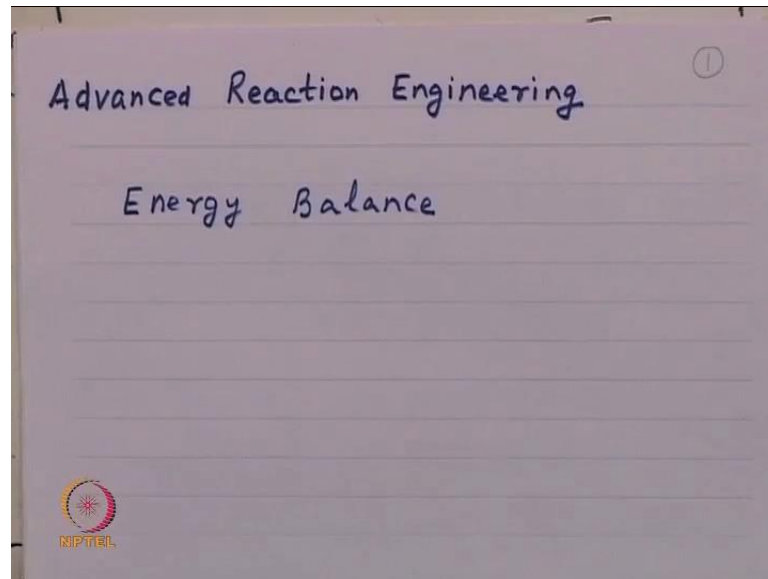


**Advanced chemical Reaction Engineering**  
**Prof. H. S. Shankar**  
**Department of Chemical Engineering**  
**Indian Institute of Technology, Bombay**

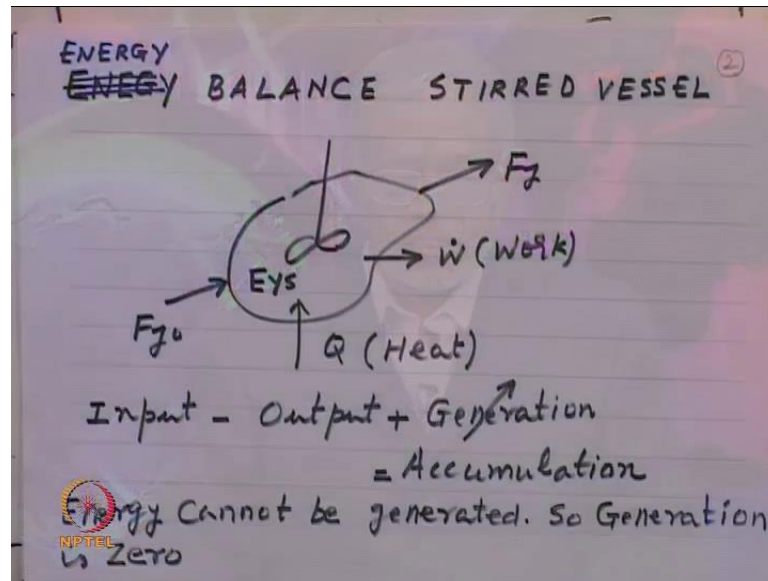
**Lecture - 14**  
**Energy Balance - I**

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Today, we will be looking chemical reactor energy balances well, energy balance is something that is, so central to chemical reaction engineering because, we have to supply heat or we have to remove heat and then we must design our systems. So, that you know the amount of area that is required for the heat addition or heat removal have to be appropriately correctly done. So, that you know the reaction proceeds as we expected to proceed. So, this is very important part of the course in chemical reaction engineering. So, this something we get started today, and then see how we can understand what happens.

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Now, we write our energy balance for vessel of our interest, I have deliberately taken a vessel of this very ((Refer Time: 01:19)) this not to indicate that the balances that we write take in to account any shape in principle any shape, but of course, we construct shape that are of you know useful to us etcetera. So, we have here a stirred equipment by stirred we mean that the intensive properties of the system at different positions in the equipment is same. So, the temperature, compositions are uniform everywhere, that is the assumption. So, that stirring means that every point inside the equipment has the same intensive property like temperature, and composition.

We have here  $F_{j0}$  moles of component  $j$  entering and because of chemical reaction  $F_j$  moles of component  $j$  comes out. So,  $F_j$  and  $F_{j0}$  if they are different to the extent that they are different that the extent to which that particular reaction is taking place. If there is 1 reaction for many reactions also we look at now, as per the first law convention, we have taken heat addition in to the system as positive. And then, heat I mean work that produced by the system we have taken as positive therefore, our statement of conservation says, input minus of output plus generation equal to accumulation, something that we have been writing for a long time our statement of conservation of mass we wrote input minus of output plus generation equal to accumulation.

And we wrote this about conservation with respect to component  $j$ , we always wrote with respect to material balance, we wrote generation of that component  $j$ . So, since the

rate of chemical reaction is known we are able to write the rate of generation of component  $j$ . Therefore, input output generation is must be equal to accumulation. Now, when it comes to energy balance since energy cannot be generated, we say that generation term in the energy balance should not be there.

That is why I have removed the generation term. So, input of energy minus output of energy must be equal to accumulation energy. So, our energy balance is different from material balance because of the fact that generation term is not there in the energy balance, but generation term is there in the mole balance not I am not saying material balance mole balance. Now, even if you are talking about material balance in terms of kilogram whatever, they generation of particular terms may be there because, it is getting converted something else. So, the overall balances there no change, but individual components balance that will be a change because of, chemical reaction.

So, what are we saying now? If we talking about energy balance in stirred vessel, there is input of energy because of, component  $j$  is bring some energy, there is output of energy because, component  $j$  is taking away some energy. Then there is input of heat, which is some energy which is some energy that is coming in, and there is output of work with some energy coming out. Therefore, input minus of output with generation equal to accumulation is a statement of conservation of energy. Let us now translate all this in terms of some symbols, just to understand what might be going on in terms of symbols is easier to work with.

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$$\frac{d}{dt}(E_{\text{sys}}) = \sum_{i=1}^n (E_i F_i)_{\text{in}} - \sum_{i=1}^n (E_i F_i)_{\text{out}} + \dot{Q} - \dot{W}$$

$n$ : number of species in the system

$$E_i = U_i + \frac{u_i^2}{2} + g z_i \quad \left( \frac{\text{J}}{\text{kg}} \equiv \frac{\text{m}^2}{\text{s}^2} \right)$$

$U_i$ : internal energy  $\left( \frac{\text{J}}{\text{kg}} \equiv \frac{\text{m}^2}{\text{s}^2} \right)$

$u_i^2$ : velocity  $\left( \frac{\text{J}}{\text{kg}} \equiv \frac{\text{m}^2}{\text{s}^2} \right)$

$g z_i$ : gravity head  $\left( \frac{\text{J}}{\text{kg}} \equiv \frac{\text{m}^2}{\text{s}^2} \right)$

So, what is mentioned here is  $d$  by  $dt$  of  $E$  system  $s y s$  refers to the energy inside the system,  $e$  is the energy  $s y s$  refers to system,  $E$  subscript system is energy of the system. So, left hand side says rate at which energy of the system changes with time. So, that is the accumulation term that you can see here, the accumulation term here this the term that is written,  $E$  system is the energy inside the system  $s$  is missing, and I put  $s$  here,  $d$  by  $dt$  of  $E$  system is rate of change of energy inside the system. So, that is left hand side now, the right hand side it is summation  $i$  equal to 1 to  $n$   $E_i F_i$ ,  $E_i$  is the energy of the component  $i$  and  $F_i$  is the molar flow of component  $i$ .

