Optical Spectroscopy and Microscopy
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Lecture – 8
Fundamentals of Optical Measurement and Instrumentation

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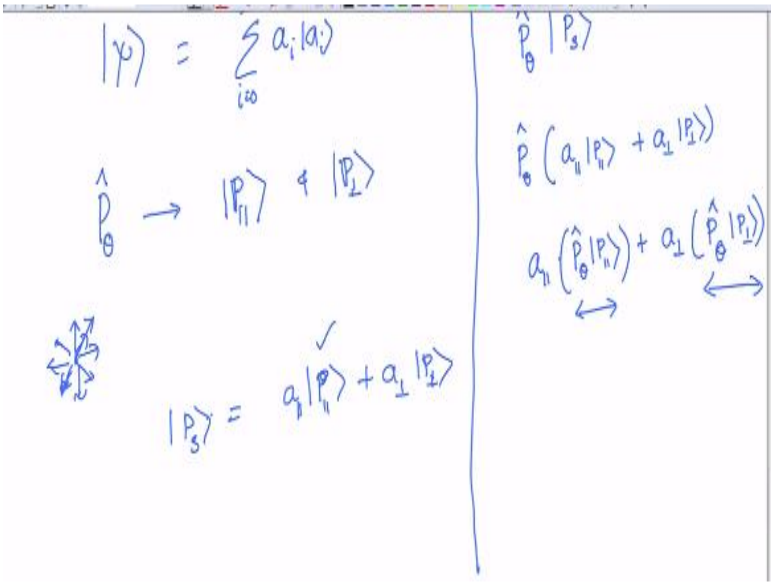

Okay, so the goal now here is that we have reduced the problem into finding what the values these P theta P parallel and P theta P perpendicular have, right. So in order to evaluate that, what we are going to do is we are going to go into the experiment itself, revisit the experiment and see if you can actually say something about the values of them. I am trying to add a new page, okay good.

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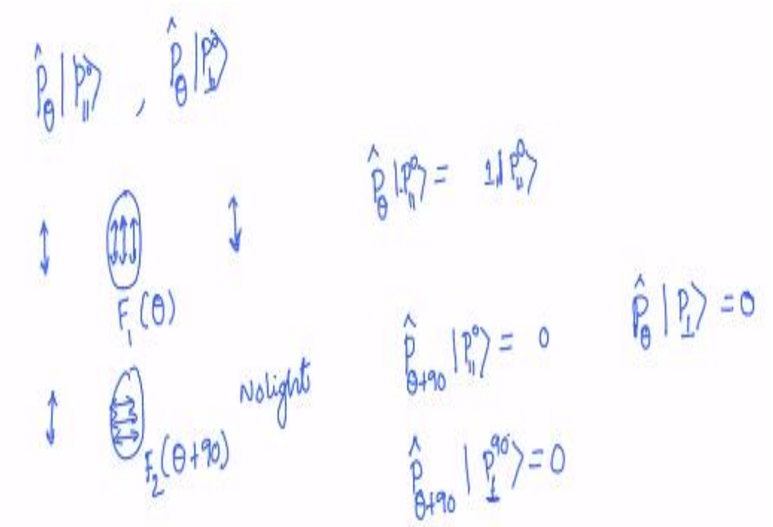

So, how do we connect the experiment and this particular things? Remember what we are after is P theta P perpendicular and P theta, sorry P parallel and P perpendicular, right. Now, how can we guess these values? Imagine that if this were to correspond the experiment, let us go back to our experiment which is we have a polarizer who is oriented at an angle theta with respect to our laboratory frame in this case it is along this axis and then the output that we see is that the light is of this polarization, right.

So, it is safe to say that if this operation that we described were to actually represent this experiment, then what we should be able to get is some value here that corresponds to this state itself or P parallel or in other words, you can think of this having an eigen value of 1, remember. So, again so if you take light which is polarized like this and then we put in a filter F1 whose orientation corresponds to theta with respect to a laboratory frame, then the output that we get is same and unaffected, and if you had to represent that, then this would correspond to an eigen state with an eigen value of 1 so that we get back the same state.

