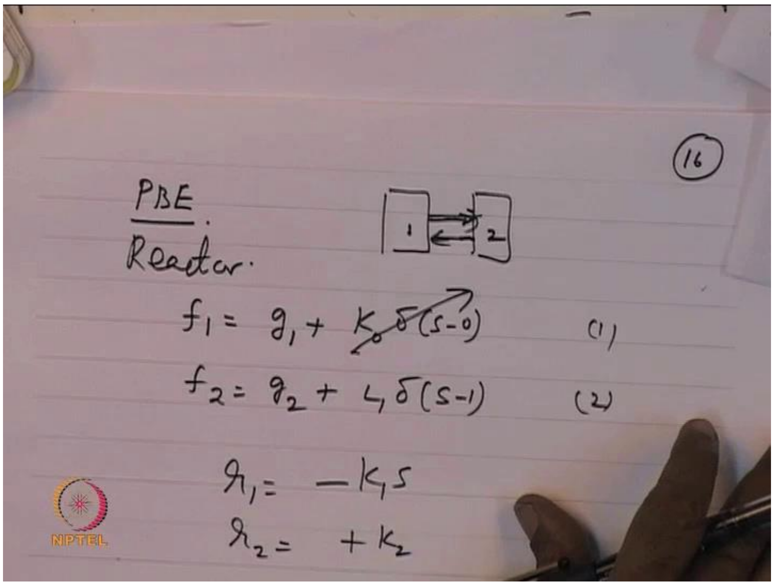

I have written the population balance like this reactor f 1 tells me whether this is right. So, this is our understanding of the distribution functions; that means, here r 1 is minus of k 1 s r 2 is plus k 2; for this case of reactor regenerator system we will find that f 1 is continuous f 2 as continuous and the discontinuous part. Why is that in f 2 there is a discontinuity at s equal to 1 because it is zero order; that means, this is 2 this is 1 therefore, whatever emerges from here we will have a discontinuity s equal to 1. But since this is first order whatever goes out of here there will be no discontinuity that why we have written we have removed this. Let us one rest of it is very straight forward.

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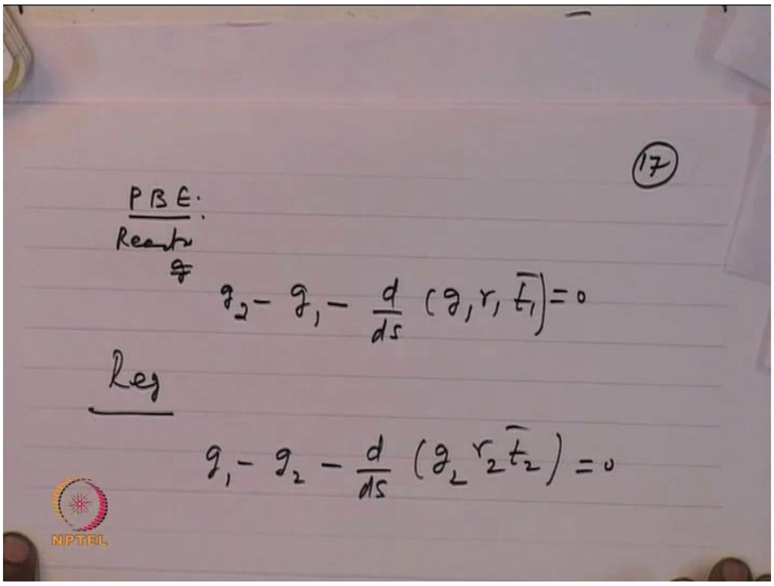

So, let us write the population balance and we said we will only write the continuous part. So, its reactor g 2 minus of g 1 minus of d by ds of g 1 r 1 t 1 bar equal to 0 regenerator g 1 minus of g 2 minus of d by ds of g 2 r two t 2 bar equal to 0. So, to solve this we said we have added this two let us add this two add this two we get d by d s of g 1 r 1 t 1 bar plus g 2 r 2 t 2 bar equal to 0; when we integrate we get g 1 r 1 t 1 bar plus g 2 r 2 t 2 bar equal to constant. You have to find the constant of integration for which we will generate a boundary condition; we generate the boundary condition at let us say between s minus ds and s in the reactor; we will do it in the reactor now.

What do we do? We write input minus of output plus generation equal to accumulation.