Therefore,  $E_i F_i$  represents energy that is associated with component  $i$  at the input; that means, here at the input here. So, summation over all components tells you the total amount of energy that these components are bringing in to the system. Similarly, if  $\sum_{i=1}^n E_i F_i$  out represents, the energy that is going out, the energy that is going out, with all these materials. So,  $E_i F_i$  summation means you are looking at the total amount of energy that is associated with all the components that is going out.

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The slide contains a handwritten energy balance equation and a schematic diagram of a stirred vessel. The equation is:

$$\frac{d(E_{\text{sys}})}{dt} = \sum_{i=1}^n (E_i F_i)_{\text{in}} - \sum_{i=1}^n (E_i F_i)_{\text{out}} + Q - \dot{W}$$

Below the equation, the text reads "ENERGY BALANCE STIRRED VESSEL". The diagram shows a stirred vessel with a central agitator. An arrow labeled  $F_{i0}$  points into the vessel, and an arrow labeled  $F_2$  points out. An arrow labeled  $Q$  (Heat) points into the vessel, and an arrow labeled  $\dot{W}$  (Work) points out. The system energy is labeled  $E_{\text{sys}}$ . There is an NPTEL logo in the bottom left corner of the slide.

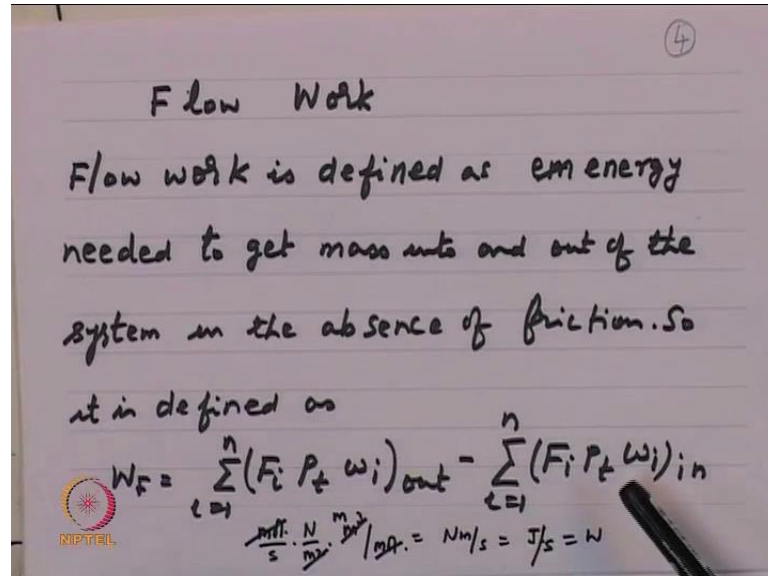
Notice here, we have not taken into account, what is called is the heat of mixing or if I mean the ((Refer Time: 06:08)) change for mixing. So, as function here is that,  $E_i F_i$  this total energy of  $E_i F_i$  and total energy of  $E_i F_i$  out with the in, there is no what is called is the loss of gain of energy because of mixing. So, that is the all right. So, that is what the assumption here we have plus  $Q$  minus  $W$ . So,  $Q$  is the amount of heat that input in to the system, and  $W$  dot is the amount of work that it take out of the system.

So, accumulation equal to input of energy minus output of energy. There is no generation of energy, but there is amount of heat that is input of this another this is energy that coming in with the materials, this is the energy putting through on your heat transfer pipes and so on. And  $W$  dot is the amount of work that you might be getting in to your shaft, that might be may be to spin, which is turning a motor and so on. So, this work, this is heat, this is the energy, that is coming in because of, the fluids that is coming in to the system.

So, you have here  $E_i$  as the energy that is associated with component  $i$  with consist of 3 component one is internal energy, kinetic energy and potential energy. So, we take total energy of component  $i$  consisting of internal energy, potential energy and kinetic energy. So,  $E_i$  represents the total energy of the system of the component  $i$ , and  $E_i F_i$  therefore, represents total amount of energy that component  $i$  brings in. And, sum it over all species

total amount of energy that is coming in with the species similarly, for the output this one term, which all of us may be familiar, but put it in the context.

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Suppose, we have some material that is going in to the system and coming out to the system. I got here  $F_i P_t \omega_i$  out minus  $F_i P_t \omega_i$  in, what is this word  $F_i P_t \omega_i$  if we look at the units of  $F_i P_t \omega_i$ .  $\omega_i$  is specific volume,  $P_t$  is pressure,  $F_i$  moles per time. So, this essentially this is work term we understand here itself. So, let us say moles per time, and this is Newton's per square meter, this is specific volume that is cubic meter per mole, cancels off. So, it is meter square meter cube, so, this meter. So, it is Newton meter per second, which is joule. So, this is per second. So, this is you know joules per second is watts.

So, essentially it is energy by unit time, is what is the. So,  $F_i P_t \omega_i$  in is kind of it is coming in. So, much of energy is coming in the form of  $F_i P_t \omega_i$ . And similarly, so much of energy coming in, so much of energy is going out. This difference, what meaning can we attach to this difference. Now, if we look carefully at this, suppose there is a chemical reaction in which  $F_i$ , this term  $\sum F_i P_t \omega_i$  and this term  $\sum F_i P_t \omega_i$  in and this out. They are not the same now, here is an instance which says that even in the absence of friction, this term  $F_i P_t \omega_i$  out is different from  $F_i P_t \omega_i$  in.

This is greater than this, it means this is positive, if it is this term is less than this the whole term negative. So, what is; that means, WF WF which we call as flow work, the flow work in to a out of a system; that means, in this out of a system, and in to the system this difference if it is positive it means that this particular system gives you some flow work. If this is a negative which means that you have to put in work to make this fluid enter the system, and come out even in the absence of friction. So, what we are trying to say is that there can be instances in which you have to put in work because, even the absence of friction this term is this minus of this is negative.

This is something that is to be understood that you may need to put in work, to be able to get system materials in, and out of the system, even in the absence of friction. So, this is the point that I am trying to get across to you. That flow work represents the difference between  $F_i P_t \omega_i$  out minus  $F_i P_t \omega_i$  in, and this term in a reacting system can be different therefore, this minus this can be positive or negative.

And, if it is negative, you have to put in work to make fluid go in and come out of the system. So, flow work is an instance where you may need to put in work to get system fluid in and out of the system. So, what are we saying then, if we looking at our system here, and we are saying the  $\dot{W}$ , is the amount of energy in the form of work, that comes out of our system.