We also know that the same light, right, and then when it passes through a filter, now this time the filter is flipped by 90 degree, right. So, the second filter we put in is basically theta + 90 degree, we will not get any light, no light at all or in other words if we were to say P theta, now this operator should correspond to this filter which is P theta + 90 degree were to operate on P parallel, then the resultant that we should get should be 0.

I mean, the only way we can do that is to actually make the eigenvalue 0, so a very simple way to do that is to make the eigenvalue 0 so that you can think of that the operator P theta +

90 when operating on a state P parallel, now this P parallel is the parallel status with respect to the first filter. So just to distinguish what we will do is that we will call the filter 1 where it corresponds to theta, the corresponding eigen states as P superscript 0, alright. So in that case, it will become much clearer.

So this would be superscript 0 would correspond to 0 or in other words, this P parallel superscript 0 in relation to this would be actually P perpendicular of 90 degree, right. So, you can think of this as nothing but we rotated the whole thing, you have rotated the states here, so you can think of this + 90 degree for its own eigen state relationship would be 90 degree of perpendicular equal to 0 or in other words you have answer to the other expression to that is actually there is nothing sacrosanct about the theta + 90, I mean I could actually have this as minus 90 to set it to 0.

So, we could generalize this into P theta the operator when operating on a state vector who corresponding to 90 degree rotated polarization with respect to the crystal axis of this would give us 0, an eigen value of 0, okay. So, armed with this, now let us go back experiment and then see how we can explain this.

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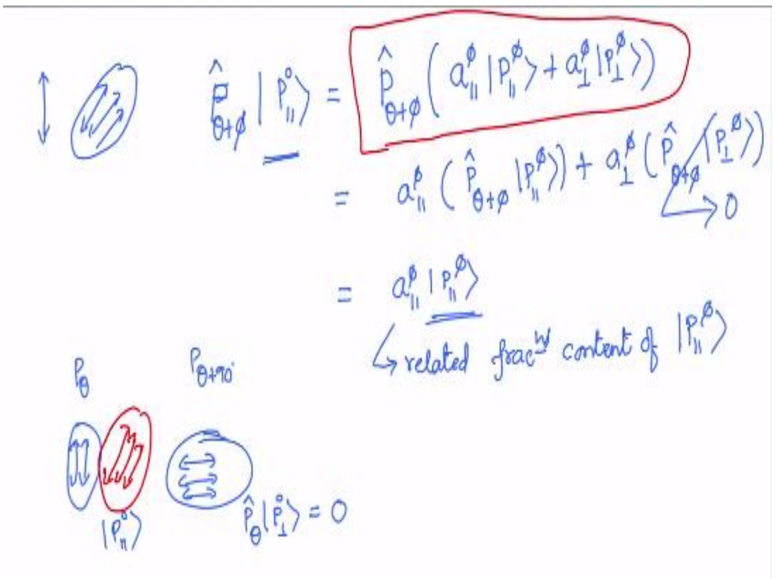

So the only thing that we have not addressed here is that suppose if I were to have a polarized light and the polarizer that is at an angle phi, then what happen, how do we describe this? Now, this I am going to state it as that we can think of this as an operator theta plus phi, from phi degrees from the original one operating on 0, then you would see that you would be able to write this state, right, this state as a linear combination.

It will be convenient to write this state as a linear combination of basis vectors of theta + phi or in other words, we can write it as some coefficient a of parallel times P phi parallel + a perpendicular P phi perpendicular. The moment we get this, now what is going to happen is that now this operator when it operates on this is going to generate a coefficient. I mean the whole thing we can write it down as remembering the last expression that we have done, so you can pull out the scalar and then a parallel times P theta phi times P parallel phi + a perpendicular times P theta + phi operating on P perpendiculars phi.

Now, this term by our definition will go to 0 leaving you and then this whole thing will become one times this according to our previous definition, I mean previous finding so that we are consistent. So, what we see is that these coefficients, the a parallel and a perpendicular in a linear combination reflects that is related to the fractional content of a particular state, okay. Of course, I am making a small jump here, assumption here which I mean we have normalized it appropriately and all that.