What are the inputs? Let me write the inputs tell me whether this is right; g 2 d s plus L 1 delta of s minus 1 this is ds minus v naught g 1 ds input output generation is W g 1 s minus of d s r 1 s minus of ds minus of W g 1 s r 1 s equal to 0 how does it look. So, let us get rid of the terms which we think is not good. So, limit as s tends to 0 ds tends to 0 it becomes this goes off this goes off this goes off this goes off. So, what do we get g 1 r 1 t 1 bar equal to 0 at s equal to 0? So, this I just write here g 1 r 1 t 1 bar 0 equal to 0. So, that is that is are boundary condition at 0.

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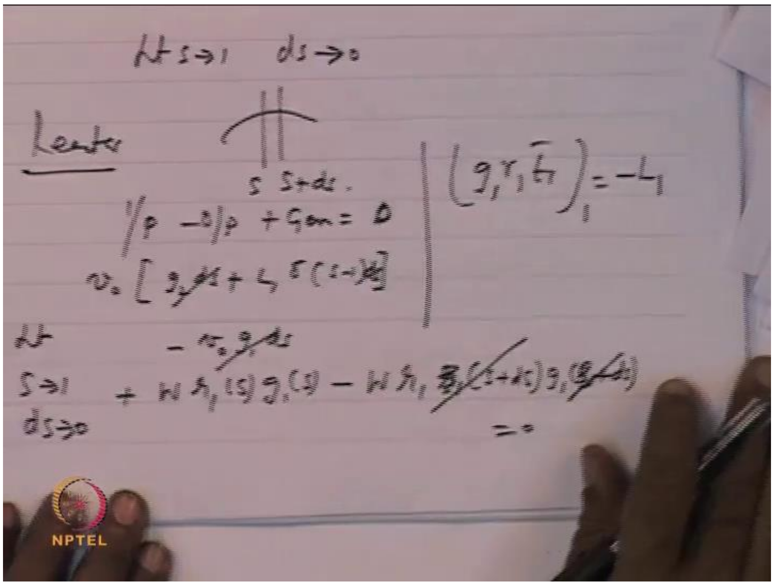

So, let us do at we repeat limit s tending to 1; ds tending to 0 it take limits between s and s plus ds once again input output generation equal to 0. Let me write all the inputs v naught g 2 ds plus L 1 delta s minus 1; I will put ds here itself minus v naught g 1 ds input output generation plus W r 1 s g 1 s in react; I am writing the react minus w r 1 r on x plus d s g 1 s s plus ds equal to 0. Now, limit as s tends to 1 ds tends to 0 s tends to 1; this goes off this goes off this does not exist and only these two terms.

So, this gives you so, I will write here. So, g 1 r 1 t 1 bar at 1 equal to minus of L 1. Now, we have two condition therefore, we should be able to find that we have done some integration; we have constant of integration. Now, tell me what is the value the conclave integration, g 1 r 1 to g 2 r 2 t 2 are not done for regenerator. Let us do for the regeneration then only we can solve the problem.

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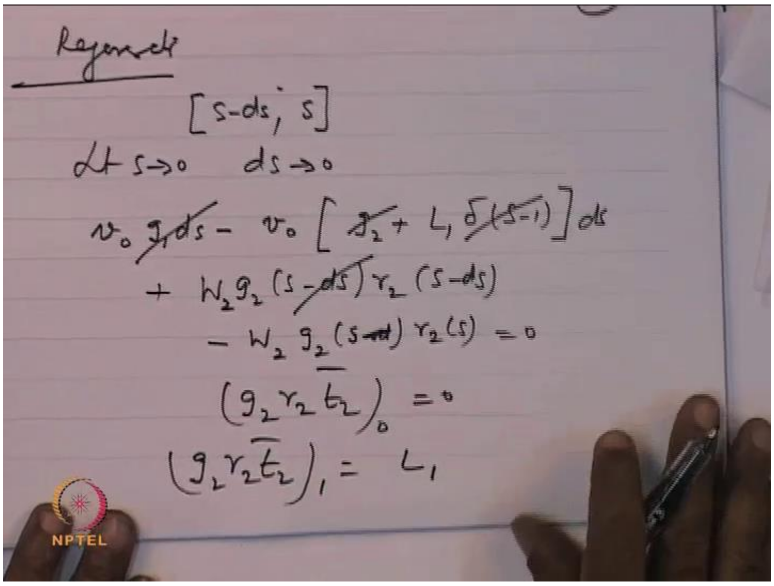