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Therefore work out of a system can be set as

$$\dot{W} = \dot{W}_S + \dot{W}_F$$

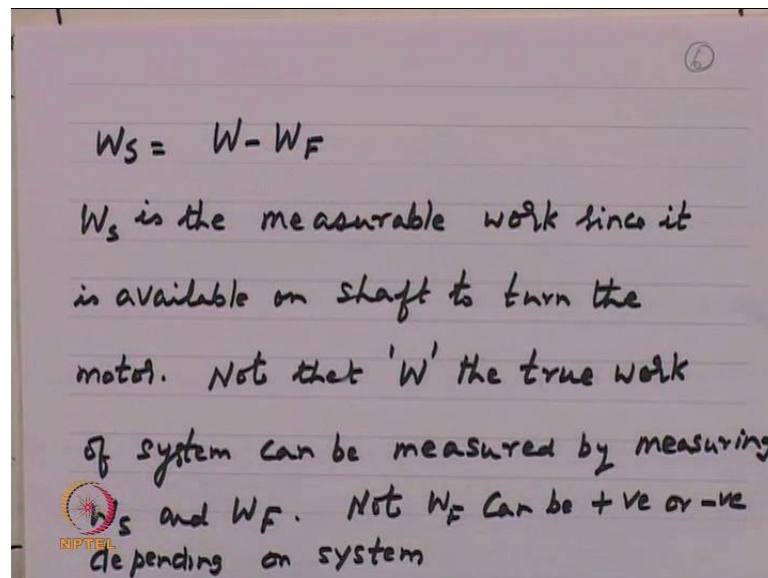
$$\dot{W}_S + \sum_{i=1}^n (F_i P_t \omega_i)_{out} - \sum_{i=1}^n (F_i P_t \omega_i)_{in}$$

$\dot{W}_S$  - shaft work,  $P_t$  total pressure  
 $\omega_i$  - specific volume species 'i'  
 number of species

We recognize that this  $\dot{W}$  actually consist of 2 types of work 1 is  $W_S$  which is shaft work and 1 is  $W_F$ , which is flow work. Now, in an instance where  $W_F$  is negative; that means, you will have to put some energy to get fluids in and out of the system  $W_S$  is negative. And therefore, the total amount of work this  $W_S$  is different from  $\dot{W}$ , that is the point we are trying to put across to you. So,  $W_S$  represents the shaft work that we will get out of the system, and therefore, it is that work, which is going to appear to us is going to turn our turbine, and this going to turn motors and so on.

That is the amount of energy that you and I will be able to make use of. So, what are we saying now? In our first law were  $\dot{W}$  represent the work that comes out of the system now, can be broken up into 2 parts; part1 is shaft work and part2, which is what is called as flow work, which is  $\sum F_i P_i \omega_i$  out minus  $\sum F_i P_i \omega_i$  in. So, the total amount of work that comes out of our system, which is  $\dot{W}$  we have broken up in to 2 parts. Part1 is shaft work, part2 is flow work. We put the flow work separately let us see, what advantages we get out of doing this what I am saying is that, this flow work, let me sort of going through this once again.

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$W_S$  is the measurable work since it is available on the shaft; that means, we can actually put in our measuring systems actually measure  $W_S$ . So, that is why is a measurable quantity and this  $W$  which is a true work can be measured only by measuring  $W_F$  and  $W_S$ . This can be measured because we know  $P_i \omega_i$  and  $F_i$  therefore, this is a known



quantity this is an experimentally measured quantity, and therefore,  $W$  become that quantity that you and i can determine.

Because,  $W_F$  is a quantity that we know because,  $F_i P_t \omega_i$  for all the components are known, we can sum over all species at inlet and outlet. And,  $W_S$  is the work that, we can measure at the motor because its turning and we can measure the work output in various ways. Therefore,  $W$  is the quantity that is measured by measuring separately both  $W_S$  and  $W_F$  all right. Let us try to illustrate this because some of these may become clearer if we look at a small example.

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FLOW WORK ⑦

In a  $SO_2 + \frac{1}{2} O_2 = SO_3$  plant processing  
 3000 tm/d  $SO_2$

$$W_F = \left( \sum_{i=1}^n F_i P_t \omega_i \right)_{out} - \left( \sum_{i=1}^n F_i P_t \omega_i \right)_{in}$$

$$\approx - P_t v_0 (0.005)$$

$P_t = 10^5 \text{ N/m}^2$      $v_0 = 250 \text{ m}^3/\text{s}$

$$W_F = - (10^5) (250) (0.005) = -125 \times 10^3 \text{ J/s} = -125 \text{ KW}$$

Energy to be supplied

Let us look at a sulphur dioxide plant, reacting with oxygen in air it gives sulphur trioxide, is well known reaction it is been going on in the process industry for last many years. And, it is a catalytic reaction and so on. Generally air is used for oxidation. Therefore, we get air oxygen contained in air is used for the reaction and so on. So, I put some numbers here just taken some data from a factory which processing about 3000 tonnes per day of sulphur dioxide.

So, which is reactive form sulphur trioxide. So, numbers look something like this, for the case of this kind of flow you can calculate all these I made some estimates the flow  $v$  not turns out to be about 250 cubic meters per second. And, this flow is measured at some standard at the reactor conditions. And, what is the pressure typically the pressure at

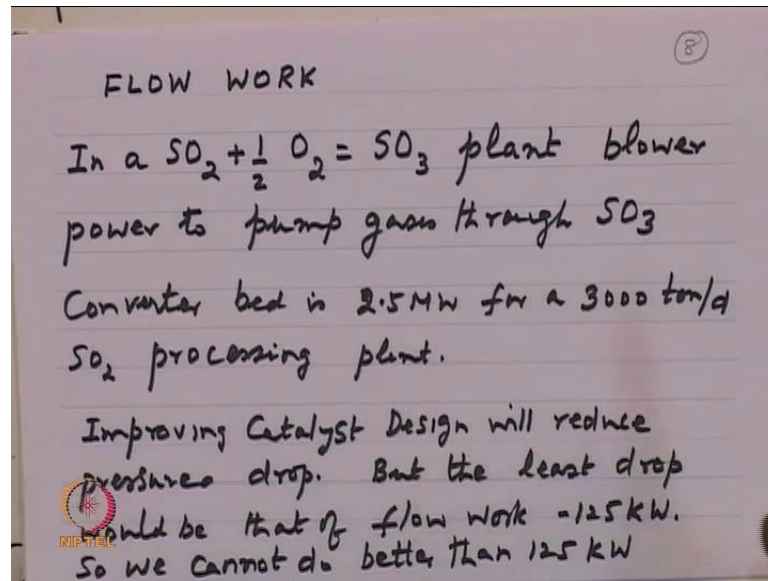
which you will pump the fluid near atmosphere or slightly near the atmosphere. So, let say 10 to the power of 5 Newton per square meter.

So, our situation here is that, we have a flow of 250 meters cubic per second, and a pressure of 10 to the power 5 per meter square is coming. So, what is to, if  $v_0 \sum F_i$  times  $\omega_i$  is a volumetric flow. So, if some over all species becomes the total volumetric flow, as in this particular term can be seen as volumetric flow multiplied by the molar flow of the component. This one way we can understand this similarly, you can understand this as flow multiplied by the number of in the sense  $F_i \omega_i$  is flow and  $P_t$  is a pressure. So, pressure multiplied by flow gives you units of energy. So, this difference is, what the flow work is.