But nevertheless, what you can actually see is that they are proportional to the a11 the coefficient here, you would get a number that is corresponding to this state P parallel state, okay good. So now, what is happening in our experiment? In our experiment, we know that when we had a first polarizer that is parallel and then another polarizer that is perpendicular, what had happened is that the act of you measuring this you asking a question of what is the polarization here even if you started with.

So this corresponds to our P theta and this corresponds to our P theta + 90. I am retaining this theta just to indicate the fact that there is nothing sacrosanct about parallel and perpendicular. What is important here is the relative orientation between these 2 filters. So, that is why the theta is important, it is just it can be oriented anywhere any direction with respect to the lab, the whole thing will follow through.

So now, what you see is that if you take in a light and then put it through a polarizing filter, what you have prepared is a pure state of P parallel state and since you have prepared this pure state P parallel state and then when you are probing for P theta + 90 which is equivalent to basically placing this operation just P theta cap on P perpendicular. When you do that, we know that the answer is going to be 0 that is why we are not getting anything.

However, the moment you introduce across polarizer here, what we see is that the resultant that is emerging out from this polarizer, it is no longer having only a P parallel state, but instead it has P parallel and P perpendicular, okay. So what you have done is that with respect to actually the 0 state, so let us write down.

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$$\left(P_{\theta+\phi} \left| P_{\parallel}^{0} \right\rangle\right) = a_{\parallel}^{\phi} \left| P_{\parallel}^{\phi} \right\rangle$$

$$a_{\parallel}^{\phi}\left(\hat{P}_{\theta+90} \left| P_{\parallel}^{\phi} \right\rangle\right) = a_{\parallel}^{\phi}\left(\hat{P}_{\theta+90}\left(b_{\parallel}^{90} \left| P_{\parallel}^{90} \right\rangle + \cancel{b_{\perp}^{90} \left| P_{\perp}^{90} \right\rangle}\right)\right)$$

$$= a_{\parallel}^{\phi} b_{\parallel}^{90} \left| P_{\parallel}^{90} \right\rangle$$

Okay, now what we have started? What we have is P parallel to the 0 and then we are operating this by we are asking plus phi. Now, what we need to do is we need to be able to express this in terms of just the way we have done, right, in terms of the eigenstates of this operator. When we do that, what we see is the end result is that we have a component here, we have prepared a state which is something like this a parallel phi, right. So, that is exactly what we have written here okay.

So this corresponds to phi, so let us keep that as phi here. Now, that is what we have written here. So, what we are going to do is that we are going to take this state and apply the theta + 90 on this. So clearly when you do that, what is going to happen is P is P theta + 90 operating on this state right, so which is basically you can write it as P parallel phi. Now you will see that this is not the eigen ket of this. So, what we do? We do the same thing, we try to expand. We like to expand in terms of the eigen kets of this or write it as a linear combination of eigen kets of this operator.

So, what we have then is phi times P theta + 90 cap this whole P parallel phi could be written as some other b parallel 90 P parallel 90 + b perpendicular 90 and P perpendicular 90. So now what it means is that when this operator operates on this, you will see while this term would go to 0 by definition because it is perpendicular, while this will yield will not be a 0, right, will be nonzero giving rise since eigenvalue is 1, you are going to get back the state, right.

Now, this you remember that this is the photon state with a polarization parallel to a filter which is rotated 90 degrees from the original that is nothing but this polarization state corresponds to light that is polarized at 90 degree. Now, our act of basically by intervening with an in between polarizer somehow seem to have generated this state that principle that idea comes from the very notion that in general, a system is thought to be existing in a state of superposition and the act of measurement collapses that superposition into one of its many eigen kets.

In this case, we took two of them P parallel and P perpendicular and then we followed it through and said that okay look you can actually see that how just following through simple algebra and defining some rules and then constantly applying that rule throughout, how we are able to actually write down in pen and paper what would have happened in an experiment. Now that is the kind of description that we are looking for, using that description, how we can actually describe a light matter interaction, alright.

I will see you in the next class on how we use these descriptions and these principles to describe light matter interaction.