Regenerator now, I will take the made balance between s minus of ds and s limit s tends to 0 ds tends to 0; v naught input output g 2 plus L 1 of delta s minus of 1 times ds plus generation g 2 s minus of ds r 2 s minus of ds minus of W 2 and W 2 g 2 s s minus of ds. So, s s r 2 s equal to 0. So, I got this half to exist to go away there is no material here therefore, I gives you g 2r 2 t 2 bar equal to 0 and you can do the same thing and show that g 2 r 2 t 2 bar at 1 equal to plus L 1 you would not do it, but in a quiet straight forward you have done this so, many times.

So, we are saying now is so, if you want to find out the constant of integration if you want to find the constant of integration you find the g 1 r 1 t 1 bar at 0 these also 0 at g 2 r 2. So, you can find constant ability integration 0 even if you take the other end it is at g 1 r 1 t 1 at 1 its plus minus L 1 plus L 1. So, it is consistency the important thing it that whether those consistencies in the way you formulate movement you make a mistake it will tell you that there is in consistency. So, that gives us the constant of integration to be zero.

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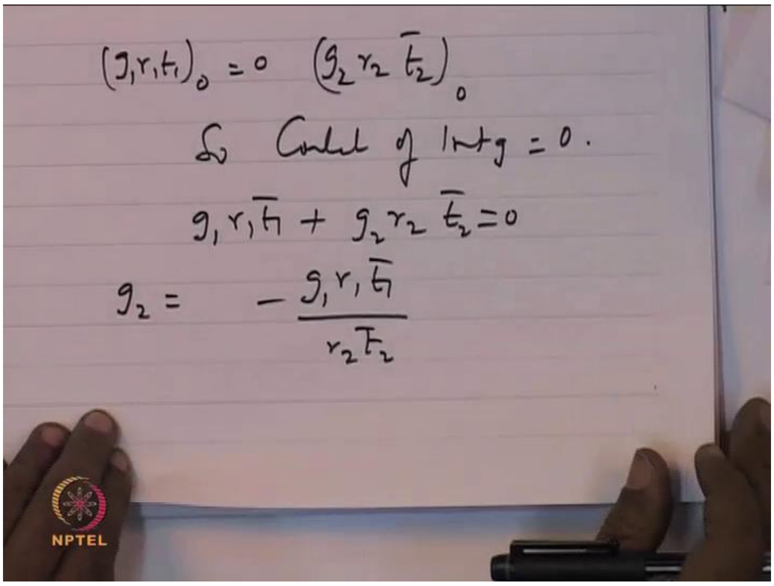

So, from g 1 r 1 t 1 bar equal to 0 g 2 r 2 t 2 bar equal to 0. So, constant of integration equal to 0. So, you get g 1 r 1 t 1 bar plus g 2 r 2 t 2 bar equal to 0. So, g 2 equal to g 1 r 1 t 1 bar divided by r 2 t 2 bar with a minus sign. We have being having getting this kind of for a long time. So, you know how to handle all this. So, now we substitute for g 2 and then integrate to get our answers. So, it is slightly messy. So, make sure you do not make any mistakes. So, in the population balance write it population balance you know what it is.

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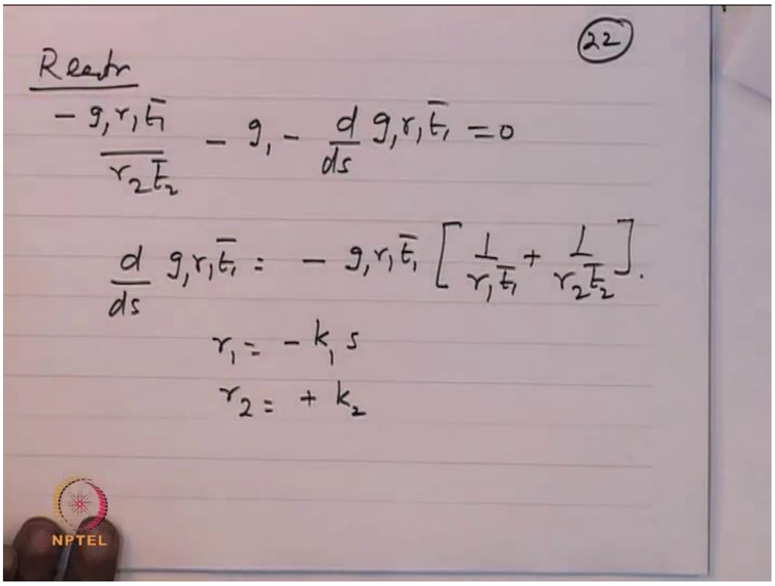