Now, if you put all the numbers  $P_t$  is given  $v_0$  is given. So, this  $P_t \omega_i$ , summation experience says that about this difference is typically about 0.5 percent; that means, this minus is this turns out to be about 0.5 percent of, what is at the out let. So, we have written this as in the form of  $P_t$  times  $v_0$  multiplied by 0.5 percent, which is 0.5 by 100 is 0.005. So, what are we try to do? We are trying to make an estimate of flow work for the case of a 3000 tonnes per day sulphur dioxide plant, and when we make the estimate for the numbers that, we have taken this turns out to be about 125 kilo watt with a minus sign.

Showing that this amount of work we will have to put in for the fluids to be put in to the system, and taken out in the absence even in the absence of friction you need, so much of work put in. So, that the fluid can go in and come out of the system, why is this? So, it is so, because  $F_i P_t \omega_i$  at the inlet and  $F_i p_t \omega_i$  at the outlet are not the same. And, this so that energy we will put in appropriately. So, just to cut the long story short what you are said here is that flow work is a term, which is energy that is associated with getting material in and out of the system in the absence of friction. And, for a plant of 3000 tonnes per day sulphur dioxide, we have made an estimate that turns out to be a 125 kilo watt with minus sign, showing that we have to put work to the system.

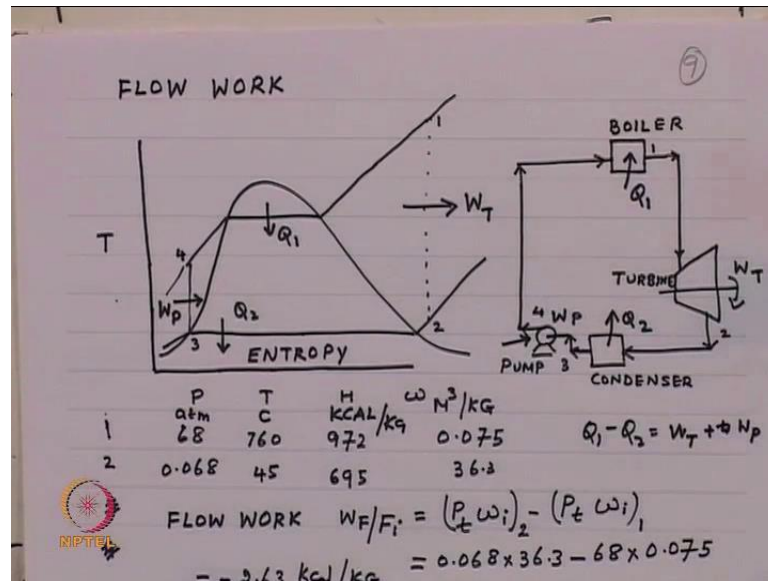
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I mean we have shown at an earlier day that, the pressure drop associated with sulphur dioxide sulphur trioxide, there we estimate that the energy required to push the material in and out of the system, in is something like 2.5 mega watt taking pressure drop in to account. So, taking pressure drop in to account the energy required is 2.5 mega watts, if we do not take pressure drop in to account, it is only 125 kilo watt, what are we saying? pressure drop is the huge activity in the process industry.

If it is properly managed, why are we saying the context is saying is that, when a pressure drop in a process is something, we cannot avoid. So, improving catalyst design of course, we reduce pressure drop understood. But, the least pressure drop would be that of flow work. So, flow work is something that is the minimum, at which you can work that mean, that amount of energy you have to put in any way otherwise fluids will not go in and come out of the system. So, flow work is a measure of minimum amount of energy that is required to put the material in and out of the system.

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So, with this, let us look at some examples to understand energy in terms of our interest. I have taken the example of steam turbine, where what do we have? We have a boiler in which fluids are coming in, and it becomes steam. That steam goes in to the turbine in here, it expands and steam finally, condenses and then comes out in to the condenser where it condenses. And then, that fluid, that liquid again pumps back in to the process. So, what is a boiler of what is power plant? Power plant is an instance of a process in which steam is generated in a boiler.

It is put in to the it is put in to turbine where it turns the wheels of the turbine, which in turns motor and so on or a generator. Now you have, so much of energy form, of heat  $Q_1$  going in to the boiler. And, so much of energy  $Q_2$ , which is thrown in to the environment. So,  $Q_1$  minus of  $Q_2$  is really the energy that is being lost by the working fluid, which is in your boiler minus condenser. So, this  $Q_1$  minus of  $Q_2$  is appear as work. So, we have got here  $W_T$  is the work for the turbine  $W_P$  is the work in the pump.  $W_T$  minus  $W_P$  is, what you will be able to produce in the system, give you where you put in  $Q_1$  of heat and then  $Q_2$  of heat output  $Q_1$  of heat input.

$Q_1$  minus of  $Q_2$  under ideal conditions  $Q_1$  minus of  $Q_2$  equal to  $W_T$  minus of  $W_P$ . Is that clear, what are we saying?  $Q_1$  minus  $Q_2$  is the total amount of energy that is absorbed by the system. And therefore, that must appear as the shaft work which is  $W_T$  minus plus  $W_P$ . So, so much of shaft work is done therefore, that difference

must equal to shaft work that is what you are said. So, what we are saying here, we also once again this also an instance of energy balance where the energy of stream is used to run a turbine. And therefore,  $Q_1$  minus of  $Q_2$  which is the energy inert energy that is absorbed by stream, must appear as turbine work which is  $W_T$  and  $W_P$ . Now, have you said all these things? Let us now, just look at what we are saying in terms of the equations that generate that describe the energy balance.

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$$\frac{d}{dt} (E_{sys}) = \left( \sum_{i=1}^n F_i \cdot E_i \right)_{in} - \left( \sum_{i=1}^n F_i \cdot E_i \right)_{out} + \dot{Q} - \left( W_s + \sum_{i=1}^n (F_i \cdot P_i \omega_i)_{out} - \sum_{i=1}^n (F_i \cdot P_i \omega_i)_{in} \right)$$

$$= \sum_{i=1}^n F_i \cdot (E_i + P_i \omega_i)_{in} - \sum_{i=1}^n F_i \cdot (E_i + P_i \omega_i)_{out} + \dot{Q} - W_s$$

$$E_i = U_i + \frac{u_i^2}{2} + gz_i$$

So, we are said that  $F_i E_i$  in and  $F_i E_i$  out this term,  $W \dot$  can be replaced as  $W_s$  plus  $W_f$  flow work that is what have been done. Here therefore,  $w \dot$  has been replaced as  $W_s$  plus flow work that is what has been done. So, essentially the same energy balance we still work with, the left hand side is the state of accumulation energy in the system. This is the amount of energy that is coming in with the fluids, the amount of energy that is going out with the fluids.

This is the amount heat that is heat transfer in to the fluid, and this term minus is the term that is associated with flow work or the work associated with getting material in out of the system in absence of friction. So, with this kind of formulation now, we can take this  $F_i P_i \omega_i$  along with the input terms, and output terms and so on. So, that we can combine this and make it might look for easier to understand now. So,  $d$  by  $d t$  of energy of the system now we have  $\sum_{i=1}^n F_i$  is the molar flow of  $E_i$  and  $P_i \omega_i$ . Notice here,  $F_i E_i$  this  $F_i P_i \omega_i$  can be combined. So, that this term now

look like  $E_i$ , which is the internal energy of the system and  $P_t \omega_i$  is it is external phenomena because of pressure and so on.

So, we have put  $E_i$  and  $P_t \omega_i$  together, because of the advantages that it will give us. So, just like we have done for the inlet term we can do the same thing for the outlet term, where  $F_i$  is multiplied by  $E_i + P_t \omega_i$ . So, what is that we are saying? What we are saying is that; if you have an energy balance, if you take flow work in to account, and then, flow work term appears in the energy balance in this form, you have  $F_i$  is the is the molar flow.

And,  $E_i + P_t \omega_i$ , you can notice here in term here is negative correct negative negative is positive. So, therefore,  $E_i + P_t \omega_i$  this is the energy of this system. Similarly,  $F_i$  energy of the system out, so you have internal energy plus  $P_t \omega_i$  is the energy in, and that is the energy out. So, since we shall we say that input energy output energy this is the amount of heat that you putting work putting. Therefore, this represents the statement of energy balance after we have made a substitution for our terms.

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$$\frac{d}{dt} \left[ \sum_{i=1}^n N_i \left( U_i + \frac{u_i^2}{2} + g z_i + P_t \omega_i \right) \right]$$

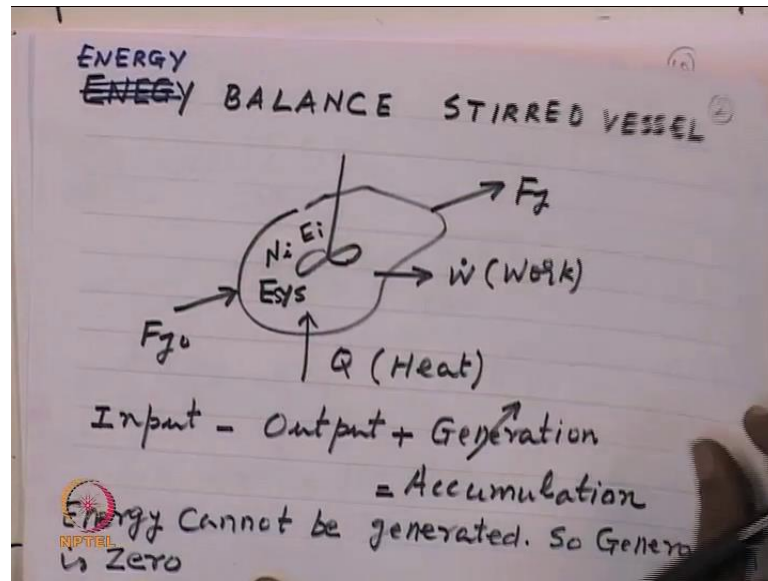
$$= \left[ \sum_{i=1}^n F_i \left( U_i + \frac{u_i^2}{2} + g z_i + P_t \omega_i \right) \right]_{in}$$

$$- \left[ \sum_{i=1}^n F_i \left( U_i + \frac{u_i^2}{2} + g z_i + P_t \omega_i \right) \right]_{out} + Q - W_s$$

$\dot{W}$   
 $W_s + W_F$

So, statement of energy balance that we are said is  $d$  by  $dt$ . Now, you can see here on your left hand side the this system on the left hand side also, I have replaced the  $e$  system by our energy for each component multiplied by the number of moles of component  $i$  inside the system. The number of moles of component  $i$  inside the system we have taken as  $N_i$ .

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See what I am saying here is that the number of moles here that we are assuming that the  $N_i$  moles of components present here each of these  $N_i$  moles has specific energy  $E_i$ . Then,  $E_i$  consist of  $u_i$   $g_i$   $z_i$  and then small  $u_i$ . So, all that taken as in to account that is how our system look like this. So, you have on the left hand side  $E$  system this is  $E$  system which consist of  $N_i$  multiplied by this is internal energy, this is kinetic energy, this is potential, this is the term associated with the flow work, which is now, we have brought in appropriately it goes from  $i$  equal to 1 to  $n$  number of species.

All these  $i$  equal to 1 to  $n$  this number of species. So, notice here that this term, this whole term  $u_i$  plus  $u_i$  plus this is the total energy. So, this is something like what is called as internal energy, and this is something like  $p v$  term. So, which is looks like our conventional nomenclature, what is called as enthalpy. So, you have inputs and you have the outputs plus  $Q$  minus  $W$  notice here, this  $W$  previously we called it as  $W$  dot which we said was equal to  $W$  plus flow work and that is how we this flow work we put in terms of  $P_t \omega_i$  and so on. And then, rearrange this equation. So, that this equation now looks slightly different from, what we started with. And, the form in which this equation now, looks is something like this. With all the simplification, what we have is this term you notice here this term  $u_i$  plus  $P_t \omega_i$  its termed as  $h_i$  that is what we have done.

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$$h_i - P_t \omega_i = U_i \quad (12)$$

$$\frac{d}{dt} \left[ \sum N_i \left\{ h_i + \frac{u_i^2}{2} + g z_i \right\} \right]$$

$$= \left[ \sum F_i \left\{ h_i + \frac{u_i^2}{2} + g z_i \right\} \right]_{in} - \left[ \sum F_i \left\{ h_i + \frac{u_i^2}{2} + g z_i \right\} \right]_{out}$$

$$+ Q - W_s$$

$\therefore h_i = U_i + P_t \omega_i$   
 If  $P_t \omega_i$  term on LHS is small in relation to  $h_i$  then we have a simplified version

Here  $h_i$  and this you have this kinetic energy this is the potential energy. Notice here, this term is not there.

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$$\frac{d}{dt} \left[ \sum_{i=1}^n N_i \left( U_i + \frac{u_i^2}{2} + g z_i + P_t \omega_i \right) \right] \quad (11)$$

$$= \left[ \sum_{i=1}^n F_i \left( U_i + \frac{u_i^2}{2} + g z_i + P_t \omega_i \right) \right]_{in}$$

$$- \left[ \sum_{i=1}^n F_i \left( U_i + \frac{u_i^2}{2} + g z_i + P_t \omega_i \right) \right]_{out} + Q - W_s$$

$$\dot{W} = W_s + W_F$$

Notice here, that on the left hand side the term  $P_t \omega_i$  does not appear that on the right hand side it appears. Is it clear? Because this term flow work is associated with flow and therefore, in the left hand side is only accumulation therefore, flow work does not appear that term does not appear. Therefore, on the left hand side I subtracted it by



putting  $P_t \omega_i$ , so that this term  $h_i$  minus  $P_t \omega_i$  by definition  $h_i$  minus  $P_t \omega_i$   $i$ .

By definition is  $u_i$  that is how it is written on the left hand side. So, this is the statement of energy balance for the system, which is well stirred, which means every position inside the equipment has same composition, same temperature. On other words all in intensive properties are the same at different positions of the system. Now, we can simplify this further which I have done. So, that we have some more advantages which I will do right now.