Let me write down minus of g 1 and for the reactor I am writing g 1 r 1 t 1 bar divided by r 2 t 2 bar that g 2 minus of g 1 minus of d by ds of g 1 r 1 t 1 bar equal to 0. So, this is the population balance for the reactor. So, that gives us d by ds of g 1 r 1 t 1 bar equal to minus g 1 t 1 bar 1 by r 1 t 1 bar plus 1 by r 2 t 2 bar. Now, please we know r 1 equal to minus of k 1 s; r 2 equal to plus k 2. I am writing the solution please I am going to write the solution and you have to integrate and tell me whether this is the right. I am simply going to substitute for r 1 and r 2 and integrate I am write in the final result.

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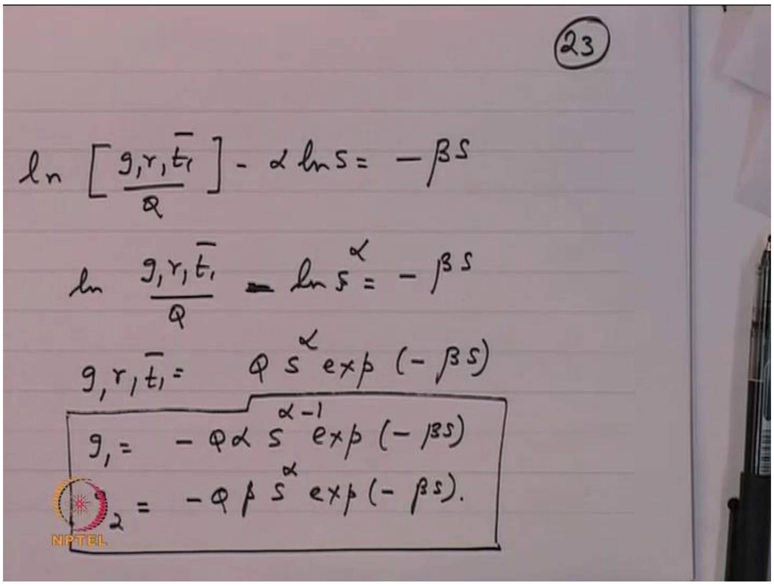

So, final solution is what I am writing please make sure there I am not wrong. This is the final result this is what I get. This is my answer we see if all of you get this answer please do it yourself please do it yourself. Spend that few minutes and make sure did you all get this result simple integration; it is not a difficult integration any way good some or shell we go forward. Anyway, I write the g 2 this is not difficult things minus of Q beta s the power of alpha exponential of minus of beta s. So, this is the g 1 and g 2. So, we have looked at various examples now just looking back there is another problem it is not. So, difficult to do you can do it yourself mixed orders were I have just change.

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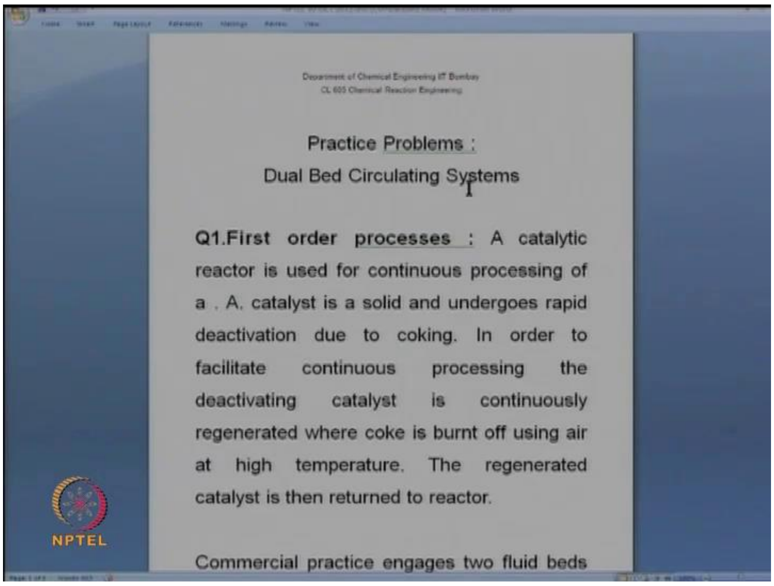