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The image shows a handwritten derivation of the energy balance equation for a well-stirred system. The equations are as follows:

$$\frac{d}{dt} \sum N_i \left\{ h_i + \frac{u_i^2}{2} + g z_i \right\}$$

$$= \left[ \sum F_i \left\{ h_i + \frac{u_i^2}{2} + g z_i \right\} \right]_{in}$$

$$- \left[ \sum F_i \left\{ h_i + \frac{u_i^2}{2} + g z_i \right\} \right]_{out} + Q - W_s$$

if  $u_i^2/2$  and  $g z_i$  terms are small which is so in chemical rxn equipment we then have

$$\frac{d}{dt} \sum N_i h_i = \sum_{in} (F_i h_i) - \sum_{out} (h_i F_i) + Q - W_s$$

So, what we have done now, is that the left hand side  $P_t \omega_i$  minus. So, this term if this term  $P_t \omega_i$  is not very large particularly in our reacting systems, that we are taking about this not a term which is not going to be important. Therefore, it is possible to look at the whole equation in a slightly simpler form by saying the following. The left hand side, this term and this term, and this 2 terms are not very important.

In the sense that the real energy that is associated with our chemical reaction is in  $h_i$  and therefore, this term  $u_i$  squared by  $2g$  is  $z_i$   $P_t \omega_i$  are terms, which may not be very important now, have you said this? So, of course, this is something that we must verify for our every case we apply these equations, we must convince ourselves, that our assumptions are correct, and if it is not you must come back, and use the general form which you have written now. So, that we do not make mistakes, so but in situations most

of our situations of chemical reaction engineering you will find, that these terms can be deleted, these terms can be deleted.

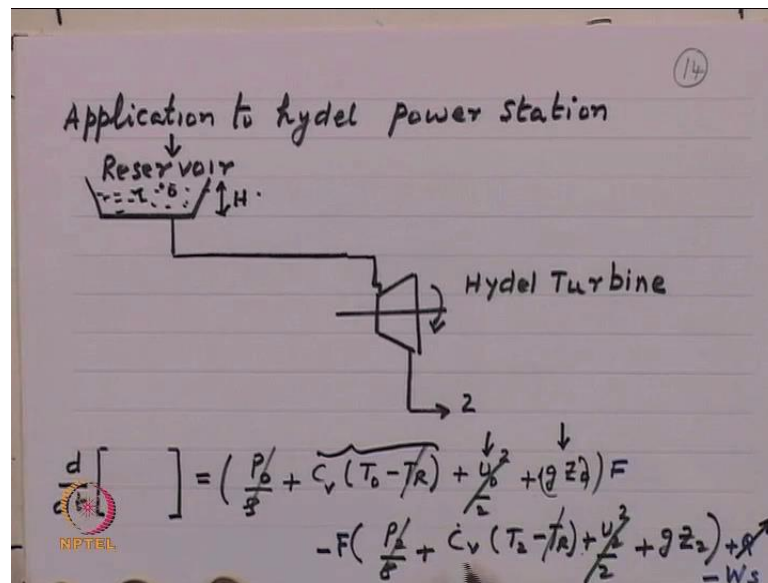
So, that the simplified form of the energy balance might look something like this  $\frac{d}{dt} \sum_{i=1}^n N_i h_i$ . So, what is that our equation look like our equations look like that left hand side is  $\sum_{i=1}^n N_i h_i$  summation over all species, the right hand side  $\sum_{i=1}^n F_i h_i$  summation over all species. And then, again the out term is  $\sum_{i=1}^n F_i h_i$  sum all species plus  $Q$  minus of  $WS$ . So, what we have done, is that is started with general statement of conservation of energy, and we made certain simplifications, and then we have written our equations now in terms of enthalpy which is term which is well tabulated etcetera.

And then,  $N_i$  is number of moles inside the system,  $h_i$  is the number of moles coming in this is the this term  $h_i F_i$  the amount of energy that is coming in, and this  $h_i F_i$  out is amount of energy that is going out. All measured in terms of  $h_i$  enthalpy, since enthalpy, is well tabulated in this representation form, there was certain advantages. Because, these can be read out of tables and therefore, our effort in trying to put the number becomes much easier. So, just put it in to the context, what we have done is that; we have set up the general balance equation and then we have written our equation in a form.

So, that we can simplify some other terms can be deleted and so on. And got the final form, in which we have on the left hand side  $\sum_{i=1}^n N_i h_i$  summed over all species, the right hand side the  $\sum_{i=1}^n F_i h_i$  summed over all species. In flow stream versus out flow stream  $Q$  and  $WS$  are heat and work that goes in to and out of the system. Now, having said this, so we would like to see how this representation, that we have done, how does it sort of explain or shall we say, how it is able to describe? What happens in different situation of our interest? We will apply our equations to variety of reacting system. So, every system that we apply our equation should hold.

So, we want to see, how our equations sort of help us to understand? What goes on in different system? Etcetera. So, what are we said? We said that our energy balance equation looks like this our energy balance equation is  $\frac{d}{dt} \sum_{i=1}^n N_i h_i$  summed over all species,  $\sum_{i=1}^n F_i h_i$  input summed over all species,  $\sum_{i=1}^n F_i h_i$  summed over all species plus  $Q$  minus  $WS$ . So, let us let us take the example of a power station let us say we have a Hydel power station.

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How does our equations help us to tell what happens in this Hydel power station. So, what is the Hydel power station? Hydel power station is an instance, where you have a reservoir containing large quantity of water. Huge many of our in our irrigation you know, dams are often used to generate energy because, water are available at very high level, and can it be made to run through a turbine.

So, you have a reservoir which holds huge quantity of water, and then it is at al level which is much higher than our Hydel turbine, which is at a lower level. Therefore, you have a difference of height there is a difference of height, so many meters, so many height differences there. And therefore, that height difference can be exploited to derive energy from that water. So, what is it? What is it doing? This water in the reservoir is going through a pipeline in to this turbine, and this turn a turbine, and this turbine turns up generator, and that is how we get electricity and so on.

So, I have taken this as position as 0 and this is position2. So, we are writing our energy balance between 2 positions 0 and position 2. So, we are said on the left hand side our equation, we can see here, that is d by dt of Ni hi, it is equal to i, equal to 1 to n Fi hi in, i equal to 1 to n Fi hi out, plus Q minus WS . So, what is Q? What is WS? Q is the amount of heat that is put in or amount of heat that is lost. Now, if you assume that this Q is not very significant, on other words, for the application of our interest, we may assume that

this  $Q$  is not very high, is not a such a bad assumption as for as, Hydel power stations are concerned.

By the fact is that; is an assumption an every assumption will have to be appropriately validated checked to make sure in our assumption are consisted with, what happens in the field. So, let us form and assume that  $Q$  is not very important; that means, heat loss is not very important therefore, that this statement of energy balance says, that left hand side accumulation of energy equal to input of energy minus output of energy plus flow energy in the form of  $Q$  that going in to the system, energy in the form of  $W$  going out of the system.