So, you can do it yourself. So, cutting the long story short the whole idea of this of looking at this dual bed circulating systems is really to sort of explosive to at very interesting developmental thing that will happen in the next number of years; particularly for gasification of colon biomass. Because of energy problems biomass is become very important around the world and if you do it in down draft gratifiers the nitrogen dilutes the gas. Therefore, the fuel value become very low; when you do it in the dual bed system you are able to keep the nitrogen away and you get much higher thermal values.

So, that is why the two bed systems have great value in the years to come. It is not yet commercial would number of reasons, but may be in your life time you might see all this. So, that is the object of looking at and giving you a way to handle this kind of problems. Now, you will take some simple problems in gas solid reactions and just put it down here.

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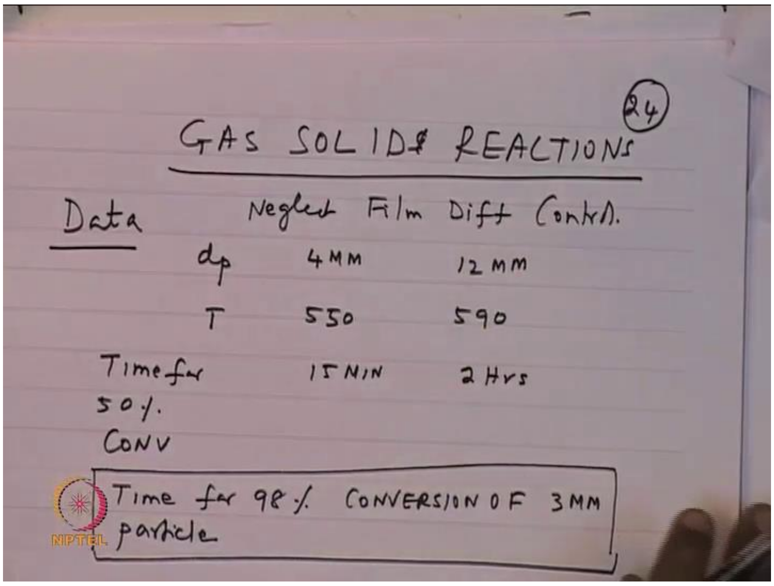

I should have brought it and then circulated gas solid reaction there some very simple problems; just to put all those things in context. I will write down the data here dp 4 mm 12 mm temperature is 550 and 590 time for 50 percent conversion 15 minutes and 2 hours. Now, what you have to find out is time for this is the question, time for 98 percent conversion of 98 percent conversion of 3 mm particle.

So, let me state the problem once again data given is this is the experimental data; particle size 4 mm and 12 mm at this temperature this is the kind of results for 50 percent conversion. What is the time required for 98 percent conversion of 3 mm particle? Now it also, says somewhere just a movement says that neglect says neglect film diffusion control. So, what did say? Now, put it in the context.

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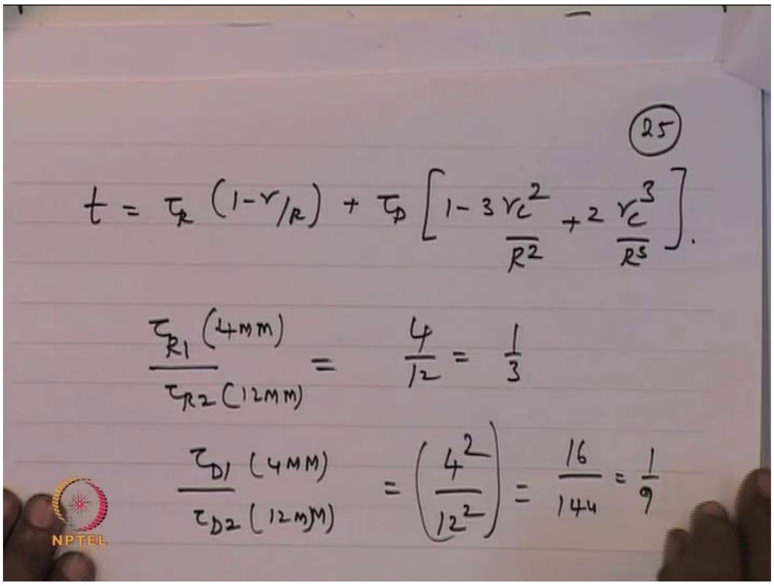