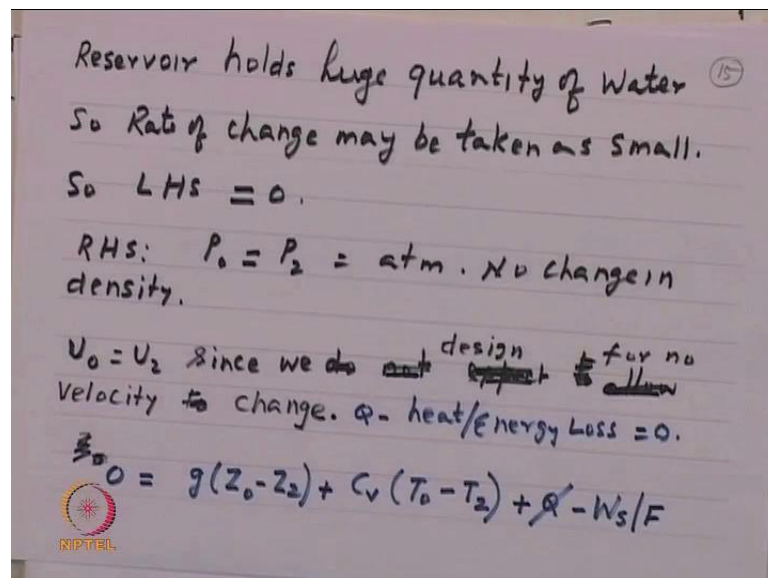
So, you notice here what I done is, I have written the energy  $F_i h_i$  and  $F_i h_i N_i$  these 2 terms. I have written it as a pressure energy  $p$  by  $\rho$ , plus  $c_v$  times  $T_0$  minus of  $T_R$  plus  $u$  not squared by  $2gz_0$ ; that means, the energy that is associated with our reservoir at this point $_0$ , what are the energy that is contain? It is there is an atmospheric pressure. So, this  $p_0$  by  $\rho$ , what is this  $T_0$  minus of  $T_R$ ?  $T_0$  is the temperature at which this fluid is there taken as  $T_R$   $T_0$ , and our reference is  $T_R$ , we always take a reference, whatever that may be  $T_R$ . Therefore, the energy associated with this fluid present in the reservoir at temperature  $T$  is given by  $c_v T_0$  minus  $T_R$ .

So, you have, so much of energy in the reservoir. Now, you also have situation where  $u_0$  is the velocity of the fluid and  $z_0$  is the potential energy of the fluid. So, in this particular case if we assume that the energy or the velocity of the fluid on the surface of the reservoir is not very high or not very, it is not very important. Therefore, this term  $u_0$  square can be neglected, why can we neglect this? Because on the top of the reservoir, where you have fluid essentially at rest are may be moving at small velocity and so on. So, for our purpose, we can take it as 0. On other words,  $u_0$  is 0, what is  $gz_0$  it is the potential energy, at which water is available  $gz_0$  is the amount of potential energy.

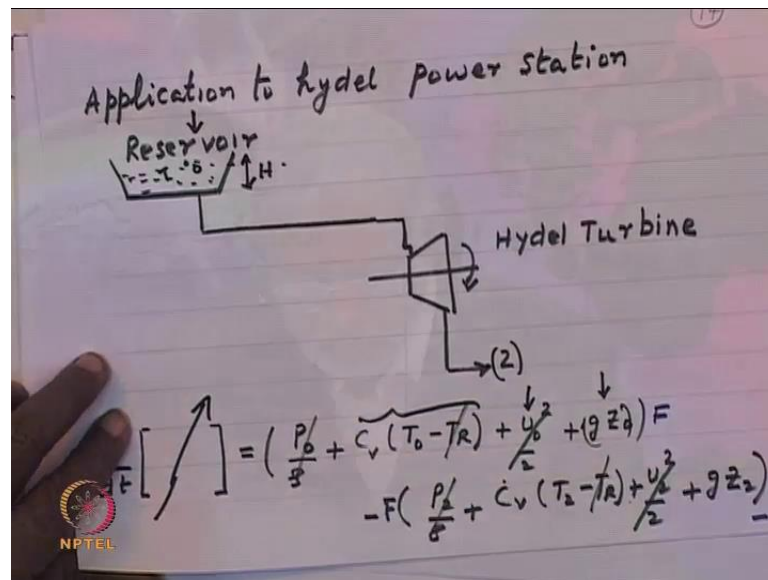
So, what is that we have? The right hand side we have pressured energy this is sensible heat energy that is associated with fluid at temperature  $T_0$ , this is kinetic energy, potential energy. You multiply with by  $f$ ,  $f$  is what? Is a molar flow of fluid that is going through the Hydel turbine. So, you have the input, what is the output? Output is pressure at  $p_2$  and then  $c_v$  which is the energy associated with the sensible heat and so on. That we written as  $c_v$  times  $T_2$  minus  $T_R$ , and this is kinetic energy, this is potential energy.

So, what we have done is that; we have use this equation, and then based on our understanding of the situation. We have converted the  $h_i$  in  $h_i$  out in terms of  $cv dt$ , so that we can take care of the change in change in the enthalpy of the fluids. So, that is taking care of  $cv T_0$  minus of  $t$ , this is for the reagents, this is for this one. Is it all right? you have pressure, you have enthalpy, you have kinetic energy, you have potential energy, this minus this, how does it look? Now, when you look carefully at this Hydel reservoir, what is the pressure at this point? It is atmospheric pressure. Similarly, at point2, which is emerging in to atmosphere therefore, pressure at position0 and position2 are atmosphere. So, what have you done here I put  $p_0$  equal to  $p_2$ .

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Second thing I have done here is that; the rate of change of reservoir capacity or reservoir fluid accumulation is not very large. Therefore, we may be delete the accumulation term in our energy balance, what we are saying is that; we can delete this term because, this term is not very large. So, putting these two simplifications, left hand side is 0, and the right hand side we knock out terms which we think is not important.

Therefore, we have left with a slightly simpler form of the energy balance, which says 0 which is on the left hand side equal to  $g(z_0 - z_2) + c_v(T_0 - T_2)$ . So, we have now said, that the energy that is associated with the height or in the with the fluid at a height. Now, able to exchange it, and converted it in to this term  $c_v(T_0 - T_2)$  are enthalpy. So, what is that our Hydel turbine doing? It is using the energy of energy of potential energy, and converting it into energy that you and I can use. This is the statement of conservation. So, let us do a small example, to illustrate how far we can go.

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(16)

$$\left(\frac{W_s}{F}\right) = g(z_0 - z_2) + C_v(T_0 - T_2)$$

a) if  $T_0 = T_2$  and  $F = 4000 \text{ kg/s}$ ;  $z_0 - z_2 = 50 \text{ M}$

$$W_s = (10) \frac{\text{m}}{\text{s}^2} (50) \frac{\text{m}}{\text{s}} (4000) \frac{\text{kg}}{\text{s}} = 2 \text{ MW}$$

b) if  $T_2 - T_0 = 0.05\text{C}$  (due to friction)

$$W_s = [(10)50(4000) - (4180)(0.05)] 4000$$

$$= (210)(4000) = 0.81 \text{ MW}$$

Frictional Loss can be serious.

This is an example where, what we have done is? We have done is that WS by f we have written our energy balance equation.

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(17)

steam Turbine

$$\frac{d}{dt} \left[ \right] = \left[ \sum F_i h_i \right]_{in} - \left[ \sum F_i h_i \right]_{out} + \overset{\downarrow}{Q} - \overset{\downarrow}{W_s}$$

LHS can be taken as zero since accumulations are not relevant at steady state.

Q can be important since heat losses are significant.

This energy balance equation, We have deleted all term excepting the term corresponding to  $gz_0$  and  $z_2$  and  $T_0$  and  $T_2$  are the terms of disappearing excepting of course, the flow work from the flow. So, when we do all this when we do all this, what is that we get? We get that  $f_s W_s$  by  $f$  is given by this relationship. It is fairly straight forward because, simply by replacing this terms simply by replacing this terms in terms

of that we know we will find that;  $WS$  by  $f$  which is the energy that is produced divided by the throughput of fluids is given by gravity plus this term.

So, what we are saying now is that; the energy in gravity term can be converted to energy in fluids in the form of  $cv$ , and then that relationship is what is given here. We have do small calculation to understand these numbers let us let us look at 1 such numbers. I have taken here  $T_0$  is  $T_2$  and  $f$  is 4000kilogram 4000tonnes per second say you large very large. And,  $z_0$  minus  $z_2$  is 5. So, these are the things some numbers, we assume to get feel for the size of the plan that you are going to building.

So, I have done here, you have 10, what is 10 50 and 4000? 4000 is the flow this is the flow. And, what is 50? This is  $m$ , this is  $g$ , this is  $m g h$ , this is  $h$  this is  $m$ . So, this is  $g$  this is  $h$ . So, what we are trying to do is that; we have a fluid a Hydel reservoir, in which you have fluid available at a height of 50 meters. Now, if it flows through a hydraulic turbine with the  $g$  value 10, what will we get? that is what is the answer we mention here. Then if you multiply all this, you get something like 2mega watts, what we are saying is that; if you had a Hydel power station, which is able to supply at a head of 50 meters. That means lift the water to that height,  $g$  is the gravitational acceleration concentration, for 4000 is the flow.

So,  $m g h$  that is the term associated with the shaft work that turns out to be 2mega watts, what we are trying to say here is that; if you had a Hydel power station in which a water is available at 50 meters height. And then, if  $kg$  per seconds and if, so many  $k g$  per second water was flowing then you will be able to generate 2mega watt power. So, this is the point we mention is that if you have a water flow of 4000kg per second, and then if you have a water flow such that the gravitational acceleration is still 10 meters per second square. And then, this  $h$  this 50 refers to the 50 refers to height at which the water available. So, now, what we have done is that we are shown that our energy balance, can explain the situation that happen in a Hydel power station.

So, we go on to see how best we may use make use of what we know for running our daily activities. Now, let us say instead of doing hydraulic turbine we do a stream turbine, what are we doing? Now instead of doing a hydraulic turbine we are doing a stream turbine, how does it help us on what difficulty does it bring us in to? Let just quickly understand this. Once when we have a stream turbine instead of hydraulic



turbine we want to apply the energy balance, our basic procedure is identical there is no change. You have a stream turbine, where d by d t left hand side equal to in minus of out  $F_i h_i - F_o h_o + Q - WS$ .

So, this  $Q$  and this is  $WS$  and this is in and this is out, this is 0, why has we taken left hand side as 0? Because, accumulation terms of quiet small in relation to the flow of energy, that is through the system. Therefore, we find taking left hand side equal to 0, is not a serious compromise on the quality of the description that we provide. Now, what might happen is that this  $Q$  this  $Q$  is the amount of energy that we put in, now we may be thinking that we have putting 100 units of energy, but what actually goes in is only 30.

This could happen to us, how do you overcome this? You overcome this by recognizing that this  $Q$  may not be a complete description of what is going on. Therefore, we must know all the heat path ways to understand, where the problems are. So, what this equation explains to you is that, how we can interpret the information to understand our system appropriately. Now, having said this, I want you to go through this exercise fully. So, that we appreciate, what we are saying? Our Hydel turbine water was coming in and going out and  $g z$  water coming out 50 meters.

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(16)

$$\left(\frac{W_s}{F}\right) = g(z_0 - z_2) + C_v(T_0 - T_2)$$

a) if  $T_0 = T_2$  and  $F = 4000 \text{ kg/s}$ ;  $z_0 - z_2 = 50 \text{ M}$

$$W_s = (10) \left( \overset{g}{\text{m/s}^2} \right) \left( \overset{h}{50} \right) \left( \overset{F}{4000} \right) \left( \frac{\text{m}}{\text{s}^2} \right) \left( \text{m} \right) \left( \frac{\text{kg}}{\text{s}} \right) = \underline{\underline{2 \text{ MW}}}$$

b) if  $T_2 - T_0 = 0.05\text{C}$  (due to friction)

$$W_s = \left[ (10) \left( 50 \right) - \left( 4180 \right) \left( 0.05 \right) \right] 4000$$

$$= (210) (4000) = \underline{\underline{0.81 \text{ MW}}}$$

Frictional Loss can be serious.

Now, you put all our numbers, let us take an example of water was coming in  $T_2$  4000 kg per second of water coming in. And, let us say  $z_0$  which is minus of  $z_2$  is 50 meter; that means, water is flowing from 50 meter height down. Now if this term if  $T_2$  minus of

$T_0$ , if  $T_2$  equal to  $T_0$ , what we are saying is that; the temperature here  $T_2$  and temperature here which is  $T_0$ , if they are same then, we would produce for this 4000k g per second of flow 2mega watt of power.

If the flow is 4000 if  $z_0$  minus of  $z_2$  is 50 then, we will produce in the shaft; that means, in the generator because shaft is shaft work is so much 2 mega watts of work. Now, if it so happens, that if  $T_2$  minus of  $T_0$  there is a slight temperature difference of 0.05degree centigrade. Suppose, which means what this term is not 0 this term is  $T_0$  minus this is actually  $T_2$  minus  $T_0$  0.05 therefore,  $T_0$  minus  $T_2$  is minus 0.05.

So, I have just put those things here; that means, our shaft work now it is not 10 multiplied by 50 multiplied by 4000, but it is subtract you have to subtract this term. Let me notice here, that this term this term is something like 290 this whole term turns out to be 290 units. So, that 500 minus of 290 is 210 multiplied by 4000. Now, you are producing 0.81 mega watt instead of 2mega watt now, what we are trying to put across here is that; in the energy balance because, we have taken in to account the effect of change in the temperature of the fluids that is going in and coming out going in and coming out.

That effect, in a Hydel turbine as per these calculation says, can be very significant. In fact, we would expect after all energy in water form of mechanical energy therefore, we would have thought mechanical energy can be can be transmitted without any loss of efficiency. Here, is an instance where you transferring mechanical energy with efficiency of only 50 percent or less there is something that should bare in mind. And therefore, we have take in to account the fact, how do we design our pipelines? Taking the water in to our turbine, without allowing it to get heated up, even by a very small margin, this is an important part, of design which we should take care. I will stop there when we come next time we will take this idea further.

Thank you very much.