Let me just write down here we have done it in class we know this. What we know from we know this at deleted film diffused here; that means, if you have a particle which is undergoing chemical reaction under this. If film diffusion is not important the time required for complete consumption is given by this expression, which you have derived in class. Now, based on this data based on this data, what are the unknown quantities here tau R and tau D, but there are two particle sizes one is 4 mm; one is 12 mm. how do we find out. So, let me just put it in the context tau R 1; that means, tau R 1 means for 4 mm particles and tau R 2 which is for 12 mm particle.

Now, this is under reaction control how are they related equal to what is this ratio, what is this ratio for reaction control time for complete consumption 4 by 12 exactly, 4 by 12 1 by 3. If it is under reaction control the time for complete consumption of 4 mm particle divided by time for complete consumption 12 mm particle is simply 4 by 12 it is 1 by 3. Similarly, tau D 1 by tau D 2 which is 4 mm by this is 12 mm equal to what 4 square by, 16 divided 144 is 1 by 9. Now, with these two results can we look at the data and tell me what you feel about this data.

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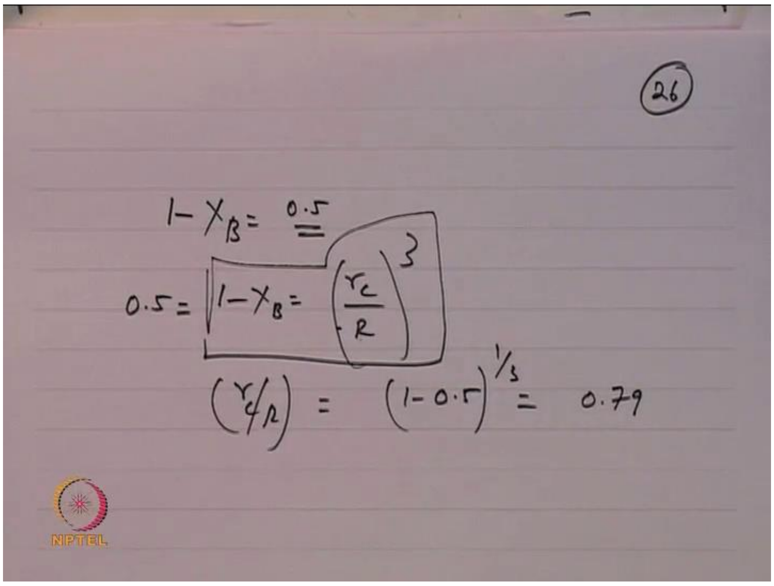

Now, what is meant by X B equal to 0.5 in terms of 1 minus of X B is what r c by R whole cube. So, if it is given as 0.5 what is r c by R value 1 minus of 0.5 to the power of 1 by is how much. If x b is given as 0.5 data says look at the data says 50 percent conversion data is given; correspondent X B could 0.5 what is the value for r c by R because everywhere r c by R is acquiring. So, what is r c by R equal to 0.79 is it. So, point i have got 0.8 so, 0.79. Now, let us now look at this equation now we have data at two temperatures; we are data for two particle size.

Now, can you tell me, what is tau R 1, tau R 2 tau D 1 tau D 2? This is the question based on this data can you tell me what is the value of tau R 1 tau R 2 tau D 1 tau D 2; data is given; 50 percent is same as r c no, what is important is what is important is this equality 1 minus of X B is this this is the conversion is X B. But if you want r c by R you have to calculate this one. So, can you please look at the data look at the model and tell me what is the values of I am taking the first data; first data is what where is the first data 15 minutes.

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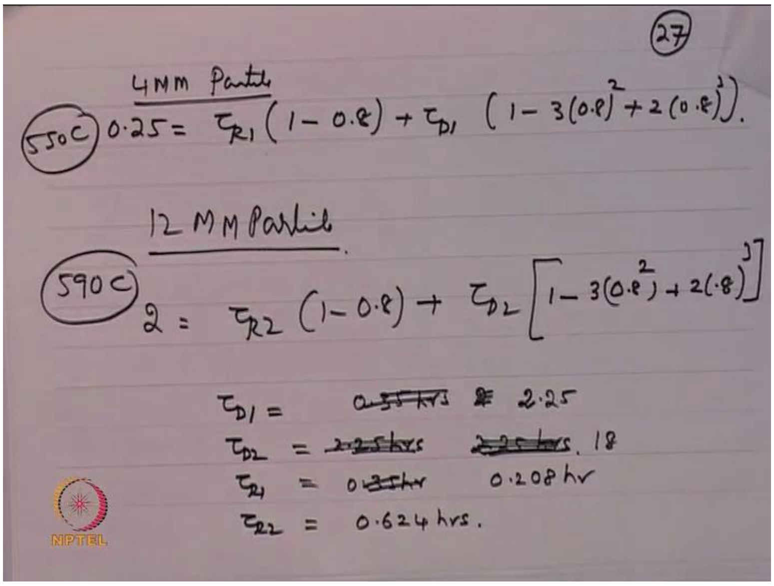

Fifteen minutes is means what 0.25 hours tau R 1 1 minus of 0.8; I have taken not 0.7 plus tau D 1 1 minus of times 0.8 squared plus 2 times 0.8 cube. Because this is one data, the second data is what; this is for 4 mm particle. For 12 mm particle, I got here what is the data for 2 hours equal to tau R 2 times 1 minus of 0.8 plus tau D 2 tau D 2 is what, where is the equation 1 minus 3 0.8 square plus 2 0.8 cube. This is for now can we solve and tell me what are the values of P R 1 tau R 1 tau R 2 and all that please solve and give me all the answers. The second data is a 590 c this is at 590 centigrade; first one is that 550 c. Now, listen to me now see we are going from 550 to 590.

So, what do we expect tau R 1 tau R 1 we are going for 590 550 it is increasing the temperature what will happen to tau R 1. What is our expectation? It would decrease. Now, if you look at the data says that we go from 4 mm to 12 mm, which means it is that the time for consumption increases by 15 to 2 hours from here are able to make any guess about the role of temperature on tau R 1 because activation is a not given data. So, data expects you to sort of look at the data and they make some guess work as to what might be appropriate. The data activation is a not given and it could happen in any real situation all these things happen. So, you have to look at the data and make your judgment.

What is your judgment regarding the effect of temperature on tau R 1? Given the data you find that you see the time for 50 percent conversion is going from 15 minutes to 2 hours is roughly how many times, if it is 9 times should I been 135 minutes, which is slightly more than 2 hours; in which case we could have said there it is all diffusion control. So, we do not have to worry about reaction much. So, it is something close to 9 times see the second data seems to suggest that the temperature dependence on tau R 1 is not all that significant. This is the judgment you have to make because data is not given you understand what I am saying data is not given what is the temperature dependence of tau R 1.

So, how do we make that judgment what I am saying is that by looking at this data it appears the tau R 1 is not a very strong function of temperature, but; that means, activation energies are not very large. See activation energy if it is large then its dependence is very strong. So, which means what you should be able to solve this quickly tau D 1 tau D 2 tau R 1 tau R 2; we need all these kind of answers I have got what are the numbers you got tau D 1 is how much point tau D 1 tau D just a minute tau D 2. Tell me all the numbers in hours I will there i write down what I have got; tau D 1 tau D 2 is 2.25 hours, tau R 1 this is what I get. What you get? tau D 1 2.25 what you get? How much are you getting? I get tau D 1 as 2.25 hours. So, tau D tau D 1 is 2 D 1 D 2 D 2 is 2.25. What are you getting this is 2. This one is 18.

Now, I have written tau D 1 is 2.25 you are getting 2; tau D 2 is 18 hours; tau R 1 0.208 you are getting. I am getting slightly higher value and how with the others, which one tau R 2 is how many 0.624 hours. You know you have all the answers to solve this time for complete consumption mm particle. How is this to be done, if i give you tau D for 12 mm you can find for 3 mm no problem, it is required only at similarly tau R if you give 4 mm you can find for 3 mm. So, you know how to do this problem because that data is not given. See, when data is not given you having to make some assumptions and to that extend our answers would be unsatisfactory.

So, we accept that till we get more data. If you would at come then we could have calculated this value of tau R at 2 because at one temperature to another temperature you can calculate; because we can use the Arrhenius factors you calculate; I stop there today.